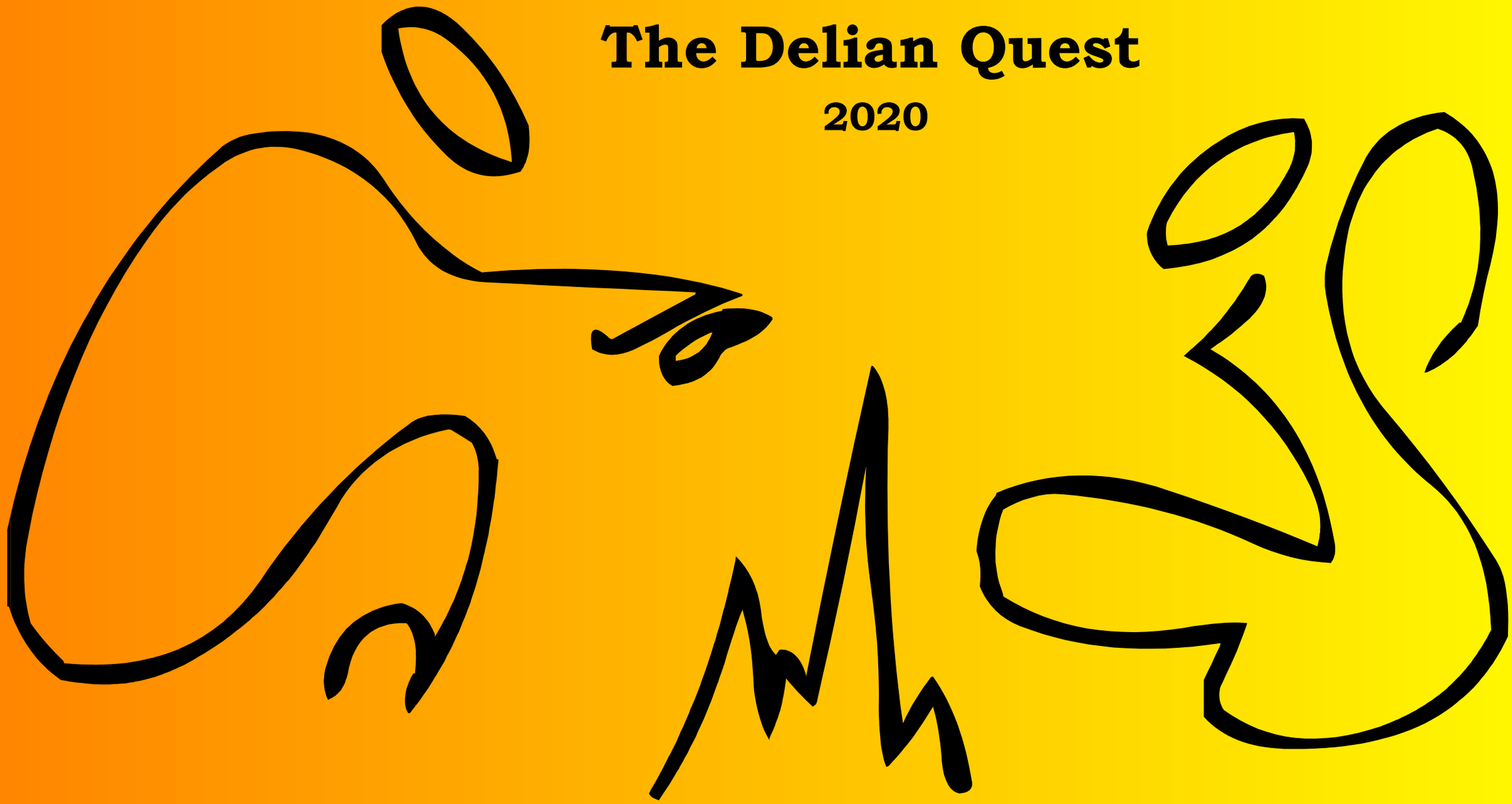
The Delian Quest

2020



John 312

The Delian Quest

John Clark

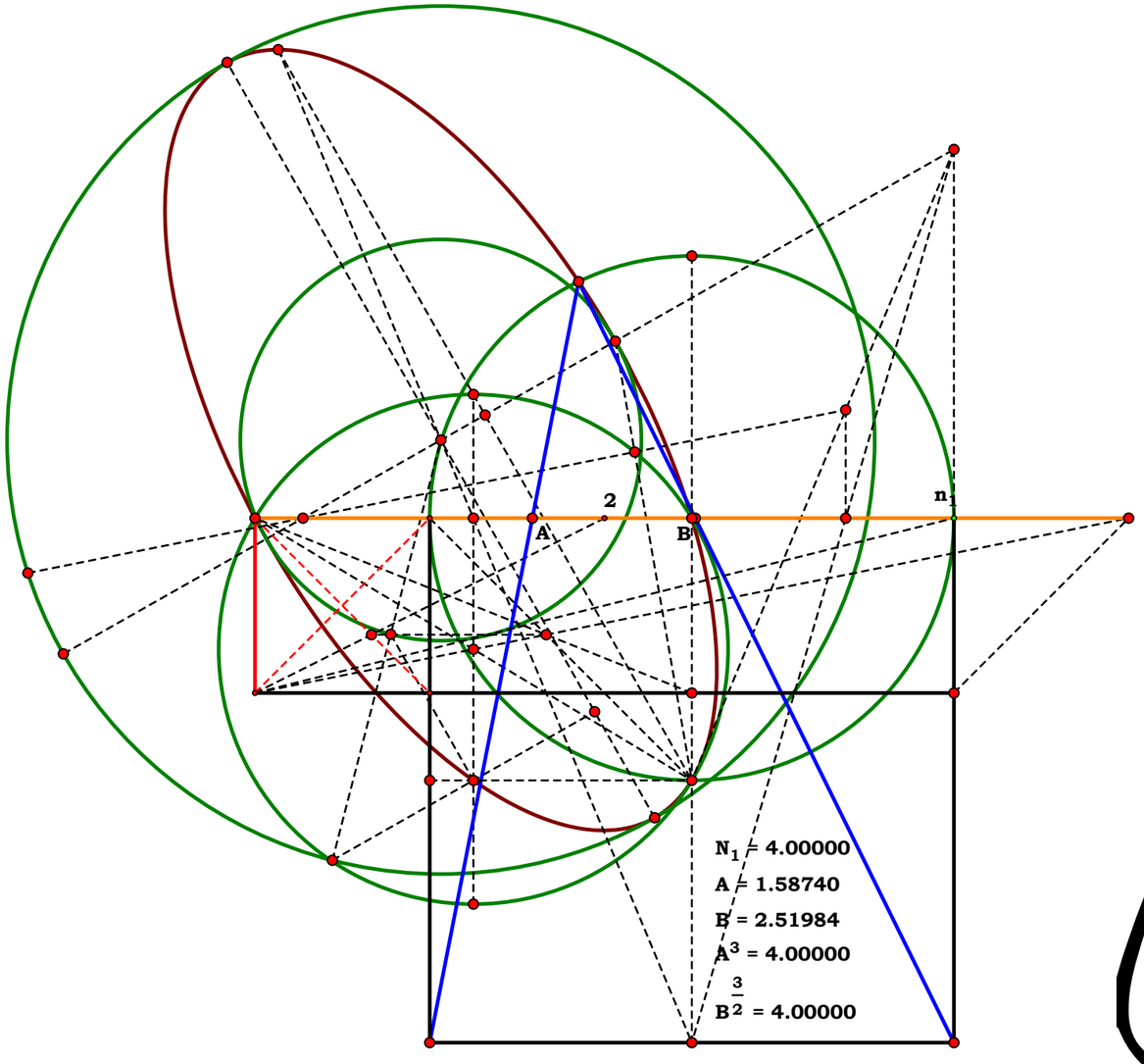
Revision Started in 2019

Monday, February 3, 2020

Essentially, the Delian Quest is a novel written in the only two divisions of grammar there are, logical and analogical (Algebra and Geometry.) What writing this novel has done, for me at least, is that in writing equations to figures long enough, I learnt to write figures to produce equations, i.e., I developed a simple and obvious analog grammar which I call BAM, or BAG; Basic Analog Mathematics, or again, Basic Analog Grammar. That grammar is inclusive of any possible thing using both logical and analogical operations. One might say, I learnt grammar in and out.

This is probably my last revision of the novel, many items which I never wrote up are included for the challenge to the reader. Unlike the 2015 revision where I was pressed for time, this one is a bit more comprehensive and may be a bit more provocative.

Still, I copied the directory over to one named 20xx just in case.



The Delian Quest

John Clark

Revision started in 2019

Current Release Date: February 3, 2020

Note on the *Delian Quest*:

2/3/2020

A mind is a symbolic information processor; it functions by transforming the intelligible of Language into the perceptibles of grammar systems. As Language is a binary, a relative constrained by correlatives, as the definition of any and every possible thing, every possible grammar is a means of using this binary for information processing. The binary of Language transforms into four specific groups of grammar, Common Grammar, Arithmetic, Algebra and Geometry.

The Bible states that man will be illiterate until he learns that a pure language is a Conjugate Binary Pair, such as exampled in Adam and Eve. Plato defined a thing as composed of two elements, neither of which could be defined and he gave them the names of noun and verb. The typical mind cannot follow this as their mind cannot group information in accordance with this binary. Instead of functioning by complete induction and deduction of a unit, they more often than not, simply learn to repeat words irrespective of grammar. There are, currently, no correct grammar books on this planet.

This also means that the typical human being is not in the least bit sane.

In geometry, we construct this binary as a simple segment. This simple binary unit affords every possible grammar, geometry included, with complete induction and deduction of that unit.

No one can teach the elements of a thing in the intelligible; most people only parrot words, manipulating the perceptible symbols independent of their intelligible associations. These people, who comprise the majority of mankind at present, are wholly unaware of when they are speaking and thinking gibberish. Many claim to be looking for a savior,

unable to relate the fact that every life support system of the human body is constructed for the salvation of man, including the mind.

One can teach the standards of behavior required for each of our four categories of grammar, however, one cannot teach the evolution required to make a person capable of comprehension.

Language is Universal and Intelligible. So far man lacks the intelligence to convert the simple binary of Language into grammar systems respecting that binary. Grammar is Particular and Perceptible. By binary recursion, there are exactly four groups of grammar, Common Grammar, Arithmetic, Algebra, and Geometry. This is achieved by the binary construct of grammar to begin with, symbol sets and how they are used to construct names. These are addressing systems for the mind. As Plato noted, words, names, these symbols, in of themselves have no meaning. This one fact is essential for data integrity; data, what we address in our mind, is our experiences. Our experiences are the meaning addressed by symbols.

What you do is what you mean, in mind and in body. If your mind is primitive and simple, that is what your life will be.

One can look at the memory addressing systems called grammar as maps to plan our life, our behavior. That is what it is for. Every possible grammar, working with the same binary unit, is predictive in nature. This predictive function is the gift of the Law of Identity.

However, all of these words denote something different in each mind commensurate with their intelligence and experience. The meaning is not the word, nor is it in the word. The meaning is addressing our memory and if we do not take pains to get that experience we remain an idiot for our entire life, thinking and acting the fool that we are.

Stupid people, who currently call themselves genius, and the intelligentsia, actually believe that they can write a new grammar, invent

words with special meaning, but that is only indicative of a mental problem which is the results of an evolutionary process leading up to a functional mind.

One of the things which the analog grammar of geometry does for one, is give one experience with the simplest universal grammar system possible, a single motion of the hand to produce the unit segment, or the binary 1. What you can recognize by that is determined by your intelligence.

So, if you believe that those who spend their time in colleges and universities as being the most intelligent people on the planet, I have to ask, if this is so, why is it that not one of them recognize that words, in of themselves, have no meaning? Why is it that not one of them recognize that there are no correct grammar books on the planet? Why is it, being industry and world leaders is the world in crisis?

The I.Q. test given to me as a child led me to understand that I would have to abandon traditional education in order to get an education. And, non-human intervention in my life made my work possible despite my stupidity. So, here it is, another product of decisions I made, and that were made for me.

The computer is just another voice, another proof that all of information processing is binary. And still, what is the major use it is put to? Games, porn, merchandising meant to defraud the public, obfuscation of grammar, and the maintenance of the mental dysfunctions of the population.

How is it that a unit can in anywise produce disunity? Or is it not a fact that it takes time to even build a computer that does not crash. Mankind is crashing.

All the years that I have been working on my grammar projects I have not received even one email from anyone who comprehended what has to

happen in history, in the mind of man, or who even cares. A mind that does not work cannot work.

Naming Convention

2/3/2020

I have always looked at the Delian Quest as a unique type of novel. But in the writing of that novel, one can say that all the work to this point is sketched out, not in a finished format. During the work, I was learning about naming conventions, and I was very aware of it, but now, in the finishing of this work, I have to lay down what I understand of that convention. For example, we have induction and deduction. Deduction is parsing what we already have, where induction is using what we have to acquire more. In geometry, it can look like this.

I can start naming relative difference, or the part, in terms of some other given system of arithmetic naming, or I can start by naming it simply as 1, making it the unit. If I start with two things, I have to name them relative to some other standard, or I can name one or the other relative and the remaining one a proportion to it. This will produce results which look different but that difference is wholly determined by the naming convention. When one renders a definition from a chain of reasoning, the resulting definitions appear different, yet that difference is wholly determined by the naming convention we started with.

I can name AB as 1, which means that point C is an induction, in ratio to AB and likewise if CB is named 1, then AC is a ratio to it. If I name AC as 36 and CB as 14, I am using an external unit. If I name AC 1, the CB is a ratio to AC.

And so to be complete with the demonstrations, I should example each of these choices and how it changes the APPEARANCE of the definition.

When starting out in exploring, one may ponder these issues, but since one is running to learn, they get put aside until one gets to where one is going, and then one has to clean up the mess in the end, just like building anything.

If one has never seen one or more of the skits, the *Anal Carpenter*, it is something to consider.

Mathcad Log

Underneath each file, put date it was reviewed followed by comment if any, highlight in red if it requires to be fixed, split, or further developed: If it is green, I reworked it.

See Naming Convention.

1992\062092\062092Z A Duplicate Ratio

1/31/2020:

1992\081292\081292Z Rusty Cube of a Sphere

1/30/2020:

1993\010893\010893Z Pythagoras Revisited

1/30/2020:

1993\060393\060393Z Exploring The Curve

1/31/2020:

1993\060793\060793Z Two Triangles with a Common Side

1/31/2020:

1993\060993\060993Z Rectangular Roots

1/31/2020:

1993\062193\062193Z Pyramid of Ratios I

1/30/2020:

1993\062793\062793Z Circle About a Triangle

2/1/2020:

1993\071593\071593Z Pyramid of Ratios II

2/1/2020:

1993\072593\072593Z Pyramid of Ratios III

2/1/2020:

1993\110693\110693Z Gruntwork I

2/2/2020:

1993\110993\110993Z Cube Root Placement

2/2/2020:

1993\111093\111093Z Gruntwork II

2/2/2020:

1993\111193\111193A\111193AZ Paper Trisector

2/2/2020:

1993\111193\111193B\111193B1Z

2/2/2020:

1993\111193\111193B\111193B2Z

2/2/2020:

1993\111193\111193B\111193B3Z

2/2/2020:

1993\111193\111193B\111193B4Z

2/2/2020:

1993\111293\111293Z Square A Circle

2/2/2020:

1993\111893\111893AZ Exploring Cube Roots

2/2/2020:

1993\111893\111893BZ Exploring Cube Roots

2/2/2020:

1993\111893\111893CZ Exploring Cube Roots

2/2/2020:

1993\111893\111893DZ Exploring Cube Roots

2/2/2020:

1993\112493\112493Z POR 4

2/2/2020:

1993\120293\120293Z POR Roots and Powers

2/2/2020:

1993\120493\120493Z Exponential Series

2/2/2020:

1993\120693\120693AZ Common Segment Common Endpoint

2/2/2020:

1993\120693\120693BZ Gruntwork IV

2/2/2020:

1993\121193\121193Z structure in red

2/3/2020:

1993\121293\121293AZ Common Segment Common Midpoint

2/3/2020:

1993\121293\121293BZ Generalize

2/3/2020:

1993\121693\121693AZ Using 120493

2/3/2020:

1993\121693\121693BZ Exponential Series

1/30/2020:

93 Revision Complete, or so it seems.

1994\040694\040694Z Inscribing A Circle

1994\042194\042194Z The Cradle

1994\042694\042694Z Tangents and Similarity Points

1994\042794\042794Z Chordal or Power Line

1994\042894\042894Z Power Point

1994\042994\042994Z what is EF

1994\043094\043094Z Division N2

1994\050194\050194Z Two Circles And A Parallel

1994\050494\050494Z Two Circles And A Tangent

1994\050694\050694Z Ratio In Trisection

1994\050794\050794Z Trisection Ratio

1994\051694\051694AZ Tangent Diameter and Circles

1994\051694\051694BZ Tangent Diameter and Circles

1994\102794\102794Z Trivial Method Square Root

1994\102894\102894Z Trivial Method Square Root

1994\103194\103194Z Square Root of a Segment

1994\110194\110194AZ Given BG and BC

1994\110194\110194BZ Given AG - AB

1994\122494\122494Z Power Line At Square Root

1994\122594\122594Z Two Prime Exponential Series

1994\122694\122694Z Is Point G on DJ

1995\010695\010695Z Alternate Method Quad Roots

1995\011295\011295Z Archamedian Trisection Revisited

1995\040195\040195Z Exponential Series

1995\091395\091395aZ A Study In Placement

1995\091395\091395bZ A Study In Placement

1995\091395\091395cZ A Study In Placement

1995\101495\101495Z Alternate Method Square Root

1995\102095\102095Z Four Times The Square

1995\110195\110195Z Gemini Roots

1995\110595\110595Z Alternate Method Gemini Roots

1995\120195\120195Z Method For Equals

1995\120795\120795Z Euler's Line

1995\121695\121695Z Descartes

1995\121895\121895Z Descartes

1995\122095\122095Z Just For Fun

1995\122195\122195Z Pascal's Triangle

1995\122995\122995Z find BC

1996\010496\010496Z Without the Numbers

1996\010796\010796Z Rusty Cubes

1996\010896\010896aZ Alternate Method Power Line

1996\010896\010896bZ quad roots

1996\011396\011396Z Pyramid of Ratios VI

1996\011696\011696Z Cube Roots

1996\011796\011796AZ Right Triangle In A Given Ratio

1996\011796\011796BZ Sides Of A Triangle In A Given Ratio

1996\012196\012196Z On Cube Roots

1996\012296\012296Z Trivial Method Square Root

1996\012496\012496Z Tangent

1996\012596\012596Z point on BD

1996\012996\012996Z Linear division

1996\013196\013196Z Gemini Roots

1996\020296\020296Z Find A Segment

1996\021496\021496Z 17 decimal place rustic

1996\041496\041496Z Method for Unequals

1996\041595\041595Z Gemini Roots

1996\041696\041696Z Given Three Radii

1996\041796\041796AZ Circle In A Crescent

1996\041796\041796BZ Circle In A Crescent

1996\042296\042296AZ BF as a ratio

1996\042296\042296BZ Place EF and GH and find JK

1996\042296\042296 REDO\100600AZ

1996\042296\042296 REDO\100600B

1996\042296\042296 REDO\100600BZ

1996\042396\042396Z CF always equal to EK

1996\042496\042496Z Three Circles

1996\042596\042596Z One Over N AND One

1996\042696\042696Z Three Bases

1996\042796\042796Z A Root Figure

1996\042896\042896Z Given one root

1996\042996\042996Z What is DJ

1996\043096\043096Z Generalize

1996\122096\122096Z Alternate Method Quad Roots

1997\040397\040397Z halve the angle

1997\040497\040497Z Triangles

1997\042897\042897Z

1997\042997\042997Z Trisection and the Cube Roots

1997\082197\082197Z Eight Circles

1997\082297\082297Z Steiner Point

1997\091197\091197AZ The Ellipse

1997\091197\091197BZ The Ellipse

1998\020298\020298Z

1998\021098\021098Z A Square In A Triangle

1998\022598\022598AZ Alternate Method Root Series

1998\022598\022598BZ Sum Divided by One Powered

1998\022598\022598CZ Doing the Math

1998\022698\022698Z

1998\042398\042398Z Square Root Figure And Triseciton

1999\072499\072499Z Gemini Roots

1999\081199\081199Z A Delian Solution

1999\081899\081899Z Promptly

1999\081999\081999Z name of the Ellipse

1999\082999\082999Z See E Go

1999\083099\083099Z

1999\100299\100299

1999\100299\100299Z

1999\100499\100499Z

1999\100799\100799Z

1999\100799\991007

1999\101799\101799Z

2000\070200\070200Z POR something

2000\070900\070900Z Alternate Method Quad Roots

2000\072000\072000AZ Quad Roots via Tangent Circles

2000\072000\072000BZ Quad Roots by equal angles

2000\080100\080100AZ Alternate Method Quad Roots

2000\080100\080100BZ Alternate Method Quad Roots

2000\080200\080200Z What Is AB

2000\080300\080300ZA On Trisection

2000\080300\080300ZB On Trisection

2000\080400\080400AZ Trisection and Square Roots

2000\080400\080400BZ Trisection and Square Roots

2000\080400\080400CZ Trisection and Square Roots

2000\080400\080400DZ Trisection and Square Roots

2000\080400\080400EZ Trisection and Square Roots

2000\080400\080400FZ Trisection and Square Roots

2000\080700\080700Z Proportion Series II

2000\082200\082200AZ Square Root and the Archimedean

2000\082200\082200BZ Show the trisection

2000\082300\082300Z Trisection In A Square Root

2000\090100\090100Z Quad Roots

2000\090300\090300AZ Ratios In Trisection

2000\090300\090300BZ

2000\091600\091600Z Goshdarn Good Pencil

2000\091800\091800AZ Midpoints and Similarity Points

2000\091800\091800BZ Midpoints and Similarity Points

2000\092000\092000AZ Squaring

2000\092000\092000BZ Squaring

2000\092000\092000CZ Cube Root Primitive

2000\111300\111300Z Two Right Triangles

2000\112700\112700Z 2N root series

2000\112800\112800Z Means On Means

2000\112900\001129

2000\112900\112900AZ Multiplication and Division-

2000\112900\112900BZ Multiplication and Division-

2000\120500\001205

2000\120500\120500Z From an observer C

2001\010101\010101Z common segment common

2001\042101\042101Z

2001\042201\042201Z

2001\042301\042301Z Counterpoint

2001\042401\042401AZ

2001\042401\042401BZ

2001\042501\042501AZ

2001\042501\042501BZ

2001\042901\042901AZ Four Lines To A Point

2001\042901\042901BZ Four Lines To A Point

2001\050201\050201Z Just an Ilustration

2001\050601\050601AZ Algebraic Names

2001\050601\050601BZ Algebraic Names

2001\050701\Z

2001\050701\01 Eliptic Progression\050701AZ Eliptic Progression

2001\050701\02 Bisection\050701BZ Bisection

2001\050701\03 Out Take Two\050701CZ Out Take Two

2001\050701\04 Trisection\050701DZ Trisection

2001\050701\05 Quadsection\050701EAZ Quadsection

2001\050701\05 Quadsection\050701EBZ Quadsection

2001\050701\050801\2001-0508

2001\050701\050801\Z

2001\050701\06 Out Take Four\050701FZ Out Take Four

2001\050701\07 Pentasection\050701GZ Pentasection

2001\050701\MCAD\AZ

2001\050701\MCAD\BZ

2001\050701\MCAD\CZ

2001\050701\MCAD\DZ

2001\050701\MCAD\EZ

2001\051101\051101Z Round Tuits

2001\051301\051301 TrisectionZ

2001\051401\010514 checker

2001\051401\010514

2001\051401\051401

2001\051401\051401Z Trisection

2001\052201\052201AZ Segment DF And HM

2001\052201\052201BZ Segment DF And HM

2001\052701\010527

2001\052701\052701AZ Point of Intersection

2001\052701\052701BZ Point of Intersection

2001\053001\053001AZ

2001\053001\053001BZ

2001\060101\010601

2001\060101\060101Z Small Extrapolation

2001\060201\060201AZ Units From Both Sides

2001\060201\060201BZ Units From Both Sides

2001\060201\060201CZ Units From Both Sides

2001\060301\060301Z Isolating A Problem

2001\060401\060401

2001\060401\060401Z

2001\061001\061001Z For Any N1 N2

2001\082601\082601Z Elipse By Parallels

2001\102201\102201Z Four Curves and Procrastination

2003\021603\021603Z

2003\021903\021903Z Parcing project

2003\022803\022803Z Fair Pencil

2003\030503\030503Z Gravitating Answer

2003\030803\030803Z Nothing Saved

2003\031503\031503Z Trisection and Square Roots

2003\032003\032003Z Parcing project

2003\032303\032303Z Figure in a Figure.

2003\032403\032403Z Parcing project

2003\032703\032703Z Parcing project

2003\040903\040903Z Parcing project

2003\072903\072903 Page 5 Z Dionysius

2003\072903\Page 5

2003\2002\010202\010202Z Proof Of Paper

2003\2002\062002\062002AZ Easy Power-Line

2003\2002\062002\062002BZ On the Other Hand

2003\2002\071902\071902Z Linear Division

2003\2002\090902\090902Z Parcing Project

2003\2002\092102\092102Z Parcing Project

2003\2002\092202\092202Z Parcing Project

2003\2002\092402\092402Z Parcing Project

2003\2002\093002\093002Z Parcing Project

2003\2002\100402\100402Z Parcing Project

2003\2002\101402\101402Z Parcing Project

2003\2002\111402\111402Z Parcing Project

2004\031604\031604AZ

2004\031604\031604BZ

2004\031704\031704Z

2004\032004\032004Z

2004\032304\032304Z

2004\040504\040504Z

2004\041904\041904AZ The Ellipse

2004\041904\041904BZ Straight Line Ellipse

2004\041904\041904CZ Straight Line Ellipse

2005\031405\031405Z Another Ellipse

2005\031605\031605Z Parcing project

2005\031705\031705Z Parcing project

2005\032305\032305Z Ellipse

2005\032405\032405Z Parcing project

2005\032905\032905AZ Elipse Projected

2005\032905\032905BZ Parcing project

2005\032905\032905EZ Writeup project

2005\033005\033005BZ

2005\033005\033005Z N cubed etc

2005\033105\033105Z Just Another Ellipse

2005\041205\041205AZ Mixing methods of naming

2005\041205\041205BZ A trip to IO

2005\041305\041305Z

2005\041405\041405Z Procrastinated Write

2005\042205\042205Z

2005\042205\050422

2005\042305\042305AZ Sum of Area

2005\042305\042305BZ Sum of Area

2005\042305\042305C Sum of AreaZ

2005\110705\110705Z Procrastinated Writeup

2006\012306\012306Z Parcing project

2006\012906\012906

2006\012906\012906Z Given AC what is CK

2006\012906\STRIP

2006\102606\102606AZ Angles and Areas

2006\102606\102606BZ Angles and Areas

2006\110706\110706aZ Going around in a circle

2006\110706\110706bZ Parcing project

2006\110706\110706uZ Parcing project

2006\110806\110806Z Really

2006\110806\2006-110806g

2007\060807\060807AZ

2007\060807\060807CZ Equation for an Ellipse

2007\061307\061307Z Procrastinated Write

2007\062007\062007CZ Tangent from Major

2007\062007\062007DZ Tangent from Minor Axis

2007\062407\062407Z Found on the Internet

2007\062507\062507Z Parcing project

2007\072707\072707Z Parcing project

2008\052108\052108Z

2008\052208\052208Z

2008\060208\060208AZ line ellipse and three givens

2008\060208\060208BZ line ellipse and three givens

2008\060208\060208CZ line ellipse and three givens

2008\060308\060308Z Procrastinated Write up

2008\061308\061308AZ Parcing project

2008\061308\061308BZ Parcing project

2008\061708\061708Z Parcing project

2008\062608\062608Z Parcing project

2010\061110\061110Z Parcing project

2011\020511\020511Z Rant

2011\052411\052411Z Plate 1

2011\052411\052411Z Plate 2

2011\052411\052411Z Plate 3

2011\052411\Lemma Pg 47\Plate 1\Plate 1

2011\052411\Lemma Pg 47\Plate 2\Plate 2

2011\052411\Lemma Pg 47\Plate 3\Plate 3

2015\080915\080915Z Pythagoras Revisited Again

2017\Circles\Plate 10\Circles-Plate 10

2017\Circles\Plate 12\Circles-Plate 12

2017\Circles\Plate 13\Circles-Plate 13

2017\Circles\Plate 14\Circles-Plate 14

2017\Circles\Plate 2\Circles-Plate 2

2017\Circles\Plate 3\Circles-Plate 3

2017\Circles\Plate 4\Circles-Plate 4

2017\Circles\Plate 5\Circles-Plate 5

2017\Circles\Plate 6\Circles-Plate 6

2017\Circles\Plate 7\Circles-Plate 7

2017\Circles\Plate 8\Circles-Plate 8

2017\Circles\Plate 9\Circles-Plate 9

2019\Z

2019\111619\111619Z

2020\012220\012220M

The Delian Quest: Original Submission

Sunday, January 19, 2020

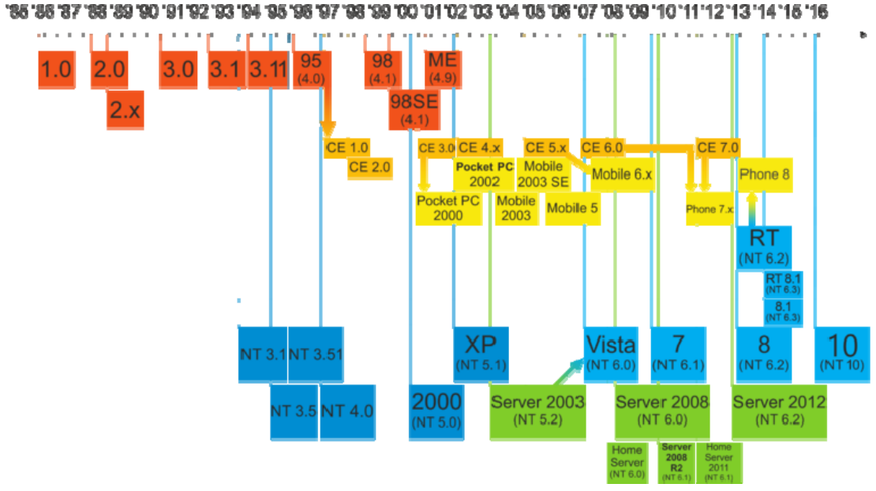
The following is a copy of the statistics of a document written long ago. I wrote it using Ami Pro, Windows Write, or Word 2, at one time had Word 6 save it another time. I eventually made Word 2003 my standard word processor. This is the first figure I started with and it eventually led not only to the Delian solution but to Basic Analog Mathematics and my understand that every possible grammar is a binary expression.

Filename: DELIAN.DOC
Directory: E:\My Documents\1989
Template: C:\Users\John\AppData\Roaming\Microsoft\Templates\Normal.dot
Title:
Subject:
Author: John J. Clark
Keywords:
Comments:
Creation Date: 5/9/1992 4:32:00 PM
Change Number: 26
Last Saved On: 10/6/1992 7:23:00 AM
Last Saved By: John J. Clark
Total Editing Time: 339 Minutes
Last Printed On: 10/2/2019 10:29:00 AM
As of Last Complete Printing
Number of Pages: 19
Number of Words: 2,019 (approx.)
Number of Characters: 9,553 (approx.)

What it tells me is that I wrote it using an operating system prior to Windows 3 and may have originally written using Ami Pro which I quickly abandoned when I purchased Word. The formatting, however, leans towards Word 2. It tells me that I started learning geometry at about the age of 38 never having taken the topic in school. It was doing my drawing by hand at the time, the first drawing program I used was TommyCad. I later found Geometer's Sketchpad. Early on, I was using Word 2 to construct data table's which I would save to a text document picked up by QBasic to do the math, and then pass it back to Word. I wrote macro's for that. I did that until Mathcad sent me an invitation to buy that program and have used it ever since.

I suspect Mathcad sent me the offer as Microsoft approached me first to become a beta tester, probably based on the machine I had just purchased which ran at a blazing 25mhz, which I later dropped out of because of the stupid way they ran the program, the original version of Dos 6 completely ate my hard drive and I lost a lot of work. I was using one of those sewing machine boxed computers at the time to carry it to and from work as I worked at G.M. I was the first G.M. factory worker to carry a pc in and out of the plant which led to a Union settlement allowing employees to bring their own computers in and out.

The following table is from Wikipedia.



I wrote the letter as I believed that a real geometer should work on the problem of cube duplication from this starting point as I have never even had geometry in school. However, I was surprised to find that these publications only expected such letters from professors instead of factory workers. No matter, once I had the combination of a drawing program and Mathcad, I could get more involved in simply learning. When I got these two together, I started putting some of my original drawing's on paper into a formal digital diary.

I have posted my work on AOL, personal free websites, even shareware CD rom's, mostly because I was looking for a kindred study flame, which never happened.

Before the Delian Quest, I pondered the issue of *Who wrote the Book of John*? I found that all evidence points to the wife of Christ, Mary and that enough still remains in the text to make a good case of it as it was deliberately altered to hide the fact. There is evidence to support the idea that Christ started to perform at his own wedding, which has a root in Law. His wife evidently became a companion prophet. There is also evidence that Peter had his eye on her which never panned out.

Then I decided to tackle the Name of the Beast 666. The depth of that solution evolved over years. After these, I decided to find another impossible problem to solve when I ran into the Delian Problem. My studies started in my childhood by putting before myself specific questions. Then for a long time just reading any book I could come up with. Then I started targeting so called impossible problems. My original approach to the Bible was a result of a sign post in a lucid dream. My first reaction was that it was rubbish, but it quickly faded when I realized, rather quickly, it was using words in a manner I had never seen before, it was deliberately testing the reader. Later, I found another writer who wrote in a similar fashion, Plato.

All of this tells me that I have been in a state of cognitive dissonance before I started my work on the Number of His Name and geometry as I should have been dead prior to this, before I came back to Michigan.

At this time, I am want to do another revision of the Delian Quest and have decided to put the opening as I had done before the 2015 release.

THE DELIAN SOLUTION

I do not view the Delian Problem in the traditional sense, that is, as the problem of duplicating a cube. I view the problem of duplication as only one application for a Geometric process of general cube root abstraction. Truisms and processes are there to be discovered, if they are at all possible, it is possible to discover them. The Delian Problem is, for the correct Geometric figure, just one application of that figure. But then I am not a Geometer. In fact I dropped out of College to work on some items that I think are of particular importance, and tracing down the solution of the Delian Problem was just an example of some of the researches for that work. I have not even studied much of applied Geometry, I have been working on the theoretical aspects of it. Therefore this is an amateurs presentation and is not anything resembling a "finished" demonstration. The preceding figures are, however, technically correct abstractions.

While doing my first once over of the Elements, I found the propositions in Book 2 relating to the equality of complements. It occurred to me that since this was the correct method in which to manipulate area, an extension of it could be used to manipulate volume. This proved, after much searching, to be true. An extension of the figure can be used to abstract cube roots as well.

To cube a three dimensional rectilineal figure one has to start with the concepts of area manipulation on the two dimensional level. Starting with the rectangle in plate 1, one can see step by step, through plate 4, how to manipulate the figure while retaining the same area. If you study the Elements you will find a more complete set of abstracts to learn from. The method of squaring two sides is an abstract from this method.

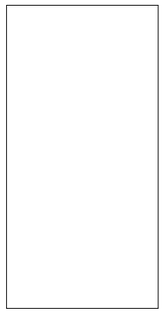


Plate 1

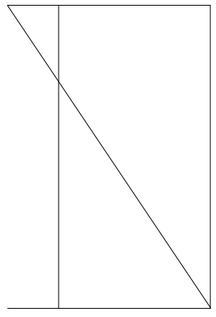


Plate 2

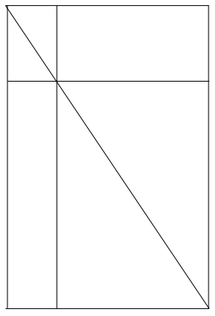


Plate 3

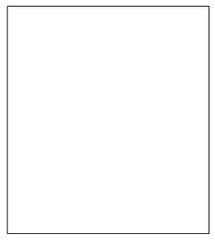


Plate 4

The question becomes- "Can the figure be extended to work with three sides and retain the same volume?"

I will take a short side trip. Turn to plate 5. Given a square and any point on the diagonal ray off the square, that is, extended outside of that square, divide two sides of that square into identical ratios with a ray from the chosen point. Plate 5 demonstrates the solution. If you play with the figure long enough you will find that it is a direct abstract from the figure that yields square roots. Length AB=CD, BC=DE. This is not a trivial matter, for to perform cubic reductions we will have to find a method to divide this square into three such segments that have a special relationship.

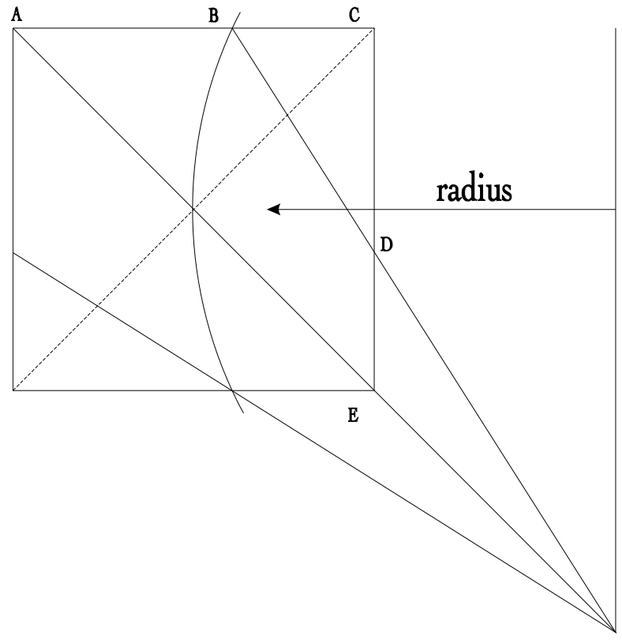
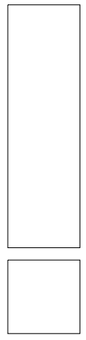
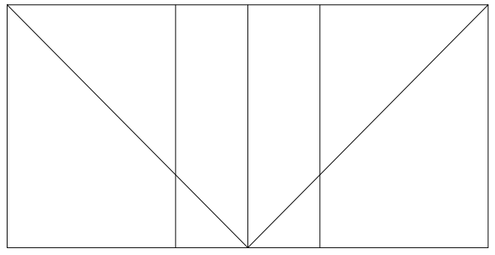


Plate 5 AB = CD. BC = DE

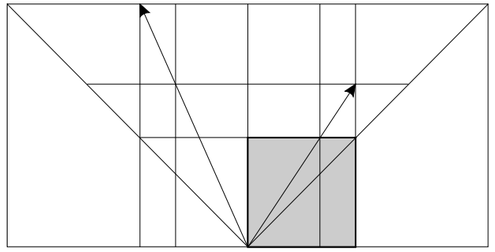
Let us take a "bar" as in P.6 and put two of its sides back to back as in P.7. Also lay out a 45 degree angle, as that is what our cube will have to end with. In P.8 we lay out where the cube is suppose to take place and we place in the two rays that will make the respective area manipulations. After several of these figures we will note that the two angles are indeed related because it always takes exactly two divisions to cube the figure, however, in this configuration I cannot gather any hint of another relationship between these two rays, or their angles.



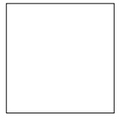
P.6



P.7

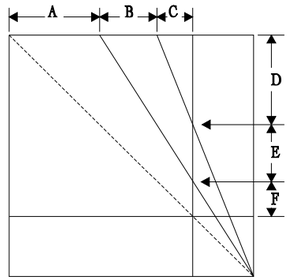


P.8

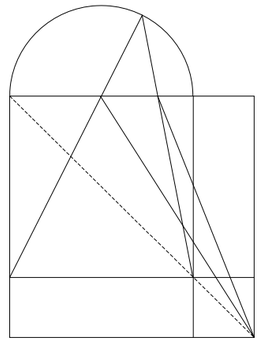


P.9

If I place both rays on the same side of the figure, I not only save paper, I also see a relationship, A=D, B=E, C=F, and by working with these segments find that the square root of AC=B.

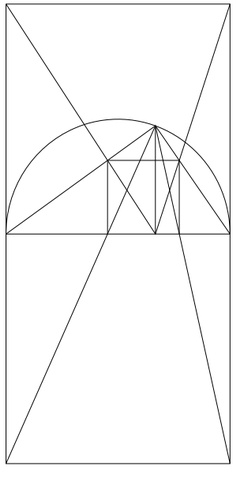


P. 10 A=D, B=E, C=F



P. 11

With a little more playing around, I note also that I can put a "hat" on the figure and that rays from the corners of the square under that figure will always converge on that semicircle. The immediate implication is that, not only is the square root related to the right angle, but the cube root as well.



Let us work with the square in a right angle for a moment. In P.12 we find the answer to the question—"How do I find the square in a right triangle?"

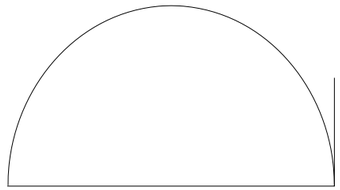


Plate 13

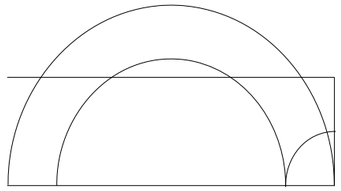


Plate 14

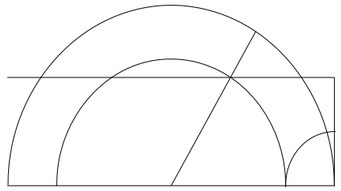


Plate 15

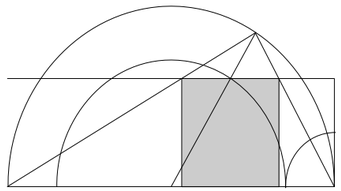
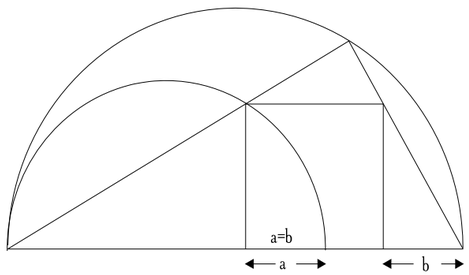
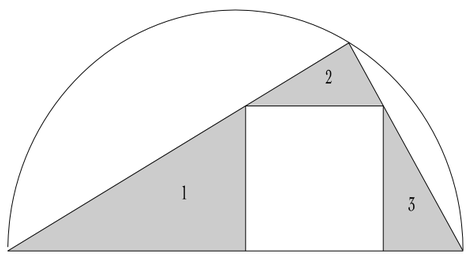


Plate 16

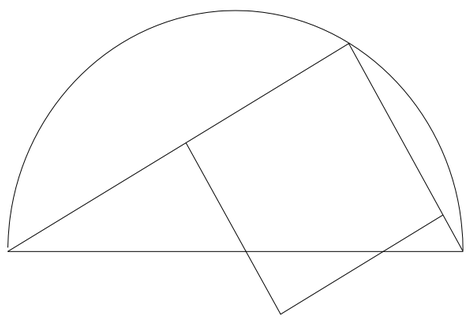
In Plates 13 through 16, we find the answer to the question—"Given a length of line, and another that must be one third or less of the first, what is the right angle which contains this segment as one side of a square?" The questions could be stated more technically than this, but—.



In P.17 We see that "The square in a right triangle is equal to the square of the remaining two segments, and in a duplicate ratio and"



P.18 "The three triangles on the sides of that square are in a triplicate ratio to those sides of that square."



P.19 Shows another square that can be formed using the same segments as the square in a right triangle. This square will be used in demonstrating another triplicate ratio, and that is the Pythagorean principle for cube roots.

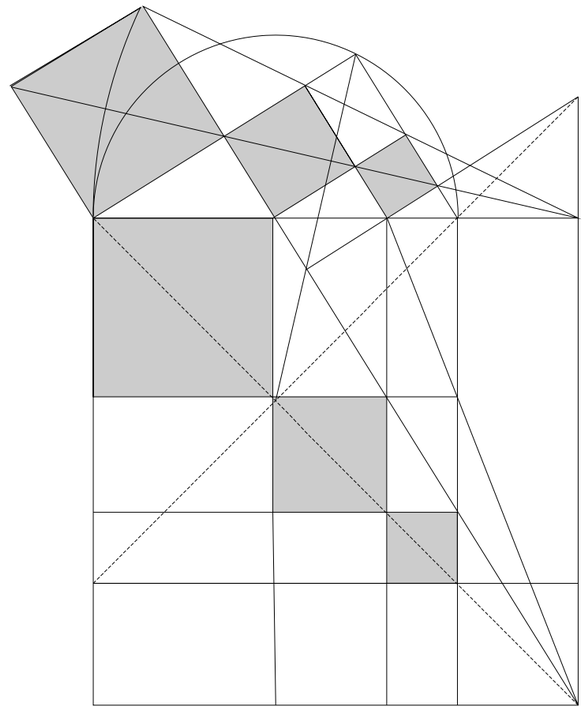
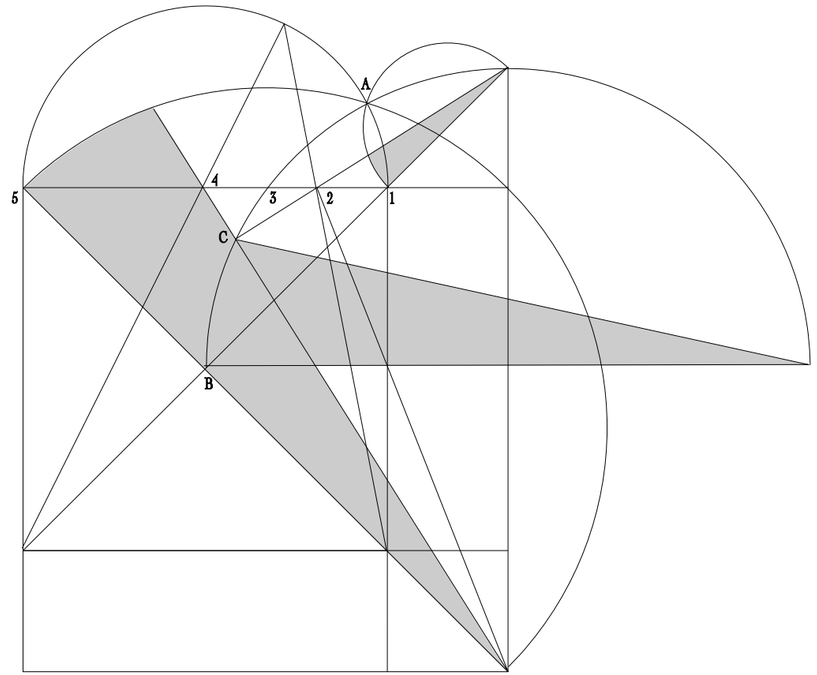


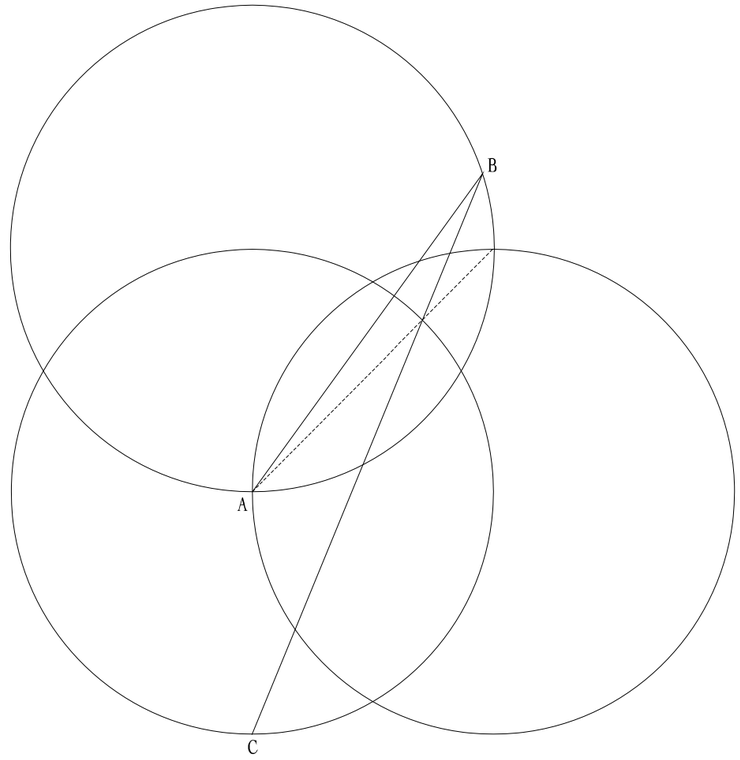
Plate 20 Demonstrates the triple implication of the Pythagorean theorem in the cubic expression. It will also do well to note that the three squares in continued proportion on the diagonal of the square under the right angle major have the same relationship as the segments on the hypotenuse of that given right angle.

There is one more triple proportion to look at. Plate 21.



All three semicircles intersect the semicircle which we draw our right angle in at the same point (A). From the center of the square under our eventual right angle (E) until they all intersect on that semicircle (A) they will produce a 90 degree angle divided into a 45 in the semicircle that is used for the square root application. I will leave it to the inquisitive reader to find the proper method of using this information to find a the method to find the intersect of the two rays under this figure. One will note that I have written in the progression along the bottom of the plate of the lengths involved.

Let us turn now to a simple but very productive diagram, Plate 22.



How close is the segment AB to the cube root of the circle given as a sphere? Is it well within pencil tolerances?

What approximate cube root figure is the segment CB a part of? (for abstract see Plate 12.)

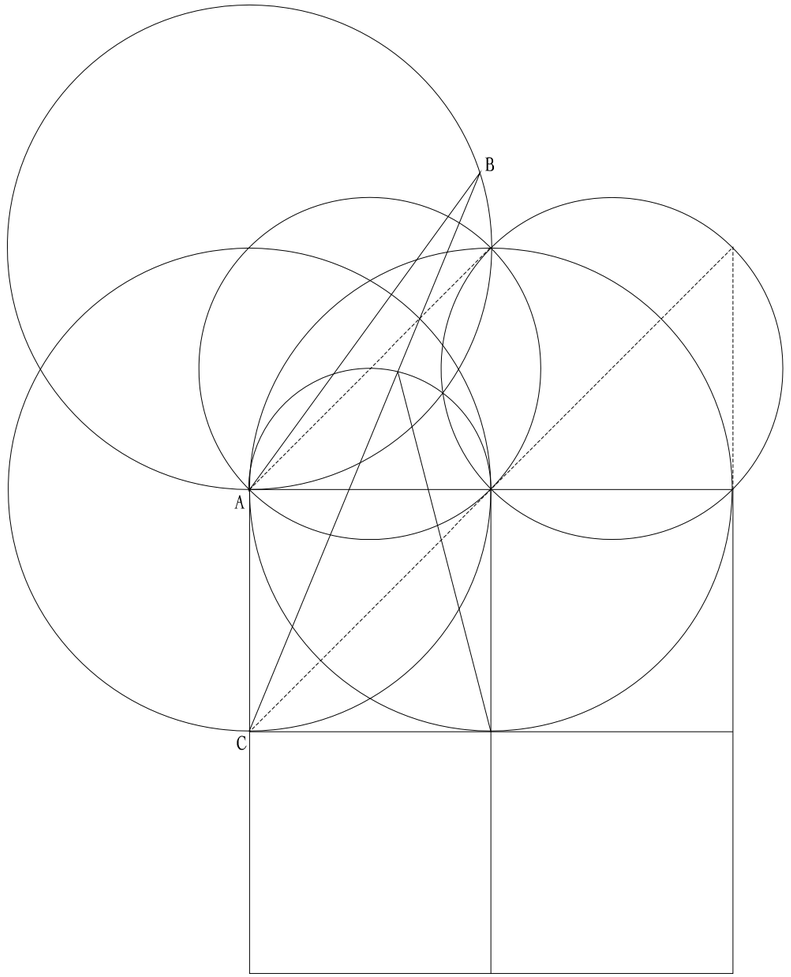
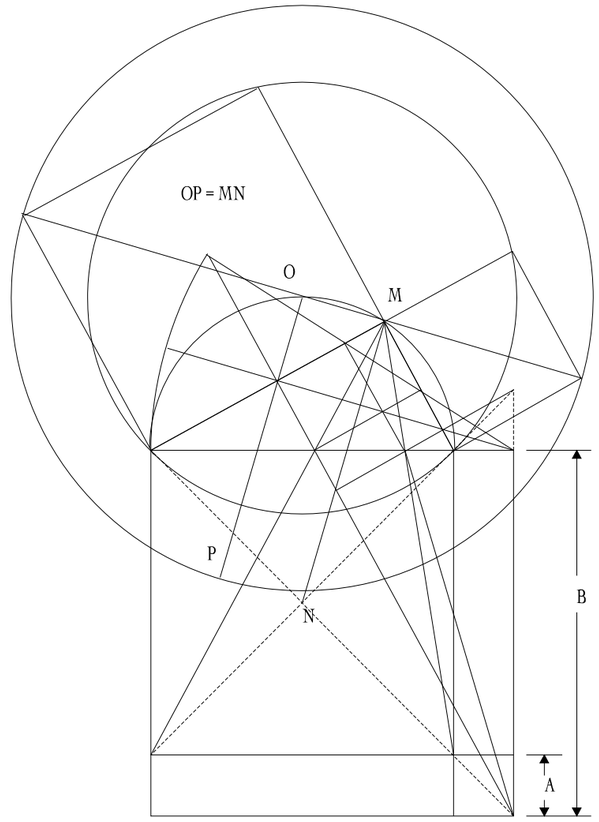


Plate 23

On Plate 24 the radius for the circle OP is given by MN.



One of the items that one will find in working with the figure is that it is infinitely recursive, by adding a couple of lines in the right place, one has another cubic relationship.

How do we test for accuracy of our cubic results? It will be noted that on the line that is the cube root of B^2A (if you have missed it, the figure gives both roots, A^2B and B^2A) there is a series of intersects, (three of them). When these intersects form a line parallel to the base of the figure then the result is accurate, this should have been abstracted, and used for proof, during one's play with P.7 and P.8.

A great number of propositions relating to the figure can be generated, of which I shall not here produce any. Doubling of the cube came very early in my search, the difficult part was in learning how to abstract any root. Now I have much work to do, and much learning to do to complete that work and I hope you have fun with the figure. J.C.

The following is a list of returns for publication of the previous material. The first one is interesting in that the writer claims not to have understood the preceding document.

GEOMETRIAE DEDICATA

Managing Editors:

K. STRAMBACH
Mathematisches Institut
Bismarckstr. 1 1/2
D-8520 Eriangen
ER. Germany
Telephone: (0)9131-8525(X)
or 852502

F. D. VELDKANW
Nblemakch Mm4uw
Postbus 80.010
3508 TA Utrecht
The Netherlands
Telephone: (0)30-531519

Utrecht, 15 December 1989

Dear Mr. Clark,

From Kluwer academic publishers I received your manuscript *The Delian Solution* which they presumed you wanted to submit for Geometriae Dedicata. It is not clear to me what these considerations on elementary Euclidean geometry are aiming at. Geometriae Dedicata is a journal for research in modern geometry and related fields. I think it is not the place to publish your manuscript, which we cannot accept therefore. I return the three copies under separated cover.

Sincerely,
F.D. Veldkamp

American Mathematical Society

PO. Box 6248, Providence, Rhode Island 02940 USA Telephone (401) 272-9500
Telex 797192, FAX 401-331-3842

Location:
201 Charles **Street**
Providence, **RI**
02904

December 8, 1989

Professor Professor John J. Clark

Dear Professor Clark,

I recently received your manuscript entitled "The Delian solution" for consideration in *BULLETIN* (*NEW SERIES*) *OF THE AMERICAN MATHEMATICAL SOCIETY*. Please note that I am forwarding your paper to Professor Roger E. Howe (Department of Matheniatics, Yale University, New Haven. CT 06520) and that you should address all your inquiries to the editor.

Sincerely yours,

Christine Vendettuoli
Publications Department

American Mathematical Society

Roger E. Howe
Bulletin
Editorial Committee

Department of Mathematics
Yale University
Box 2155, Yale Station
New Haven, CT 06520

December 14, 1989

Dear Professor Clark:

I regret to inform you that we are unable to accept your paper, "The Delian solution" for publication in the Bulletin. Our reviewers felt that the results are not of sufficient general interest or significance to warrant publication in the Bulletin.

Yours truly,
Roger E. Howe
Editor
Research Bulletin

REH/med

JOURNAL OF GEOMETRY

Editor's Office

Prof. Dr. H.-J. Kroll
Mathematisches Institut
Technische Universitiit MUnchen
Arcisstr. 21

D-8000 MUnchen 2

January 17, 1990

Dear Professor Clark,

Thank you very much for your manuscript on "THE DELIAN SOLUTION".

Date of receipt is January 12, 1990. As soon as we have the referee's report at hand, you will receive further information.

Yours sincerely,
H.-J. Kroll

Expert opinion on the article "The Delian Solution" of John J. Clark:

You can find some interesting statements in the submitted version of this article but exact constructions are missing. Mostly it's not clear where some points or lines come from. Therefore the proofs also cannot be examined very good. And for the more complicated constructions it's very difficult to follow the ideas of the author. Moreover the relation between the single statements and the opening problem doesn't come out very good.

All together the article in the given version is not understandable.

JOURNAL OF GEOM TRY

Editor's Office

München, 1 June 1990

Dear Professor Clark,

Your paper "The Delian Solution" submitted to the Journal of Geometry is hereby regrettably returned by the decision of the reviewers.

We are very sorry that we could not be of any help to you.

Sincerely yours,
H.-J. Kroll

(This one is a form letter.)

société mathématique de france

paris, le
BULLETIN

n. réf. a l'attention de
v. réf.

Cher(e) collègue,

Le Comité de Rédacton du Bulletin de la Société Mathématique de France n'a pas pu accepter votre article intitulé

the Detian Solution

Malgré l'intérêt de votre travail, celui-ci ne correspond pas à l'esprit de notre Revue destinée à un large public de mathématiciens et non à des lecteurs trop spécialisés.

Nous vous prions d'agréer, cher(e) collégue, l'expression de nos sentiments les meilleurs.

P. SCHAPIRA
Directeur de la Publication

P.J. : Manuscrit

I was expecting at least some type of guidance, or cogent response, other than, after stating that I was not a geometer, everyone insisted on rubbing it in by calling me a professor, which I found very rude.

062092

Given DE, AB, BC, place DE on AC such that with some point J, as AB : AD :: AE : AC and as AD : AJ :: AJ : AE and as AB : AJ :: AJ : AC.

Descriptions.

Given.

$N_1 := .74206$

$N_2 := 3 \qquad BC := N_2$

$N_3 := 1.05 \qquad DE := N_3$

$AC := N_1 + BC$

$AF := N_1 \qquad AG := AC \qquad AJ := \sqrt{AF \cdot AG}$

$AL := \frac{DE}{2} \qquad JL := \sqrt{AJ^2 + AL^2}$

$AD := JL - AL \qquad AE := JL + AL$

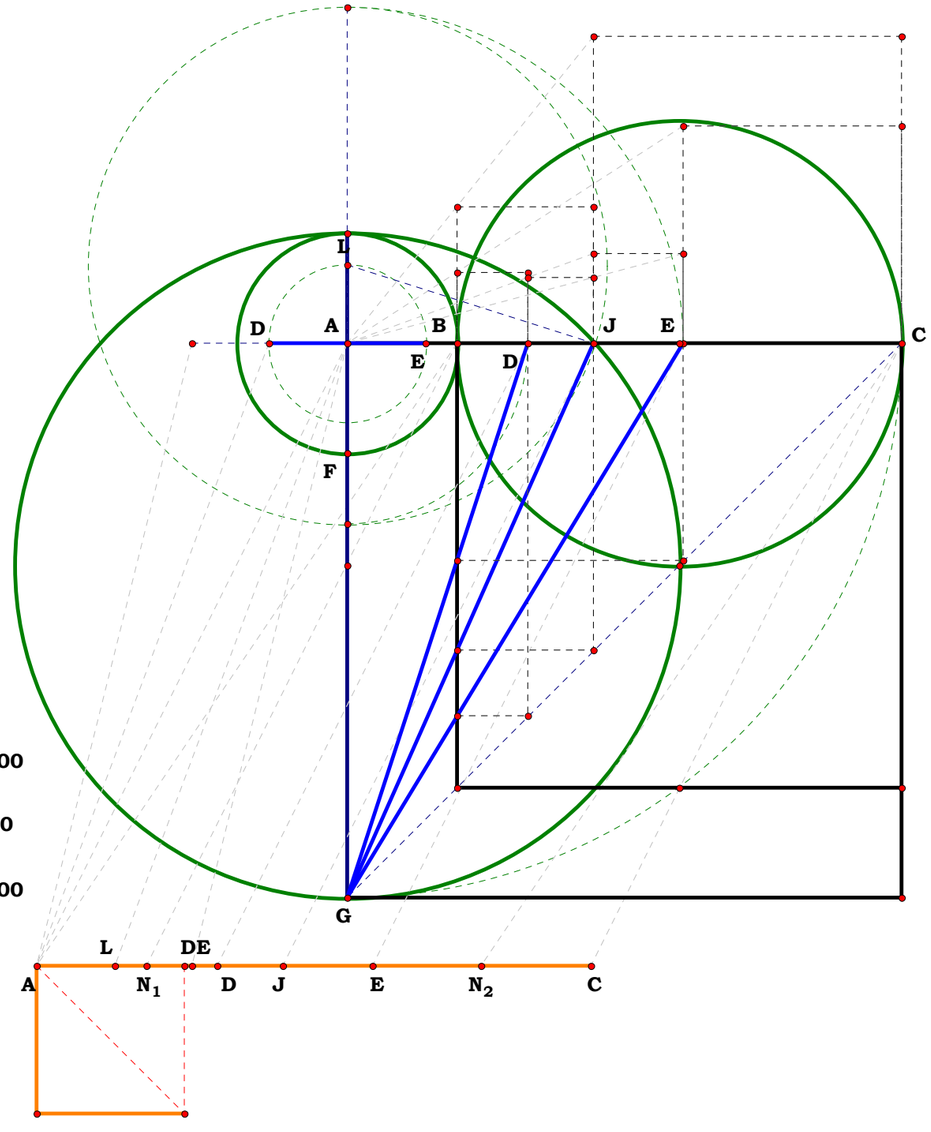
$\frac{N_1}{AD} - \frac{AE}{AC} = 0 \qquad \frac{AD}{AJ} - \frac{AJ}{AE} = 0 \qquad \frac{N_1}{AJ} - \frac{AJ}{AC} = 0$ etc.,

Definitions

AC = 3.74206 AJ = 1.666383 AL = 0.525

AD = 1.222129 AE = 2.272129

A Duplicate Ratio



N_1 = 0.74206
N_2 = 3.00000
DE = 1.05000
X/Y = 0.35000
X = 7.00000
Y = 20.00000
C = 3.74206
J = 1.66638
L = 0.52500
D = 1.22212
E = 2.27212
$\frac{N_1}{D} - \frac{E}{C}$ = 0.00000
$\frac{D}{J} - \frac{J}{E}$ = 0.00000
$\frac{N_1}{J} - \frac{J}{C}$ = 0.00000

Definitions

$$AC - \left(N_1 + N_2\right) = 0$$

$$AF - N_1 = 0$$

$$AG - \left(N_1 + N_2\right) = 0$$

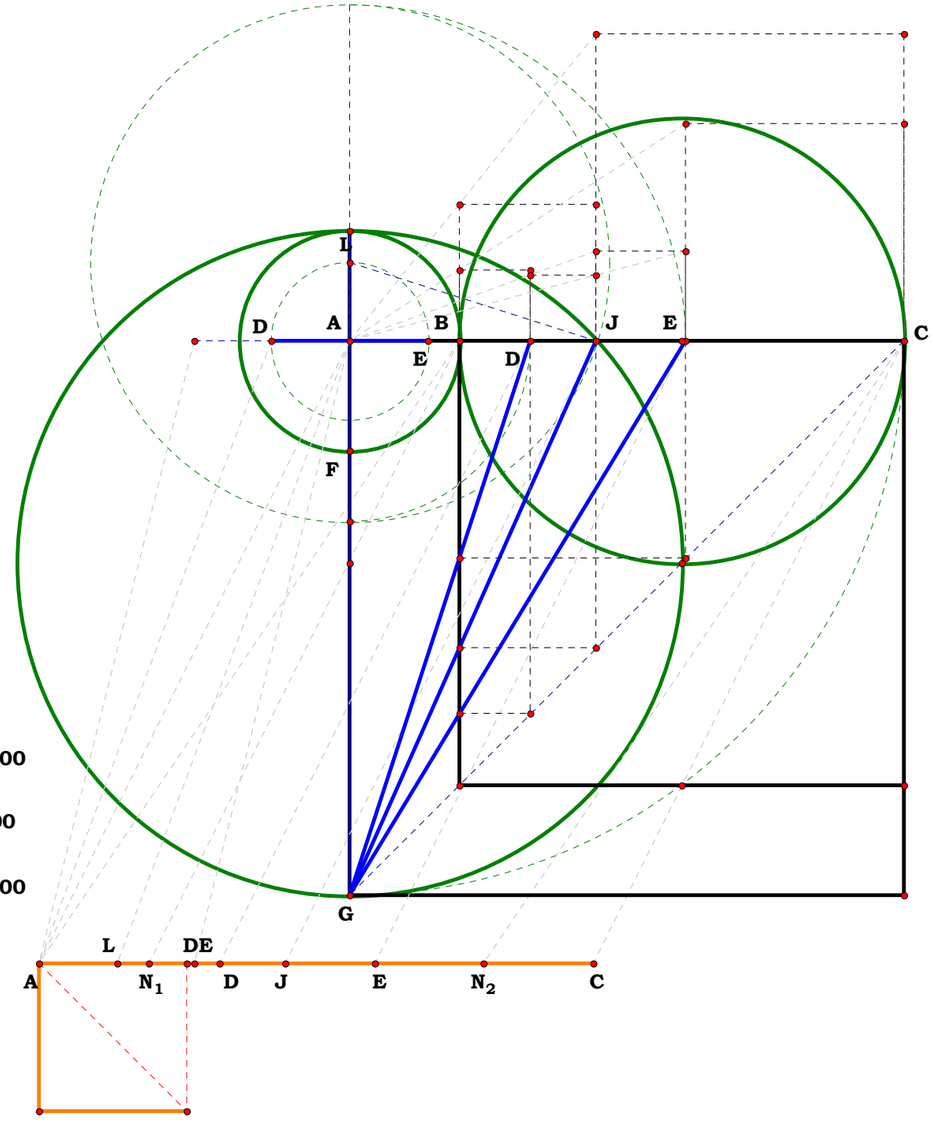
$$AJ - \sqrt{N_1 \cdot \left(N_1 + N_2\right)} = 0$$

$$AL - \frac{N_3}{2} = 0$$

$$JL - \frac{\sqrt{\left(4 \cdot N_1^{\,2} + 4 \cdot N_2 \cdot N_1 + N_3^{\,2}\right)}}{2} = 0$$

$$AD - \frac{\sqrt{4 \cdot N_1^{\,2} + 4 \cdot N_1 \cdot N_2 + N_3^{\,2}} - N_3}{2} = 0$$

$$AE - \frac{N_3 + \sqrt{4 \cdot N_1^{\,2} + 4 \cdot N_1 \cdot N_2 + N_3^{\,2}}}{2} = 0$$

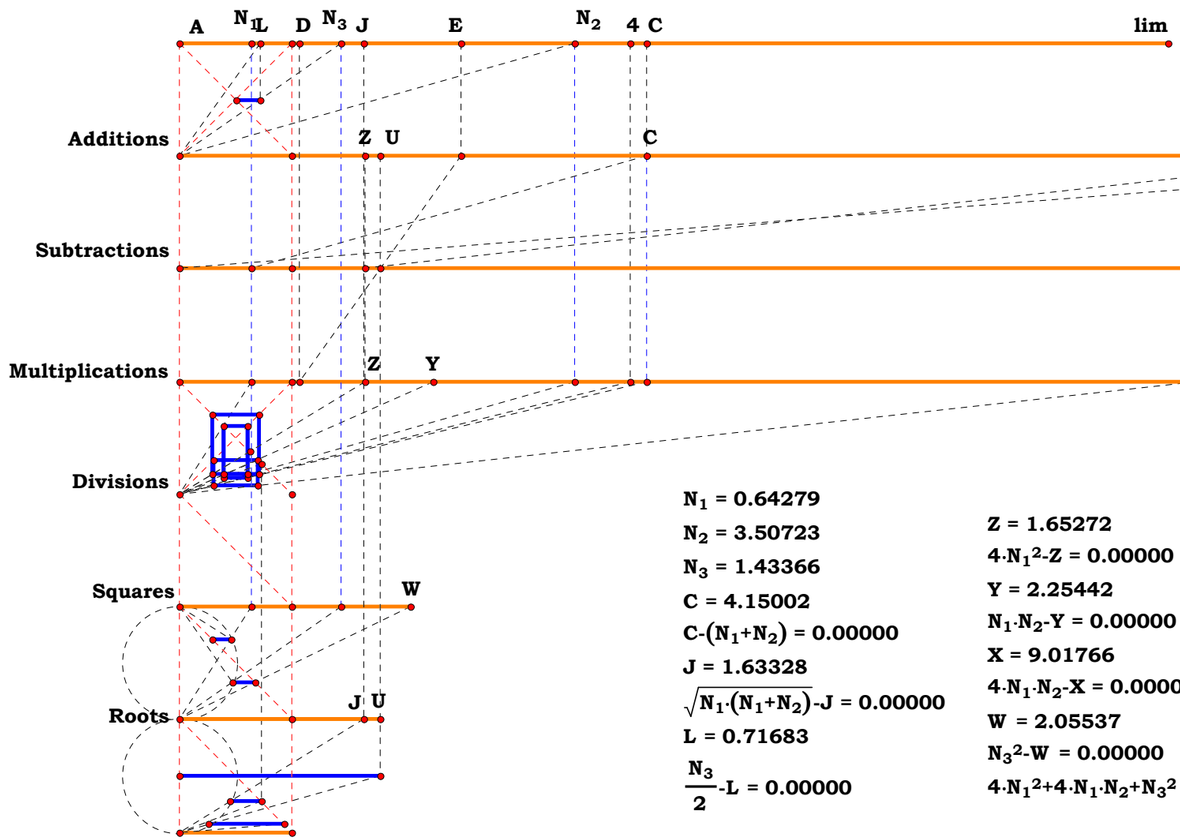


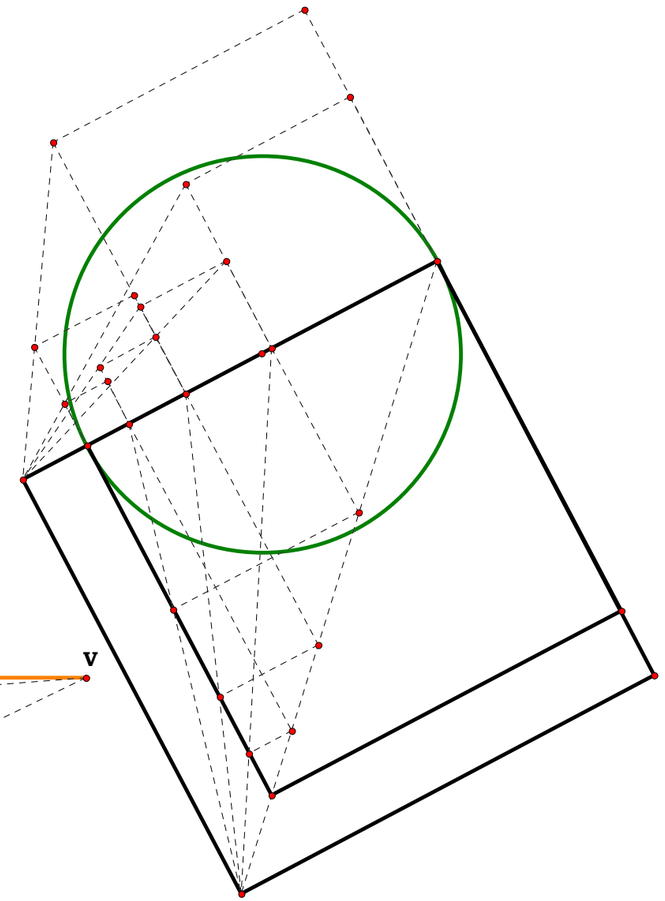
N_1 = 0.74206
N_2 = 3.00000
DE = 1.05000
X/Y = 0.35000
X = 7.00000
Y = 20.00000
C = 3.74206
J = 1.66638
L = 0.52500
D = 1.22212
E = 2.27212

$$\frac{N_1}{D} - \frac{E}{C} = 0.00000$$

$$\frac{D}{J} - \frac{J}{E} = 0.00000$$

$$\frac{N_1}{J} - \frac{J}{C} = 0.00000$$





$N_1 = 0.64279$

$N_2 = 3.50723$

$N_3 = 1.43366$

$C = 4.15002$

$C-(N_1+N_2) = 0.00000$

$J = 1.63328$

$\sqrt{N_1 \cdot (N_1+N_2)}-J = 0.00000$

$L = 0.71683$

$\frac{N_3}{2}-L = 0.00000$

$Z = 1.65272$

$4 \cdot N_1{}^2-Z = 0.00000$

$Y = 2.25442$

$N_1 \cdot N_2-Y = 0.00000$

$X = 9.01766$

$4 \cdot N_1 \cdot N_2-X = 0.00000$

$W = 2.05537$

$N_3{}^2-W = 0.00000$

$4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2 = 12.72575$

$V = 12.72575$

$U = 1.78366$

$\frac{\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}}{2}-U = 0.00000$

$D = 1.06683$

$\frac{\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}-N_3}{2}-D = 0.00000$

$E = 2.50049$

$\frac{N_3+\sqrt{(4 \cdot N_1{}^2+4 \cdot N_1 \cdot N_2+N_3{}^2)}}{2}-E = 0.00000$

Unit.

$AB := 1$

Given.

$N_1 := AB$

081292

Given AB, how close is BJ to the cube root of AB taken as a sphere?

$$CUBE\_ROOT := \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}$$

Descriptions.

$$BH := \sqrt{2\cdot AB^2} \qquad CG := \frac{AB^2}{BH}$$

$$AG := \sqrt{CG^2 + (AB + CG)^2} \qquad DG := CG\cdot\frac{2AB}{AG}$$

$$GJ := \sqrt{AB^2 - DG^2} \qquad AE := \frac{(AB + CG)\cdot(AG + GJ)}{AG}$$

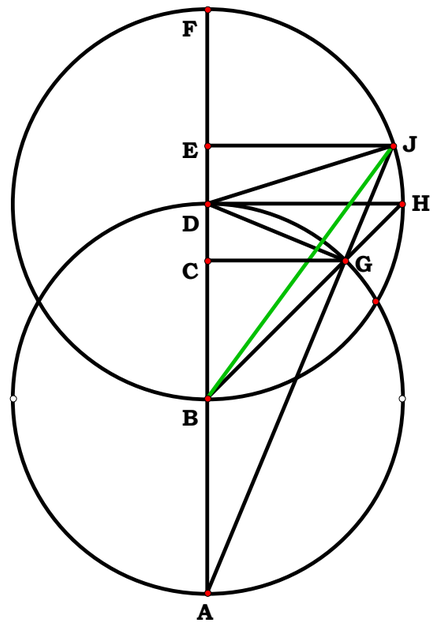
$$EJ := \frac{CG\cdot AE}{AB + CG} \qquad BJ := \sqrt{EJ^2 + (AE - AB)^2}$$

$$\frac{BJ}{\left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}}} = 1.000943 \qquad BJ - \left(\frac{4}{3}\cdot\pi\cdot N_1^{\,3}\right)^{\frac{1}{3}} = 0.00152$$

Definitions.

$$BJ - N_1\cdot\sqrt{\sqrt{2} + 2^{\frac{1}{4}}} = 0$$

Rusty Cube of a Sphere



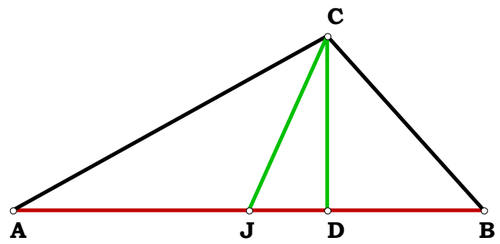
010893

Pythagoras Revisited

$AB := 7.89517$ $S_1 := AB$

$AC := 6.02581$ $S_2 := AC$

$BC := 3.92697$ $S_3 := BC$



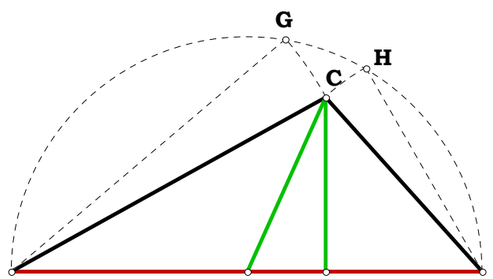
Descriptions.

Given just the three sides of any triangle, find its heighth from the perpendicular CD, DJ and the medial bisector CJ.

$$AE := \frac{AC^2}{AB} \qquad BF := \frac{BC^2}{AB} \qquad EF := AB - (AE + BF) \qquad DE := \frac{EF}{2}$$

$$AD := AE + DE \qquad BD := AB - AD \qquad CD := \sqrt{S_2^2 - AD^2}$$

$$AJ := \frac{AB}{2} \qquad DJ := AD - AJ \qquad CJ := \sqrt{CD^2 + DJ^2}$$

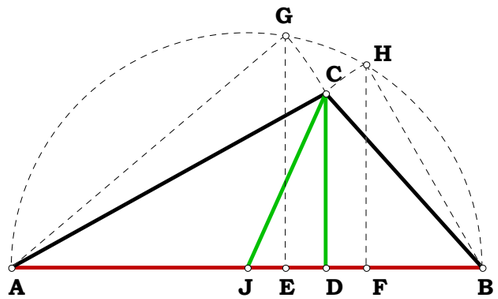


Definitions.

$$EF - \frac{AB^2 - AC^2 - BC^2}{AB} = 0 \qquad DE - \frac{AB^2 - AC^2 - BC^2}{2 \cdot AB} = 0$$

$$AD - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0 \qquad BD - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

$$DJ - \frac{\sqrt{\left(AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0 \qquad CJ - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0$$



$$EF - \frac{S_1^2 - S_2^2 - S_3^2}{S_1} = 0 \qquad DE - \frac{S_1^2 - S_2^2 - S_3^2}{2 \cdot S_1} = 0$$

$$AD - \frac{S_1^2 + S_2^2 - S_3^2}{2 \cdot S_1} = 0 \qquad BD - \frac{S_1^2 - S_2^2 + S_3^2}{2 \cdot S_1} = 0$$

$$DJ - \frac{\sqrt{\left(S_2^2 - S_3^2\right)^2}}{2 \cdot S_1} = 0 \qquad CJ - \frac{\sqrt{2 \cdot S_2^2 - S_1^2 + 2 \cdot S_3^2}}{2} = 0$$

$$CD - \frac{\sqrt{[(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)]}}{2 \cdot AB} = 0$$

$$CD - \frac{\sqrt{\left[\left(S_1 + S_2 - S_3\right) \cdot \left(S_1 - S_2 + S_3\right) \cdot \left(S_2 - S_1 + S_3\right) \cdot \left(S_1 + S_2 + S_3\right)\right]}}{2 \cdot S_1} = 0$$

Unit is external.

Given.

$N_1 := 3$

$N_2 := 4$

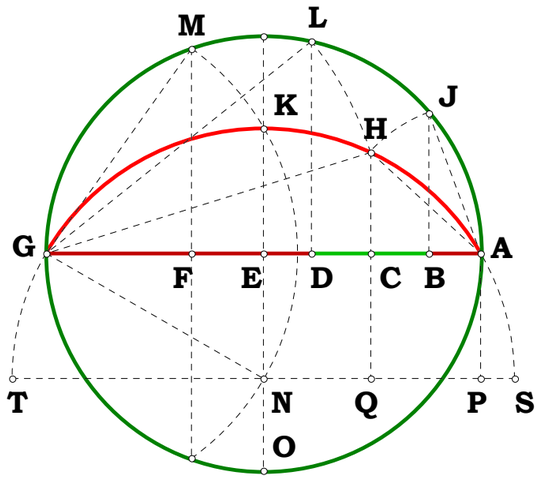
060393R1

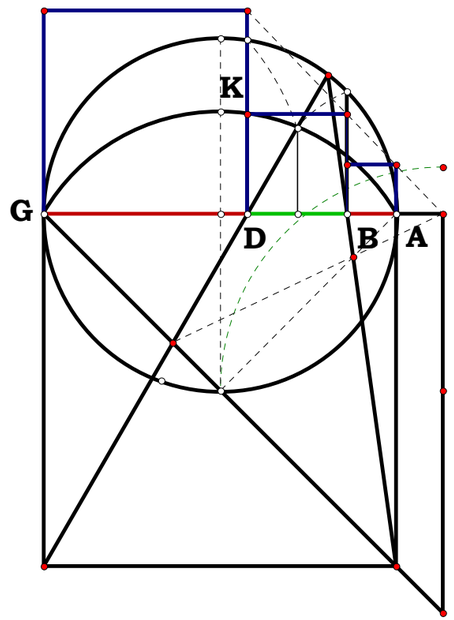
The curve AK is derived from the cube root figure as demonstrated.

Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?
Divide a segment twice such that the mean segment is the root of the extreems.

Descriptions.

Exploring The Curve AK





$AG := N_1 \qquad AC := \frac{AG}{N_2}$

$GF := \frac{AG}{3} \qquad FM := \sqrt{GF \cdot (AG - GF)}$

$GM := \sqrt{GF^2 + FM^2} \qquad ST := 2 \cdot GM \qquad EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$

$PS := \frac{ST - AG}{2} \qquad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$

$CH := HQ - EN \qquad AH := \sqrt{AC^2 + CH^2} \qquad GH := \sqrt{(AG - AC)^2 + CH^2}$

$AB := \frac{AH^2}{AG} \qquad DG := \frac{GH^2}{AG} \qquad BD := AG - (AB + DG)$

$BD - \sqrt{AB \cdot DG} = 0 \qquad AB = 0.348612 \qquad BD = 0.802776 \qquad DG = 1.848612$

Definitions.

$AC - \frac{N_1}{N_2} = 0 \qquad GF - \frac{N_1}{3} = 0 \qquad FM - \frac{\sqrt{2} \cdot N_1}{3} = 0 \qquad GM - \frac{\sqrt{3} \cdot N_1}{3} = 0 \qquad ST - \frac{2 \cdot \sqrt{3} \cdot N_1}{3} = 0 \qquad EN - \frac{N_1}{\sqrt{12}} = 0$

$PS - \frac{N_1 \cdot (2 \cdot \sqrt{3} - 3)}{6} = 0 \qquad HQ - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12}}{N_2 \cdot \sqrt{12}} = 0 \qquad CH - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{N_2 \cdot \sqrt{12}} = 0 \qquad AH - \frac{N_1 \cdot \sqrt{N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6}}{\sqrt{6 \cdot N_2}} = 0$

$GH - \frac{N_1 \cdot \sqrt{7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6}}{\sqrt{6 \cdot N_2}} = 0 \qquad AB - \frac{N_1 \cdot \left(N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} + 6\right)}{6 \cdot N_2} = 0 \qquad DG - \frac{N_1 \cdot \left(7 \cdot N_2 - \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{6 \cdot N_2} = 0$

$BD - \frac{N_1 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - N_1 \cdot N_2}{3 \cdot N_2} = 0 \qquad BD^2 - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0 \qquad AB \cdot DG - \frac{2 \cdot N_1^2 \cdot \left(6 \cdot N_2 + N_2^2 - N_2 \cdot \sqrt{N_2^2 + 12 \cdot N_2 - 12} - 6\right)}{9 \cdot N_2^2} = 0$

Unit.

$AG := 1$

Given.

$N_1 := 4$

060393R2

The curve AK is derived from the cube root figure as demonstrated.

Given AG and that GF equals one third of AG, for any AC is BD the square root of AB multiplied by DG?
Divide a segment twice such that the mean segment is the root of the extreems.

Exploring The Curve AK

Descriptions.

$$AC := \frac{AG}{N_1} \qquad GF := \frac{AG}{3} \qquad FM := \sqrt{GF \cdot (AG - GF)}$$

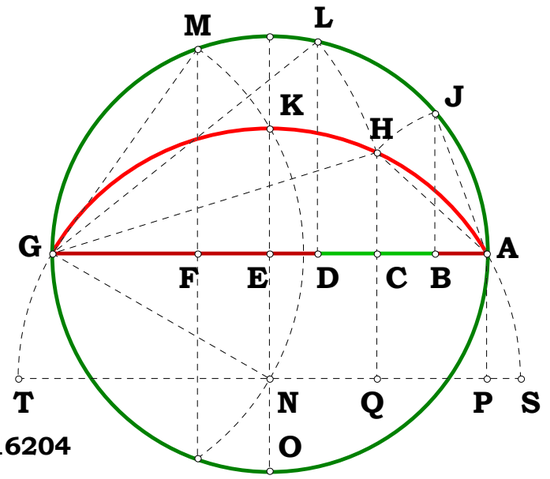
$$GM := \sqrt{GF^2 + FM^2} \qquad ST := 2 \cdot GM \qquad EN := \sqrt{GM^2 - \left(\frac{AG}{2}\right)^2}$$

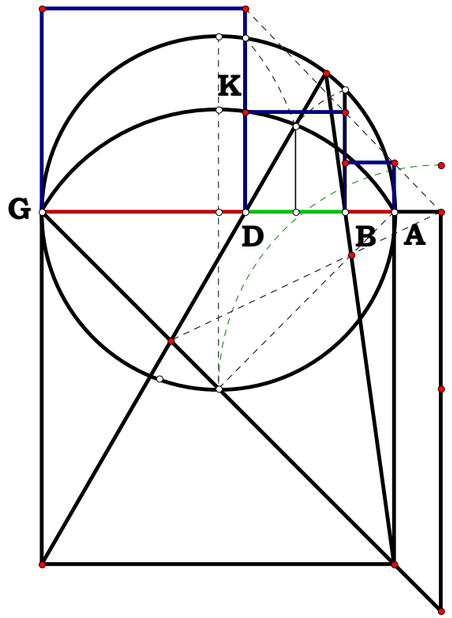
$$PS := \frac{ST - AG}{2} \qquad HQ := \sqrt{(AC + PS) \cdot (AG - AC + PS)}$$

$$CH := HQ - EN \qquad AH := \sqrt{AC^2 + CH^2} \qquad GH := \sqrt{(AG - AC)^2 + CH^2}$$

$$AB := \frac{AH^2}{AG} \qquad DG := \frac{GH^2}{AG} \qquad BD := AG - (AB + DG)$$

$$BD - \sqrt{AB \cdot DG} = 0 \qquad AB = 0.116204 \qquad BD = 0.267592 \qquad DG = 0.616204$$





Definitions.

$$AC - \frac{1}{N_1} = 0 \qquad GF - \frac{1}{3} = 0 \qquad FM - \frac{\sqrt{2}}{\sqrt{9}} = 0 \qquad GM - \frac{1}{\sqrt{3}} = 0 \qquad ST - \frac{2 \cdot \sqrt{3}}{3} = 0$$

$$EN - \frac{1}{\sqrt{12}} = 0 \qquad PS - \left(\frac{\sqrt{3}}{3} - \frac{1}{2}\right) = 0 \qquad HQ - \frac{\sqrt{N_1^2 + 12 \cdot N_1 - 12}}{\sqrt{12} \cdot N_1} = 0 \qquad CH - \frac{\left(\sqrt{N_1^2 + 12 \cdot N_1 - 12} - N_1\right) \cdot \sqrt{3}}{6 \cdot N_1} = 0$$

$$AH - \frac{\sqrt{N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} + 6}}{\sqrt{6 \cdot N_1}} = 0 \qquad GH - \frac{\sqrt{7 \cdot N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} - 6}}{\sqrt{6 \cdot N_1}} = 0 \qquad AB - \frac{N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} + 6}{6 \cdot N_1} = 0$$

$$DG - \frac{7 \cdot N_1 - \sqrt{N_1^2 + 12 \cdot N_1 - 12} - 6}{6 \cdot N_1} = 0 \qquad BD - \frac{\sqrt{N_1^2 + 12 \cdot N_1 - 12} - N_1}{3 \cdot N_1} = 0 \qquad BD - \frac{\sqrt{2 \cdot \left(N_1^2 + 6 \cdot N_1 - 6\right) - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 12 \cdot N_1 - 12}}}{3 \cdot N_1} = 0$$

060793R1

Unit is external.

Given.

$AD := 2.17506$ $AB := 3.14654$ $AC := 1.74732$

$BD := 2.61333$ $CD := 1.38168$

Two Triangles with a Common Side.

Descriptions.

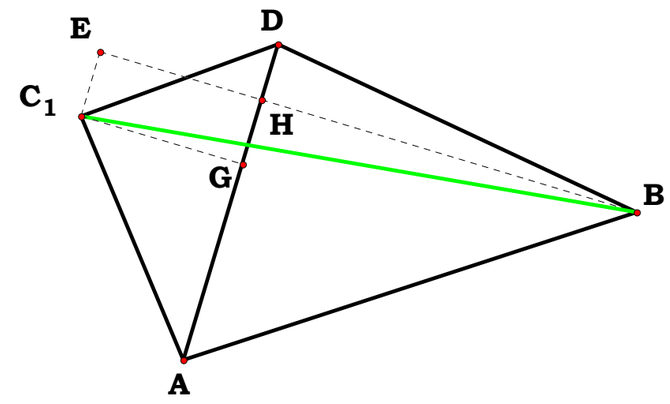
Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$CG := \frac{\sqrt{(AD + CD + AC)\cdot(-AD + CD + AC)\cdot(AD - CD + AC)(AD + CD - AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD + AB + BD)\cdot(-AD + AB + BD)\cdot(AD - AB + BD)(AD + AB - BD)}}{2\cdot AD}$$

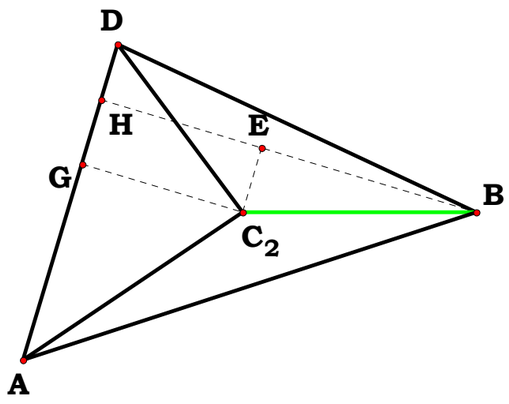
Greatest Disance:



$$BC_1 := \sqrt{GH^2 + (CG + BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH - CG)^2}$$

Least Distance



$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + AB^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$

Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}\ \ldots}{2\cdot AD} = 0$$

$$+\ -AD^4 - AB^2\cdot AC^2 + AB^2\cdot AD^2 + AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 + AD^2\cdot BD^2 + AB^2\cdot CD^2 + AD^2\cdot CD^2 - BD^2\cdot CD^2 + AB^2\cdot AD^2 - AD^4 - AB^2\cdot AC^2}\ \ldots}{2\cdot AD} = 0$$

$$+\ -\sqrt{(AB + AD - BD)\cdot(AB - AD + BD)\cdot(AD - AB + BD)\cdot(AB + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}$$

060793R2

Unit

$AB := 1$

Given.

$AD := 2.17506$ $\quad AC := 1.74732$

$BD := 2.61333$ $\quad CD := 1.38168$

Descriptions.

Let the two triangles ABD and ACD be given.

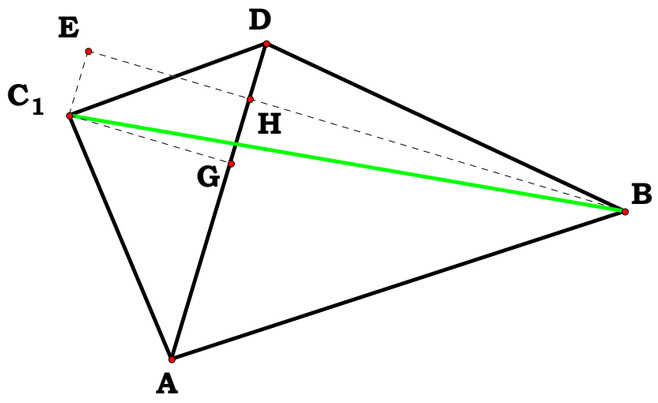
Given two triangles with a common side, find the difference between their free vertices from opposing sides.

Two Triangles with a Common Side.

$$CG := \frac{\sqrt{(AD+CD+AC)\cdot(-AD+CD+AC)\cdot(AD-CD+AC)(AD+CD-AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD+BD-1)\cdot(AD+BD+1)\cdot(AD-BD+1)\cdot(BD-AD+1)}}{2\cdot AD}$$

Greatest Disance:



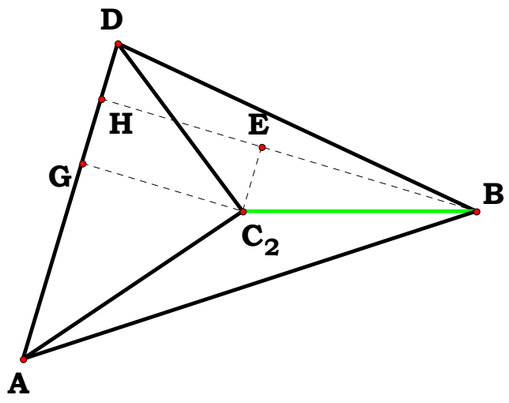
$$BC_1 := \sqrt{GH^2 + (CG+BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH-CG)^2}$$

Least Distance

$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + 1^2 - BD^2}{2\cdot AD}$$

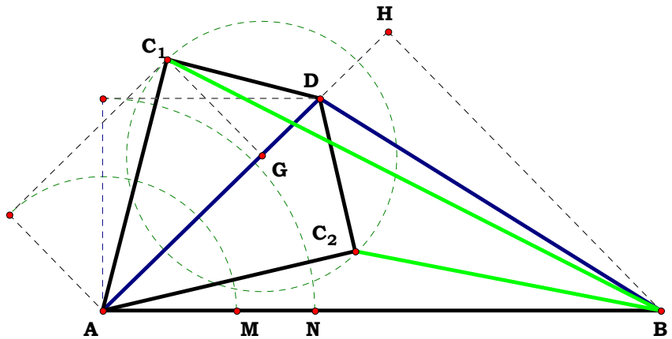
$$GH := AH - AG$$



Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{(AC\cdot AD)^2 + (AC\cdot BD)^2 - AC^2 - AD^4 + (AD\cdot BD)^2 + (AD\cdot CD)^2 - (BD\cdot CD)^2 + AD^2 + CD^2 \ldots + \sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)\cdot(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}}}{2\cdot AD} = 0$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{(AC\cdot AD)^2 + (AC\cdot BD)^2 - AC^2 - AD^4 + (AD\cdot BD)^2 + (AD\cdot CD)^2 - (BD\cdot CD)^2 + AD^2 + CD^2 \ldots + -\sqrt{(1+AD-BD)\cdot(1-AD+BD)\cdot(AD-1+BD)\cdot(1+AD+BD)(AC+AD-CD)\cdot(AC-AD+CD)\cdot(AD-AC+CD)\cdot(AC+AD+CD)}}}{2\cdot AD} = 0$$



Unit = 1.00000
XY = 0.24024
X = 3.20765
Y = 13.35191

XY = 0.38041
X = 7.60821
Y = 20.00000

AB = 9.36083 cm
AD = 5.09446 cm
AC_1 = 4.34404 cm
BD = 6.73584 cm
C_1D = 2.63736 cm
AG = 3.71664 cm
AH = 6.69425 cm
$\frac{AB}{AB}$ = 1.00000
$\frac{AD}{AB}$ = 0.54423
$\frac{AC_1}{AB}$ = 0.46407
$\frac{BD}{AB}$ = 0.71958
$\frac{C_1D}{AB}$ = 0.28174
$\frac{AG}{AB}$ = 0.39704
$\frac{AH}{AB}$ = 0.71513

BC_1 = 9.28248 cm
BC_2 = 5.22560 cm

(1+AD)-BD = 0.82465
(1-AD)+BD = 1.17535
(AD-1)+BD = 0.26381
1+AD+BD = 2.26381

(AC+AD)-CD = 0.72655
(AC-AD)+CD = 0.20158
(AD-AC)+CD = 0.36191
AC+AD+CD = 1.29004

AB = 1.00000
AD = 0.54423
AC = 0.46407
CD = 0.28174
BD = 0.71958
AG = 0.39704
AH = 0.71513

$\frac{(AD^2+AC^2)-CD^2}{2 \cdot AD}$ -AG = 0.00000

$\frac{(AD^2+1)-BD^2}{2 \cdot AD}$ -AH = 0.00000

BC_1 = 0.99163

BC_2 = 0.55824

$$\frac{(((((AC \cdot AD)^2+(AC \cdot BD)^2)-AC^2-AD^4)+(AD \cdot BD)^2+(AD \cdot CD)^2)-(BD \cdot CD)^2)+AD^2+CD^2}{\sqrt{((1+AD)-BD) \cdot ((1-AD)+BD) \cdot ((AD-1)+BD) \cdot (1+AD+BD) \cdot ((AC+AD)-CD) \cdot ((AC-AD)+CD) \cdot ((AD-AC)+CD) \cdot (AC+AD+CD)}} = 0.38355 \div 0.19895$$

$$\frac{\sqrt{2} \cdot \sqrt{(((((AC \cdot AD)^2+(AC \cdot BD)^2)-AC^2-AD^4)+(AD \cdot BD)^2+(AD \cdot CD)^2)-(BD \cdot CD)^2)+AD^2+CD^2)+\sqrt{((1+AD)-BD) \cdot ((1-AD)+BD) \cdot ((AD-1)+BD) \cdot (1+AD+BD) \cdot ((AC+AD)-CD) \cdot ((AC-AD)+CD) \cdot ((AD-AC)+CD) \cdot (AC+AD+CD)}}{2 \cdot AD} - BC_1 = 0.00000$$

$$\frac{\sqrt{2} \cdot \sqrt{(((((AC \cdot AD)^2+(AC \cdot BD)^2)-AC^2-AD^4)+(AD \cdot BD)^2+(AD \cdot CD)^2)-(BD \cdot CD)^2)+AD^2+CD^2)-\sqrt{((1+AD)-BD) \cdot ((1-AD)+BD) \cdot ((AD-1)+BD) \cdot (1+AD+BD) \cdot ((AC+AD)-CD) \cdot ((AC-AD)+CD) \cdot ((AD-AC)+CD) \cdot (AC+AD+CD)}}{2 \cdot AD} - BC_2 = 0.00000$$

060793R3

Unit

$AB := 1$

Given.

$AD := 2.17506$ $AC := 1.74732$

$BD := 2.61333$ $CD := 1.38168$

Two Triangles with a Common Side.

Descriptions.

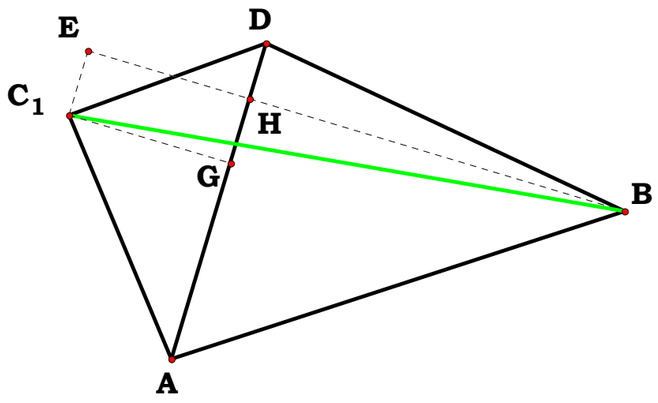
Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$CG := \frac{\sqrt{(AD + CD + AC)\cdot(-AD + CD + AC)\cdot(AD - CD + AC)(AD + CD - AC)}}{2\cdot AD}$$

$$BH := \frac{\sqrt{(AD + BD - 1)\cdot(AD + BD + 1)\cdot(AD - BD + 1)\cdot(BD - AD + 1)}}{2\cdot AD}$$

Greatest Disance:



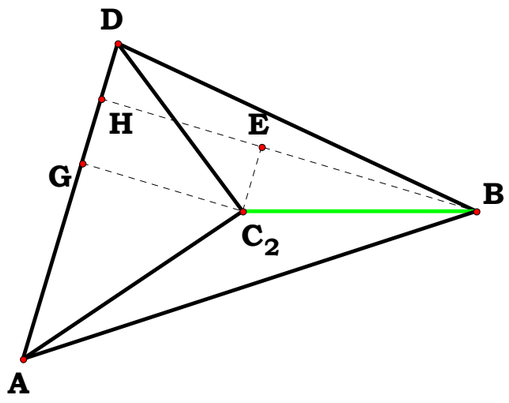
$$BC_1 := \sqrt{GH^2 + (CG + BH)^2}$$

$$BC_2 := \sqrt{GH^2 + (BH - CG)^2}$$

Least Distance

$$AG := \frac{AD^2 + AC^2 - CD^2}{2\cdot AD} \qquad AH := \frac{AD^2 + 1^2 - BD^2}{2\cdot AD}$$

$$GH := AH - AG$$



Definitions.

$$BC_1 - \frac{\sqrt{2}\cdot\sqrt{\sqrt{(1 + AD - BD)\cdot(1 - AD + BD)\cdot(AD - 1 + BD)\cdot(1 + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}\ \ldots}{2\cdot AD} = 0$$

$$+\ AC^2\cdot AD^2 + AC^2\cdot BD^2 - AC^2 - AD^4 + AD^2\cdot BD^2 + AD^2\cdot CD^2 + AD^2 - BD^2\cdot CD^2 + CD^2$$

$$BC_2 - \frac{\sqrt{2}\cdot\sqrt{AC^2\cdot AD^2 + AC^2\cdot BD^2 - AC^2 - AD^4 + AD^2\cdot BD^2 + AD^2\cdot CD^2 + AD^2 - BD^2\cdot CD^2 + CD^2\ \ldots}}{2\cdot AD} = 0$$

$$+\ -\sqrt{(1 + AD - BD)\cdot(1 - AD + BD)\cdot(AD - 1 + BD)\cdot(1 + AD + BD)}\cdot\sqrt{(AC + AD - CD)\cdot(AC - AD + CD)\cdot(AD - AC + CD)\cdot(AC + AD + CD)}$$

060793R4

Unit

AB := 1

Given.

Z := 10 U := 5

V := 2 W := 3 X := 7

Descriptions.

Let the two triangles ABD and ACD be given.

Given two triangles with a common side, find the difference between their free vertices from opposing sides.

$$AM := \frac{U}{Z} \quad AN := \frac{V}{Z} \quad AO := \frac{W}{Z} \quad AP := \frac{X}{Z} \quad AD := \sqrt{AN^2 + AM^2}$$

$$BN := AB - AN \quad BD := \sqrt{BN^2 + AM^2} \quad AC := \sqrt{AP^2 + AO^2}$$

$$DH := AP - AD \quad CD := \sqrt{DH^2 + AO^2} \quad AG := \frac{AD^2 + AB^2 - BD^2}{2 \cdot AD}$$

$$BG := \frac{\sqrt{(AD + BD - AB) \cdot (AD - BD + AB) \cdot (BD - AD + AB) \cdot (AD + AB + BD)}}{2 \cdot AD}$$

$$BJ := BG + AO \quad BF := BG - AO \quad AH := AD + DH \quad CJ := AH - AG$$

$$BC := \sqrt{BJ^2 + CJ^2} \quad BE := \sqrt{BF^2 + CJ^2}$$

Definitions.

$$AM - \frac{U}{Z} = 0 \quad AN - \frac{V}{Z} = 0 \quad AO - \frac{W}{Z} = 0 \quad AP - \frac{X}{Z} = 0 \quad AD - \frac{\sqrt{U^2 + V^2}}{Z} = 0$$

$$BN - \frac{Z - V}{Z} = 0 \quad BD - \frac{\sqrt{U^2 + V^2 - 2 \cdot V \cdot Z + Z^2}}{Z} = 0 \quad AC - \frac{\sqrt{W^2 + X^2}}{Z} = 0 \quad DH - \frac{X - \sqrt{U^2 + V^2}}{Z} = 0 \quad CD - \frac{\sqrt{U^2 + V^2 + W^2 + X^2 - 2 \cdot X \cdot \sqrt{U^2 + V^2}}}{Z} = 0$$

$$AG - \frac{2 \cdot V \cdot Z}{2 \cdot Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BG - \frac{U}{\sqrt{U^2 + V^2}} = 0 \quad BJ - \frac{U \cdot Z + W \cdot \sqrt{U^2 + V^2}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BF - \frac{U \cdot Z - W \cdot \sqrt{U^2 + V^2}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad AH - \frac{X}{Z} = 0 \quad CJ - \frac{X \cdot \sqrt{U^2 + V^2} - V \cdot Z}{Z \cdot \sqrt{U^2 + V^2}} = 0$$

$$BC - \frac{\sqrt{2 \cdot Z \cdot (U \cdot W - V \cdot X) \cdot \sqrt{U^2 + V^2} + (W^2 + X^2 + Z^2) \cdot (U^2 + V^2)}}{Z \cdot \sqrt{U^2 + V^2}} = 0 \quad BE - \frac{\sqrt{(W^2 + X^2 + Z^2) \cdot (U^2 + V^2) - 2 \cdot Z \cdot (U \cdot W + V \cdot X) \cdot \sqrt{U^2 + V^2}}}{Z \cdot \sqrt{U^2 + V^2}} = 0$$

Two Triangles with a Common Side.

Unit = 1.00000
M = 0.50000
U = 5.00000
Z = 10.00000

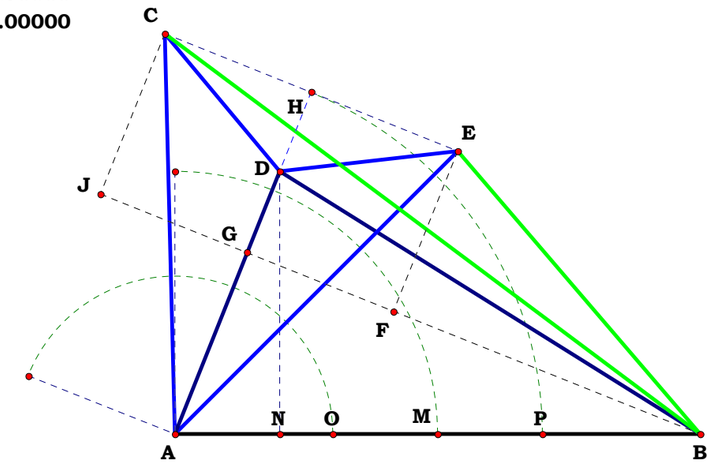
N = 0.20000
V = 2.00000

O = 0.30000
W = 3.00000

P = 0.70000
X = 7.00000

AB = 1.00000
AC = 0.76158
AD = 0.53852
BD = 0.94340
CD = 0.34070
AG = 0.37139

BG = 0.92848
BJ = 1.22848
BF = 0.62848
BC = 1.27167
BE = 0.70920



060993R1 Rectangular Roots.

Given.

$AB := 3$

$AC := 1$

Given any value N_1, any other value, N_2, greater than twice the square root of N_1 can be divided such that the resulting pair of values equals N_1.

Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.

Descriptions.

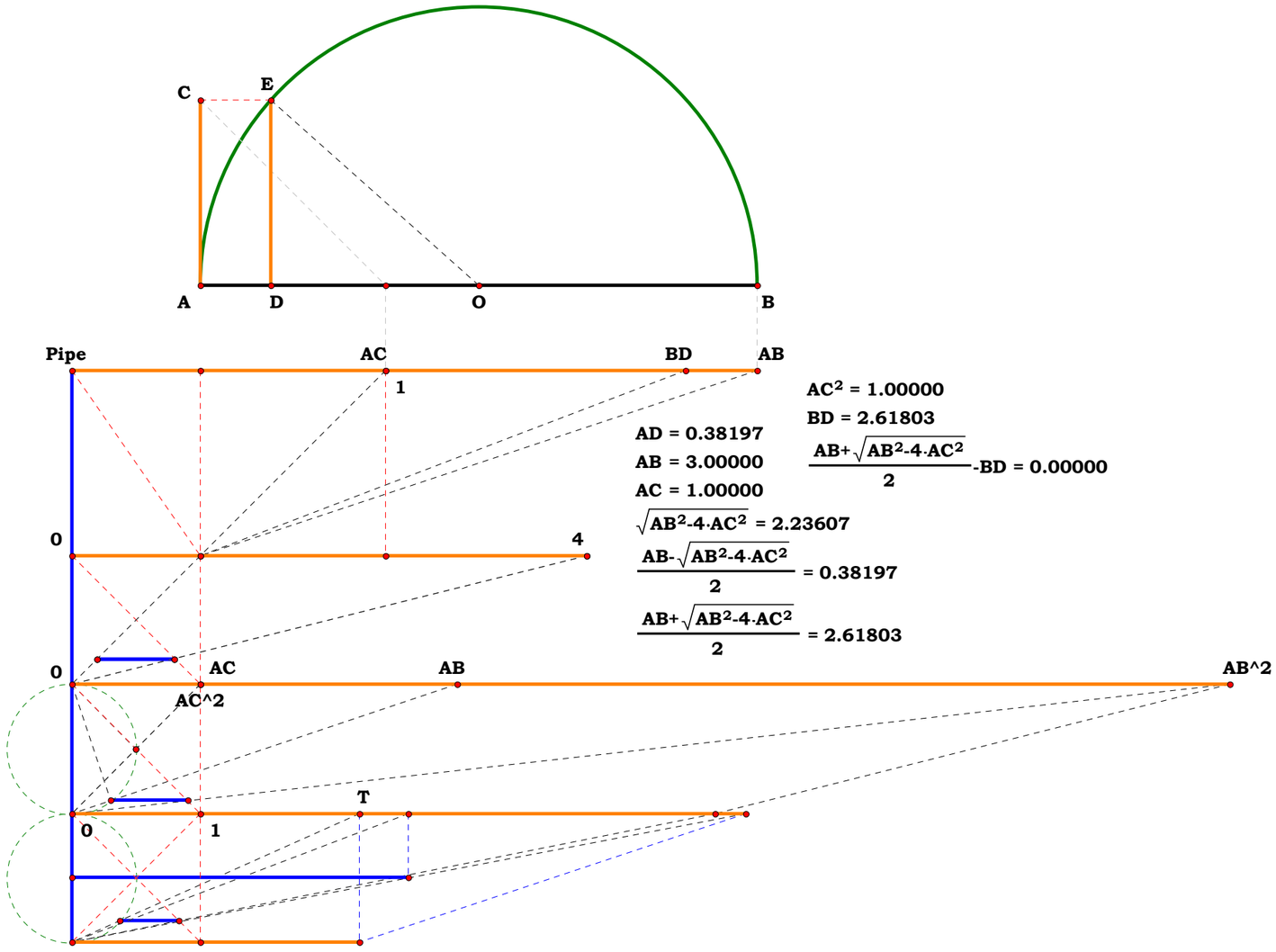
$DE := AC \qquad EO := \frac{AB}{2} \qquad DO := \sqrt{EO^2 - DE^2} \qquad BD := EO + DO$

$AD := AB - BD \qquad AD \cdot BD - AC^2 = 0$

Definitions.

$AD - \frac{AB - \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \qquad AD = 0.381966$

$BD - \frac{AB + \sqrt{AB^2 - 4 \cdot AC^2}}{2} = 0 \qquad BD = 2.618034 \qquad AD + BD = 3$



Unit.

$AB := 1$

Given.

$Y := 20$

$X := 8$

060993R2 Rectangular Roots.

Descriptions.

$$AC := \frac{X}{Y} \qquad DE := AC \qquad EO := \frac{AB}{2}$$

$$DO := \sqrt{EO^2 - DE^2} \qquad BD := EO + DO$$

$$AD := AB - BD \qquad AD \cdot BD - AC^2 = 0$$

Definitions.

$$AC - \frac{X}{Y} = 0 \qquad DE - \frac{X}{Y} = 0 \qquad EO - \frac{1}{2} = 0$$

$$DO - \frac{\sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0 \qquad BD - \frac{Y + \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0$$

$$AD - \frac{Y - \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} = 0$$

$$\frac{Y - \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} \cdot \frac{Y + \sqrt{Y^2 - 4 \cdot X^2}}{2 \cdot Y} - AC^2 = 0 \qquad \frac{X^2}{Y^2} - AC^2 = 0$$

Given any value N_1, any other value, N_2, greater than twice the square root of N_1 can be divided such that the resulting pair of values equals N_1.

Given DE as a square, and some AD equal to or greater than twice the square root of DE, divide AD into rectangluar roots of DE.

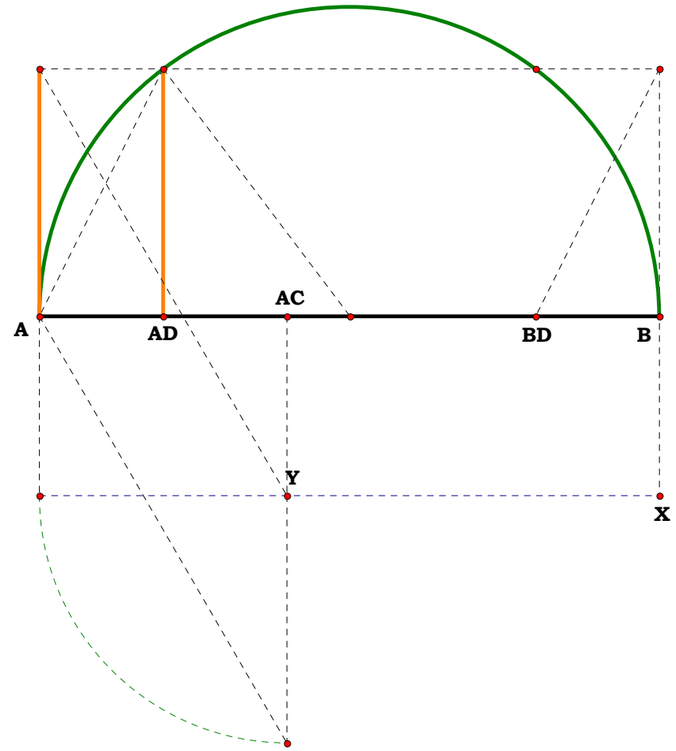
XY = 0.40000
X = 8.00000
Y = 20.00000

AD = 0.20000
AC = 0.40000
BD = 0.80000
AB = 1.00000

Unit = 1.00000

$\frac{Y-\sqrt{Y^2-4 \cdot X^2}}{2 \cdot Y}$-AD = 0.00000

$\frac{Y+\sqrt{Y^2-4 \cdot X^2}}{2 \cdot Y}$-BD = 0.00000



Pyramid of Rations 1

If you just draw the figure, and measure and compare the numbers, you start to see it. Now one should simply take it and all its treasures.

Definitions for the unit we use for division. This will give us a Cardinal result. The buttons will put the N1 or 2 on a number, but you have to define its actual ratio to the figure.

Hide Points

0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10

Unit = 1.00000

X = 0.40000

Y = 0.30000

01 = 2.32038 cm

$0N_1$ = 9.28153 cm

$0N_2$ = 6.96115 cm

$\frac{0N_1}{01}$ = 4.00000

N_1 = 4.00000

$\frac{0N_2}{01}$ = 3.00000

N_2 = 3.00000

AC = 16.40759 cm

AF = 6.56304 cm

$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1}$ = 2.50000

$\frac{AC}{AF}$ = 2.50000

$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1} - \frac{AC}{AF}$ = 0.00000

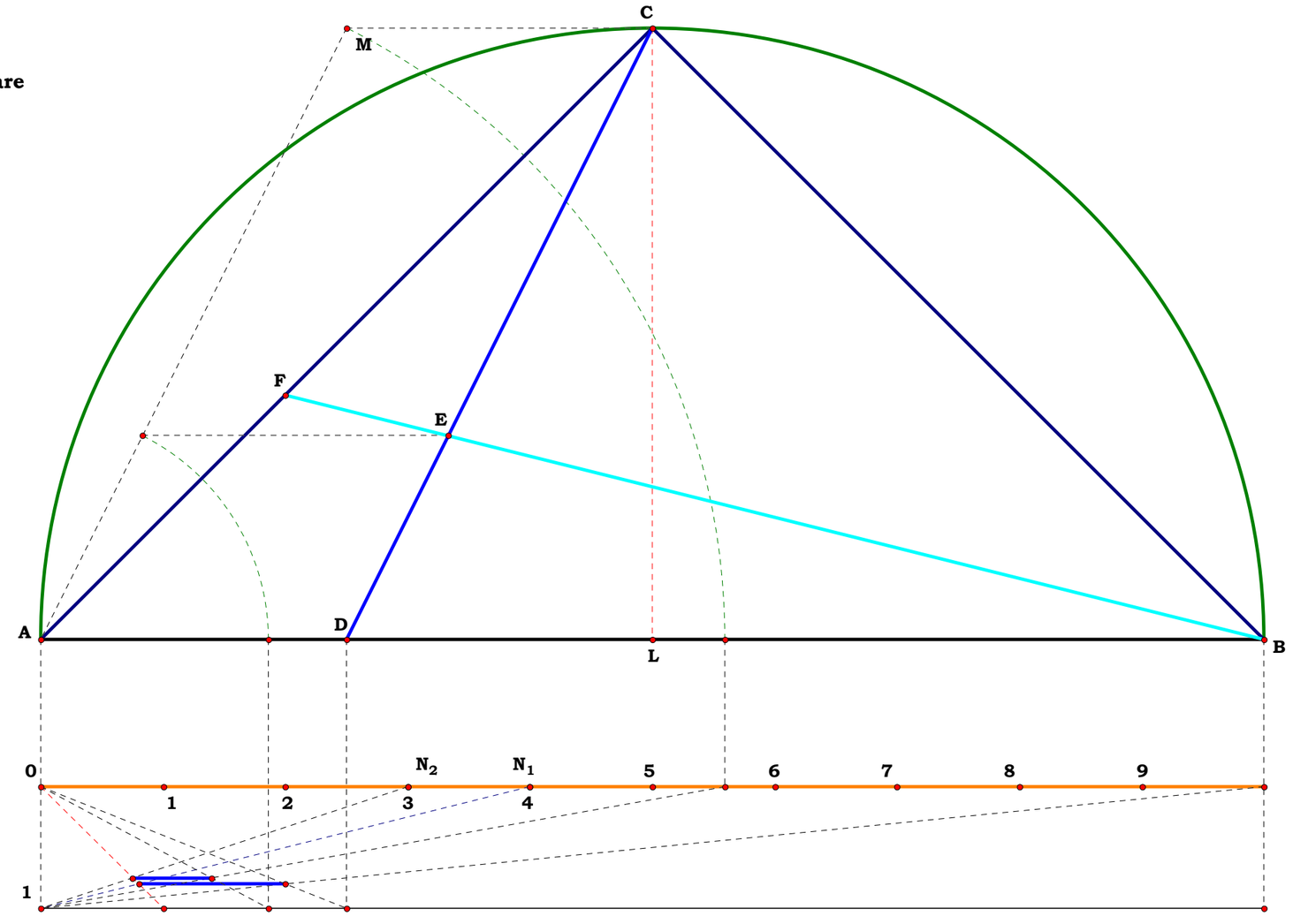
BF = 19.13437 cm

EF = 3.18906 cm

$\frac{BF}{EF}$ = 6.00000

$\frac{N_1 \cdot N_2}{N_2 - 1}$ = 6.00000

$\frac{N_1 \cdot N_2}{N_2 - 1} - \frac{BF}{EF}$ = 0.00000



062193

Unit.
Given.

Pyramid of Ratios I

Divide AB by N1 then divide CD by N2, what are BF/EF and AC/AF?

Descriptions.
Definitions.

$N1 := 3 \quad N2 := 5 \quad \delta := 1 .. N2$

$\delta =$

1
2
3
4
5

$AB := 1 \quad AD := \frac{AB}{N1} \quad AL := \frac{AB}{2}$

$DL := AL - AD \quad AC := \sqrt{\frac{AB^2}{2}}$

$CL := AL \quad CD := \sqrt{DL^2 + CL^2}$

$DE_\delta := \frac{CD \cdot \delta}{N2} \quad DK_\delta := \frac{DL \cdot DE_\delta}{CD} \quad AK_\delta := AD + DK_\delta \quad BK_\delta := AB - AK_\delta \quad EK_\delta := \frac{CL \cdot DK_\delta}{DL}$

$BE_\delta := \sqrt{(EK_\delta)^2 + (BK_\delta)^2} \quad HK_\delta := \frac{AL \cdot DK_\delta}{DL} \quad BH_\delta := BK_\delta + HK_\delta \quad EH_\delta := \frac{AC \cdot DK_\delta}{DL}$

$AF_\delta := \frac{EH_\delta \cdot AB}{BH_\delta} \quad BF_\delta := \frac{BE_\delta \cdot AB}{BH_\delta} \quad EF_\delta := BF_\delta - BE_\delta$

$if\left(EF_\delta, \frac{BF_\delta}{EF_\delta}, 0\right) =$

3.75
5
7.5
15
0

$if\left(N2 - \delta, \frac{N1 \cdot N2}{N2 - \delta}, 0\right) =$

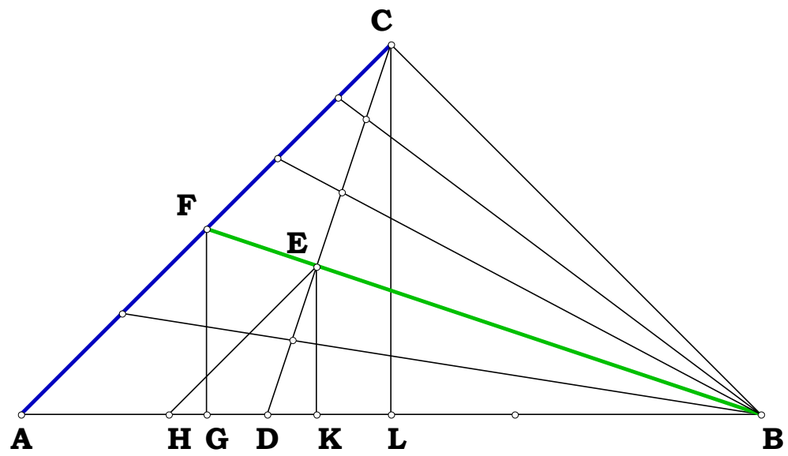
3.75
5
7.5
15
0

$\frac{AC}{AF_\delta} =$

3.666667
2
1.444444
1.166667
1

$\frac{(N1 - 1) \cdot N2 + \delta}{N1 \cdot \delta} =$

3.666667
2
1.444444
1.166667
1



This was my original write up, long ago, and it takes advantage of some functions of Mathcad. For the final product of The Delian Quest, it will become a series of demonstrations. However, the root figure does not change, nor what it does.

I never had Geometry in school, nor did I ever do well in algebra because like everything else, those who teach it really do not know what in the hell they are doing. Too much mythology, and too much insistence on traditional carping. I am a simple person and I like things simple, honest, and organized.

062193R3

Descriptions.

Unit.

$AB := 1$

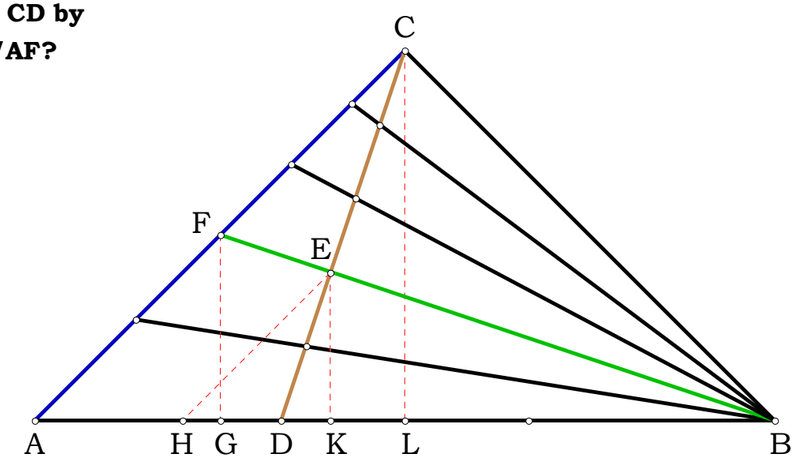
Given.

$N_1 := 4$

$N_2 := 2$

Pyramid of Ratios I

Divide AB by N_1 then divide CD by N_2, what are BF/EF and AC/AF?



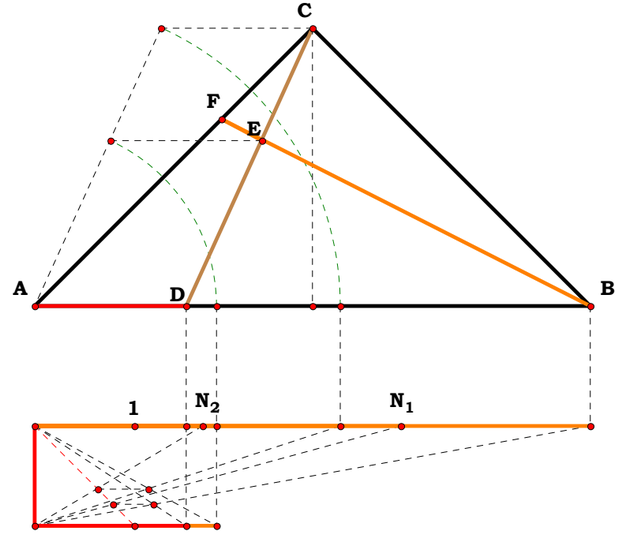
$AD := \frac{AB}{N_1}$ $AL := \frac{AB}{2}$ $AC := \sqrt{2 \cdot AL^2}$ $CL := AL$ $DL := AL - AD$

$CD := \sqrt{DL^2 + CL^2}$ $DE := \frac{CD}{N_2}$ $EK := \frac{CL \cdot DE}{CD}$ $DK := \frac{DL \cdot EK}{CL}$

$AK := AD + DK$ $BK := AB - AK$ $BE := \sqrt{BK^2 + EK^2}$

$HK := \frac{AL \cdot EK}{CL}$ $BH := BK + HK$ $EH := \frac{AC \cdot EK}{CL}$ $AF := \frac{EH \cdot AB}{BH}$

$BF := \frac{BE \cdot AB}{BH}$ $EF := BF - BE$



$\frac{BF}{EF} = 8$ $\frac{AC}{AF} = 1.75$ $\frac{CD}{DE} = 2$

$\frac{BF}{EF} = 9.08799$ $\frac{N_1 \cdot N_2}{N_2 - 1} = 9.08799$

$\frac{AC}{AF} = 1.49057$ $\frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 1.49057$

$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0.00000$ $\frac{AC}{AF} - \frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 0.00000$

Definitions.

$$\frac{AC}{AF} - \frac{N_1 \cdot N_2 - N_2 + 1}{N_1} = 0$$

$$\frac{BF}{EF} - \frac{N_1 \cdot N_2}{N_2 - 1} = 0$$

Pyramid of Ratios I

The buttons on the macro are divided into two columns, one for the number of divisions I want for a given unit, and the other a particular point in that range. This means that if I set the points equal to what I am working with, I do not have to draw anything further to play with the line to examine it. Thus, althoght N1 and N2 are working with two different lengths of line, one always constant and the other variable, it is transparent in the the equation. We are dividing each of them in a Cardinal fashion, just rendering an arithmetic name for its point in a ratio of that particular line. And, this allows me to write up the equations for the figure which reduce, in the end, to w, x, y and z and always giving an exact answer no matter what original system of measurement one started with, inches, meters, or pixels, or even mystical particals of quantum dust. A thing, is a thing, is a thing, which is always independent of any convention of names and what one is looking for, is a simple, universal method of notation without using Einstein's rubbrer bands and crazy glue.

Just the fact that one is working with four names in the equation tells me that I have two different things using two different systems of measurement, each one bringing itself into the equation as it is.

Y -> 1
Y -> 2
Y -> 3
Y -> 4
Y -> 5
Y -> 6
Y -> 7
Y -> 8
Y -> 9
Y -> 10
Y -> 11
Y -> 12
Y -> 13
Y -> 14
Y -> 15
Y -> 16
Y -> 17
Y -> 18
Y -> 19
Y -> 20

X -> 0
X -> 1
X -> 2
X -> 3
X -> 4
X -> 5
X -> 6
X -> 7
X -> 8
X -> 9
X -> 10
X -> 11
X -> 12
X -> 13
X -> 14
X -> 15
X -> 16
X -> 17
X -> 18
X -> 19
X -> 20
Show Points

Y -> 1
Y -> 2
Y -> 3
Y -> 4
Y -> 5
Y -> 6
Y -> 7
Y -> 8
Y -> 9
Y -> 10
Y -> 11
Y -> 12
Y -> 13
Y -> 14
Y -> 15
Y -> 16
Y -> 17
Y -> 18
Y -> 19
Y -> 20

X -> 0
X -> 1
X -> 2
X -> 3
X -> 4
X -> 5
X -> 6
X -> 7
X -> 8
X -> 9
X -> 10
X -> 11
X -> 12
X -> 13
X -> 14
X -> 15
X -> 16
X -> 17
X -> 18
X -> 19
X -> 20
Show Points

$$\frac{(N_1 \cdot N_2 - N_2)+1}{N_1} = 1.53333$$

$$\frac{X \cdot (W-Z)+Y \cdot Z}{Y \cdot W} = 1.53333$$

$$\frac{AC}{AF} = 1.53333$$

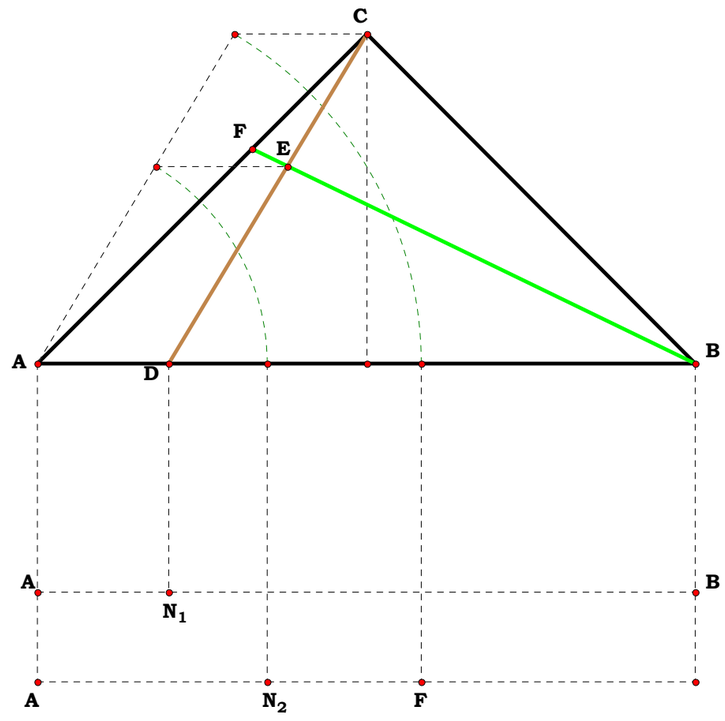
$$\frac{N_1 \cdot N_2}{N_2 - 1} = 12.50000$$

$$\frac{Y \cdot Z}{X \cdot (Z-W)} = 12.50000$$

$$\frac{BF}{EF} = 12.50000$$

AC = 7.80151 cm
AF = 5.08794 cm
BF = 8.25996 cm
EF = 0.66080 cm

Unit = 1.00000
X = 4.00000
Y = 20.00000
$\frac{Y}{X} = 5.00000$
$N_1 = 5.00000$
W = 6.00000
Z = 10.00000
$\frac{Z}{W} = 1.66667$
$N_2 = 1.66667$



062193R5

Unit.

$AB := 1$

Given.

$W := 6$

$X := 4$

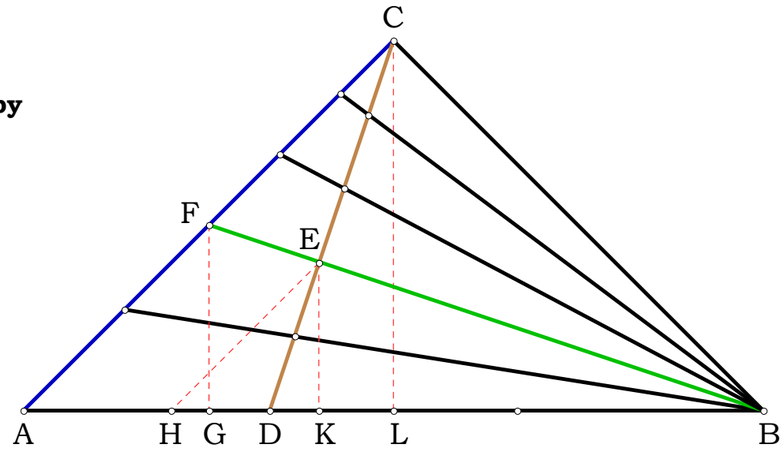
$Y := 20$

$Z := 10$

Descriptions.

Pyramid of Ratios I

Divide AB by N_1 then divide CD by N_2, what are BF/EF and AC/AF?



$$N_1 := \frac{Y}{X} \qquad N_2 := \frac{Z}{W}$$

$$AD := \frac{AB}{N_1} \qquad AL := \frac{AB}{2} \qquad AC := \sqrt{2 \cdot AL^2} \qquad CL := AL \qquad DL := AL - AD$$

$$CD := \sqrt{DL^2 + CL^2} \qquad DE := \frac{CD}{N_2} \qquad EK := \frac{CL \cdot DE}{CD} \qquad DK := \frac{DL \cdot EK}{CL}$$

$$AK := AD + DK \qquad BK := AB - AK \qquad BE := \sqrt{BK^2 + EK^2}$$

$$HK := \frac{AL \cdot EK}{CL} \qquad BH := BK + HK \qquad EH := \frac{AC \cdot EK}{CL} \qquad AF := \frac{EH \cdot AB}{BH}$$

$$BF := \frac{BE \cdot AB}{BH} \qquad EF := BF - BE$$

$$\frac{BF}{EF} = 12.5 \qquad \frac{AC}{AF} = 1.533333 \qquad \frac{CD}{DE} = 1.666667$$

$$\frac{(N_1 \cdot N_2 - N_2) + 1}{N_1} = 1.53333$$

$$\frac{X \cdot (W - Z) + Y \cdot Z}{Y \cdot W} = 1.53333$$

$$\frac{AC}{AF} = 1.53333$$

$$\frac{N_1 \cdot N_2}{N_2 - 1} = 12.50000$$

$$\frac{Y \cdot Z}{X \cdot (Z - W)} = 12.50000$$

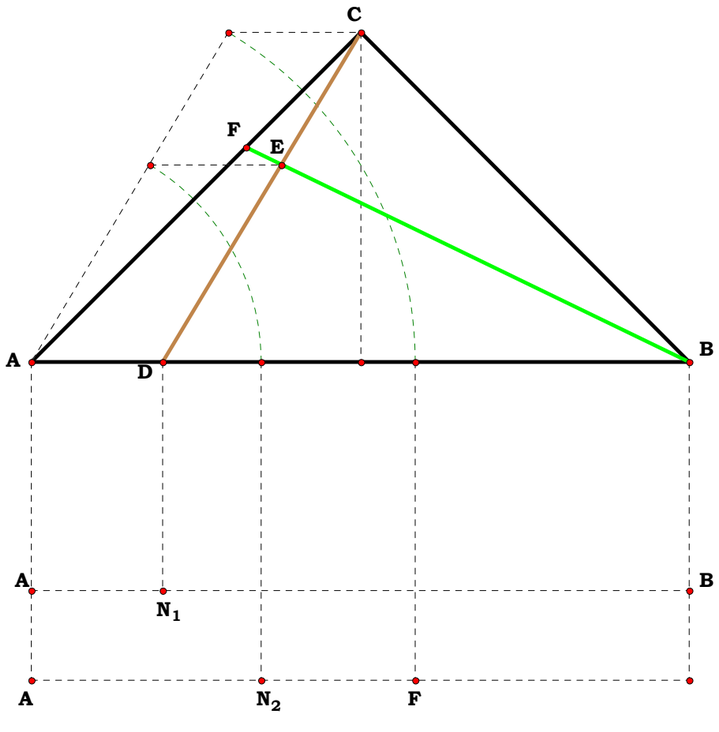
$$\frac{BF}{EF} = 12.50000$$

$AC = 7.80151$ cm

$AF = 5.08794$ cm

$BF = 8.25996$ cm

$EF = 0.66080$ cm



Unit = 1.00000

X = 4.00000

Y = 20.00000

$\frac{Y}{X} = 5.00000$

$N_1 = 5.00000$

W = 6.00000

Z = 10.00000

$\frac{Z}{W} = 1.66667$

$N_2 = 1.66667$

Definitions.

$$\frac{AC}{AF}-\frac{X\cdot(W-Z)+Y\cdot Z}{Y\cdot W}=0 \qquad \frac{BF}{EF}-\frac{Y\cdot Z}{X\cdot(Z-W)}=0$$

$$N_1-\frac{Y}{X}=0 \quad N_2-\frac{Z}{W}=0 \quad AD-\frac{X}{Y}=0 \quad AL-\frac{1}{2}=0 \quad AC-\frac{1}{\sqrt{2}}=0$$

$$CL-\frac{1}{2}=0 \quad DL-\frac{Y-2\cdot X}{2\cdot Y}=0 \quad CD-\frac{\sqrt{2\cdot X^2-2\cdot X\cdot Y+Y^2}}{\sqrt{2}\cdot Y}=0 \quad DE-\frac{\sqrt{2}\cdot W\cdot\sqrt{2\cdot X^2-2\cdot X\cdot Y+Y^2}}{2\cdot Y\cdot Z}=0$$

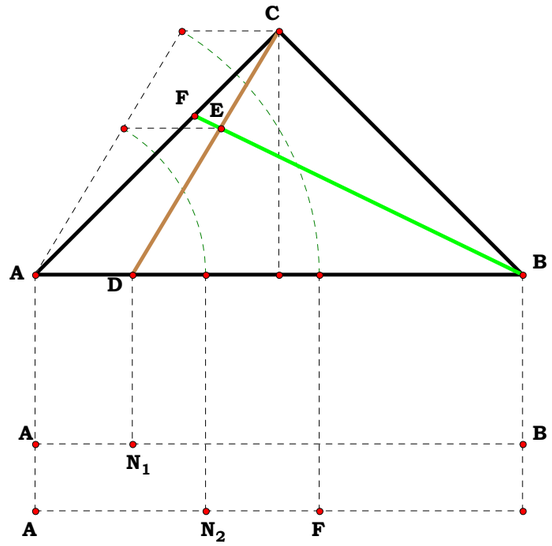
$$EK-\frac{W}{2\cdot Z}=0 \quad DK-\frac{W\cdot(Y-2\cdot X)}{2\cdot Y\cdot Z}=0 \quad AK-\frac{[W\cdot(Y-2\cdot X)+2\cdot X\cdot Z]}{2\cdot Y\cdot Z}=0 \quad BK-\frac{2\cdot(W\cdot X+Y\cdot Z)-(W\cdot Y+2\cdot X\cdot Z)}{2\cdot Y\cdot Z}=0$$

$$BE-\frac{\sqrt{2\cdot(X-Y)^2\cdot Z^2+[2\cdot W\cdot(Y-X)\cdot(2\cdot X-Y)]\cdot Z+W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)}}{\sqrt{2}\cdot Y\cdot Z}=0 \quad HK-\frac{W}{2\cdot Z}=0$$

$$BH-\frac{W\cdot X-X\cdot Z+Y\cdot Z}{Y\cdot Z}=0 \qquad EH-\frac{\sqrt{2}\cdot W}{2\cdot Z}=0 \qquad AF-\frac{\sqrt{2}\cdot W\cdot Y}{2\cdot(W\cdot X-X\cdot Z+Y\cdot Z)}=0$$

$$BF-\frac{\sqrt{2\cdot W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)+4\cdot Z^2\cdot(X-Y)^2+4\cdot W\cdot Z\cdot(X-Y)\cdot(Y-2\cdot X)}}{2\cdot(W\cdot X-X\cdot Z+Y\cdot Z)}=0$$

$$EF-\frac{X\cdot(W-Z)\cdot\sqrt{2\cdot W^2\cdot\left(2\cdot X^2-2\cdot X\cdot Y+Y^2\right)+4\cdot Z^2\cdot(X-Y)^2+4\cdot W\cdot Z\cdot(X-Y)\cdot(Y-2\cdot X)}}{2\cdot Y\cdot Z\cdot(X\cdot Z-W\cdot X-Y\cdot Z)}=0$$



Unit = 1.00000
X = 4.00000
Y = 20.00000
$\frac{Y}{X}$ = 5.00000
N_1 = 5.00000
W = 6.00000
Z = 10.00000
$\frac{Z}{W}$ = 1.66667
N_2 = 1.66667

$\frac{(N_1\cdot N_2-N_2)+1}{N_1}$ = 1.53333
$\frac{X\cdot(W-Z)+Y\cdot Z}{Y\cdot W}$ = 1.53333
$\frac{AC}{AF}$ = 1.53333
$\frac{N_1\cdot N_2}{N_2-1}$ = 12.50000
$\frac{Y\cdot Z}{X\cdot(Z-W)}$ = 12.50000
$\frac{BF}{EF}$ = 12.50000
AC = 5.78095 cm
AF = 3.77019 cm
BF = 6.12067 cm
EF = 0.48965 cm

062793R1

Given.

$N_1 := 3 \quad AB := N_1$

$N_2 := 6 \quad BC := N_2$

$N_3 := 4 \quad AC := N_3$

Describe A Circle About a Triangle

Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

Descriptions.

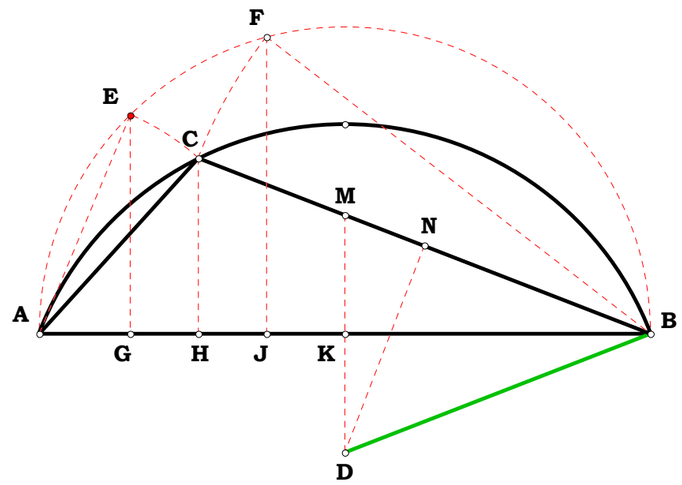
$\Delta := (AB + AC > BC) \cdot (AB + BC > AC) \cdot (BC + AC > AB) \quad NOT(X) := X = \delta := 0 .. 2$

$BK := \frac{AB}{2} \quad AE := AC \quad BF := BC \quad AG := \frac{AE^2}{AB} \quad BJ := \frac{BF^2}{AB}$

$GJ := AB - (AG + BJ) \quad HJ := \frac{GJ}{2} \quad BH := BJ + HJ$

$CH := \sqrt{BC^2 - BH^2} \quad BN := \frac{BC}{2} \quad BM := \frac{BC \cdot BK}{BH}$

$MN := BM - BN \quad DN := \frac{BH \cdot MN}{CH} \quad BD := \sqrt{BN^2 + DN^2}$



Definitions.

$radius := if(\Delta, BD, 0) \quad imaginary\_radius := if(NOT(\Delta), BD, 0)$

$radius = 3.375412$

$imaginary\_radius = 0$

The construction is independent of the side one starts with.

$\Delta = 1$

$S_1 := \begin{pmatrix} AB \\ AC \\ BC \end{pmatrix} \quad S_2 := \begin{pmatrix} AC \\ BC \\ AB \end{pmatrix} \quad S_3 := \begin{pmatrix} BC \\ AB \\ AC \end{pmatrix} \quad R_\delta := \frac{S1_\delta \cdot S2_\delta \cdot S3_\delta}{\sqrt{S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}$

The name of the Radius in terms of the givens.

$R^T = (3.375412 \quad 3.375412 \quad 3.375412)$ **The equation is a statement in regard to the relationship between each side of a triangle.**

$$BK - \frac{N_1}{2} = 0 \qquad AE - N_3 = 0 \qquad BF - N_2 = 0 \qquad AG - \frac{N_3^2}{N_1} = 0 \qquad BJ - \frac{N_2^2}{N_1} = 0$$

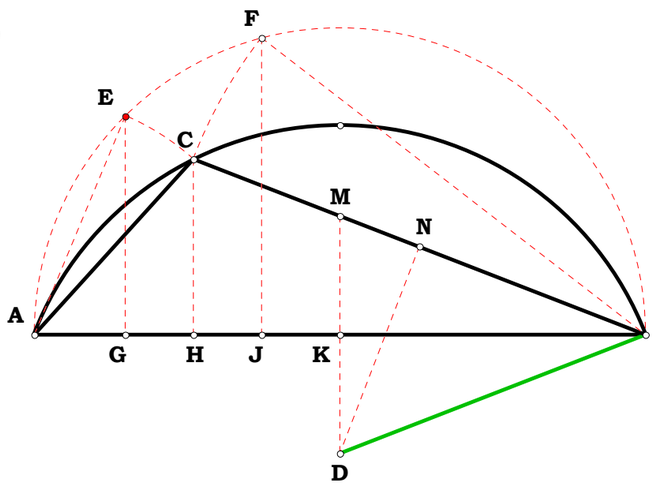
$$GJ - \frac{N_1^2 - N_2^2 - N_3^2}{N_1} = 0 \qquad HJ - \frac{N_1^2 - N_2^2 - N_3^2}{2 \cdot N_1} = 0 \qquad BH - \frac{N_1^2 + N_2^2 - N_3^2}{2 \cdot N_1} = 0$$

$$CH - \frac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot N_1} = 0 \qquad BN - \frac{N_2}{2} = 0$$

$$BM - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2 - N_3^2} = 0 \qquad MN - \frac{N_2 \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}{2 \cdot \left(N_1^2 + N_2^2 - N_3^2\right)} = 0$$

$$DN - \frac{N_2 \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}{2 \cdot \sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}} = 0$$

$$BD - \frac{N_1 \cdot N_2 \cdot N_3}{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}} = 0$$



AB = 4.03775 in. N_1 = 4.03775 in.
BC = 3.20102 in. N_2 = 3.20102 in.
AC = 1.55963 in. N_3 = 1.55963 in.
BD = 2.16667 in.

$$\frac{N_1 \cdot N_2 \cdot N_3}{\sqrt{(N_1+N_2+N_3) \cdot ((N_1-N_2)+N_3) \cdot ((N_2-N_1)+N_3) \cdot ((N_1+N_2)-N_3)}} - BD = 0.00000 \text{ in.}$$

$\frac{XO}{01} = 1.35962$ $\frac{WO}{01} = 1.35962$ $\frac{ZO}{01} = 1.35962$

X = 1.35962 XO = 1.18110 in. 01 = 0.86870 in.

M = 1.35962 WO = 1.18110 in. OZ = 1.18110 in.

X-M = 0.00000 ZO = 1.18110 in.

PO = 1.18110 in.

TR = 1.18110 in.

$S_1 = 1.55042$

$S_2 = 2.69740$

$S_3 = 2.41211$

A = 10.08768 $S_1 \cdot S_2 \cdot S_3 - A = 0.00000$

$\frac{S_1 \cdot S_2 \cdot S_3}{\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3}} - M = 0.00000$

B = 6.65993 $(S_1+S_2+S_3)-B = 0.00000$

C = 2.58068 $\sqrt{S_1+S_2+S_3}-C = 0.00000$

D = 3.55909 $(S_2+S_3)-S_1-D = 0.00000$

E = 1.88655 $\sqrt{(S_2+S_3)-S_1}-E = 0.00000$

F = 1.26512 $(S_1+S_3)-S_2-F = 0.00000$

G = 1.12478 $\sqrt{(S_1+S_3)-S_2}-G = 0.00000$

H = 1.83571 $(S_1+S_2)-S_3-H = 0.00000$

J = 1.35489 $\sqrt{(S_1+S_2)-S_3}-J = 0.00000$

K = 7.41948

$\sqrt{S_1+S_2+S_3} \cdot \sqrt{(S_2+S_3)-S_1} \cdot \sqrt{(S_1+S_3)-S_2} \cdot \sqrt{(S_1+S_2)-S_3} - K = 0.00000$

062793R3

Unit.

$AB := 1$

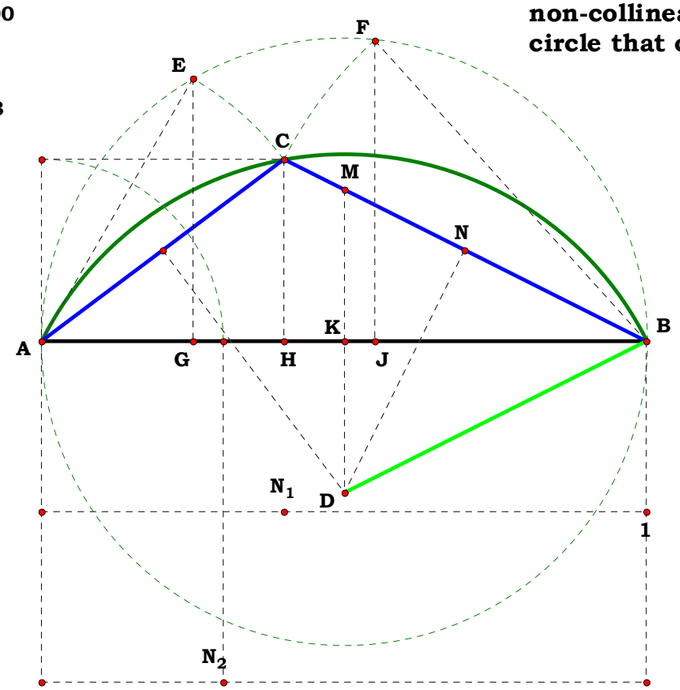
Given.

$W := 4$ $Y := 3$

$X := 10$ $Z := 10$

Describe A Circle About a Triangle

Unit = 1.00000
X/W = 2.50000
W = 4.00000
X = 10.00000
Z/Y = 3.33333
Y = 3.00000
Z = 10.00000
N_1 = 2.50000
N_2 = 3.33333



Given the difference between three non-collinear points, find the radius of the circle that circumscribes them.

Descriptions.

$AH := \frac{W}{X}$ $CH := \frac{Y}{Z}$ $AC := \sqrt{AH^2 + CH^2}$ $BH := AB - AH$

$BC := \sqrt{BH^2 + CH^2}$ $BK := \frac{AB}{2}$ $AE := AC$ $BF := BC$

$AG := \frac{AE^2}{AB}$ $BJ := \frac{BF^2}{AB}$ $GJ := AB - (AG + BJ)$ $HJ := \frac{GJ}{2}$

$BN := \frac{BC}{2}$ $BM := \frac{BC \cdot BK}{BH}$ $MN := BM - BN$ $DN := \frac{BH \cdot MN}{CH}$

$BD := \sqrt{BN^2 + DN^2}$

Definitions.

$AH - \frac{W}{X} = 0$ $CH - \frac{Y}{Z} = 0$ $AC - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BH - \frac{X - W}{X} = 0$

$BC - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0$ $BK - \frac{1}{2}$ $AE - \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z} = 0$ $BF - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{X \cdot Z} = 0$

$AG - \frac{W^2 \cdot Z^2 + X^2 \cdot Y^2}{X^2 \cdot Z^2} = 0$ $BJ - \frac{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}{X^2 \cdot Z^2} = 0$ $GJ - \frac{2 \cdot (W \cdot X \cdot Z^2 - W^2 \cdot Z^2 - X^2 \cdot Y^2)}{X^2 \cdot Z^2} = 0$ $HJ - \frac{(W \cdot X \cdot Z^2 - W^2 \cdot Z^2 - X^2 \cdot Y^2)}{X^2 \cdot Z^2} = 0$

$BN - \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{2 \cdot X \cdot Z} = 0$ $BM - \frac{\sqrt{X^2 \cdot (Y^2 + Z^2) + W \cdot Z^2 \cdot (W - 2 \cdot X)}}{2 \cdot Z \cdot (X - W)} = 0$ $MN - \frac{W \cdot \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{2 \cdot X \cdot Z \cdot (X - W)} = 0$

$DN - \frac{W \cdot \sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot (Y^2 + Z^2)}}{2 \cdot X^2 \cdot Y} = 0$ $BD - \frac{\sqrt{(W^2 \cdot Z^2 + X^2 \cdot Y^2) \cdot (W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2 + X^2 \cdot Y^2 + X^2 \cdot Z^2)}}{2 \cdot X^2 \cdot Y \cdot Z} = 0$

Giving any side of a triangle as unity, then each of the remaing two sides are named as the algebraic names assigned to A and B like such:

$$A := \frac{\sqrt{W^2 \cdot Z^2 + X^2 \cdot Y^2}}{X \cdot Z}$$

$$B := \frac{\sqrt{W \cdot Z^2 \cdot (W - 2 \cdot X) + X^2 \cdot \left(Y^2 + Z^2\right)}}{X \cdot Z}$$

$\delta := 0 .. 2$ **The result is independent of the side one starts assignes to unity**

$$S_1 := \begin{pmatrix} 1 \\ A \\ B \end{pmatrix} \quad S_2 := \begin{pmatrix} A \\ B \\ 1 \end{pmatrix} \quad S_3 := \begin{pmatrix} B \\ 1 \\ A \end{pmatrix}$$

$$R_\delta := \frac{S_{1_\delta} \cdot S_{2_\delta} \cdot S_{3_\delta}}{\sqrt{S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{-S_{1_\delta} + S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} - S_{2_\delta} + S_{3_\delta}} \cdot \sqrt{S_{1_\delta} + S_{2_\delta} - S_{3_\delta}}}$$

The Arithmetic name of the Radius in terms of the givens.

$R_\delta =$

0.559017
0.559017
0.559017

BD = 0.559017

The only time this process fails in in the Mythological Great Circle, which Shamen (sic) claim is a line. The real significance of the Great Circle is that in looking forward to one's future, these people actually end up where they started. I believe Dodson called it a Caucus Race of which he had plenty of experience. It is a whole lot more flattering to claim we have moved when all we did is stand still. the Couch Potato Philosopher.

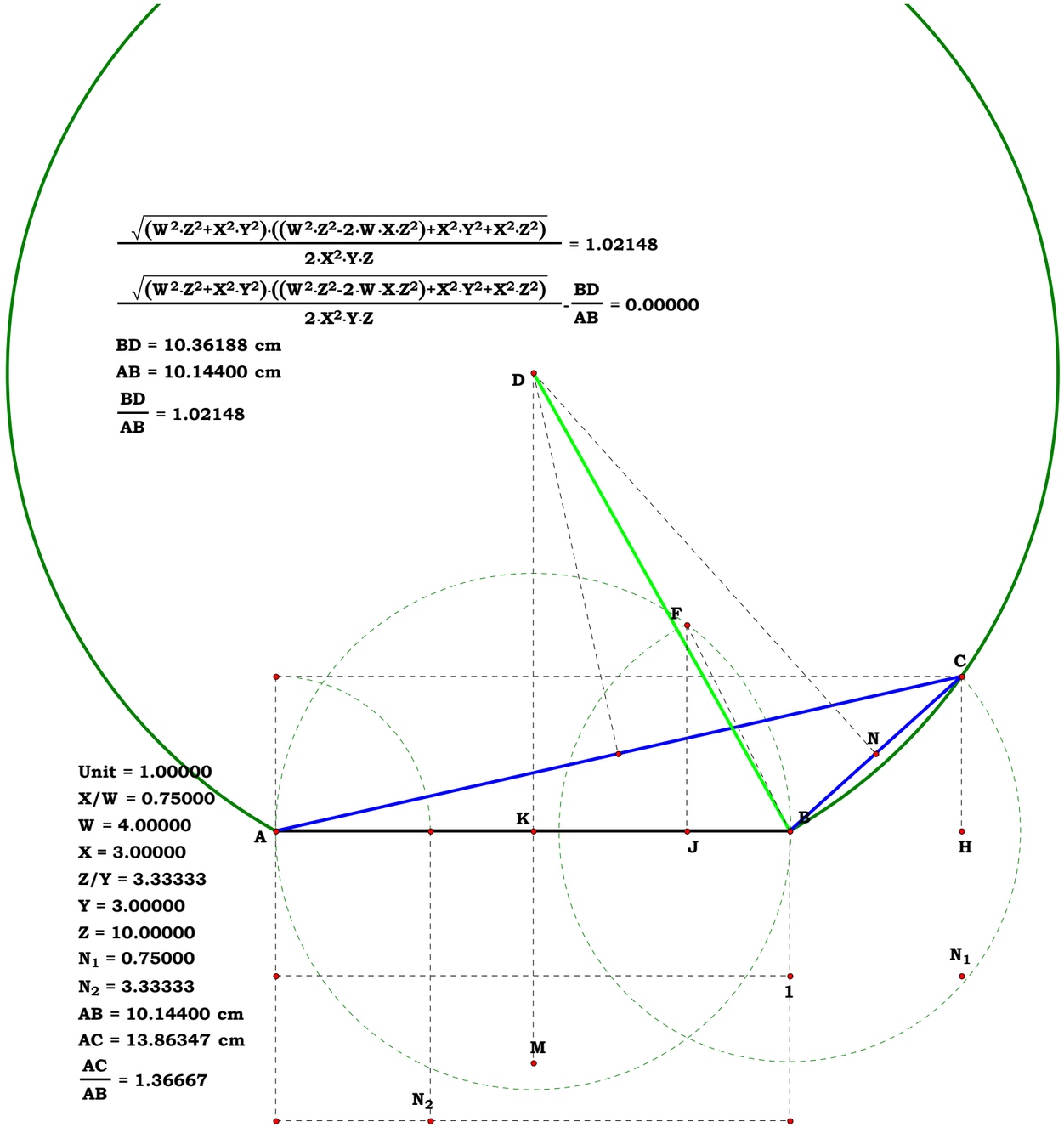
$$\frac{\sqrt{(W^2 \cdot Z^2 + X^2 \cdot Y^2) \cdot ((W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2) + X^2 \cdot Y^2 + X^2 \cdot Z^2)}}{2 \cdot X^2 \cdot Y \cdot Z} = 1.02148$$

$$\frac{\sqrt{(W^2 \cdot Z^2 + X^2 \cdot Y^2) \cdot ((W^2 \cdot Z^2 - 2 \cdot W \cdot X \cdot Z^2) + X^2 \cdot Y^2 + X^2 \cdot Z^2)}}{2 \cdot X^2 \cdot Y \cdot Z} - \frac{BD}{AB} = 0.00000$$

BD = 10.36188 cm

AB = 10.14400 cm

$$\frac{BD}{AB} = 1.02148$$



Unit = 1.00000
X/W = 0.75000
W = 4.00000
X = 3.00000
Z/Y = 3.33333
Y = 3.00000
Z = 10.00000
N_1 = 0.75000
N_2 = 3.33333
AB = 10.14400 cm
AC = 13.86347 cm

$$\frac{AC}{AB} = 1.36667$$

071593R1

Pyramid of Ratios II

Unit.

$AB := 1$

Given.

$N_1 := 3$

$N_2 := 5$

$\delta := 1 .. N_2$

Back in the day, I used to write shit up like this, but then I forgot just what in the hell I was doing, so, I had to invent other ways to write these up and to understand them.

Descriptions.

$AD := \frac{AB}{N_1}$ $AC := \sqrt{\frac{AB^2}{2}}$ $BD := AB - AD$ $DE_\delta := \frac{BD \cdot \delta}{N_2}$ $AG_\delta := \frac{AC \cdot \delta}{N_2}$

$AE_\delta := AD + DE_\delta$ $AH_\delta := \sqrt{\frac{(AG_\delta)^2}{2}}$ $GH_\delta := AH_\delta$ $EH_\delta := AE_\delta - AH_\delta$

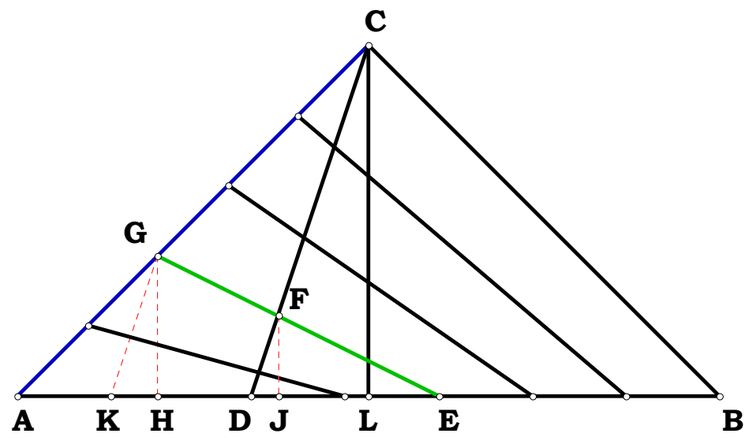
$EG_\delta := \sqrt{(EH_\delta)^2 + (GH_\delta)^2}$ $AL := \frac{AB}{2}$ $DL := AL - AD$ $CL := \sqrt{\frac{AC^2}{2}}$

$HK_\delta := \frac{DL \cdot GH_\delta}{CL}$ $EK_\delta := EH_\delta + HK_\delta$ $DJ_\delta := \frac{HK_\delta \cdot DE_\delta}{EK_\delta}$ $FJ_\delta := \frac{GH_\delta \cdot DE_\delta}{EK_\delta}$

$DF_\delta := \sqrt{(DJ_\delta)^2 + (FJ_\delta)^2}$ $EF_\delta := \frac{EG_\delta \cdot DE_\delta}{EK_\delta}$ $FG_\delta := EG_\delta - EF_\delta$

$CD := \sqrt{CL^2 + DL^2}$

AB is divided by N_1 and AC and BD is divided by N_2, what are EG/FG and CD/DF?



$\text{if}\left(\delta, \frac{CD}{DF_\delta}, 0\right) =$

15
4.375
2.222222
1.40625
1

$\text{if}\left[\delta, N_2 \cdot \frac{\left[\left(N_2 + \delta \cdot N_1\right) - 2 \cdot \delta\right]}{\left[\delta^2 \cdot \left(N_1 - 1\right)\right]}, 0\right] =$

15
4.375
2.222222
1.40625
1

$\text{if}\left(FG_\delta, \frac{EG_\delta}{FG_\delta}, 0\right) =$

1.5
2.333333
4
9
0

$\text{if}\left[N_2 - \delta, \frac{N_2 + \delta \cdot \left(N_1 - 2\right)}{N_2 - \delta}, 0\right] =$

1.5
2.333333
4
9
0

071593R1

Pyramid of Ratios II

Unit.

$AB := 1$

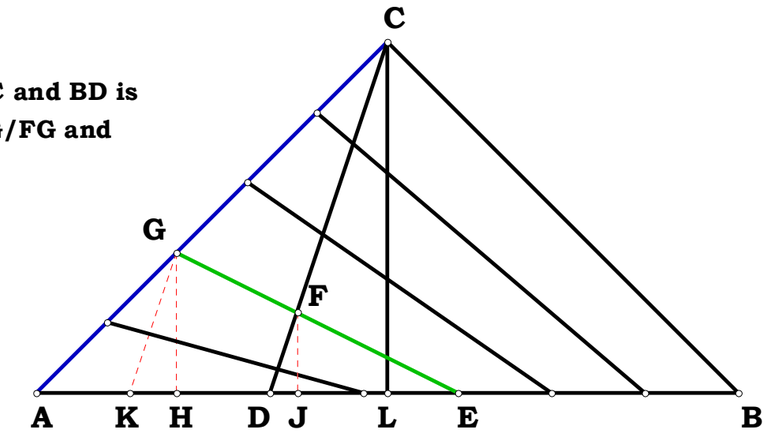
Given.

$N_1 := 3$

$N_2 := 5$

When I got to this point, I thought I had the bull gonads, but, I was not satisfied.

AB is divided by N_1 and AC and BD is divided by N_2, what are EG/FG and CD/DF?



Descriptions.

$AD := \frac{AB}{N_1}$ $AC := \sqrt{\frac{AB^2}{2}}$ $BD := AB - AD$ $AL := \frac{AB}{2}$

$DE := \frac{BD}{N_2}$ $AG := \frac{AC}{N_2}$ $AE := AD + DE$ $AH := \sqrt{\frac{AG^2}{2}}$

$GH := AH$ $EH := AE - AH$ $EG := \sqrt{EH^2 + GH^2}$ $DL := AL - AD$

$CL := AL$ $HK := \frac{DL \cdot AH}{AL}$ $EK := EH + HK$

$DJ := \frac{HK \cdot DE}{EK}$ $EF := \frac{EG \cdot DE}{EK}$ $FG := EG - EF$

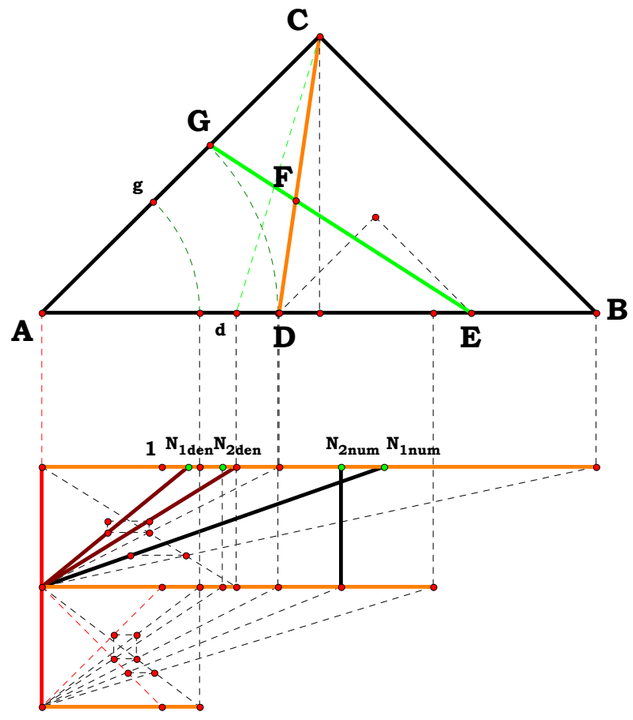
$FJ := \frac{GH \cdot DE}{EK}$ $CD := \sqrt{CL^2 + DL^2}$ $DF := \sqrt{DJ^2 + FJ^2}$

$\frac{EG}{FG} = 1.5$ $\frac{CD}{DF} = 15$

Definitions.

$\frac{N_2 + N_1 - 2}{N_2 - 1} = 1.5$ $\frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} = 15$

$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0$ $\frac{CD}{DF} - \frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1}$



$N_1 = 2.32900$

$N_2 = 1.65606$

$\frac{N_{1\,num}}{N_{1\,den}} = 2.32900$

$\frac{N_{2\,num}}{N_{2\,dem}} = 1.65606$

$\frac{EG}{FG} = 3.02573$ $\frac{(N_2+N_1)-2}{N_2-1} = 3.02573$ $\frac{EG}{FG} - \frac{(N_2+N_1)-2}{N_2-1} = 0.00000$

$\frac{CD}{DF} = 2.47357$ $\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 2.47357$ $\frac{CD}{DF} - \frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 0.00000$

071593R3

Unit.

$AB := 1$

Given.

$W := 3$

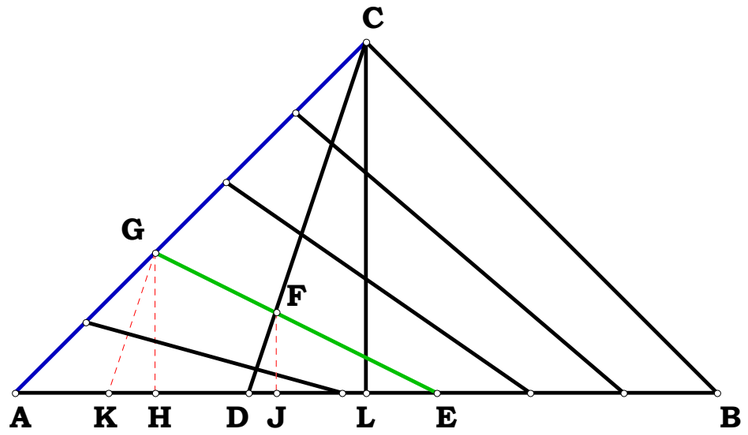
$X := 10$

$Y := 8$

$Z := 20$

Pyramid of Ratios II

What we have, is a way to understand how to write something, which shows a very intimate account of the ratios which, if we stop to think about it, we had set all along. We are learning the results of our own behavior applied to information.



Descriptions.

$N_1 := \frac{X}{W}$ $N_2 := \frac{Z}{Y}$ $AD := \frac{AB}{N_1}$ $AC := \sqrt{\frac{AB^2}{2}}$ $BD := AB - AD$

$AL := \frac{AB}{2}$ $DE := \frac{BD}{N_2}$ $AG := \frac{AC}{N_2}$ $AE := AD + DE$ $AH := \sqrt{\frac{AG^2}{2}}$

$GH := AH$ $EH := AE - AH$ $EG := \sqrt{EH^2 + GH^2}$ $DL := AL - AD$

$CL := AL$ $HK := \frac{DL \cdot AH}{AL}$ $EK := EH + HK$ $DJ := \frac{HK \cdot DE}{EK}$

$EF := \frac{EG \cdot DE}{EK}$ $FG := EG - EF$ $FJ := \frac{GH \cdot DE}{EK}$

$CD := \sqrt{CL^2 + DL^2}$ $DF := \sqrt{DJ^2 + FJ^2}$

$\frac{EG}{FG} - \frac{N_2 + N_1 - 2}{N_2 - 1} = 0$ $\frac{CD}{DF} - \left(\frac{N_2^2 + N_1 \cdot N_2 - 2 \cdot N_2}{N_1 - 1} \right) = 0$

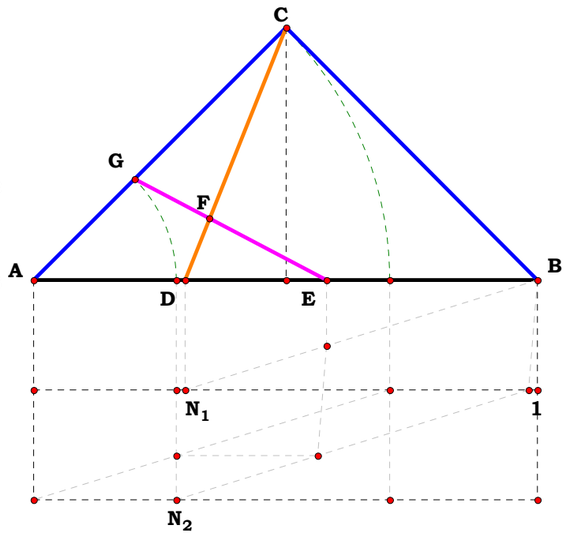
Definitions.

$N_1 - \frac{X}{W} = 0$ $N_2 - \frac{Z}{Y} = 0$ $AD - \frac{W}{X} = 0$ $AC - \frac{1}{\sqrt{2}} = 0$ $BD - \frac{(X - W)}{X} = 0$

$AL - \frac{1}{2} = 0$ $DE - \frac{Y \cdot (X - W)}{X \cdot Z} = 0$ $AG - \frac{\sqrt{2} \cdot Y}{2 \cdot Z} = 0$ $AE - \frac{W \cdot (Z - Y) + X \cdot Y}{X \cdot Z} = 0$ $AH - \frac{Y}{2 \cdot Z} = 0$

$GH - \frac{Y}{2 \cdot Z} = 0$ $EH - \frac{2 \cdot W \cdot (Z - Y) + X \cdot Y}{2 \cdot X \cdot Z} = 0$ $EG - \frac{\sqrt{Y^2 \cdot X^2 + [2 \cdot W \cdot Y \cdot (Z - Y)] \cdot X + 2 \cdot W^2 \cdot (Y - Z)^2}}{\sqrt{2} \cdot X \cdot Z} = 0$

$DL - \frac{(X - 2 \cdot W)}{2 \cdot X} = 0$ $CL - \frac{1}{2} = 0$ $HK - \frac{Y \cdot (X - 2 \cdot W)}{2 \cdot X \cdot Z} = 0$ $EK - \frac{(W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)}{X \cdot Z} = 0$



Unit = 1.00000
W/X = 0.30000
W = 3.00000
X = 10.00000
X/W = 3.33333
N_1 = 3.33333

Y/Z = 0.40000
Y = 8.00000
Z = 20.00000
Z/Y = 2.50000
N_2 = 2.50000

EG = 3.62887 cm
FG = 1.41999 cm
$\frac{EG}{FG} = 2.55556$
$\frac{(N_1+N_2)-2}{N_2-1} = 2.55556$
$\frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{W \cdot (Y - Z)} = 2.55556$

CD = 4.55082 cm
DF = 1.10803 cm
$\frac{CD}{DF} = 4.10714$
$\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 4.10714$
$\frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W - X)} = 4.10714$

$$DJ - \frac{Y^2 \cdot (W - X) \cdot (X - 2 \cdot W)}{2 \cdot X \cdot Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)} = 0$$

$$EF - \frac{Y \cdot (X - W) \cdot \sqrt{2 \cdot X^2 \cdot Y^2 + 4 \cdot W^2 \cdot (Y - Z)^2 - 4 \cdot W \cdot X \cdot Y \cdot (Y - Z)}}{2 \cdot X \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

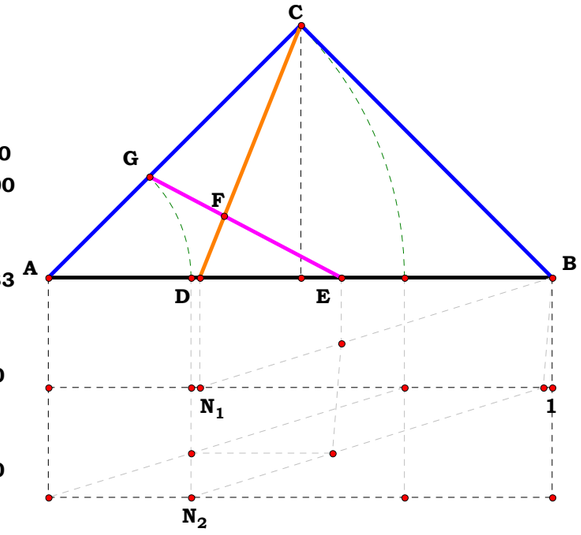
$$FG - \frac{W \cdot (Z - Y) \cdot \sqrt{2 \cdot X^2 \cdot Y^2 + 4 \cdot W^2 \cdot (Y - Z)^2 - 4 \cdot W \cdot X \cdot Y \cdot (Y - Z)}}{2 \cdot X \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

$$FJ - \frac{Y^2 \cdot (X - W)}{2 \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

$$CD - \frac{\sqrt{2 \cdot W^2 - 2 \cdot W \cdot X + X^2}}{\sqrt{2} \cdot X} = 0$$

$$DF - \frac{Y^2 \cdot (X - W) \cdot \sqrt{2 \cdot W^2 - 2 \cdot W \cdot X + X^2}}{\sqrt{2} \cdot X \cdot Z \cdot (W \cdot Z - 2 \cdot W \cdot Y + X \cdot Y)} = 0$$

$$\frac{EG}{FG} - \frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{(Y - Z) \cdot W} = 0 \qquad \frac{CD}{DF} - \left[\frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W - X)} \right] = 0$$



Unit = 1.00000
W/X = 0.30000
W = 3.00000
X = 10.00000
X/W = 3.33333
N_1 = 3.33333

Y/Z = 0.40000
Y = 8.00000
Z = 20.00000
Z/Y = 2.50000
N_2 = 2.50000

EG = 3.62887 cm
FG = 1.41999 cm

$\frac{EG}{FG} = 2.55556$

$\frac{(N_1+N_2)-2}{N_2-1} = 2.55556$

$\frac{2 \cdot W \cdot Y - W \cdot Z - X \cdot Y}{W \cdot (Y - Z)} = 2.55556$

CD = 4.55082 cm
DF = 1.10803 cm

$\frac{CD}{DF} = 4.10714$

$\frac{(N_2^2+N_1 \cdot N_2)-2 \cdot N_2}{N_1-1} = 4.10714$

$\frac{Z \cdot (2 \cdot W \cdot Y - W \cdot Z - X \cdot Y)}{Y^2 \cdot (W - X)} = 4.10714$

Unit.

$AB := 1$

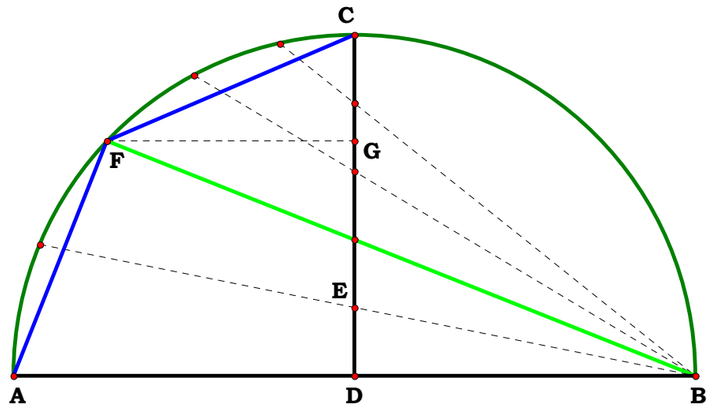
Given.

$N := 6$

$\delta := 1 .. N$

Pyramid of Ratios III

Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?



072593R1

Descriptions.

$AD := \frac{AB}{2}$ $BD := AD$ $CD := BD$ $BC := \sqrt{\frac{BD^2}{2}}$ $DE_\delta := \frac{CD \cdot \delta}{N}$

$BE_\delta := \sqrt{BD^2 + (DE_\delta)^2}$ $BF_\delta := \frac{BD \cdot AB}{BE_\delta}$ $AF_\delta := \frac{DE_\delta \cdot AB}{BE_\delta}$ $EF_\delta := BF_\delta - BE_\delta$

$EG_\delta := \frac{DE_\delta \cdot EF_\delta}{BE_\delta}$ $FG_\delta := \sqrt{(EF_\delta)^2 - (EG_\delta)^2}$ $DG := DE + EG$ $CG_\delta := CD - DG_\delta$

$CF_\delta := \sqrt{(FG_\delta)^2 + (CG_\delta)^2}$

$\frac{DE_\delta}{DG_\delta} =$

0.514
0.556
0.625
0.722
0.847
1

$\frac{\left(\frac{\delta}{N}\right)^2 + 1}{2} =$

0.513889
0.555556
0.625
0.722222
0.847222
1

$\frac{BE_\delta}{BF_\delta} =$

0.513889
0.555556
0.625
0.722222
0.847222
1

$\frac{BF_\delta}{BE_\delta} =$

1.945946
1.8
1.6
1.384615
1.180328
1

$\frac{2}{\left[\left(\frac{\delta}{N}\right)^2 + 1\right]} =$

1.945946
1.8
1.6
1.384615
1.180328
1

$\text{if}\left(N - \delta, \frac{AF_\delta}{CF_\delta}, 0\right) =$

0.283
0.707
1.414
2.828
7.071
0

$\text{if}\left(N - \delta, \frac{\delta \cdot \sqrt{2}}{N - \delta}, 0\right) =$

0.282843
0.707107
1.414214
2.828427
7.071068
0

Unit.

$AB := 1$

Given.

$N := 5$

072593R2

Descriptions.

$$CD := \frac{AB}{2} \qquad DE := \frac{CD}{N} \qquad BE := \sqrt{CD^2 + DE^2}$$

$$BF := \frac{CD \cdot AB}{BE} \qquad AF := \frac{DE \cdot BF}{CD} \qquad DG := \frac{DE \cdot BF}{BE}$$

$$CG := CD - DG \qquad FG := \frac{CD \cdot (DG - DE)}{DE} \qquad CF := \sqrt{FG^2 + CG^2}$$

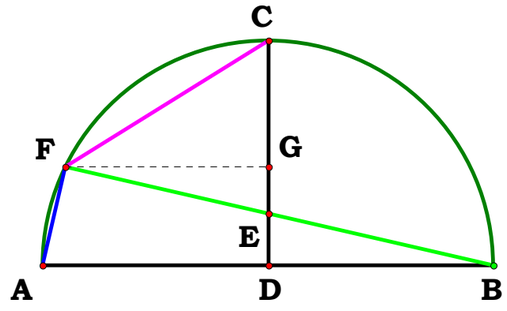
Definitions.

$$\frac{BE}{BF} = 0.52 \qquad \frac{N^2 + 1}{2 \cdot N^2} = 0.52$$

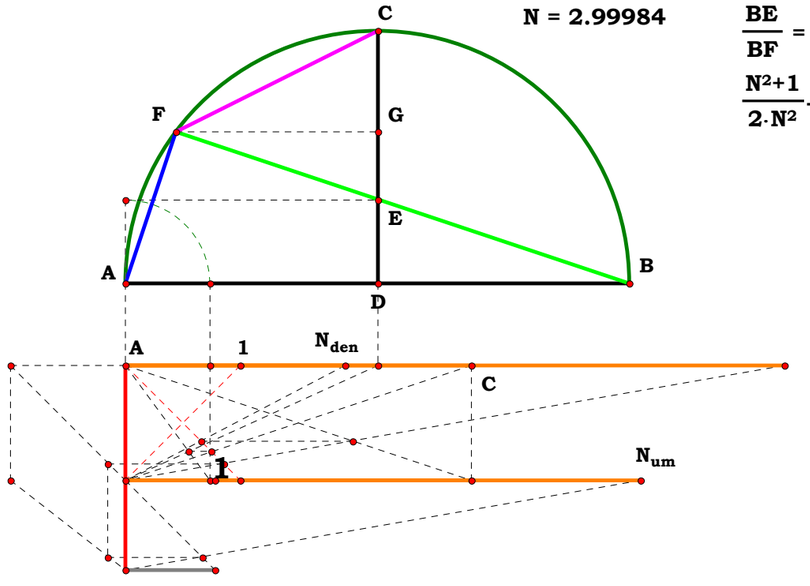
$$\frac{AF}{CF} = 0.353553 \qquad \frac{\sqrt{2}}{N - 1} = 0.353553$$

$$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0 \qquad \frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$$

Pyramid of Ratios III



Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?



$N = 2.99984$

$\frac{N_{um}}{N_{den}} = 2.99984$

$BE = 1.75445$ in.

$BF = 3.15797$ in.

$\frac{BE}{BF} = 0.55556$

$\frac{N^2+1}{2 \cdot N^2} - \frac{BE}{BF} = 0.00000$

$AF = 1.05271$ in.

$CF = 1.48864$ in.

$\frac{AF}{CF} = 0.70716$

$\frac{\sqrt{2}}{N-1} - \frac{AF}{CF} = 0.00000$

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 4$

072593R3

Descriptions.

$$N := \frac{Y}{X} \quad CD := \frac{AB}{2} \quad DE := \frac{CD}{N} \quad BE := \sqrt{CD^2 + DE^2}$$

$$BF := \frac{CD \cdot AB}{BE} \quad AF := \frac{DE \cdot BF}{CD} \quad DG := \frac{DE \cdot BF}{BE}$$

$$CG := CD - DG \quad FG := \frac{CD \cdot (DG - DE)}{DE} \quad CF := \sqrt{FG^2 + CG^2}$$

$$\frac{BE}{BF} - \frac{N^2 + 1}{2 \cdot N^2} = 0 \quad \frac{AF}{CF} - \frac{\sqrt{2}}{N - 1} = 0$$

Definitions.

$$N - \frac{Y}{X} = 0 \quad CD - \frac{1}{2} \quad DE - \frac{X}{2 \cdot Y} = 0 \quad BE - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$$

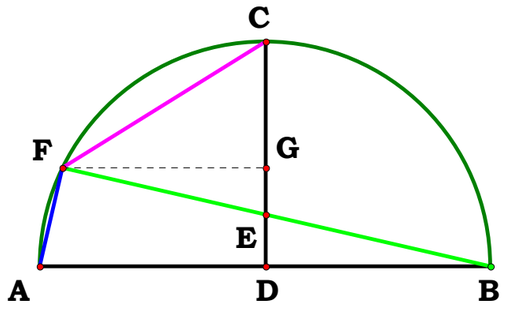
$$BF - \frac{Y}{\sqrt{X^2 + Y^2}} = 0 \quad AF - \frac{X}{\sqrt{X^2 + Y^2}} = 0 \quad DG - \frac{X \cdot Y}{X^2 + Y^2} = 0$$

$$CG - \frac{(X - Y)^2}{2 \cdot (X^2 + Y^2)} = 0 \quad FG - \frac{(Y - X) \cdot (X + Y)}{2 \cdot (X^2 + Y^2)} = 0$$

$$CF - \frac{(Y - X)}{\sqrt{2 \cdot (X^2 + Y^2)}} = 0$$

$$\frac{BE}{BF} - \frac{X^2 + Y^2}{2 \cdot Y^2} = 0 \quad \frac{AF}{CF} - \frac{\sqrt{2} \cdot X}{Y - X} = 0$$

Pyramid of Ratios III



Dividing DC into a ratio provides what in terms of BE/BF and AF/CF?

Unit = 1.00000

X = 4.00000

Y = 20.00000

X/Y = 0.20000

Y/X = 5.00000

N = 5.00000

$\frac{N^2+1}{2 \cdot N^2}$ = 0.52000

$\frac{\sqrt{2}}{N-1}$ = 0.35355

BE = 4.78390 cm

BF = 9.19981 cm

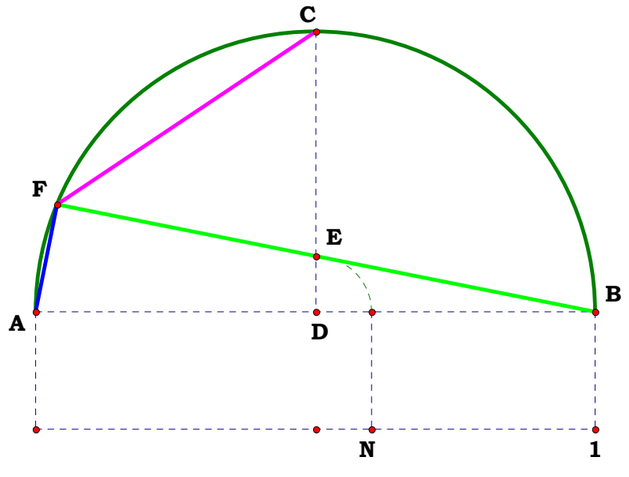
$\frac{BE}{BF}$ = 0.52000

AF = 1.83996 cm

CF = 5.20420 cm

$\frac{X^2+Y^2}{2 \cdot Y^2}$ = 0.52000

$\frac{\sqrt{2} \cdot X}{Y-X}$ = 0.35355



110693R1

Gruntwork I on the Delian Solution

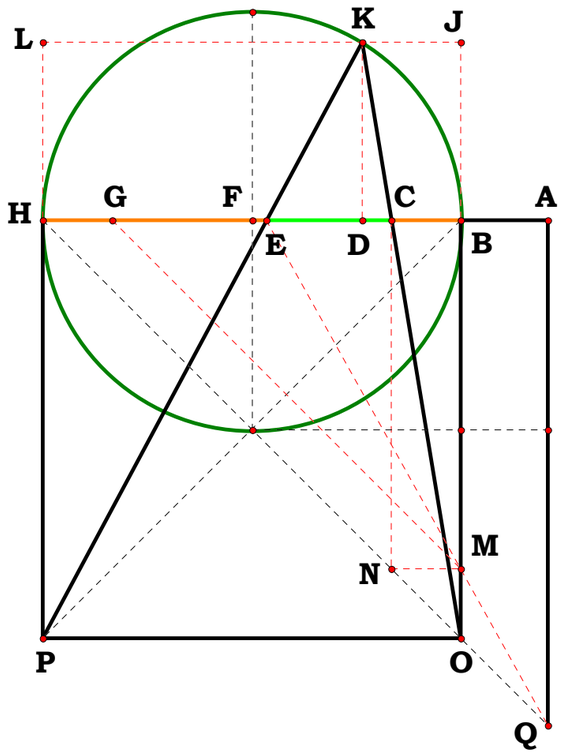
Unit.

$BH := 1$

Given.

$N := 5$

Does $(AB^2 \times AH)^{1/3} = AC$
and $(AB \times AH^2)^{1/3} = AE$?



Descriptions.

$$BF := \frac{BH}{2} \qquad BD := \frac{BF}{N} \qquad DH := BH - BD$$

$$DK := \sqrt{BD \cdot DH} \qquad JO := BH + DK \qquad BC := \frac{BD \cdot BH}{JO}$$

$$BG := BH - BC \qquad EH := \frac{DH \cdot BH}{JO} \qquad BE := BH - EH \qquad EG := EH - BC$$

$$GM := \sqrt{2 \cdot BG^2} \qquad HO := \sqrt{2 \cdot BH^2} \qquad HQ := \frac{GM \cdot EH}{EG} \qquad OQ := HQ - HO$$

$$AB := \frac{OQ}{\sqrt{2}} \qquad AC := AB + BC \qquad AE := AB + BE \qquad AH := AB + BH$$

$$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$$

Definitions.

$$BF - \frac{1}{2} = 0 \qquad BD - \frac{1}{2 \cdot N} = 0 \qquad DH - \frac{(2 \cdot N - 1)}{2 \cdot N} = 0 \qquad DK - \frac{\sqrt{2 \cdot N - 1}}{(2 \cdot N)} = 0$$

$$JO - \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)}{2 \cdot N} = 0 \qquad BC - \frac{1}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \qquad BG - \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$$

$$EH - \frac{(2 \cdot N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \qquad BE - \frac{\left(\sqrt{2 \cdot N - 1} + 1\right)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0 \qquad EG - \frac{2 \cdot (N - 1)}{2 \cdot N + \sqrt{2 \cdot N - 1}} = 0$$

$$GM - \sqrt{2} \cdot \frac{\left(2 \cdot N + \sqrt{2 \cdot N - 1} - 1\right)}{\left(2 \cdot N + \sqrt{2 \cdot N - 1}\right)} = 0 \qquad HO - \sqrt{2} = 0$$

$$HQ - \frac{\sqrt{2}\cdot(2\cdot N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

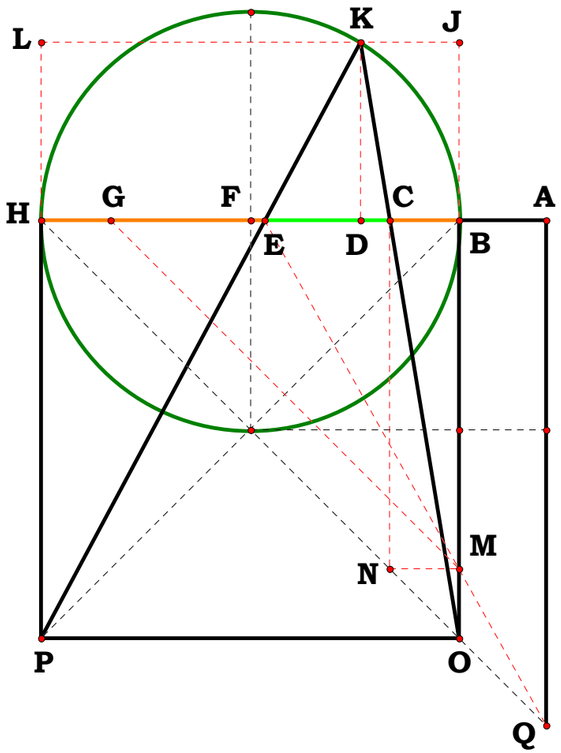
$$OQ - \frac{\sqrt{2}\cdot\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{\left(2\cdot N+\sqrt{2\cdot N-1}\right)\cdot(2\cdot N-2)} = 0$$

$$AB - \frac{\left[2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AC - \frac{\left[2\cdot N+2\cdot\sqrt{2\cdot N-1}+(2\cdot N-1)^{\frac{3}{2}}-2\cdot N\cdot\sqrt{2\cdot N-1}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AE - \frac{\left[2\cdot N+(2\cdot N-1)^{\frac{3}{2}}-1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$

$$AH - \frac{\left[(2\cdot N-1)^{\frac{3}{2}}-4\cdot N+4\cdot N^2+1\right]}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)} = 0$$



BF = 1.37917 in.

BD = 0.65310 in.

$\frac{BF}{BD} = 2.11173$

N = 2.11173

BH = 2.75833 in.

AE = 1.85725 in.

$\frac{BH\cdot\left(\left(2\cdot N+(2\cdot N-1)^{\frac{3}{2}}\right)-1\right)}{2\cdot(N-1)\cdot\left(2\cdot N+\sqrt{2\cdot N-1}\right)}$-AE = 0.00000 in.

110693R2

Gruntwork I on the Delian Solution

Unit.

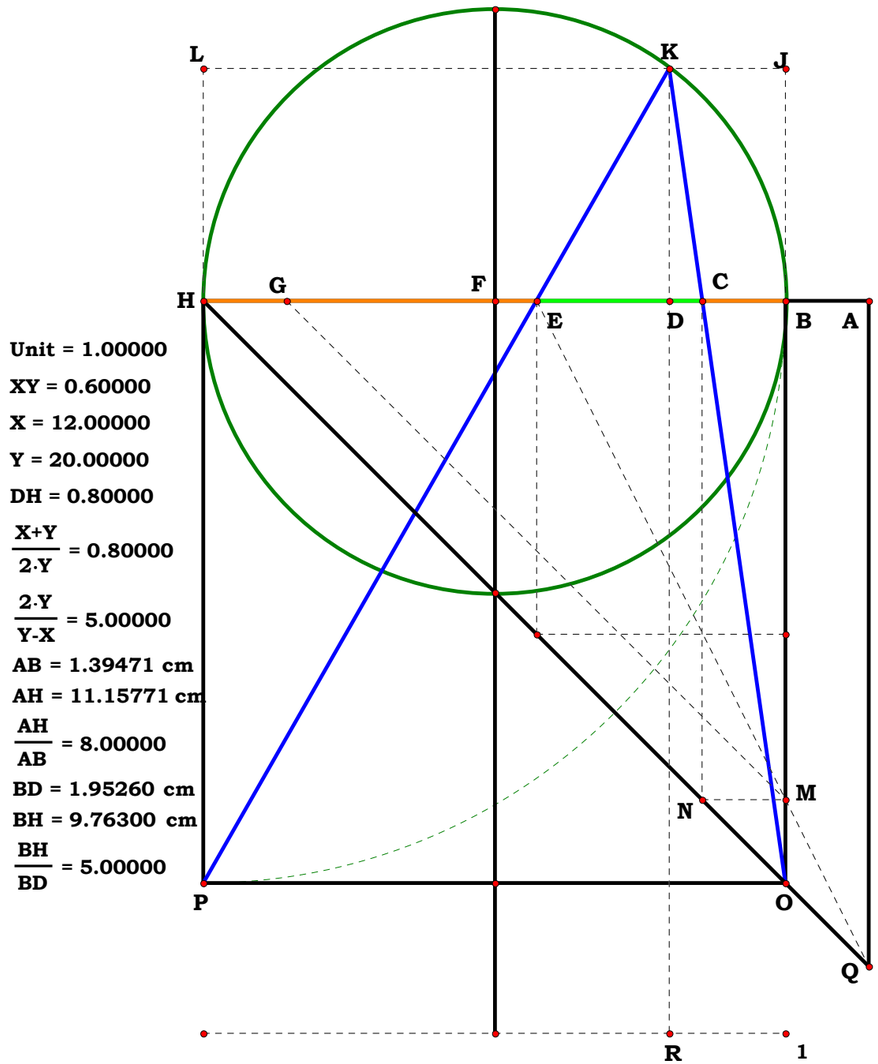
$BH := 1$

Given.

$Y := 20$

$X := 13$

Does $(AB^2 \times AH)^{1/3} = AC$ and $(AB \times AH^2)^{1/3} = AE$?



Unit = 1.00000

XY = 0.60000

X = 12.00000

Y = 20.00000

DH = 0.80000

$\frac{X+Y}{2 \cdot Y}$ = 0.80000

$\frac{2 \cdot Y}{Y-X}$ = 5.00000

AB = 1.39471 cm

AH = 11.15771 cm

$\frac{AH}{AB}$ = 8.00000

BD = 1.95260 cm

BH = 9.76300 cm

$\frac{BH}{BD}$ = 5.00000

Descriptions.

$$BF := \frac{BH}{2} \quad BD := \frac{Y - X}{2 \cdot Y} \quad DH := BH - BD \quad DK := \sqrt{BD \cdot DH} \quad JO := BH + DK$$

$$BC := \frac{BD \cdot BH}{JO} \quad BG := BH - BC \quad EH := \frac{DH \cdot BH}{JO} \quad BE := BH - EH \quad EG := EH - BC$$

$$GM := \sqrt{2 \cdot BG^2} \quad HO := \sqrt{2 \cdot BH^2} \quad HQ := \frac{GM \cdot EH}{EG} \quad OQ := HQ - HO$$

$$AB := \frac{OQ}{\sqrt{2}} \quad AC := AB + BC \quad AE := AB + BE \quad AH := AB + BH$$

$$\frac{BF}{BD} = 2.857143 \quad \frac{AH}{AB} = 10.235849 \quad \left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AE = 0$$

Definitions.

$$BF - \frac{1}{2} = 0 \quad BD - \frac{Y - X}{2 \cdot Y} = 0 \quad DH - \frac{X + Y}{2 \cdot Y} = 0 \quad DK - \frac{\sqrt{Y^2 - X^2}}{2 \cdot Y} = 0$$

$$JO - \frac{2 \cdot Y + \sqrt{Y^2 - X^2}}{2 \cdot Y} = 0 \quad BC - \frac{Y - X}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0 \quad BG - \frac{X + Y + \sqrt{Y^2 - X^2}}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$$

$$EH - \frac{X + Y}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0 \quad BE - \frac{(X + Y) \cdot \sqrt{Y^2 - X^2} + (X - Y)^2}{X^2 + 3 \cdot Y^2} = 0 \quad EG - \frac{2 \cdot X}{2 \cdot Y + \sqrt{Y^2 - X^2}} = 0$$

$$GM - \frac{\sqrt{2} \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{\left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad HO - \sqrt{2} = 0 \quad HQ - \frac{\sqrt{2} \cdot (X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad OQ - \frac{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right) \cdot \sqrt{2}}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$$

$$AB - \frac{(X - Y) \cdot \left(X - Y - \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AC - \frac{(Y - X) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AE - \frac{(X + Y) \cdot \left(\sqrt{Y^2 - X^2} + Y - X\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0 \quad AH - \frac{(X + Y) \cdot \left(X + Y + \sqrt{Y^2 - X^2}\right)}{2 \cdot X \cdot \left(2 \cdot Y + \sqrt{Y^2 - X^2}\right)} = 0$$

Unit.

$BH := 1$

Given.

$N_1 := 5$

110993R1

Descriptions.

$BG := \frac{BH}{2}$ $CF := \frac{BH}{N_1}$ $BL := CF$ $GP := BG$

$BK := \frac{BL}{2}$ $BD := BK$ $NP := BD$ $GN := GP - NP$ $EN := BL$

$GE := \sqrt{GN^2 - EN^2}$ $CE := BD$ $BC := BG - (GE + CE)$

$GH := BG$ $EF := BD$ $FH := GH + GE - EF$ $FQ := FH$

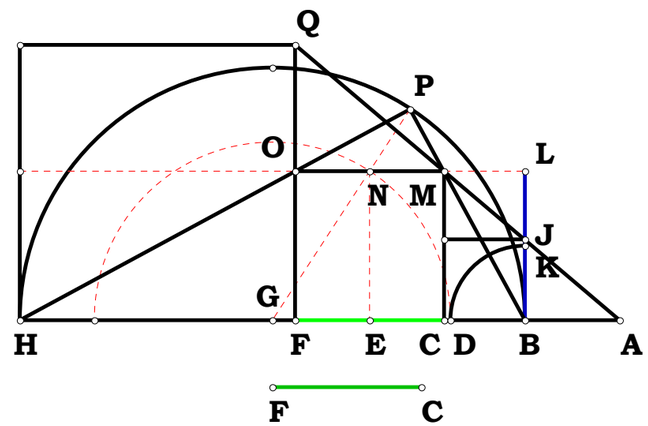
$FO := BL$ $OQ := FQ -]MO := CF$ $AF := \frac{MO \cdot FQ}{OQ}$ $AC := AF - CF$

$AH := AF + FH$ $AB := AH - BH$

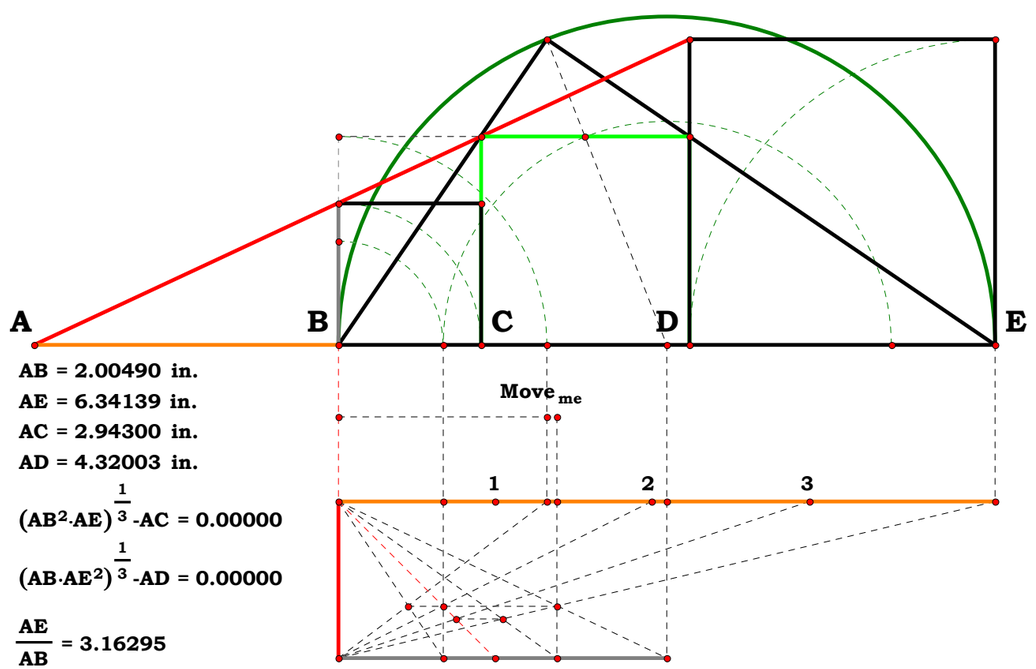
$(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0$ $(AB \cdot AH^2)^{\frac{1}{3}} - AF = 0$ $\frac{AH}{AB} = 51.980762113532$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

Solve For Cube Root Placement



With straight edge and compass only, solve the given problem.
BH is the difference between the segments AH and AB.
CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



AB = 2.00490 in.

AE = 6.34139 in.

AC = 2.94300 in.

AD = 4.32003 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

$\frac{AE}{AB} = 3.16295$

Definitions.

$$BH - 1 = 0 \qquad BG - \frac{1}{2} = 0 \qquad CF - \frac{1}{N_1} = 0 \qquad BK - \frac{1}{2 \cdot N_1} = 0 \qquad GN - \frac{N_1 - 1}{2 \cdot N_1} = 0$$

$$GE - \frac{\sqrt{(N_1 + 1) \cdot (N_1 - 3)}}{2 \cdot N_1} = 0 \qquad BC - \frac{N_1 - \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{2 \cdot N_1} = 0$$

$$FH - \frac{N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{2 \cdot N_1} = 0$$

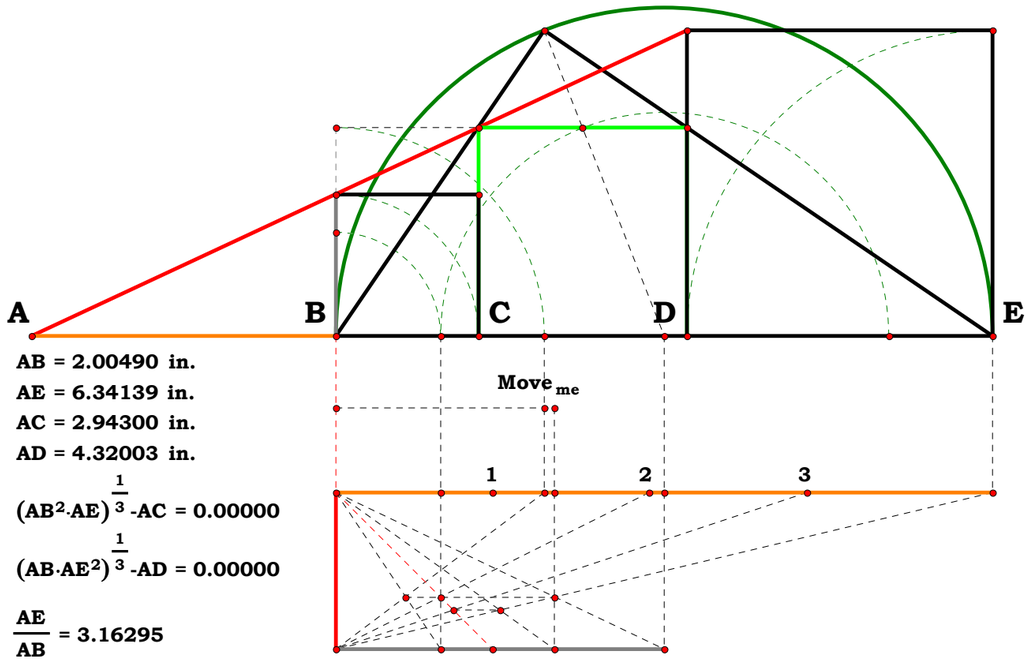
$$OQ - \frac{N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3}{2 \cdot N_1} = 0$$

$$AF - \frac{N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$

$$AC - \frac{2}{N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$

$$AH - \frac{\left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1\right]^2}{2 \cdot N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$

$$AB - \frac{N_1 - \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 1}{N_1 \cdot \left[N_1 + \sqrt{(N_1 + 1) \cdot (N_1 - 3)} - 3\right]} = 0$$



AB = 2.00490 in.
AE = 6.34139 in.
AC = 2.94300 in.
AD = 4.32003 in.

$(AB^2 \cdot AE)^{\frac{1}{3}} - AC = 0.00000$

$(AB \cdot AE^2)^{\frac{1}{3}} - AD = 0.00000$

$\frac{AE}{AB} = 3.16295$

Unit.

$BG := 1$

Given.

$Y := 20$

$X := 15$

110993R2

Descriptions.

$BH := 2 \cdot BG \quad CF := \frac{2 \cdot X}{3 \cdot Y} \quad BL := CF \quad GP := BG$

$BK := \frac{BL}{2} \quad BD := BK \quad NP := BD \quad GN := GP - NP \quad EN := BL$

$GE := \sqrt{GN^2 - EN^2} \quad CE := BD \quad BC := BG - (GE + CE)$

$GH := BG \quad EF := BD \quad FH := GH + GE - EF \quad FQ := FH$

$FO := BL \quad OQ := FQ -]MO := CF \quad AF := \frac{MO \cdot FQ}{OQ} \quad AC := AF - CF$

$AH := AF + FH \quad AB := AH - BH$

$CH := CF + FH$

Arithmetic Names:

$CF = 0.5$

$FH = 1.309017$

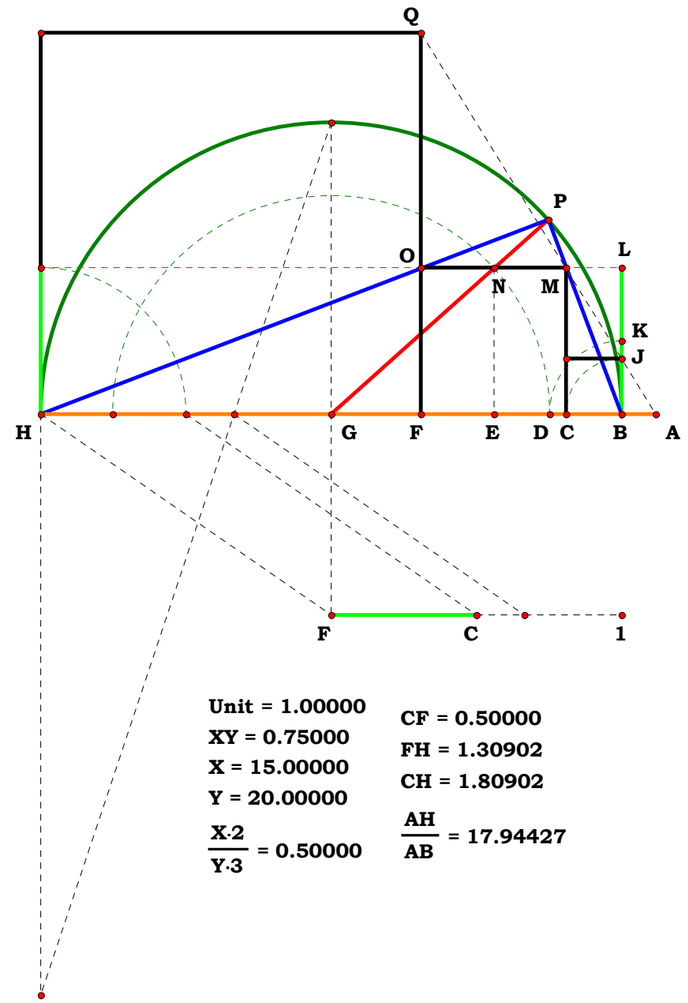
$CH = 1.809017$

$\left(AB^2 \cdot AH\right)^{\frac{1}{3}} - AC = 0 \quad \left(AB \cdot AH^2\right)^{\frac{1}{3}} - AF = 0 \quad \frac{AH}{AB} = 17.944271909999$

The square inside a right triangle on the hypotenuse is equal to the square of the remaining two segments and all three squares taken to the point of similarity form a cube root relationship.

Solve For Cube Root Placement

With straight edge and compass only, solve the given problem. BH is the difference between the segments AH and AB. CF is the difference between the cube root of AB squared by AH and the cube root of AH squared by AB. Find AB and place the roots.



Unit = 1.00000
XY = 0.75000
X = 15.00000
Y = 20.00000
$\frac{X \cdot 2}{Y \cdot 3} = 0.50000$

CF = 0.50000
FH = 1.30902
CH = 1.80902
$\frac{AH}{AB} = 17.94427$

Definitions.

$$BH - 2 = 0 \quad CF - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad BL - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad GP - 1 = 0$$

$$BK - \frac{X}{3 \cdot Y} = 0 \quad BD - \frac{X}{3 \cdot Y} = 0 \quad NP - \frac{X}{3 \cdot Y} = 0 \quad GN - \frac{3 \cdot Y - X}{3 \cdot Y} = 0$$

$$EN - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad GE - \frac{\sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{\sqrt{3} \cdot Y} = 0 \quad CE - \frac{X}{3 \cdot Y} = 0$$

$$BC - \frac{3 \cdot Y - X - \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0 \quad GH - 1 = 0$$

$$EF - \frac{X}{3 \cdot Y} = 0 \quad FH - \frac{3 \cdot Y - X + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0$$

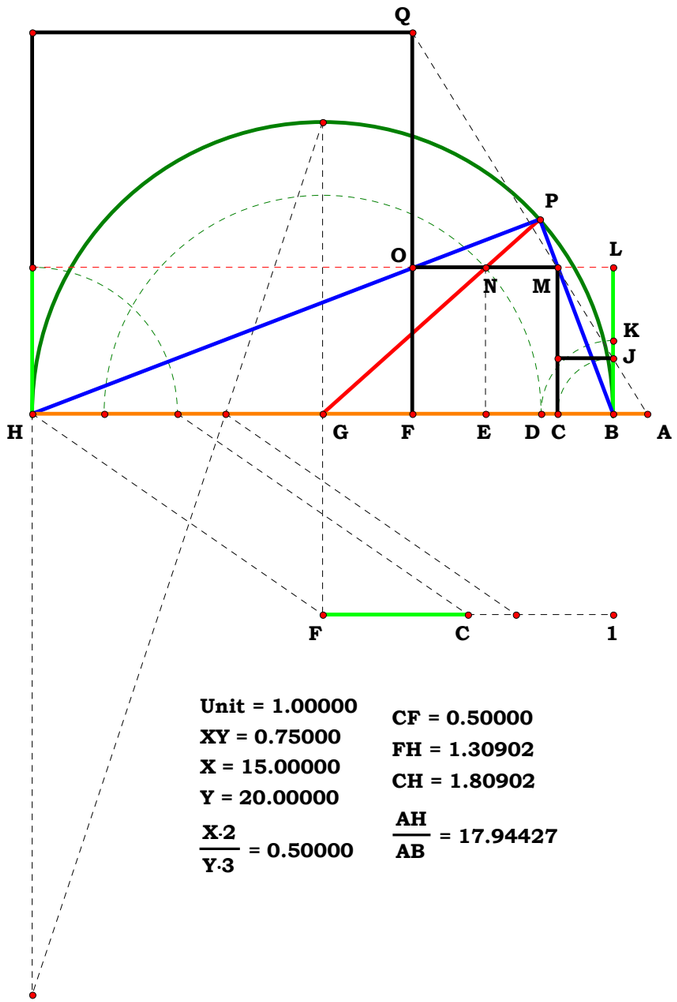
$$CH - \frac{X + 3 \cdot Y + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0 \quad FQ - \frac{3 \cdot Y - X + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0$$

$$FO - \frac{2 \cdot X}{3 \cdot Y} = 0 \quad OQ - \frac{3 \cdot (Y - X) + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}}{3 \cdot Y} = 0 \quad MO - \frac{2 \cdot X}{3 \cdot Y} = 0$$

$$AF - \frac{2 \cdot X \cdot \left(3 \cdot Y - X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)} = 0 \quad AC - \frac{4 \cdot X^2}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)} = 0$$

$$AH - \frac{18 \cdot Y^2 - 12 \cdot X \cdot Y - 2 \cdot X^2 - 2 \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2} \cdot (X - 3 \cdot Y) \cdot \sqrt{3}}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}\right)} = 0 \quad AB - \frac{2 \cdot X \cdot \left(3 \cdot Y - X - \sqrt{3} \cdot \sqrt{3 \cdot Y^2 - 2 \cdot X \cdot Y - X^2}\right)}{3 \cdot Y \cdot \left(3 \cdot Y - 3 \cdot X + \sqrt{9 \cdot Y^2 - 6 \cdot X \cdot Y - 3 \cdot X^2}\right)} = 0$$

$$\frac{AH}{AB} = 17.944272$$



Unit = 1.00000
XY = 0.75000
X = 15.00000
Y = 20.00000
$\frac{X \cdot 2}{Y \cdot 3} = 0.50000$

CF = 0.50000
FH = 1.30902
CH = 1.80902
$\frac{AH}{AB} = 17.94427$

111093R

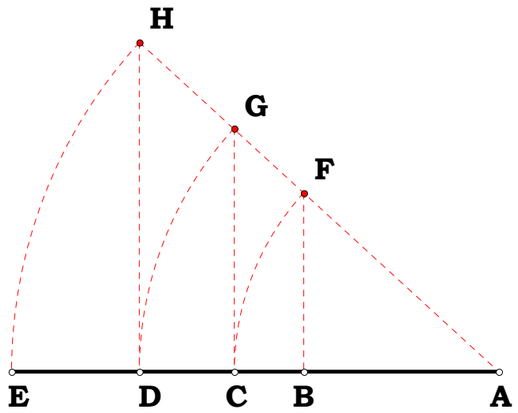
Gruntwork II on the Delian Solution

Unit.

$AE := 1$

Given.

$N := 4$



Descriptions.

$DE := \frac{AE}{N}$ $AD := AE - DE$

$AH := AE$ $AG := AD$

$AC := \frac{AD \cdot AD}{AE}$ $AF := AC$

$AB := \frac{AC \cdot AC}{AD}$

Definitions.

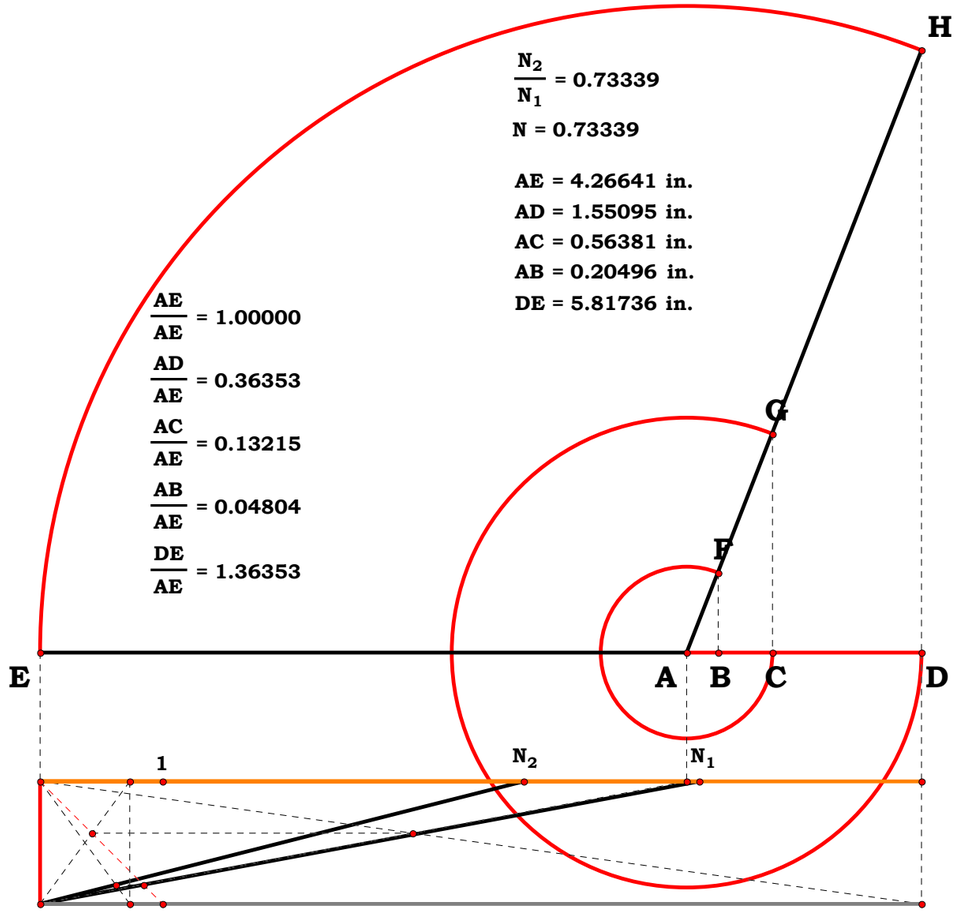
$\left(AB^2 \cdot AE\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AE^2\right)^{\frac{1}{3}} - AD = 0$

$\frac{AE}{AB} = 2.37$ $\frac{AD}{AB} = 1.777778$ $\frac{AC}{AB} = 1.333333$

Albebraic Names:

$\frac{1}{N} - DE = 0$ $1 - \frac{1}{N} - AD = 0$ $\frac{(N-1)^2}{N^2} - AC = 0$

$\frac{(N-1)^3}{N^3} - AB = 0$



$\frac{N_2}{N_1} = 0.73339$

$N = 0.73339$

$AE = 4.26641$ in.
$AD = 1.55095$ in.
$AC = 0.56381$ in.
$AB = 0.20496$ in.
$DE = 5.81736$ in.

$\frac{AE}{AE} = 1.00000$
$\frac{AD}{AE} = 0.36353$
$\frac{AC}{AE} = 0.13215$
$\frac{AB}{AE} = 0.04804$
$\frac{DE}{AE} = 1.36353$

$\sqrt{\left(\frac{1}{N}\right)^2} = 1.36353$ $\sqrt{\left(1-\frac{1}{N}\right)^2} = 0.36353$ $\frac{(N-1)^2}{N^2} = 0.13215$ $\sqrt{\frac{(N-1)^{3\,2}}{N^3}} = 0.04804$

$\sqrt{\left(\frac{1}{N}\right)^2} - \frac{DE}{AE} = 0.00000$ $\sqrt{\left(1-\frac{1}{N}\right)^2} - \frac{AD}{AE} = 0.00000$ $\frac{(N-1)^2}{N^2} - \frac{AC}{AE} = 0.00000$ $\sqrt{\frac{(N-1)^{3\,2}}{N^3}} - \frac{AB}{AE} = 0.00000$

Unit.

$AB := 1$

Given.

$N := 3$

111193AR

Descriptions.

The Archimedean Paper Trisector

When I looked up the Archimedean Paper Trisector, which is all I found. I did not find where anyone had bothered to complete the figure, for it was obvious to me that the figure was simply not complete. The first task then in writing up the figure is to simply complete the figure.

Once one understands that the angle on the center is twice the angle from the circumference one can then start to work filling in the figure to include the APT. One can see, not only here, but in other figures that trisection is involved with the right triangle and square roots.

$AD := \frac{AB}{N}$ $BD := AB - AD$ $CD := \sqrt{AD \cdot BD}$ $BC := \sqrt{CD^2 + BD^2}$

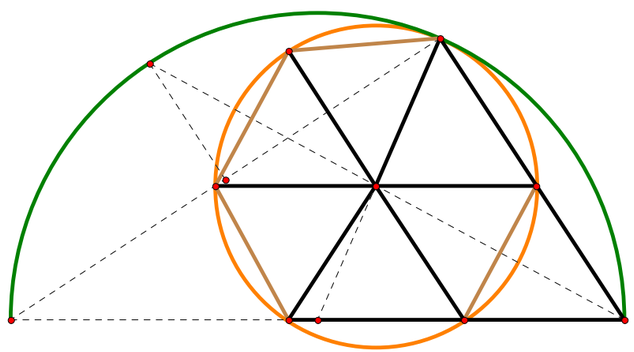
$AC := \sqrt{AD^2 + CD^2}$ $BH := \frac{BC}{2}$ $AO := \frac{AB}{2}$ $HO := \frac{AC}{2}$ $HL := \frac{CD}{2}$ $LO := \frac{AD}{2}$

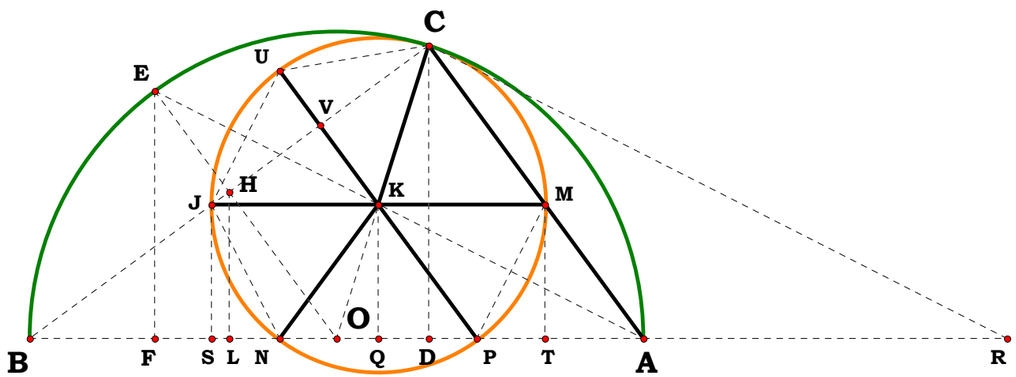
$OF := \frac{LO \cdot AO}{HO}$ $AF := AO + OF$ $BF := AB - AF$ $EF := \sqrt{BF \cdot AF}$ $DR := \frac{AF \cdot CD}{EF}$

$DO := AO - AD$ $OR := DR + DO$ $KQ := \frac{CD \cdot AO}{OR}$ $OK := \frac{AO \cdot KQ}{CD}$

$CK := AO - OK$ $QP := \sqrt{CK^2 - KQ^2}$ $OQ := \frac{DO \cdot KQ}{CD}$ $EH := AO - HO$

$AP := AO - (OQ + QP)$ $AP - CK = 0$





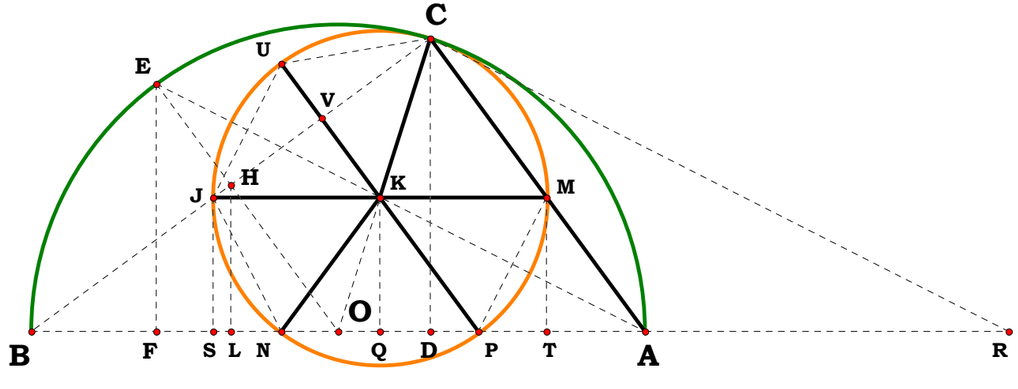
$$CJ := \frac{BC \cdot CK}{AO} \qquad AQ := AO - OQ \qquad SO := CK - OQ \qquad BS := AO - SO$$

$$BJ := BC - CJ \qquad JS := \sqrt{BJ^2 - BS^2} \qquad JS - KQ = 0 \qquad SN := SO + OQ - QP$$

$$JN := \sqrt{JS^2 + SN^2} \qquad UV := \frac{EH \cdot CJ}{BC} \qquad JV := \frac{CJ}{2} \qquad JU := \sqrt{JV^2 + UV^2} \qquad CU := JU$$

$$AT := \frac{AD \cdot CK}{AC} \qquad PT := AP - AT \qquad MT := \frac{CD \cdot AP}{AC} \qquad MP := \sqrt{PT^2 + MT^2}$$

$$MP - JN = 0 \qquad MP - JU = 0 \qquad MP - CU = 0$$



Definitions.

$$AD - \frac{1}{N} = 0 \qquad BD - \frac{N-1}{N} = 0 \qquad CD - \frac{\sqrt{N-1}}{N} = 0 \qquad BC - \frac{\sqrt{N-1}}{\sqrt{N}} = 0 \qquad AC - \frac{1}{\sqrt{N}} = 0 \qquad BH - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0$$

$$AO - \frac{1}{2} = 0 \qquad HO - \frac{1}{2 \cdot \sqrt{N}} = 0 \qquad HL - \frac{\sqrt{N-1}}{2 \cdot N} = 0 \qquad LO - \frac{1}{2 \cdot N} = 0 \qquad OF - \frac{1}{2 \cdot \sqrt{N}} = 0 \qquad AF - \frac{\sqrt{N}+1}{2 \cdot \sqrt{N}} \qquad BF - \frac{\sqrt{N}-1}{2 \cdot \sqrt{N}} = 0$$

$$EF - \frac{\sqrt{N-1}}{2 \cdot \sqrt{N}} = 0 \qquad DR - \frac{\sqrt{N-1} \cdot \left(\sqrt{N}+1\right)}{N \cdot \sqrt{\left(\sqrt{N}-1\right) \cdot \left(\sqrt{N}+1\right)}} = 0 \qquad DO - \frac{\sqrt{(N-2)^2}}{2 \cdot N} = 0 \qquad OR - \frac{\sqrt{N}+2}{2 \cdot \sqrt{N}} = 0 \qquad KQ - \frac{\sqrt{N-1}}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0$$

$$OK - \frac{\sqrt{N}}{2 \cdot \left(\sqrt{N}+2\right)} = 0 \qquad QP - \frac{1}{\sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad OQ - \frac{\sqrt{(N-2)^2}}{2 \cdot \sqrt{N} \cdot \left(\sqrt{N}+2\right)} = 0 \qquad EH - \frac{\sqrt{N}-1}{2 \cdot \sqrt{N}} = 0$$

$$AP - \frac{1}{\sqrt{N}+2} = 0 \qquad CK - \frac{1}{\sqrt{N}+2} = 0$$

In logic, things which have the same name are equal. CK equals AP.

$$CJ - \frac{2\cdot\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AQ - \frac{\sqrt{N}+1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SO - -\frac{N-2\cdot\sqrt{N}-2}{2\cdot\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$BS - \frac{1\cdot(N-1)}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad BJ - \frac{1\cdot\sqrt{N-1}}{\sqrt{N}+2} = 0 \qquad JS - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

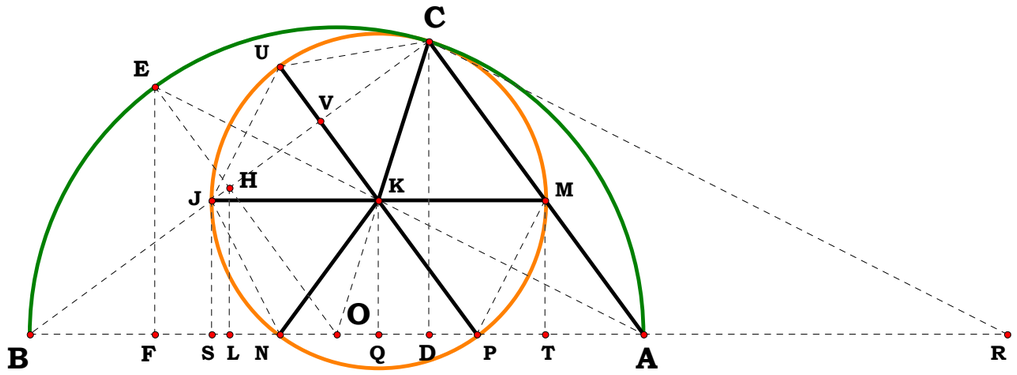
$$JS - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad SN - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JN - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad UV - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad JV - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$JU - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad CU - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad AT - \frac{1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$PT - \frac{\sqrt{N}-1}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MT - \frac{\sqrt{N-1}}{\sqrt{N}\cdot\left(\sqrt{N}+2\right)} = 0$$

$$MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0 \qquad MP - \frac{\sqrt{2}\cdot\sqrt{\sqrt{N}-1}}{N^{\frac{1}{4}}\cdot\left(\sqrt{N}+2\right)} = 0$$



Unit := 1

Given.

N := 4

111193BR1

Descriptions.

$A := N - Unit \quad AB := \frac{A}{2} \quad AN := N - A$

$NK := \sqrt{N \cdot AN} \quad BK := N - (NK + AB)$

$DK := \sqrt{AB^2 + BK^2} \quad KM := \frac{BK \cdot NK}{DK}$

$CE := \frac{N - AB}{2} \quad MP := \frac{CE}{2} \quad CK := CE - BK$

$KP := \frac{CK}{2} \quad NP := NK - KP \quad CF := \frac{MP \cdot CE}{NP}$

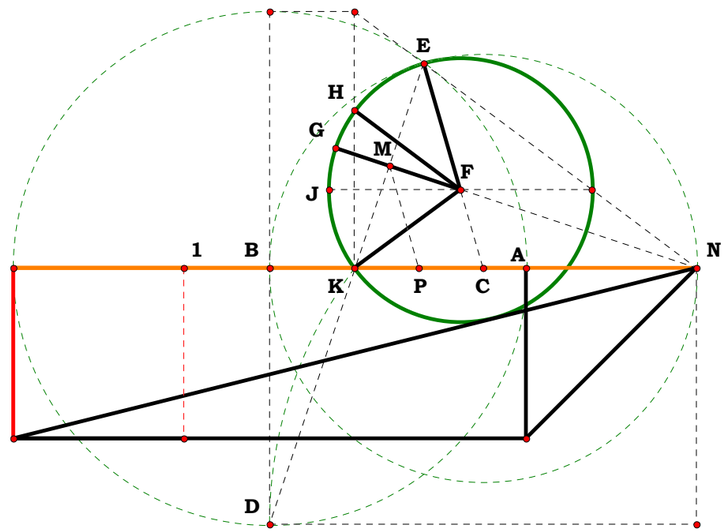
$EF := CE - CF \quad EM := KM$

EF = 0.769231 KM = 0.632456

Definitions.

$EF - \frac{\sqrt{N} \cdot (N + 1)}{N + 4 \cdot \sqrt{N} + 1} = 0 \quad EM - \frac{\sqrt{2} \cdot \sqrt{N} \cdot \left(\sqrt{N} - 1\right)^2}{2 \cdot \sqrt{(N + 1) \cdot \left(\sqrt{N} - 1\right)^2}} = 0$

$\frac{EF}{EM} - \frac{(N + 1) \cdot \sqrt{(2 \cdot N + 2) \cdot \left(\sqrt{N} - 1\right)^2}}{\left(\sqrt{N} - 1\right)^2 \cdot \left(N + 4 \cdot \sqrt{N} + 1\right)} = 0$



N = 4.00000

EM = 1.81051 cm

GM = 0.94862 cm

m∠EFG = 55.30485°

m∠HFG = 18.43495°

m∠EFK = 110.60969°

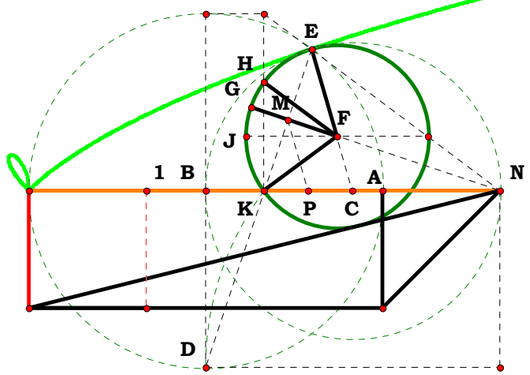
m∠HFJ = 36.86990°

EF = 0.76923

EM = 0.63246

$\frac{m\angle EFG}{m\angle HFG} = 3.00000 \quad \frac{m\angle EFK}{m\angle HFJ} = 3.00000$

$\frac{\sqrt{N} \cdot (N+1)}{N+4 \cdot \sqrt{N}+1} - EF = 0.00000 \quad \frac{\sqrt{2 \cdot N} \cdot \left(\sqrt{N}-1\right)^2}{2 \cdot \sqrt{(N+1) \cdot \left(\sqrt{N}-1\right)^2}} - EM = 0.00000$



N = 4.00000

EM = 1.24826 cm

GM = 0.65403 cm

m∠EFG = 55.30485°

m∠HFG = 18.43495°

m∠EFK = 110.60969°

m∠HFJ = 36.86990°

EF = 0.76923

EM = 0.63246

$\frac{m\angle EFG}{m\angle HFG} = 3.00000 \quad \frac{m\angle EFK}{m\angle HFJ} = 3.00000$

$\frac{\sqrt{N} \cdot (N+1)}{N+4 \cdot \sqrt{N}+1} - EF = 0.00000 \quad \frac{\sqrt{2 \cdot N} \cdot \left(\sqrt{N}-1\right)^2}{2 \cdot \sqrt{(N+1) \cdot \left(\sqrt{N}-1\right)^2}} - EM = 0.00000$

Unit := 1

Given.

N := 3

111193BR2

Descriptions.

$H := \frac{N}{2}$ $J := N - Unit$ $F := N - \sqrt{(N + J)}$

$FK := \sqrt{F^2 + J^2}$ $FN := N - F$

$EF := \frac{F \cdot FN}{FK}$ $CF := 2 \cdot EF$ $EG := \frac{H}{2}$

$FH := H - F$ $FG := \frac{FH}{2}$ $GN := FN - FG$

$BH := \frac{EG \cdot H}{GN}$ $BC := H - BH$ $BC = 0.897763$

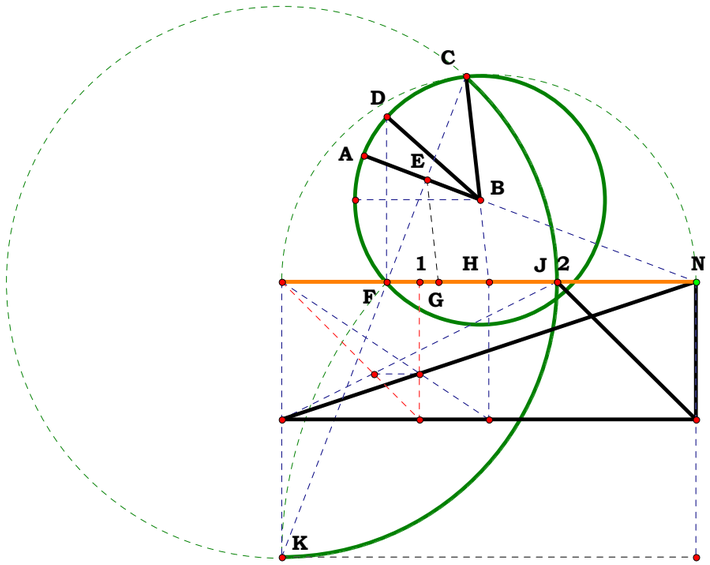
$CE := EF$ $CE = 0.797878$

Definitions.

$$BC - \frac{N \cdot \sqrt{2 \cdot N - 1}}{N + 2 \cdot \sqrt{2 \cdot N - 1}} = 0$$

$$CE - \frac{\left(N \cdot \sqrt{2 \cdot N - 1} - 2 \cdot N + 1\right)}{\sqrt{2 \cdot N \cdot \left(N - \sqrt{2 \cdot N - 1}\right)}} = 0$$

$$\frac{BC}{CE} - \frac{\sqrt{2} \cdot \sqrt{2 \cdot N - 1} \cdot N \cdot \sqrt{N^2 - \sqrt{2 \cdot N - 1} \cdot N}}{\sqrt{2 \cdot N - 1} \cdot \left(N^2 - 4 \cdot N + 2\right) - N + 2 \cdot N^2} = 0$$



m∠ABC = 62.71547°

m∠ABD = 20.90516°

$\frac{m\angle ABC}{m\angle ABD}$ = 3.00000

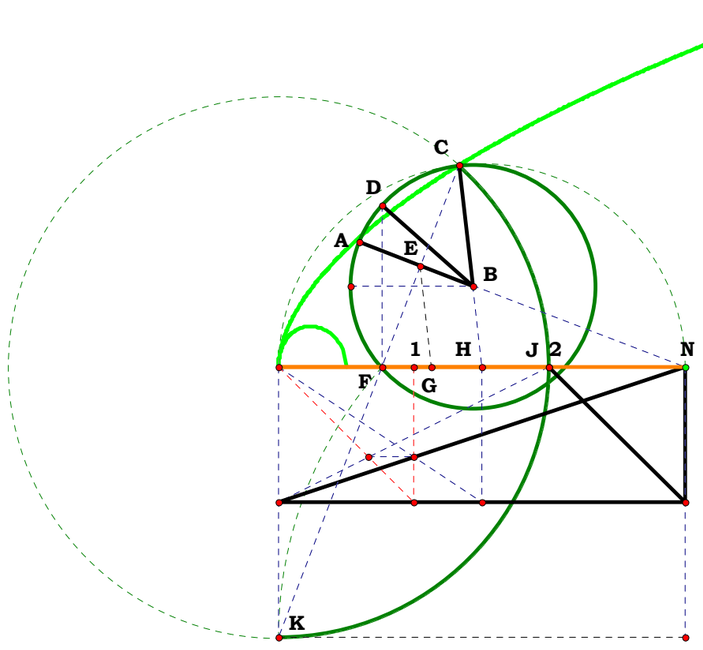
N = 3.00000

BC = 0.89776

CE = 0.79788

$\frac{N \cdot \sqrt{2 \cdot N - 1}}{N + 2 \cdot \sqrt{2 \cdot N - 1}}$ - BC = 0.00000

$\frac{\left(N \cdot \sqrt{2 \cdot N - 1} - 2 \cdot N\right) + 1}{\sqrt{2 \cdot N \cdot \left(N - \sqrt{2 \cdot N - 1}\right)}}$ - CE = 0.00000



m∠ABC = 62.71547°

m∠ABD = 20.90516°

$\frac{m\angle ABC}{m\angle ABD}$ = 3.00000

N = 3.00000

BC = 0.89776

CE = 0.79788

$\frac{N \cdot \sqrt{2 \cdot N - 1}}{N + 2 \cdot \sqrt{2 \cdot N - 1}}$ - BC = 0.00000

$\frac{\left(N \cdot \sqrt{2 \cdot N - 1} - 2 \cdot N\right) + 1}{\sqrt{2 \cdot N \cdot \left(N - \sqrt{2 \cdot N - 1}\right)}}$ - CE = 0.00000

Unit := 1

Given.

N := 5

111193BR3

Descriptions.

$P := N + Unit \qquad F := P - \sqrt{P} \qquad FP := P - F$

$O := \frac{N}{2} \qquad FO := F - O \qquad OP := P - O$

$AJ := \frac{OP}{2} \qquad FK := \sqrt{FO^2 + O^2} \qquad FG := \frac{FO \cdot FP}{FK}$

$GH := \frac{AJ}{2} \qquad FJ := AJ - FO$

$FH := \frac{FJ}{2} \qquad PH := FP - FH \qquad BJ := \frac{GH \cdot AJ}{PH}$

$AB := AJ - BJ \qquad AG := FG$

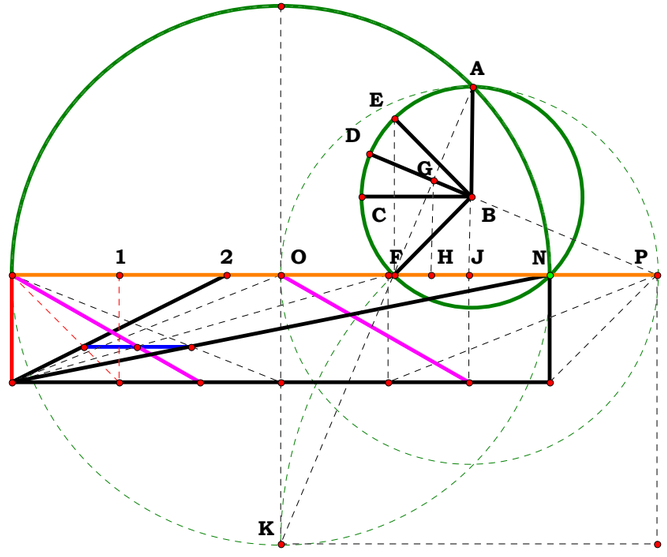
$AB = 1.020745 \qquad AG = 0.948914$

Definitions.

$$AB - \frac{\sqrt{N+1} \cdot (N+2)}{N + 4 \cdot \sqrt{N+1} + 2} = 0$$

$$AG - \frac{2 \cdot \sqrt{2} \cdot \left(\sqrt{N+1}\right)^3 - 2 \cdot \sqrt{2} \cdot N - 2 \cdot \sqrt{2} - \sqrt{2} \cdot N \cdot \sqrt{N+1}}{2 \cdot \sqrt{4 \cdot N + N^2 + 2 \cdot N \cdot \sqrt{N+1} - 4 \cdot \left(\sqrt{N+1}\right)^3 + 4}} = 0$$

$$\frac{AB}{AG} - \frac{\sqrt{2} \cdot \sqrt{N+1} \cdot (N+2) \cdot \sqrt{4 \cdot N + N^2 + 2 \cdot N \cdot \sqrt{N+1} - 4 \cdot \left(\sqrt{N+1}\right)^3 + 4}}{\left(N + 4 \cdot \sqrt{N+1} + 2\right) \cdot \left[2 \cdot \left(\sqrt{N+1}\right)^3 - N \cdot \sqrt{N+1} - 2 \cdot N - 2\right]} = 0$$



N = 5.00000

m∠ABD = 68.37704°

m∠DBE = 22.79235°

m∠ABF = 136.75407°

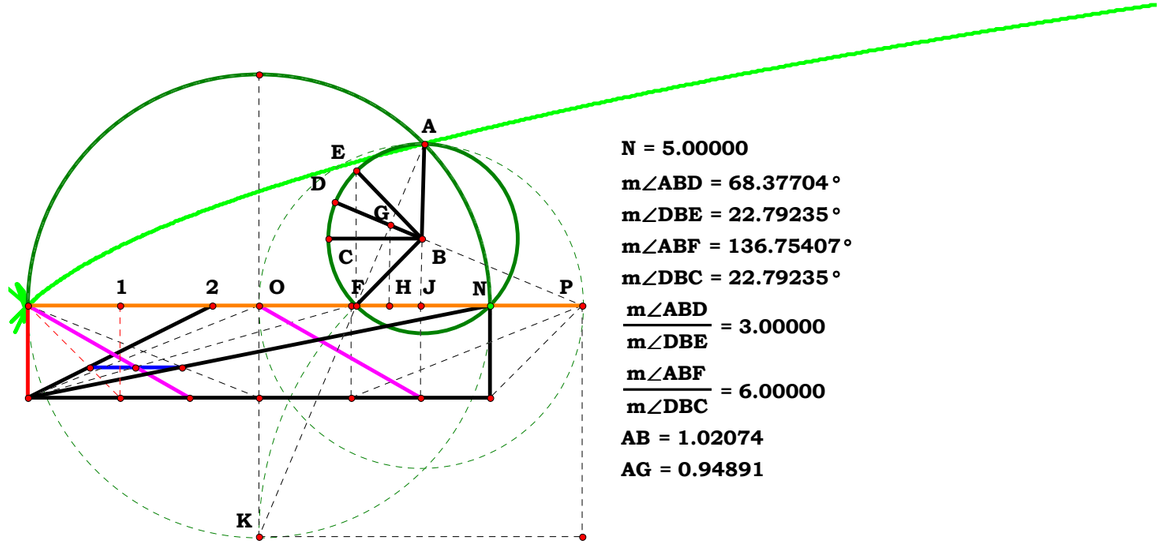
m∠DBC = 22.79235°

$\frac{m\angle ABD}{m\angle DBE} = 3.00000$

$\frac{m\angle ABF}{m\angle DBC} = 6.00000$

AB = 1.02074

AG = 0.94891



N = 5.00000

m∠ABD = 68.37704°

m∠DBE = 22.79235°

m∠ABF = 136.75407°

m∠DBC = 22.79235°

$\frac{m\angle ABD}{m\angle DBE} = 3.00000$

$\frac{m\angle ABF}{m\angle DBC} = 6.00000$

AB = 1.02074

AG = 0.94891

Unit.

$AC := 1$

Given.

$N := 5$

111193BR4

Descriptions.

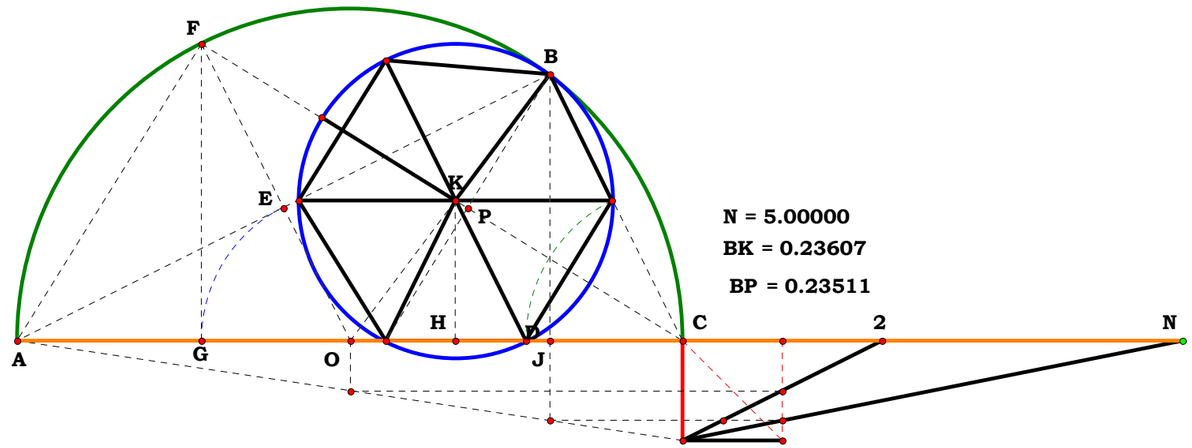
$AO := \frac{AC}{2}$ $CO := AO$ $CD := \frac{AC}{N}$ $AD := AC - CD$ $BD := \sqrt{AD \cdot CD}$

$AB := \sqrt{AD^2 + BD^2}$ $BE := \frac{AB}{2}$ $FG := BE$ $EO := \sqrt{AO^2 - BE^2}$

$AG := AO - EO$ $CG := AC - AG$ $DO := AD - AO$ $GM := \frac{DO \cdot FG}{BD}$

$CM := CG + GM$ $FM := \sqrt{GM^2 + FG^2}$ $KO := \frac{FM \cdot AO}{CM}$

$BK := AO - KO$ $BK = 0.236068$



Definitions.

$BK - \frac{1}{\sqrt{N} + 2} = 0$ $N - \left(\frac{1}{BK} - 2\right)^2 = 0$ $BC := \sqrt{CD^2 + BD^2}$ $CF := \sqrt{FG^2 + CG^2}$

$CP := \frac{CF \cdot BC}{AC}$ $BP := \sqrt{BC^2 - CP^2}$ $KP := \sqrt{BK^2 - BP^2}$

$KP - \frac{\sqrt{2} \cdot \sqrt{N^3 + 4 \cdot N^{\frac{3}{2}} - 3 \cdot N^{\frac{5}{2}}}}{2 \cdot \sqrt{4 \cdot N^3 + N^4 + 4 \cdot N^{\frac{7}{2}}}} = 0$ $KP - \frac{\sqrt{\left(\sqrt{N} + 1\right) \cdot \left(\sqrt{N} - 2\right)^2}}{\sqrt{2 \cdot N^{\frac{3}{2}} \cdot \left(\sqrt{N} + 2\right)^2}} = 0$

$\frac{BK}{KP} = 11.135164$ $\frac{\sqrt{2} \cdot \sqrt{\left[\left(\sqrt{N}\right)^3 \cdot \left(\sqrt{N} + 2\right)^2\right]}}{\left(\sqrt{N} + 2\right) \cdot \sqrt{\left(\sqrt{N} + 1\right) \cdot \left(\sqrt{N} - 2\right)^2}} = 11.135164$ $\frac{BK}{KP} - \frac{\sqrt{2} \cdot \sqrt{\left[\left(\sqrt{N}\right)^3 \cdot \left(\sqrt{N} + 2\right)^2\right]}}{\left(\sqrt{N} + 2\right) \cdot \sqrt{\left(\sqrt{N} + 1\right) \cdot \left(\sqrt{N} - 2\right)^2}} = 0$

Unit.

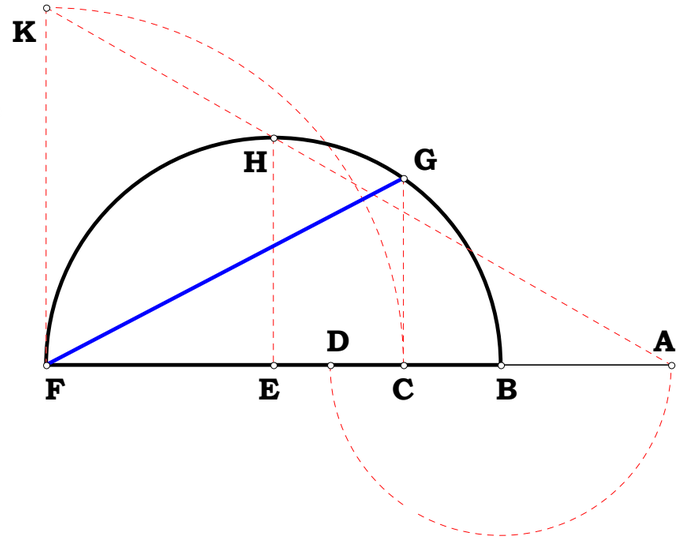
$BF := 1$

To Square A Circle Off The Base Of A Right Triangle.

Using the approximation, π = 22/7, square the circle off the base of a right triangle.

111293

Sometime in 1992, I remembered reading that some man spent some time in prison and learned the process for squaring a circle off the base of a right triangle but then history lost the figure, so I set out to find it - or something that could pass for it. It took a couple hours so I wonder what he did with the rest of his time?



Descriptions.

$BE := \frac{BF}{2}$ $EH := BE$ $BD := \frac{3}{4} \cdot BE$ $AB := BD$

$AE := AB + BE$ $AF := AB + BF$ $FK := \frac{EH \cdot AF}{AE}$ $CF := FK$

$BC := BF - CF$ $CG := \sqrt{BC \cdot CF}$ $FG := \sqrt{CF^2 + CG^2}$

$\pi\_A := \frac{FG^2}{BE^2}$

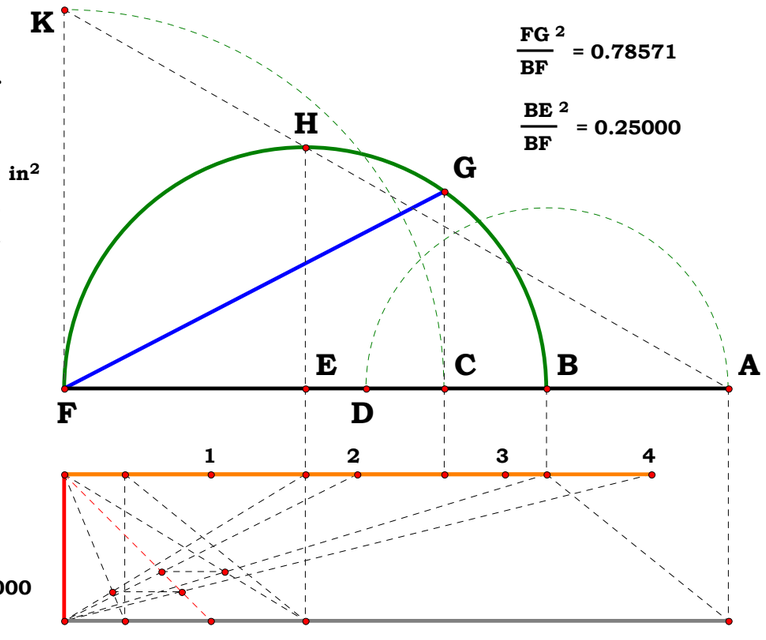
Definitions.

$\pi\_A - \frac{22}{7} = 0$

$\pi = 3.14159265359$

$\pi\_A = 3.142857142857$

$\frac{\pi}{\pi\_A} = 0.999597662505843$



DE = 0.39839 in.

BF = 3.18715 in.

AB = 1.19518 in.

AB·BF = 3.80922 in^2

FG = 2.82511 in.

BE = 1.59357 in.

BF = 3.18715 in.

$\frac{FG^2}{BF} = 0.78571$

$\frac{BE^2}{BF} = 0.25000$

$\frac{22}{\frac{FG^2}{BF}} = 28.00000$

$\frac{7}{\frac{BE^2}{BF}} = 28.00000$

FK = 2.50419 in.

$\left(\frac{FK}{BF}\right) \cdot 28 = 22.00000$

Unit.

$BJ := 1$

Given.

$N_1 := 2$

Exploring Cube Roots Plate A

111893AR

Descriptions. describe AB.

$$BH := \frac{BJ}{2} \qquad BD := \frac{BH}{N_1} \qquad HJ := BH$$

$$DH := BH - BD \qquad HR := BJ \qquad DJ := DH + HJ$$

$$DL := \sqrt{BD \cdot DJ} \qquad DF := \frac{DH \cdot DL}{DL + HR} \qquad FO := BH \qquad BF := BD + DF$$

$$MO := FO - DL \qquad LM := DF \qquad AF := \frac{LM \cdot FO}{MO} \qquad AB := AF - BF$$

Definitions.

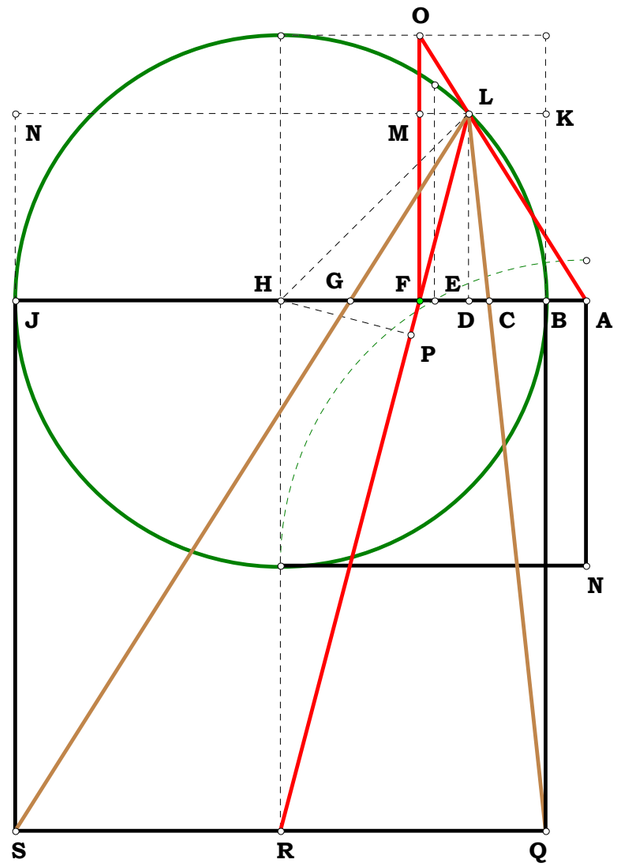
$$BJ - 1 = 0 \qquad BH - \frac{1}{2} = 0 \qquad BD - \frac{1}{2 \cdot N_1} = 0 \qquad HJ - \frac{1}{2} = 0 \qquad DH - \frac{N_1 - 1}{2 \cdot N_1} = 0$$

$$HR - 1 = 0 \qquad DJ - \frac{2 \cdot N_1 - 1}{2 \cdot N_1} = 0 \qquad DL - \frac{\sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$$

$$DF - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right)} = 0 \qquad FO - \frac{1}{2} = 0 \qquad BF - \frac{\sqrt{2 \cdot N_1 - 1} + 2}{2 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right)} = 0$$

$$MO - \frac{N_1 - \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0 \qquad LM - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right)} = 0$$

$$AF - \frac{(N_1 - 1) \cdot \sqrt{2 \cdot N_1 - 1}}{2 \cdot \left(2 \cdot N_1 + \sqrt{2 \cdot N_1 - 1}\right) \cdot \left(N_1 - \sqrt{2 \cdot N_1 - 1}\right)} = 0 \qquad AB - \frac{\sqrt{2 \cdot N_1 - 1} - 1}{2 \cdot \left(2 \cdot {N_1}^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{2 \cdot N_1 - 1} + 1\right)} = 0$$



BH- $\frac{N_1}{2}$ = 0.00000 in.

BD- $\frac{N_1}{2 \cdot N_2}$ = 0.00000 in.

HJ- $\frac{N_1}{2}$ = 0.00000 in.

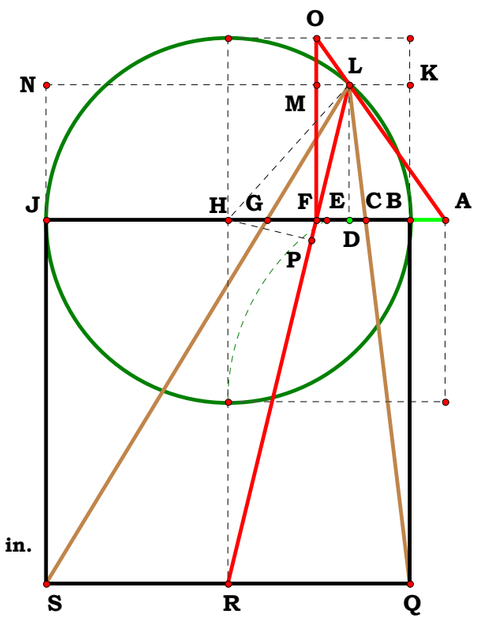
DH- $\frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2}$ = 0.00000 in.

HR- N_1 = 0.00000 in.

DJ- $\frac{N_1 \cdot (2 \cdot N_2 - 1)}{2 \cdot N_2}$ = 0.00000 in.

DL- $\frac{N_1 \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2}$ = 0.00000 in.

DF- $\frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.



FO- $\frac{N_1}{2}$ = 0.00000 in.

BF- $\frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} + 2\right)}{2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.

MO- $\frac{N_1 \cdot \left(N_2 - \sqrt{2 \cdot N_2 - 1}\right)}{2 \cdot N_2}$ = 0.00000 in.

LM- $\frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{2 \cdot N_2 \cdot \left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.

AF- $\frac{N_1 \cdot (N_2 - 1) \cdot \sqrt{2 \cdot N_2 - 1}}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_2 - 1}\right) \cdot \left(2 \cdot N_2 - 2 \cdot \sqrt{2 \cdot N_2 - 1}\right)}$ = 0.00000 in.

AB- $\frac{N_1 \cdot \left(\sqrt{2 \cdot N_2 - 1} - 1\right)}{2 \cdot \left(\left(2 \cdot N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{2 \cdot N_2 - 1}\right) + 1\right)}$ = 0.00000 in.

N_1 = 2.40000 in.

N_2 = 3.01550

BJ = 2.40000 in.

BH = 1.20000 in.

BD = 0.39794 in.

HJ = 1.20000 in.

$\frac{BH}{BD}$ = 3.01550

DH = 0.80206 in.

HR = 2.40000 in.

DJ = 2.00206 in.

DL = 0.89258 in.

DF = 0.21743 in.

FO = 1.20000 in.

BF = 0.61537 in.

MO = 0.30742 in.

LM = 0.21743 in.

AF = 0.84873 in.

AB = 0.23336 in.

Given.

$BJ := 1$

Given.

$N_1 := 3.01550$

111893BR

Descriptions.

Exploring Cube Roots Plate B

Using the parallel FO to project to the point of similarity for the square root, point L is used for the cube root. Notice in this write-up I chose the wrong point to proportion. I get the right answers, but the equations are more complicated. Compare features to plate A.

$BH := \frac{BJ}{2}$ $HL := BH$ $BF := \frac{BH}{N_1}$ $FH := BH - BF$ $HR := BJ$ $FR := \sqrt{FH^2 + HR^2}$

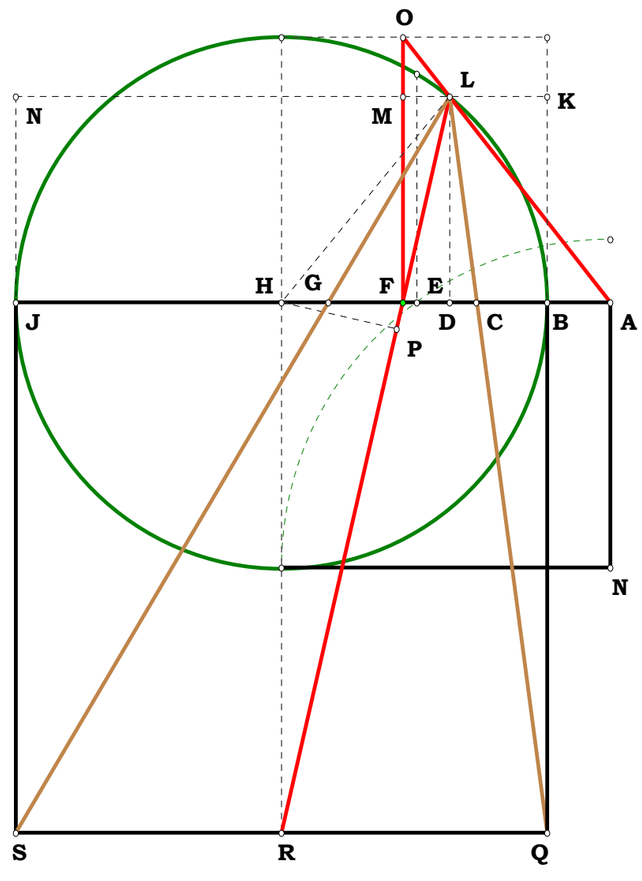
$FP := \frac{FH^2}{FR}$ $PH := \frac{HR \cdot FP}{FH}$ $LP := \sqrt{HL^2 - PH^2}$ $FL := LP - FP$ $DF := \frac{FH \cdot FL}{FR}$

$DL := \frac{HR \cdot FL}{FR}$ $FO := BH$ $FM := DL$ $MO := FO - FM$ $LM := DF$ $AF := \frac{LM \cdot FO}{MO}$

$AB := AF - BF$ $BQ := BJ$ $BK := DL$ $BD := BF - DF$ $KQ := BQ + BK$ $KL := BD$

$BC := \frac{KL \cdot BQ}{KQ}$ $DJ := BJ - BD$ $LN := DJ$ $JS := BJ$ $JN := DL$ $NS := JS + JN$

$GJ := \frac{LN \cdot JS}{NS}$ $BG := BJ - GJ$ $AC := AB + BC$ $AG := AB + BG$ $AJ := AB + BJ$



Definitions.

$\left(AB^2 \cdot AJ\right)^{\frac{1}{3}} - AC = 0$ $\left(AB \cdot AJ^2\right)^{\frac{1}{3}} - AG = 0$

$BJ - 1 = 0$ $BH - \frac{1}{2} = 0$ $HL - \frac{1}{2} = 0$ $BF - \frac{1}{2 \cdot N_1} = 0$ $FH - \frac{N_1 - 1}{2 \cdot N_1} = 0$ $HR - 1 = 0$

$FR - \frac{\sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}}{2 \cdot N_1} = 0$ $FP - \frac{\left(N_1 - 1\right)^2}{2 \cdot N_1 \cdot \sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0$ $PH - \frac{N_1 - 1}{\sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0$

$$LP - \frac{\sqrt{N_1^2 + 6 \cdot N_1 - 3}}{2 \cdot \sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0 \qquad FL - -\frac{N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1}{2 \cdot N_1 \cdot \sqrt{5 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0$$

$$DF - -\frac{(N_1 - 1) \cdot \left(N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1\right)}{2 \cdot N_1 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0 \qquad DL - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} - 1}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0$$

$$FO - \frac{1}{2} = 0 \qquad FM - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} - 1}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0 \qquad MO - \frac{7 \cdot N_1^2 - 6 \cdot N_1 - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$

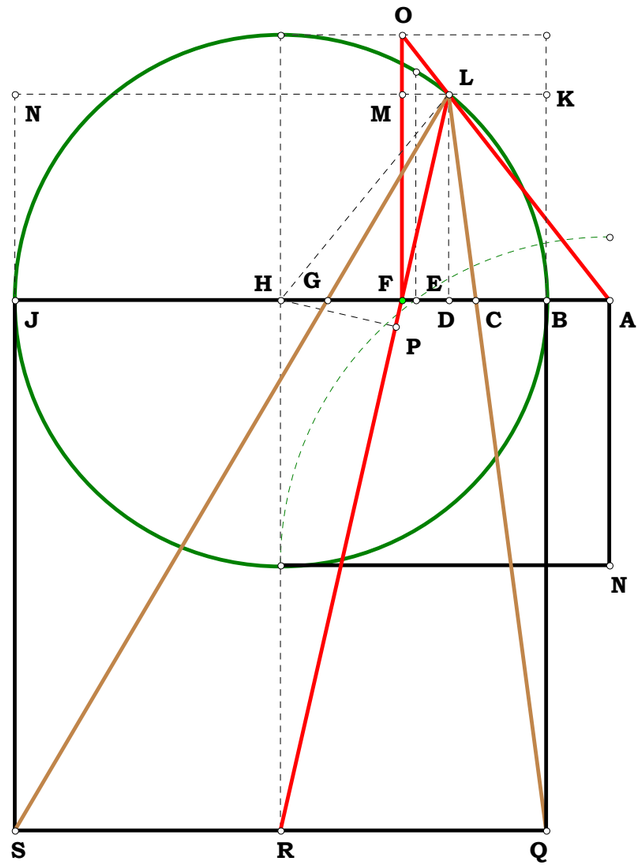
$$LM - -\frac{(N_1 - 1) \cdot \left(N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1\right)}{2 \cdot N_1 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0 \qquad AF - \frac{(1 - N_1) \cdot \left(N_1^2 - 2 \cdot N_1 - N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1\right)}{2 \cdot N_1 \cdot \left(7 \cdot N_1^2 - 6 \cdot N_1 - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3\right)} = 0$$

$$AB - \frac{\left(N_1^2 + N_1\right) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3 \cdot N_1 - 4 \cdot N_1^2 - N_1^3 - 2}{2 \cdot N_1 \cdot \left(7 \cdot N_1^2 - 6 \cdot N_1 - 2 \cdot N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 3\right)} = 0 \qquad BQ - 1 = 0$$

$$BK - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} - 1}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0 \qquad BD - \frac{(1 - N_1) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + N_1^2 + 2 \cdot N_1 + 1}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$

$$KQ - \frac{N_1 \cdot \left(4 \cdot N_1 + \sqrt{N_1^2 + 6 \cdot N_1 - 3}\right)}{5 \cdot N_1^2 - 2 \cdot N_1 + 1} = 0 \qquad KL - \frac{2 \cdot N_1 + N_1^2 + (1 - N_1) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 1}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$

$$BC - \frac{2 \cdot N_1 + N_1^2 - \sqrt{N_1^2 + 6 \cdot N_1 - 3} \cdot (N_1 - 1) + 1}{2 \cdot N_1 \cdot \left(4 \cdot N_1 + \sqrt{N_1^2 + 6 \cdot N_1 - 3}\right)} = 0 \qquad DJ - \frac{(N_1 - 1) \cdot \sqrt{N_1^2 + 6 \cdot N_1 - 3} + 9 \cdot N_1^2 - 6 \cdot N_1 + 1}{2 \cdot \left(5 \cdot N_1^2 - 2 \cdot N_1 + 1\right)} = 0$$



$$LN - \frac{(N_1 - 1)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} + 9\cdot N_1^2 - 6\cdot N_1 + 1}{2\cdot(5\cdot N_1^2 - 2\cdot N_1 + 1)} = 0 \qquad JS - 1 = 0$$

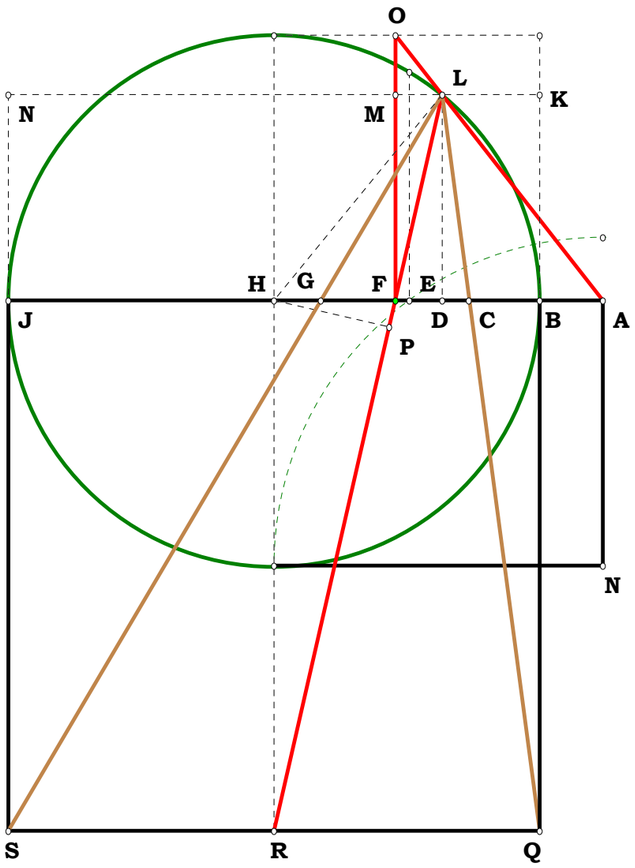
$$JN - \frac{2\cdot N_1 - N_1^2 + N_1\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} - 1}{5\cdot N_1^2 - 2\cdot N_1 + 1} = 0 \qquad NS - \frac{N_1\cdot(4\cdot N_1 + \sqrt{N_1^2 + 6\cdot N_1 - 3})}{5\cdot N_1^2 - 2\cdot N_1 + 1} = 0$$

$$GJ - \frac{(N_1 - 1)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} + 9\cdot N_1^2 - 6\cdot N_1 + 1}{2\cdot N_1\cdot(4\cdot N_1 + \sqrt{N_1^2 + 6\cdot N_1 - 3})} = 0 \qquad BG - \frac{3 - N_1 + \sqrt{N_1^2 + 6\cdot N_1 - 3}}{6\cdot N_1} = 0$$

$$AC - \frac{(9\cdot N_1^2 - 6\cdot N_1^3 - 8\cdot N_1 + 1)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3} + (N_1 + 1)\cdot(6\cdot N_1^3 + 3\cdot N_1^2 - 8\cdot N_1 + 3)}{2\cdot N_1\cdot(26\cdot N_1^3 - 36\cdot N_1^2 + 18\cdot N_1) - 2\cdot N_1\cdot(N_1^2 + 6\cdot N_1 - 3)\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3}} = 0$$

$$AG - \frac{N_1^2 - 4\cdot N_1^3 + 1 + \sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot(4\cdot 1\cdot N_1^2 - 3\cdot 1\cdot N_1 + 1) - 2\cdot N_1}{6\cdot N_1 - 12\cdot N_1^2 + 14\cdot N_1^3 - 4\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot N_1^2} = 0$$

$$AJ - \frac{13\cdot N_1^3 - 16\cdot N_1^2 - 2 + \sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot(1\cdot N_1 - 3\cdot 1\cdot N_1^2) + 9\cdot N_1}{6\cdot N_1 - 12\cdot N_1^2 + 14\cdot N_1^3 - 4\cdot\sqrt{N_1^2 + 6\cdot N_1 - 3}\cdot N_1^2} = 0$$



Etc.

BF- $\frac{N_1}{2 \cdot N_2}$ = 0.00000 in.

FH- $\frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2}$ = 0.00000 in.

FR- $\frac{N_1 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}{2 \cdot N_2}$ = 0.00000 in.

FP- $\frac{N_1 \cdot (N_2 - 1)^2}{2 \cdot N_2 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

PH- $\frac{N_1 \cdot (N_2 - 1)}{\sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

LP- $\frac{N_1 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}}{2 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

FL- $\frac{N_1 \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{2 \cdot N_2 \cdot \sqrt{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}}$ = 0.00000 in.

DF- $\frac{N_1 \cdot (N_2 - 1) \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{2 \cdot N_2 \cdot ((5 \cdot N_2^2 - 2 \cdot N_2) + 1)}$ = 0.00000 in.

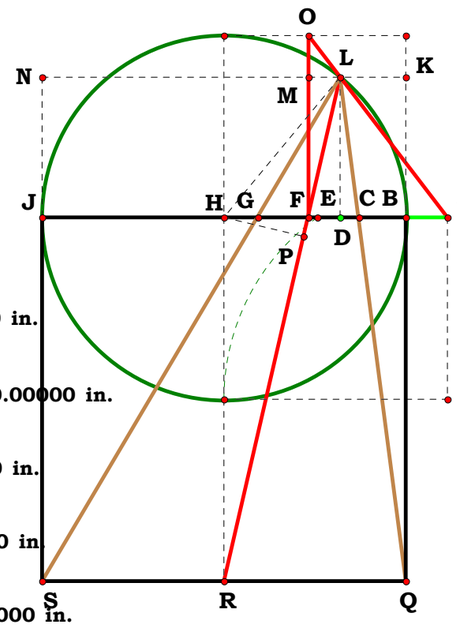
DL- $\frac{N_1 \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}$ = 0.00000 in.

FM- $\frac{N_1 \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{(5 \cdot N_2^2 - 2 \cdot N_2) + 1}$ = 0.00000 in.

MO- $\frac{N_1 \cdot ((7 \cdot N_2^2 - 6 \cdot N_2 - 2 \cdot N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) + 3)}{2 \cdot ((5 \cdot N_2^2 - 2 \cdot N_2) + 1)}$ = 0.00000 in.

LM- $\frac{N_1 \cdot (N_2 - 1) \cdot (((2 \cdot N_2 - N_2^2) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 1)}{2 \cdot N_2 \cdot ((5 \cdot N_2^2 - 2 \cdot N_2) + 1)}$ = 0.00000 in.

AF- $\frac{N_1 \cdot (N_2 - 1) \cdot ((N_2^2 - 2 \cdot N_2 - N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) + 1)}{2 \cdot N_2 \cdot (((6 \cdot N_2 - 7 \cdot N_2^2) + 2 \cdot N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 3)}$ = 0.00000 in.



AB- $\frac{N_1 \cdot ((((3 \cdot N_2 + N_2^2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 4 \cdot N_2^2 - N_2^3) + N_2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3}) - 2)}{2 \cdot ((3 \cdot N_2 - 2 \cdot N_2^2 \cdot \sqrt{(N_2^2 + 6 \cdot N_2) - 3} - 6 \cdot N_2^2) + 7 \cdot N_2^3)}$ = 0.00000 in

N_1 = 2.40000 in.

N_2 = 1.86107

Animate Point

BJ = 2.40000 in.

BH = 1.20000 in.

BF = 0.64479 in.

$\frac{BH}{BF}$ = 1.86107

FH = 0.55521 in.

FR = 2.46338 in.

FP = 0.12514 in.

PH = 0.54092 in.

LP = 1.07117 in.

FL = 0.94603 in.

DF = 0.21322 in.

DL = 0.92169 in.

FM = 0.92169 in.

MO = 0.27831 in.

LM = 0.21322 in.

AF = 0.91936 in.

AB = 0.27457 in.

Unit.

$BK := 1$

Given.

$N_1 := 4$

111893CR

Descriptions.

$BH := \frac{BK}{2}$ $BD := \frac{BH}{N_1}$ $DK := BK - BD$

$DN := \sqrt{BD \cdot DK}$ $BQ := BK$ $KS := BK$ $HR := BK$

$BC := \frac{BD \cdot BQ}{BQ + DN}$ $GK := \frac{DK \cdot KS}{KS + DN}$ $BG := BK - GK$

$DH := BH - BD$ $FH := \frac{DH \cdot HR}{HR + DN}$ $BF := BH - FH$

$CF := BF - BC$ $AL := CF$ $DF := BF - BD$

$NO := DF$ $FP := BH$ $PO := FP - DN$

$AD := \frac{NO \cdot DN}{PO}$ $AB := AD - BD$

$AF := AB + BF$ $LM := AF$ $EL := AF$ $AK := AD + DK$

$AE_1 := \sqrt{EL^2 - AL^2}$ $AE_2 := \sqrt{AB \cdot AK}$ $AE_1 - AE_2 = 0$

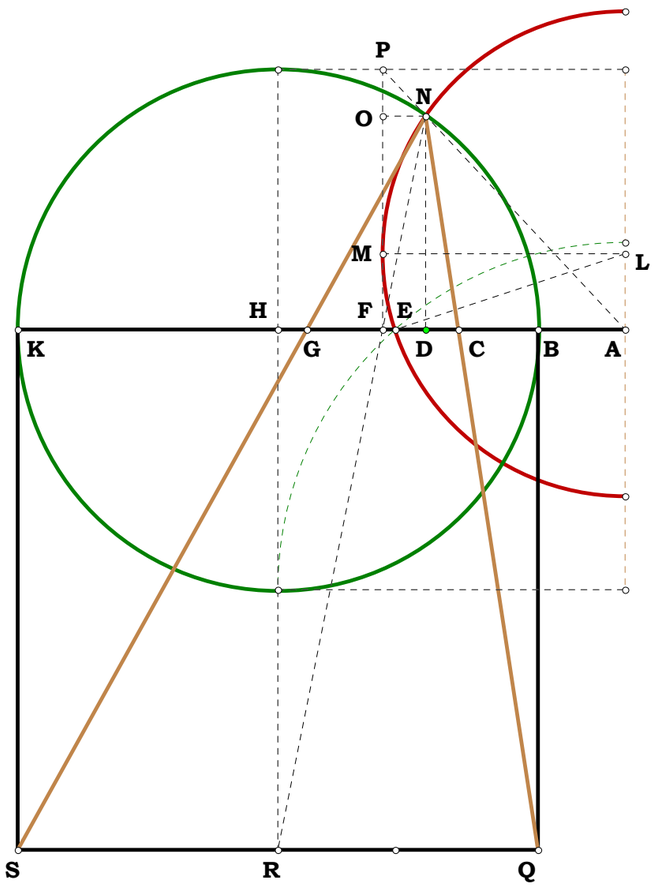
Definitions.

$BH - \frac{1}{2} = 0$ $BD - \frac{1}{2 \cdot N_1} = 0$ $DK - \frac{(2 \cdot N_1 - 1)}{2 \cdot N_1} = 0$

$DN - \frac{\sqrt{2 \cdot N_1 - 1}}{2 \cdot N_1} = 0$ $BQ - 1 = 0$ $KS - 1 = 0$

Exploring Cube Roots Plate C

If AL = 1/2 of CG, then the circle LM passes through the square root of AB x AK, being point E.

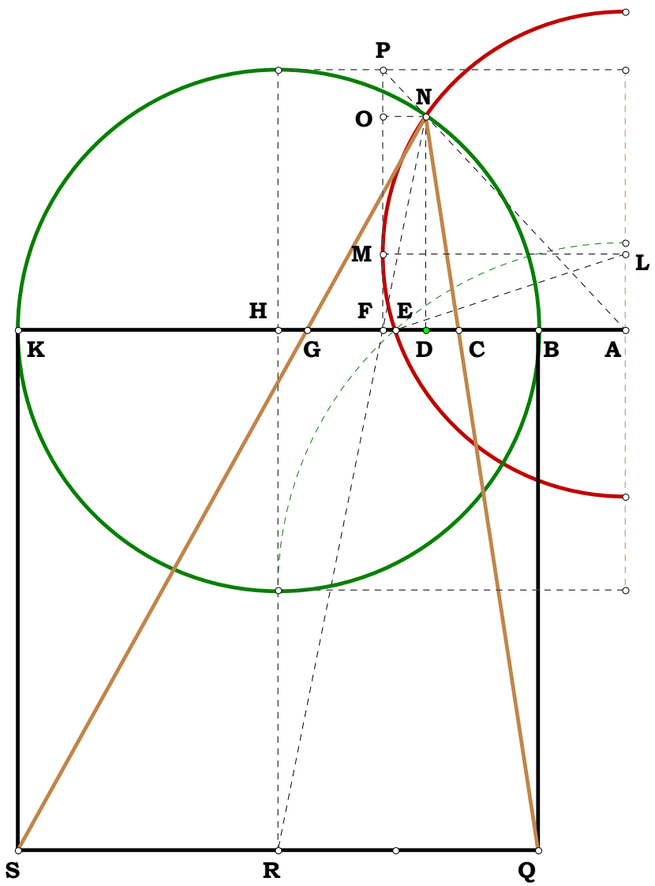


$$HR - 1 = 0 \qquad BC - \frac{1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0 \qquad GK - \frac{2\cdot N_1 - 1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0$$

$$BG - \frac{\sqrt{2\cdot N_1 - 1} + 1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0 \qquad DH - \frac{N_1 - 1}{2\cdot N_1} = 0 \qquad FH - \frac{N_1 - 1}{2\cdot N_1 + \sqrt{2\cdot N_1 - 1}} = 0$$

$$BF - \frac{\sqrt{2\cdot N_1 - 1} + 2}{2\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad CF - \frac{\sqrt{2\cdot N_1 - 1}}{2\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad AL - \frac{\sqrt{2\cdot N_1 - 1}}{2\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0$$

$$DF - \frac{\left(N_1 - 1\right)\cdot\sqrt{2\cdot N_1 - 1}}{2\cdot N_1\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad NO - \frac{\left(N_1 - 1\right)\cdot\sqrt{2\cdot N_1 - 1}}{2\cdot N_1\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0$$



$$FP - \frac{1}{2} = 0 \qquad PO - \frac{N_1 - \sqrt{2\cdot N_1 - 1}}{2\cdot N_1} = 0 \qquad AD - \frac{\left(N_1 - 1\right)\cdot\left(\sqrt{2\cdot N_1 - 1}\right)^2}{2\cdot N_1\cdot\left(N_1 - \sqrt{2\cdot N_1 - 1}\right)\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)} = 0$$

$$AB - \frac{\left(\sqrt{2\cdot N_1 - 1} - 1\right)}{2\cdot\left(2\cdot {N_1}^2 - 2\cdot N_1 - N_1\cdot\sqrt{2\cdot N_1 - 1} + 1\right)} = 0 \qquad AF - \frac{\left(N_1 - 1\right)\cdot\left(2\cdot N_1 + 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1} - 1\right)}{2\cdot\left(3\cdot N_1 + \sqrt{2\cdot N_1 - 1} - 6\cdot {N_1}^2 + 4\cdot {N_1}^3 - 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1}\right)} = 0 \qquad EL - AF = 0 \qquad LM - AF = 0$$

$$AK - \frac{\left[4\cdot {N_1}^2 - \left(2\cdot N_1 - 1\right)^{\frac{3}{2}} - 4\cdot N_1 + 1\right]}{2\cdot\left(2\cdot {N_1}^2 - 2\cdot N_1 - N_1\cdot\sqrt{2\cdot N_1 - 1} + 1\right)} = 0 \qquad AE_1 - \sqrt{\left[\frac{\left(N_1 - 1\right)^2\cdot\left(2\cdot N_1 + 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1} - 1\right)^2}{4\cdot\left(3\cdot N_1 + \sqrt{2\cdot N_1 - 1} - 6\cdot {N_1}^2 + 4\cdot {N_1}^3 - 2\cdot N_1\cdot\sqrt{2\cdot N_1 - 1}\right)^2} - \frac{\left(2\cdot N_1 - 1\right)}{4\cdot\left(2\cdot N_1 + \sqrt{2\cdot N_1 - 1}\right)^2}\right]} = 0$$

$$AE_2 - \sqrt{\left[\frac{\left(\sqrt{2\cdot N_1 - 1} - 1\right)\cdot\left[4\cdot {N_1}^2 - \left(2\cdot N_1 - 1\right)^{\frac{3}{2}} - 4\cdot N_1 + 1\right]}{4\cdot\left(2\cdot N_1 - 2\cdot {N_1}^2 + N_1\cdot\sqrt{2\cdot N_1 - 1} - 1\right)^2}\right]} = 0 \qquad AE_1 - AE_2 = 0$$

111893DR

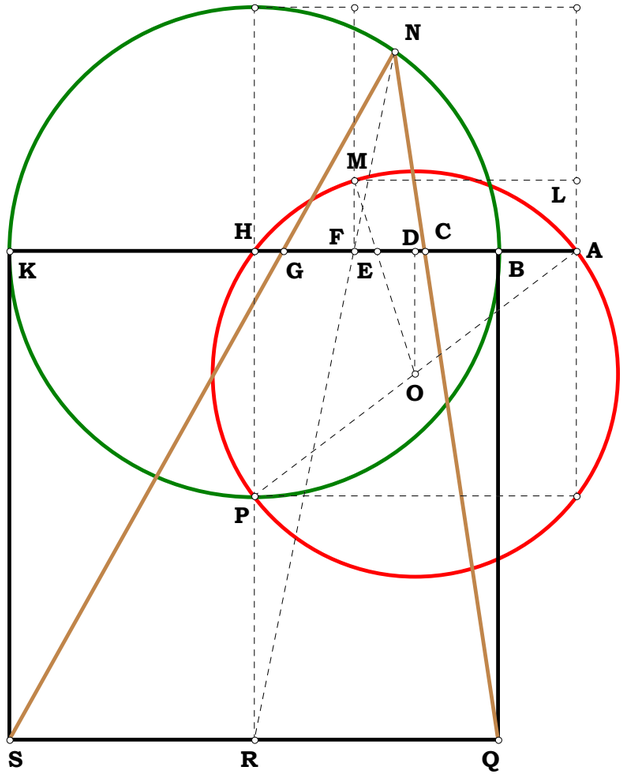
$AB := 1$

Given.

$N_1 := 3$

Exploring Cube Roots Plate D

The circle AO passes through point M. FM equals half of CG.



Descriptions.

$BK := N_1 \quad AK := BK + AB$

$AC := \left(AB^2 \cdot AK\right)^{\frac{1}{3}} \quad AG := \left(AB \cdot AK^2\right)^{\frac{1}{3}}$

$CG := AG - AC \quad CF := \frac{CG}{2} \quad BH := \frac{BK}{2}$

$AH := AB + BH \quad HP := BH \quad AP := \sqrt{AH^2 + HP^2}$

$AO := \frac{AP}{2} \quad DO := \frac{HP}{2} \quad AF := AC + CF \quad AD := \frac{AH}{2}$

$DF := AF - AD \quad FM := CF \quad MO := AO$

Definitions.

$MO^2 - \left[DF^2 + (DO + FM)^2\right] = 0 \quad AK - \left(1 + N_1\right) = 0$

$AC - \left(N_1 + 1\right)^{\frac{1}{3}} = 0 \quad AG - \left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} = 0 \quad CG - \left[\left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - \left(N_1 + 1\right)^{\frac{1}{3}}\right] = 0$

$CF - \frac{\left[\left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - \left(N_1 + 1\right)^{\frac{1}{3}}}{2} = 0 \quad BH - \frac{N_1}{2} = 0 \quad AH - \frac{2 + N_1}{2} = 0 \quad HP - \frac{N_1}{2} = 0 \quad AP - \frac{\sqrt{N_1^2 + 2 \cdot N_1 + 2}}{\sqrt{2}} = 0$

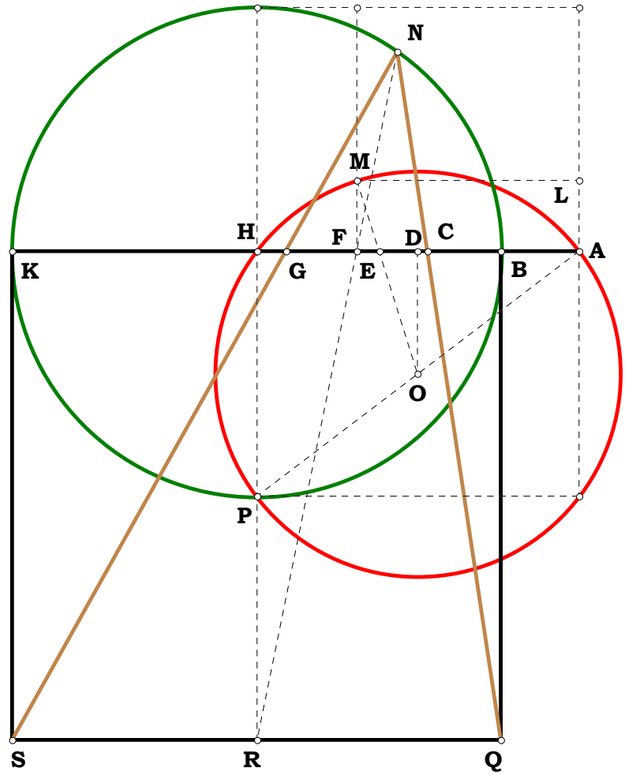
$$AO - \frac{\sqrt{2}}{4}\cdot\sqrt{\left(N_1{}^2 + 2\cdot N_1 + 2\right)} = 0 \qquad DO - \frac{N_1}{4} = 0 \qquad AF - \frac{\left[\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} + \left(N_1+1\right)^{\frac{1}{3}}\right]}{2} = 0$$

$$AD - \frac{2+N_1}{4} = 0 \qquad DF - \frac{\left[2\cdot\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} - N_1 - 2 + 2\cdot\left(N_1+1\right)^{\frac{1}{3}}\right]}{4} = 0$$

$$FM - \frac{\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}} - \left(N_1+1\right)^{\frac{1}{3}}}{2} = 0 \qquad MO - \frac{\sqrt{2}}{4}\cdot\sqrt{2\cdot 1^2 + 2\cdot 1\cdot N_1 + N_1{}^2} = 0$$

$$MO^2 - \frac{\left[\begin{array}{l} 2\cdot N_1 + N_1{}^2 - 4\cdot N_1\cdot\left(N_1+1\right)^{\frac{1}{3}} - 4\cdot\left(N_1+1\right)^{\frac{1}{3}} \ \ldots \\ +\ 4\cdot\left(N_1+1\right)^{\frac{2}{3}} - 4\cdot\left(N_1{}^2 + 2\cdot N_1 + 1\right)^{\frac{1}{3}} + 4\cdot\left(N_1{}^2 + 2\cdot N_1 + 1\right)^{\frac{2}{3}} + 2 \end{array}\right]}{8} = 0$$

$$\frac{\left[\left(N_1+1\right)^2\right]^{\frac{2}{3}}}{2} + \frac{\left(N_1+1\right)^{\frac{2}{3}}}{2} - \frac{\left[\left(N_1+1\right)^2\right]^{\frac{1}{3}}}{2} - \frac{\left(N_1+1\right)^{\frac{1}{3}}}{2} - \frac{N_1\cdot\left(N_1+1\right)^{\frac{1}{3}}}{2} = 0$$



112493R

POR Series IV

Unit.

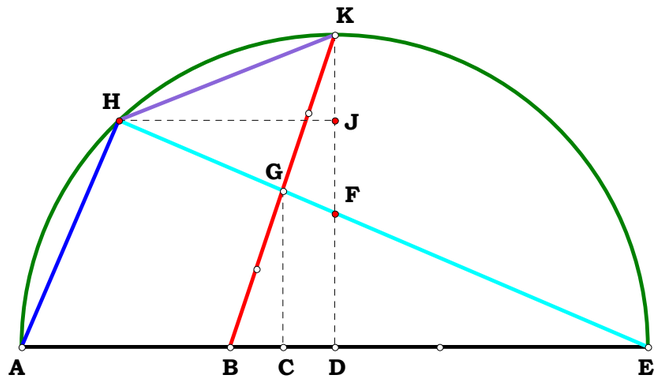
$AE := 1$

Given.

$N_1 := 3$ $\alpha := 1 .. N_1 - 1$

$N_2 := 5$ $\beta := 1 .. N_2 - 1$

Generalize the work of 07/25/93 for dividing the base AE with K constant.



Descriptions.

$AB := \frac{AE}{N_1}$ $AD := \frac{AE}{2}$ $DK := AD$ $DE := AD$

$BD := AD - AB$ $BK := \sqrt{BD^2 + DK^2}$ $BG := \frac{BK}{N_2}$ $BC := \frac{BD \cdot BG}{BK}$

$CG := \frac{DK \cdot BG}{BK}$ $AC := AB + BC$ $CE := AE - AC$ $DF := \frac{CG \cdot DE}{CE}$ $EG := \sqrt{CE^2 + CG^2}$

$EF := \sqrt{DE^2 + DF^2}$ $AH := \frac{DF \cdot AE}{EF}$ $EH := \frac{DE \cdot AE}{EF}$ $GH := EH - EG$ $FH := EH - EF$

$FJ := \frac{DF \cdot FH}{EF}$ $HJ := \frac{DE \cdot FH}{EF}$ $DJ := DF + FJ$ $JK := DK - DJ$ $HK := \sqrt{HJ^2 + JK^2}$

Definitions.

$\frac{AH}{HK} = 0.265165$ $\frac{\sqrt{2} \cdot N_1}{2 \cdot (N_1 - 1) \cdot (N_2 - 1)} = 0.265165$

$\frac{EH}{GH} = 2.85$ $N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 - N_1 + 2}{(N_2 - 1) \cdot (2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 + N_1^2 - 2 \cdot N_1 + 2)} = 2.85$

$SeriesAH_{\alpha, \beta} := \frac{\sqrt{2} \cdot N_1 \cdot \beta}{2 \cdot (N_1 - \alpha) \cdot (N_2 - \beta)}$

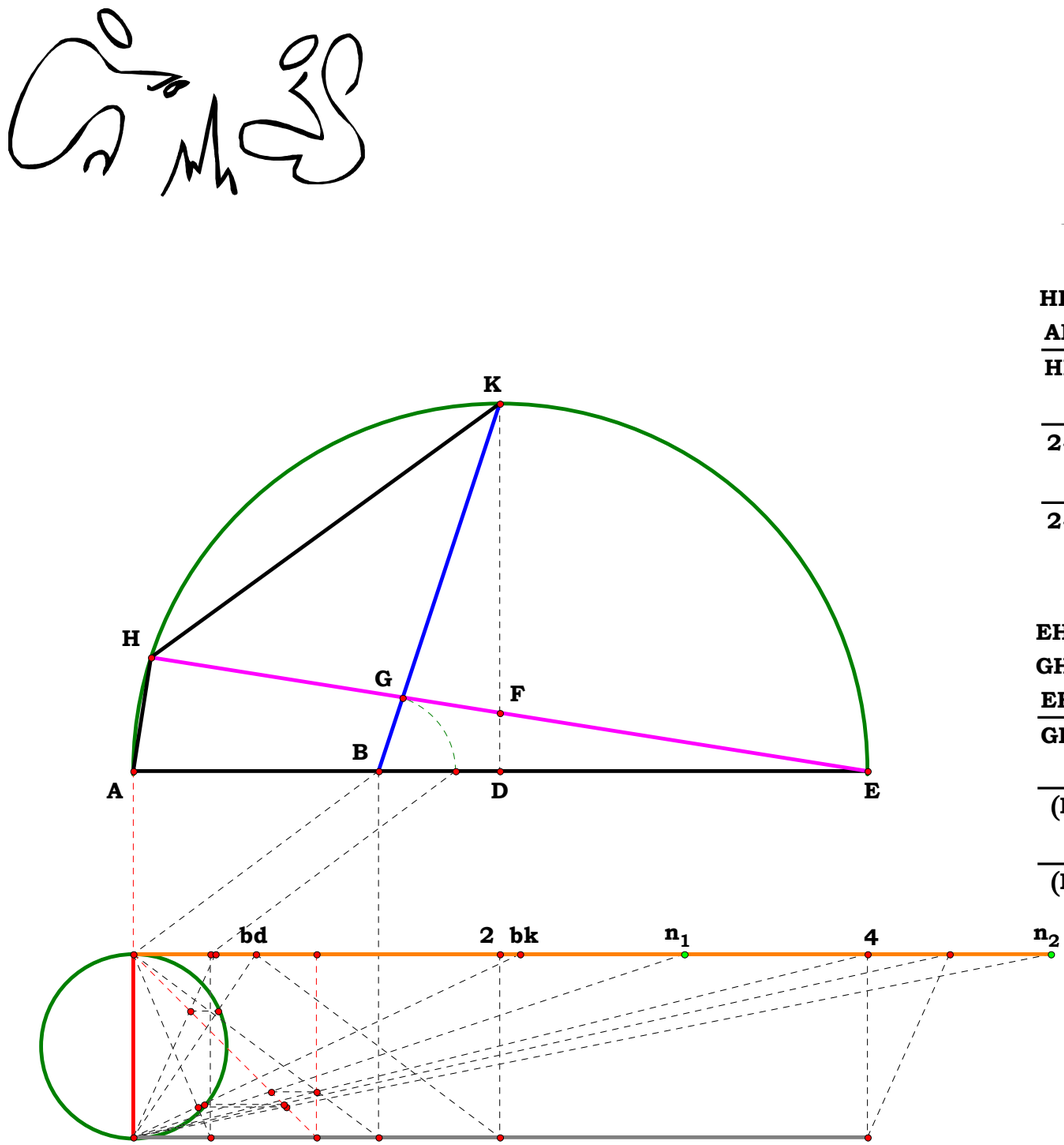
$SeriesEH_{\alpha, \beta} := N_1 \cdot N_2 \cdot \frac{2 \cdot N_1 \cdot N_2 - 2 \cdot N_2 \cdot \alpha - N_1 \cdot \beta + 2 \cdot \alpha \cdot \beta}{(N_2 - \beta) \cdot (2 \cdot N_1 \cdot N_2 \cdot \alpha - 2 \cdot N_2 \cdot \alpha^2 + N_1^2 \cdot \beta - 2 \cdot N_1 \cdot \alpha \cdot \beta + 2 \cdot \alpha^2 \cdot \beta)}$

$SeriesAH =$

	1	2	3	4
1	0.265165	0.707107	1.59099	4.242641
2	0.53033	1.414214	3.181981	8.485281

$SeriesEH =$

	1	2	3	4
1	2.85	3	3.643	6
2	1.65	2	2.786	5.25



$$HK = 2.35269$$

$$\frac{AH}{HK} = 0.26517$$

$$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} = 0.26517$$

$$\frac{\sqrt{2}\cdot N_1}{2\cdot(N_1-1)\cdot(N_2-1)} - \frac{AH}{HK} = 0.00000$$

$$EH = 3.95105$$

$$GH = 1.38633$$

$$\frac{EH}{GH} = 2.85000$$

$$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} = 2.85000$$

$$\frac{N_1\cdot N_2\cdot((2\cdot N_1\cdot N_2-2\cdot N_2-N_1)+2)}{(N_2-1)\cdot((((2\cdot N_1\cdot N_2-2\cdot N_2)+N_1^2)-2\cdot N_1)+2)} - \frac{EH}{GH} = 0.00000$$

Unit.

$AH := 1$

Given.

$\delta := 1 .. 7$

$\Delta := 1 .. 6$

120293R

Descriptions.

$AP_\delta := \frac{AH}{\delta}$ $\quad AG_\delta := \frac{(AP_\delta)^2}{AH}$ $\quad AO_\delta := AG_\delta$

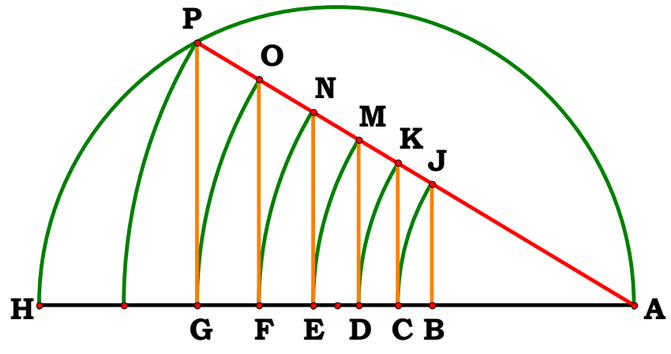
$AF_\delta := \frac{(AG_\delta)^2}{AP_\delta}$ $\quad AE_\delta := \frac{(AF_\delta)^2}{AO_\delta}$ $\quad AN_\delta := AF_\delta$

$AD_\delta := \frac{(AE_\delta)^2}{AN_\delta}$ $\quad AM_\delta := AE_\delta$

$AC_\delta := \frac{(AD_\delta)^2}{AM_\delta}$ $\quad AK_\delta := AD_\delta$ $\quad AB_\delta := \frac{(AC_\delta)^2}{AK_\delta}$

POR Roots and Powers (Pyramid of Ratio Series V)

Is the progression noticed in 112993 a continuous phenomenon?



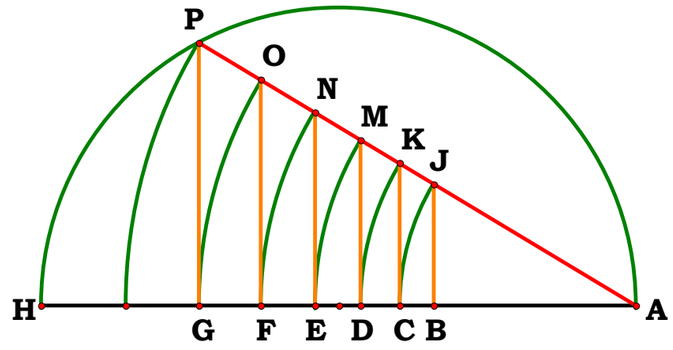
Definitions.

$\frac{AH}{AP_\Delta} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} =$

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6

$$\left(\frac{AP_\Delta}{AB_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AG_\Delta}{AB_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AF_\Delta}{AB_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AE_\Delta}{AB_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AD_\Delta}{AB_\Delta}\right)^{\frac{1}{2}} = \frac{AC_\Delta}{AB_\Delta} =$$

1	1	1	1	1	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6



$$\left(\begin{array}{ccccc}
\frac{AH}{1}, & \frac{AH}{1^2}, & \frac{AH}{1^3}, & \frac{AH}{1^4}, & \text{etc} \\
\frac{AH}{2}, & \frac{AH}{2^2}, & \frac{AH}{2^3}, & \frac{AH}{2^4}, & \text{etc} \\
\frac{AH}{3}, & \frac{AH}{3^2}, & \frac{AH}{3^3}, & \frac{AH}{3^4}, & \text{etc} \\
\frac{AH}{4}, & \frac{AH}{4^2}, & \frac{AH}{4^3}, & \frac{AH}{4^4}, & \text{etc} \\
\frac{AH}{5}, & \frac{AH}{5^2}, & \frac{AH}{5^3}, & \frac{AH}{5^4}, & \text{etc}
\end{array}\right)$$

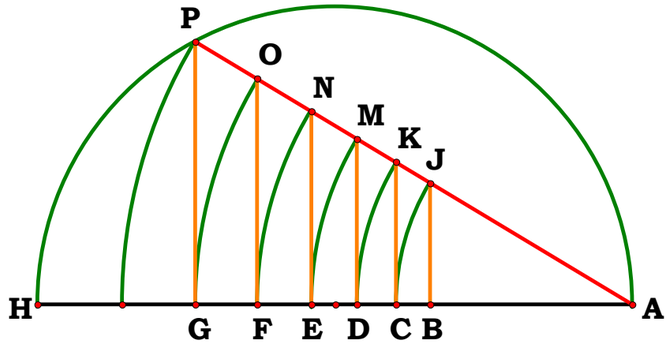
$AP_\Delta =$	$AG_\Delta =$	$AF_\Delta =$	$AE_\Delta =$	$AD_\Delta =$	$AC_\Delta =$	$AB_\Delta =$
1	1	1	1	1	1	1
0.5	0.25	0.125	0.0625	0.03125	0.015625	$7.8125 \cdot 10^{-3}$
0.333333	0.111111	0.037037	0.012346	$4.115226 \cdot 10^{-3}$	$1.371742 \cdot 10^{-3}$	$4.572474 \cdot 10^{-4}$
0.25	0.0625	0.015625	$3.90625 \cdot 10^{-3}$	$9.765625 \cdot 10^{-4}$	$2.441406 \cdot 10^{-4}$	$6.103516 \cdot 10^{-5}$
0.2	0.04	$8 \cdot 10^{-3}$	$1.6 \cdot 10^{-3}$	$3.2 \cdot 10^{-4}$	$6.4 \cdot 10^{-5}$	$1.28 \cdot 10^{-5}$
0.166667	0.027778	$4.62963 \cdot 10^{-3}$	$7.716049 \cdot 10^{-4}$	$1.286008 \cdot 10^{-4}$	$2.143347 \cdot 10^{-5}$	$3.572245 \cdot 10^{-6}$

$$AP_\Delta = \quad AG_\Delta \cdot \frac{AH}{AP_\Delta} = \quad AF_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^2 = \quad AE_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^3 = \quad AD_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^4 = \quad AC_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^5 = \quad AB_\Delta \cdot \left(\frac{AH}{AP_\Delta}\right)^6 =$$

1	1	1	1	1	1	1
0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.333333	0.333333	0.333333	0.333333	0.333333	0.333333	0.333333
0.25	0.25	0.25	0.25	0.25	0.25	0.25
0.2	0.2	0.2	0.2	0.2	0.2	0.2
0.166667	0.166667	0.166667	0.166667	0.166667	0.166667	0.166667

$$\frac{AH^2}{AP_\Delta} = \left(\frac{AH^3}{AG_\Delta}\right)^{\frac{1}{2}} = \left(\frac{AH^4}{AF_\Delta}\right)^{\frac{1}{3}} = \left(\frac{AH^5}{AE_\Delta}\right)^{\frac{1}{4}} = \left(\frac{AH^6}{AD_\Delta}\right)^{\frac{1}{5}} = \left(\frac{AH^7}{AC_\Delta}\right)^{\frac{1}{6}} = \left(\frac{AH^8}{AB_\Delta}\right)^{\frac{1}{7}} = \left(\frac{AH}{AB_\Delta}\right)^{\frac{1}{7}} \cdot AH =$$

1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6



$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{3}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{3}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{3}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{3}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{3}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{3}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{3} =$$

1	1	1	1	1	1	1
8	8	8	8	8	8	8
27	27	27	27	27	27	27
64	64	64	64	64	64	64
125	125	125	125	125	125	125
216	216	216	216	216	216	216

$$\left(\frac{AH}{AB_\Delta}\right)^{\frac{7}{7}} = \left(\frac{AH}{AC_\Delta}\right)^{\frac{7}{6}} = \left(\frac{AH}{AD_\Delta}\right)^{\frac{7}{5}} = \left(\frac{AH}{AE_\Delta}\right)^{\frac{7}{4}} = \left(\frac{AH}{AF_\Delta}\right)^{\frac{7}{3}} = \left(\frac{AH}{AG_\Delta}\right)^{\frac{7}{2}} = \left(\frac{AH}{AP_\Delta}\right)^{\frac{7}{1}} =$$

1	1	1	1	1	1	1
128	128	128	128	128	128	128
$2.187 \cdot 10^3$	$2.187 \cdot 10^3$	$2.187 \cdot 10^3$	$2.187 \cdot 10^3$	$2.187 \cdot 10^3$	$2.187 \cdot 10^3$	$2.187 \cdot 10^3$
$1.6384 \cdot 10^4$	$1.6384 \cdot 10^4$	$1.6384 \cdot 10^4$	$1.6384 \cdot 10^4$	$1.6384 \cdot 10^4$	$1.6384 \cdot 10^4$	$1.6384 \cdot 10^4$
$7.8125 \cdot 10^4$	$7.8125 \cdot 10^4$	$7.8125 \cdot 10^4$	$7.8125 \cdot 10^4$	$7.8125 \cdot 10^4$	$7.8125 \cdot 10^4$	$7.8125 \cdot 10^4$
$2.79936 \cdot 10^5$	$2.79936 \cdot 10^5$	$2.79936 \cdot 10^5$	$2.79936 \cdot 10^5$	$2.79936 \cdot 10^5$	$2.79936 \cdot 10^5$	$2.79936 \cdot 10^5$

$$GH_\delta := AH - AG_\delta \qquad GP_\delta := \sqrt{AG_\delta \cdot GH_\delta} \qquad FO_\delta := \frac{GP_\delta \cdot AF_\delta}{AG_\delta} \qquad EN_\delta := \frac{GP_\delta \cdot AE_\delta}{AG_\delta}$$

$$DM_\delta := \frac{GP_\delta \cdot AD_\delta}{AG_\delta} \qquad CK_\delta := \frac{GP_\delta \cdot AC_\delta}{AG_\delta} \qquad BJ_\delta := \frac{GP_\delta \cdot AB_\delta}{AG_\delta}$$

$\frac{GP_\Delta}{\Delta^5} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{FO_\Delta}{\Delta^4} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{EN_\Delta}{\Delta^3} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{DM_\Delta}{\Delta^2} =$

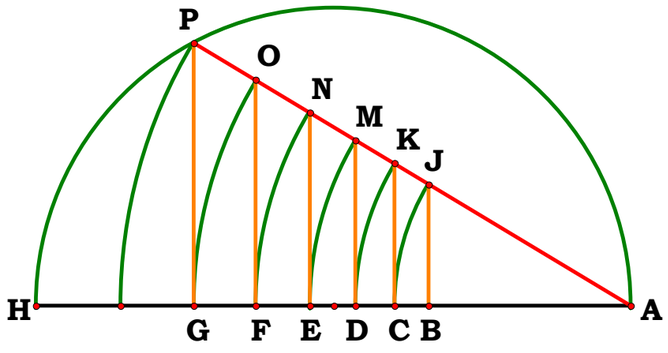
0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{CK_\Delta}{\Delta} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$BJ_\Delta =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$



$GP_\Delta =$

0
0.433013
0.31427
0.242061
0.195959
0.164336

$FO_\Delta =$

0
0.216506
0.104757
0.060515
0.039192
0.027389

$EN_\Delta =$

0
0.108253
0.034919
0.015129
$7.838367 \cdot 10^{-3}$
$4.564876 \cdot 10^{-3}$

$DM_\Delta =$

0
0.054127
0.01164
$3.78221 \cdot 10^{-3}$
$1.567673 \cdot 10^{-3}$
$7.608127 \cdot 10^{-4}$

$CK_\Delta =$

0
0.027063
$3.879873 \cdot 10^{-3}$
$9.455526 \cdot 10^{-4}$
$3.135347 \cdot 10^{-4}$
$1.268021 \cdot 10^{-4}$

$BJ_\Delta =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{GP_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^5} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{FO_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^4} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{EN_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^3} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{DM_\Delta}{\left(\frac{AH}{AP_\Delta}\right)^2} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$\frac{CK_\Delta}{\frac{AH}{AP_\Delta}} =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

$BJ_\Delta =$

0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$

GP_{Δ} =
0
0.433013
0.31427
0.242061
0.195959
0.164336

FO_{Δ} =
0
0.216506
0.104757
0.060515
0.039192
0.027389

EN_{Δ} =
0
0.108253
0.034919
0.015129
$7.838367 \cdot 10^{-3}$
$4.564876 \cdot 10^{-3}$

DM_{Δ} =
0
0.054127
0.01164
$3.78221 \cdot 10^{-3}$
$1.567673 \cdot 10^{-3}$
$7.608127 \cdot 10^{-4}$

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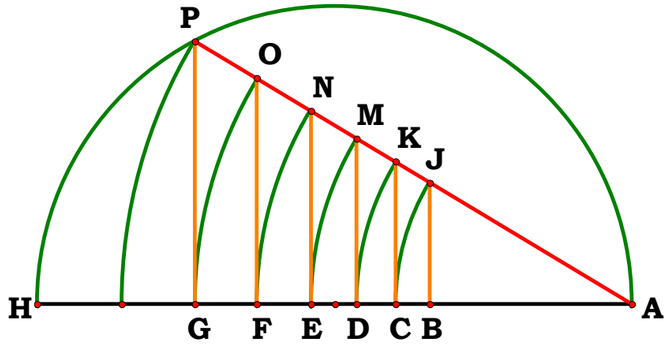
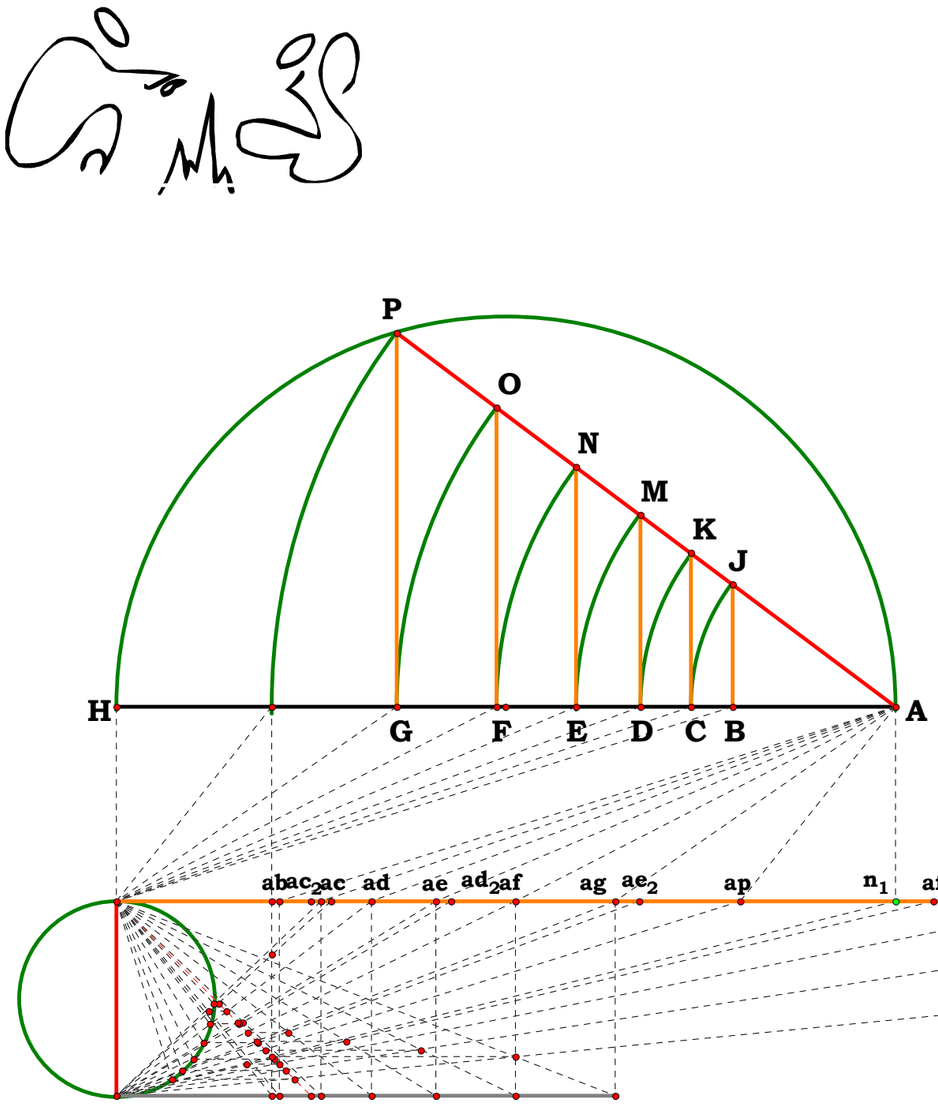
$1.268021 \cdot 10^{-4}$

BJ_{Δ} =
0
0.013532
$1.293291 \cdot 10^{-3}$
$2.363881 \cdot 10^{-4}$
$6.270694 \cdot 10^{-5}$
$2.113369 \cdot 10^{-5}$



$$\frac{\sqrt{AG_2 \cdot GH_2}}{2} = 0.216506 \quad \frac{\sqrt{AG_2 \cdot GH_2}}{2^2} = 0.108253 \quad \frac{\sqrt{AG_2 \cdot GH_2}}{2^3} = 0.054127$$

$$\frac{\sqrt{AG_3 \cdot GH_3}}{3} = 0.104757 \quad \frac{\sqrt{AG_3 \cdot GH_3}}{3^2} = 0.034919 \quad \frac{\sqrt{AG_3 \cdot GH_3}}{3^3} = 0.01164$$



ab = 0.83886
ac = 1.04858
ad = 1.31072
ae = 1.63840
af = 2.04800
ag = 2.56000
ap = 3.20000

$\frac{N_1}{ap} = 1.25000$

$\frac{N_1\,\frac{1}{7}}{ab} = 1.25000$

$\frac{N_1\,\frac{1}{6}}{ac} = 1.25000$

$\frac{N_1\,\frac{1}{5}}{ad} = 1.25000$

$\frac{N_1\,\frac{1}{4}}{ae} = 1.25000$

$\frac{N_1\,\frac{1}{3}}{af} = 1.25000$

$\frac{N_1\,\frac{1}{2}}{ag} = 1.25000$

$\frac{ap\,\frac{1}{6}}{ab} = 1.25000$

$\frac{ag\,\frac{1}{5}}{ab} = 1.25000$

$\frac{af\,\frac{1}{4}}{ab} = 1.25000$

$\frac{ae\,\frac{1}{3}}{ab} = 1.25000$

$\frac{ad\,\frac{1}{2}}{ab} = 1.25000$

$\frac{ac}{ab} = 1.25000$

Exponential Series M^(1/2^N)

Unit.

$AB := 1$

Given.

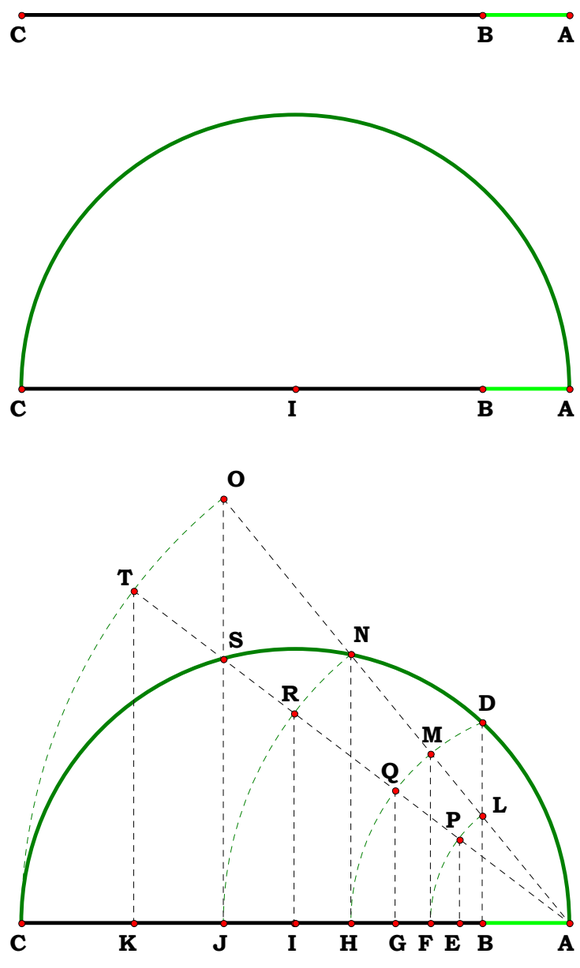
$N_1 := 5 \qquad AC := N_1$

120493R

Descriptions.

To use the digital indexing system to apply names, let AC be the thing with which we seek to name an exponential series on. AB is our unit. As a number is a ratio, numbers are two dimensional.

A number is no more than a digital name used with the stipulation that the indexing system is further qualified by using as standard difference. In other words that the concept of ratio will employ a name called a number. Ratio, however, is independent of the naming convention. Given any ration, say, M, describe a two prime exponential series, M^1/2^N, where N is any whole ratio.



The circle is a two dimensional object which is capable of producing every ratio between two differences.

$BC := AC - AB \qquad BD := \sqrt{BC \cdot AB} \qquad AH := \sqrt{AB^2 + BD^2} \qquad CH := AC - AH$

$HN := \sqrt{CH \cdot AH} \qquad AJ := \sqrt{AH^2 + HN^2} \qquad CJ := AC - AJ \qquad JS := \sqrt{CJ \cdot AJ}$

$AK := \sqrt{JS^2 + AJ^2} \qquad AI := \frac{AJ^2}{AK} \qquad AG := \frac{AH^2}{AI} \qquad AF := \frac{AH^2}{AJ} \qquad AE := \frac{AF^2}{AG}$

Definitions.

$AC^0 - AB = 0$

$AC^{\frac{1}{8}} - AE = 0 \qquad AC^{\frac{2}{8}} - AF = 0 \qquad AC^{\frac{3}{8}} - AG = 0 \qquad AC^{\frac{4}{8}} - AH = 0$

$AC^{\frac{5}{8}} - AI = 0 \qquad AC^{\frac{6}{8}} - AJ = 0 \qquad AC^{\frac{7}{8}} - AK = 0 \qquad AC^{\frac{8}{8}} - AC = 0$

$AC^{\frac{1}{8}} - AE = 0$

$AC^{\frac{2}{8}} - AF = 0$

$AC^{\frac{3}{8}} - AG = 0$

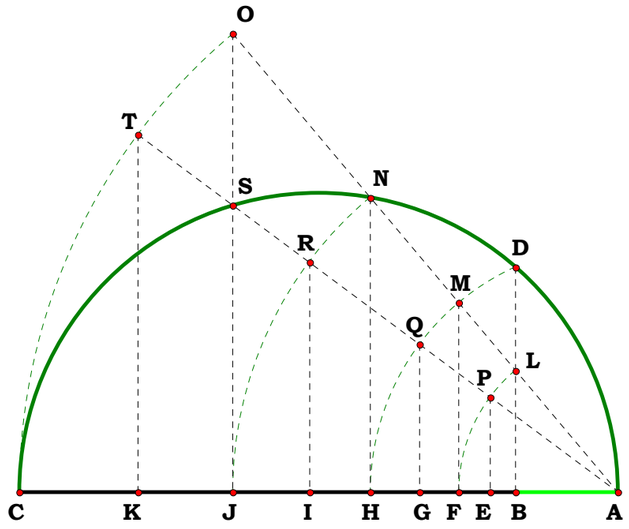
$AC^{\frac{4}{8}} - AH = 0$

$AC^{\frac{5}{8}} - AI = 0$

$AC^{\frac{6}{8}} - AJ = 0$

$AC^{\frac{7}{8}} - AK = 0$

$AC^{\frac{8}{8}} - AC = 0$



$AC^0 = 1.00000$	$N_1 = 1.00000$	$\frac{AB}{AB} = 1.00000$	AC = 3.95000 in.
$AC^{\frac{1}{8}} = 1.24752$	AE = 1.24752	$\frac{AE}{AB} = 1.24752$	AB = 0.67333 in.
$AC^{\frac{2}{8}} = 1.55630$	AF = 1.55630	$\frac{AF}{AB} = 1.55630$	AE = 0.83998 in.
$AC^{\frac{3}{8}} = 1.94151$	AG = 1.94151	$\frac{AG}{AB} = 1.94151$	AF = 1.04790 in.
$AC^{\frac{4}{8}} = 2.42207$	AH = 2.42207	$\frac{AH}{AB} = 2.42207$	AG = 1.30727 in.
$AC^{\frac{5}{8}} = 3.02157$	AI = 3.02157	$\frac{AI}{AB} = 3.02157$	AH = 1.63084 in.
$AC^{\frac{6}{8}} = 3.76946$	AJ = 3.76946	$\frac{AJ}{AB} = 3.76946$	AI = 2.03450 in.
$AC^{\frac{7}{8}} = 4.70247$	AK = 4.70247	$\frac{AK}{AB} = 4.70247$	AJ = 2.53807 in.
$AC^{\frac{8}{8}} = 5.86641$	AC = 5.86641	$\frac{AC}{AB} = 5.86641$	AK = 3.16629 in.

$AC^0 - N_1 = 0.00000$

$AC^{\frac{1}{8}} - AE = 0.00000$

$AC^{\frac{2}{8}} - AF = 0.00000$

$AC^{\frac{3}{8}} - AG = 0.00000$

$AC^{\frac{4}{8}} - AH = 0.00000$

$AC^{\frac{5}{8}} - AI = 0.00000$

$AC^{\frac{6}{8}} - AJ = 0.00000$

$AC^{\frac{7}{8}} - AK = 0.00000$

$AC^{\frac{8}{8}} - AC = 0.00000$

Alternate method of creating an exponential series.

$N_1 = 2.97175$

$N_1^{\frac{8}{8}} = 2.97175$

$N_1^{\frac{7}{8}} = 2.59350$

$N_1^{\frac{6}{8}} = 2.26339$

$N_1^{\frac{5}{8}} = 1.97530$

$N_1^{\frac{4}{8}} = 1.72388$

$N_1^{\frac{3}{8}} = 1.50446$

$N_1^{\frac{2}{8}} = 1.31296$

$N_1^{\frac{1}{8}} = 1.14585$

$N_1^{\frac{9}{8}} = 3.40517$

$N_1^{\frac{10}{8}} = 3.90181$

$N_1^{\frac{11}{8}} = 4.47087$

$N_1^{\frac{-1}{8}} = 0.87272$

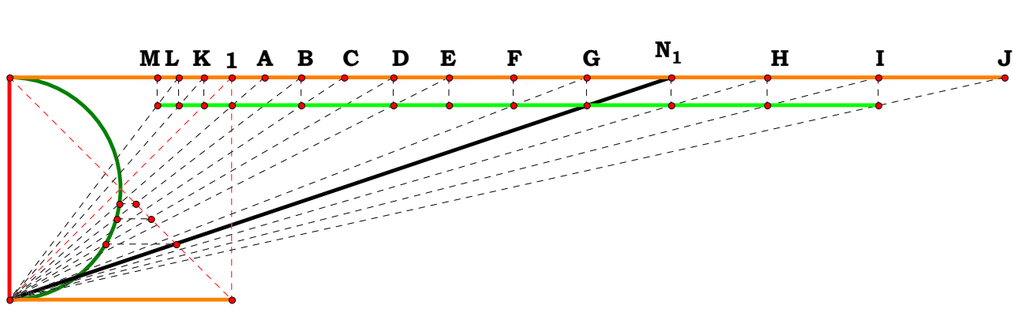
$N_1^{\frac{-2}{8}} = 0.76163$

$N_1^{\frac{-3}{8}} = 0.66469$

A = 1.14585
B = 1.31296
C = 1.50446
D = 1.72388
E = 1.97530
F = 2.26339
G = 2.59350
H = 3.40517
I = 3.90181
J = 4.47087
K = 0.87272
L = 0.76163
M = 0.66469

$N_1^0 = 1.00000$

$N_1^{\frac{8}{8}} - N_1 = 0.00000$



$N_1^{\frac{-1}{8}} - K = 0.00000$

$N_1^{\frac{-2}{8}} - L = 0.00000$

$N_1^{\frac{-3}{8}} - M = 0.00000$

$N_1^{\frac{1}{8}} - A = 0.00000$

$N_1^{\frac{2}{8}} - B = 0.00000$

$N_1^{\frac{3}{8}} - C = 0.00000$

$N_1^{\frac{4}{8}} - D = 0.00000$

$N_1^{\frac{5}{8}} - E = 0.00000$

$N_1^{\frac{6}{8}} - F = 0.00000$

$N_1^{\frac{7}{8}} - G = 0.00000$

$N_1^{\frac{9}{8}} - H = 0.00000$

$N_1^{\frac{10}{8}} - I = 0.00000$

$N_1^{\frac{11}{8}} - J = 0.00000$

Unit.

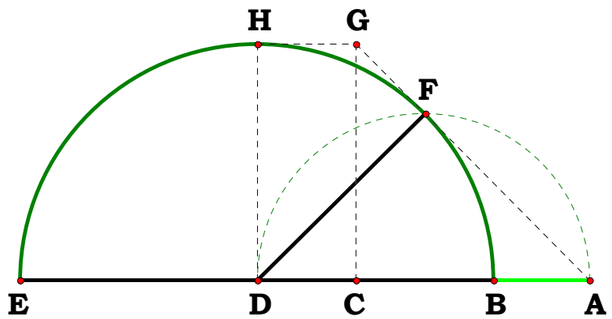
$BE := 1$

Given.

$N_1 := 6$

120693AR

Alternate Method: Square Root
Common Segment Common Endpoint



Descriptions.

$AB := N_1 \qquad AE := AB + BE \qquad BD := \frac{BE}{2}$

$AE - (N_1 + BE) = 0 \qquad BD - \frac{BE}{2} = 0 \qquad DF := BD$

$AD := BD + AB \qquad DF - \frac{BE}{2} = 0 \qquad AD - \frac{2 \cdot N_1 + BE}{2} = 0$

$AF := \sqrt{AD^2 - DF^2} \qquad AC := AF$

Definitions.

$\sqrt{AB \cdot AE} - AC = 0$

$AF - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

$AC - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

120693BR

Unit.

$CJ := 1$

Given.

$N_1 := 5$

Are A, P and Q collinear? Are A, K and N collinear?

Descriptions.

$AC := N_1$ $\quad AJ := AC + CJ$ $\quad AE := \left(AC^2 \cdot AJ\right)^{\frac{1}{3}}$ $\quad AG := \left(AC \cdot AJ^2\right)^{\frac{1}{3}}$

$CG := AG - AC$ $\quad GJ := CJ - CG$ $\quad GN := \sqrt{CG \cdot GJ}$ $\quad AB := \frac{AE}{2}$ $\quad CE := AE - AC$

$CH := \frac{CJ}{2}$ $\quad BK := AB$ $\quad HK := CH$ $\quad HJ := CH$ $\quad AH := AJ - HJ$ $\quad BH := AH - AB$

$BD := \frac{BK^2 + BH^2 - HK^2}{2 \cdot BH}$ $\quad AD := AB + BD$ $\quad DE := AE - AD$ $\quad DK := \sqrt{AD \cdot DE}$

$GQ := \sqrt{AG \cdot GJ}$ $\quad CP := \sqrt{AC \cdot CE}$ $\quad \frac{AG}{GN} - \frac{AD}{DK} = -8.526$ $\quad \frac{AG}{GQ} - \frac{AC}{CP} = 0$

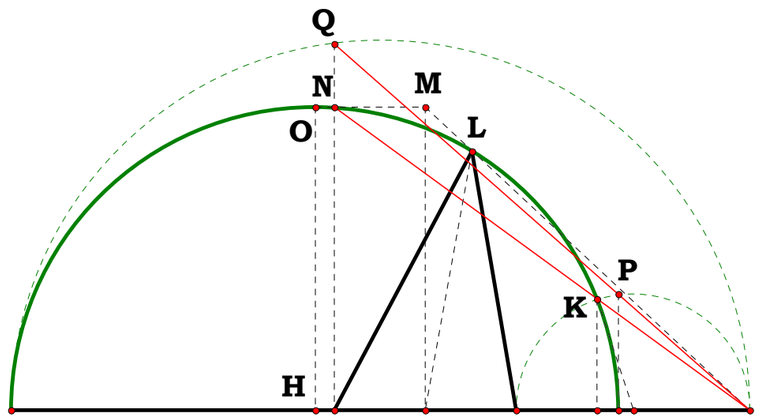
Definitions.

$AJ - \left(N_1 + 1\right) = 0$ $\quad AE - \left(N_1^3 + N_1^2\right)^{\frac{1}{3}} = 0$ $\quad AG - \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{1}{3}} = 0$

$CG - \left[\left[N_1 \cdot \left(N_1 + 1\right)^2\right]^{\frac{1}{3}} - N_1\right] = 0$ $\quad GJ - \left[N_1 - \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{1}{3}} + 1\right] = 0$

$GN - \sqrt{\left(2 \cdot N_1 + 1\right) \cdot \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{1}{3}} - \left[N_1 + N_1^2 + \left(N_1^3 + 2 \cdot N_1^2 + N_1\right)^{\frac{2}{3}}\right]} = 0$

Gruntwork IV on the Delian Solution.



121193R1

Unit.

$AB := 1$

Given.

$N := 6$

Descriptions.

$AL := AB \cdot N$ $\quad BL := AL - AB$ $\quad BK := \frac{BL}{2}$ $\quad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$ $\quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB$ $\quad BJ := AJ - AB$ $\quad JL := BL - BJ$ $\quad EJ := AJ - AE$ $\quad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $\quad BF := BL - FL$ $\quad FP := \sqrt{BF \cdot FL}$ $\quad KR := BK$ $\quad KL := BK$

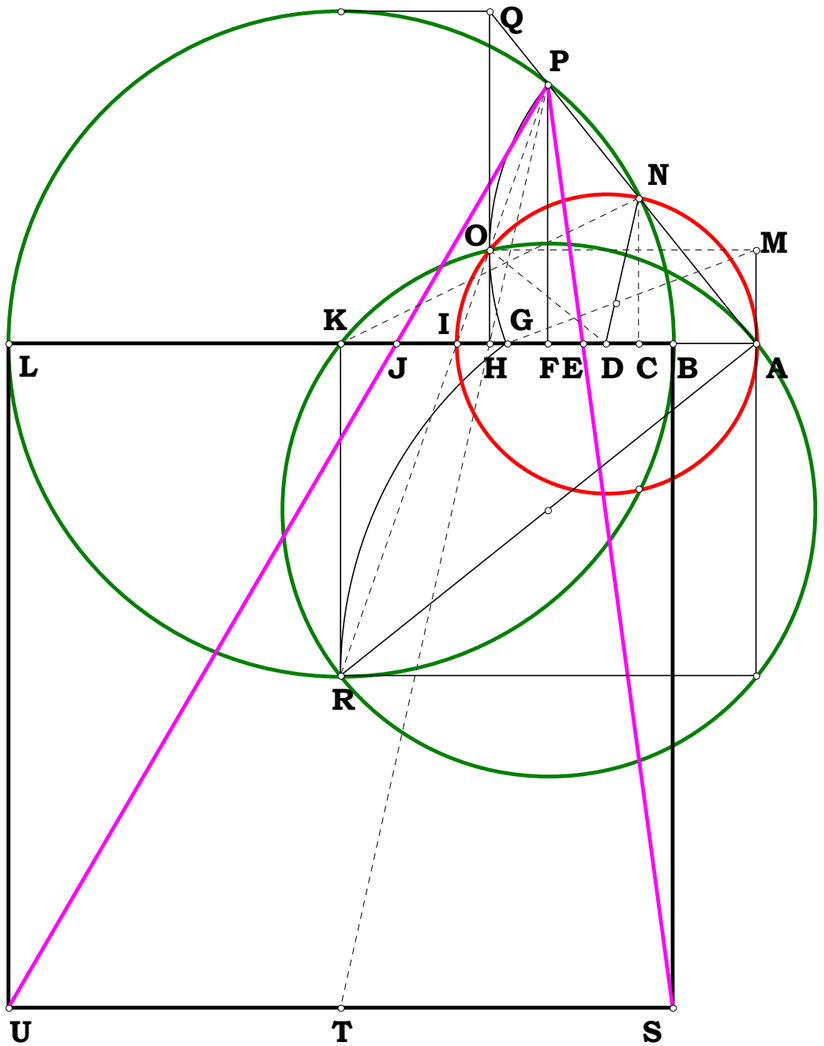
$FK := FL - KL$ $\quad IK := \frac{FK \cdot KR}{KR + FP}$ $\quad AK := BK + AB$ $\quad AI := AK - IK$ $\quad AD := \frac{AI}{2}$

$KT := BL$ $\quad FH := \frac{FK \cdot FP}{KT + FP}$ $\quad AF := BF + AB$ $\quad AH := AF + FH$ $\quad HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $\quad DN := AD$ $\quad KN := BK$ $\quad DK := AK - AD$ $\quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $\quad CI := AI - AC$ $\quad CN := \sqrt{AC \cdot CI}$ $\quad \frac{KR}{IK} - \frac{HO}{HI} = 0$ $\quad \frac{AF}{FP} - \frac{AC}{CN} = 0$

The structure in red appears to be a constant.

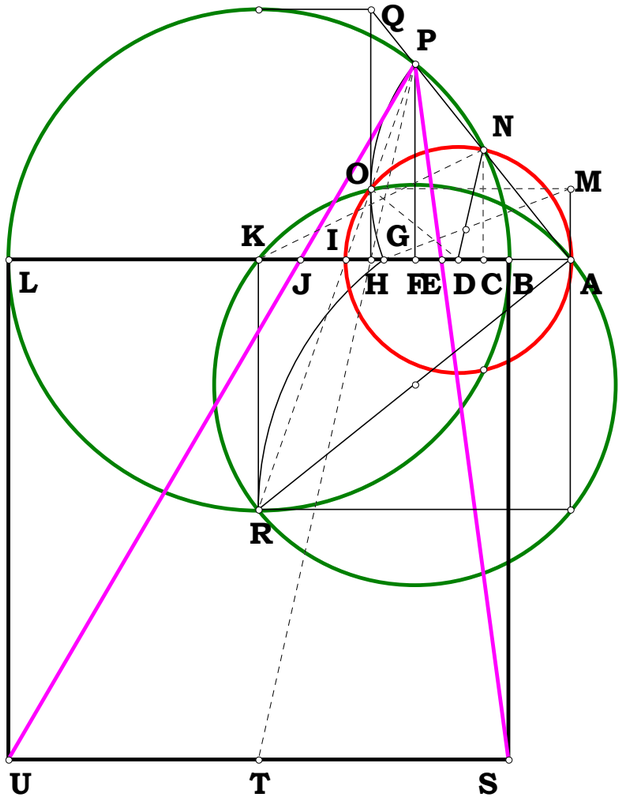


Definitions.

$$AL - N = 0 \qquad BL - (N-1) = 0 \qquad BK - \frac{N-1}{2} = 0 \qquad AE - N^{\frac{1}{3}} = 0 \qquad AJ - N^{\frac{2}{3}} = 0$$

$$BE - \left(N^{\frac{1}{3}} - 1\right) = 0 \qquad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0 \qquad JL - \left(N - N^{\frac{2}{3}}\right) = 0 \qquad EJ - N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) = 0$$

$$FJ - \frac{N \cdot \left(N^{\frac{1}{3}} - 1\right)}{N^{\frac{2}{3}} + 1} = 0 \qquad FL - \frac{N^{\frac{2}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0 \qquad BF - \frac{N-1}{N^{\frac{2}{3}} + 1} = 0 \qquad FP - \frac{N^{\frac{1}{3}} \cdot (N-1)}{N^{\frac{2}{3}} + 1} = 0$$



$$FK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \qquad IK - \frac{\left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$AK - \frac{N+1}{2} = 0 \qquad AI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{2}{3}} + 1\right)}{N^{\frac{1}{3}} + 1} = 0 \qquad AD - \frac{N + N^{\frac{1}{3}}}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0 \qquad FH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{2 \cdot \left(N^{\frac{2}{3}} + 1\right)} = 0 \qquad AF - \frac{\left(N^{\frac{1}{3}}\right)^2 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{2}{3}} + 1} = 0$$

$$AH - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{2} = 0$$

$$HI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$HO - \frac{\sqrt{N^{\frac{2}{3}} - 2 \cdot N + N^{\frac{4}{3}}}}{2} = 0$$

$$DK - \frac{N^{\frac{4}{3}} + 1}{2 \cdot \left(N^{\frac{1}{3}} + 1\right)} = 0$$

$$CK - \frac{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right) \cdot \left(N^{\frac{1}{3}} - 1\right)^2}{2 \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

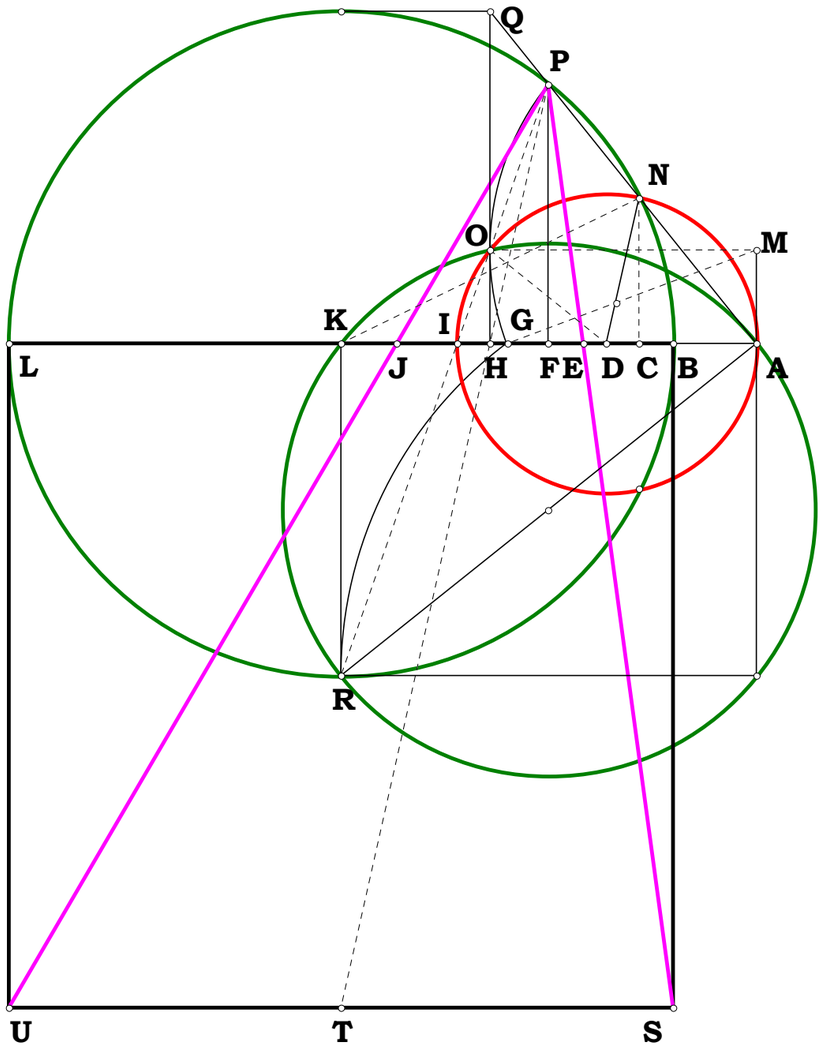
$$AC - \frac{\left(N^{\frac{1}{3}}\right)^3 \cdot \left(N^{\frac{1}{3}} + 1\right)}{N^{\frac{4}{3}} + 1} = 0$$

$$CI - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right)^2 \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)^2}{\left(N^{\frac{1}{3}} + 1\right) \cdot \left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$CN - \frac{N^{\frac{2}{3}} \cdot \left(N^{\frac{1}{3}} - 1\right) \cdot \left(N^{\frac{1}{3}} + N^{\frac{2}{3}} + 1\right)}{\left(N^{\frac{4}{3}} + 1\right)} = 0$$

$$\frac{KR}{IK} - \frac{N^{\frac{1}{3}} + 1}{N^{\frac{1}{3}} - 1} = 0$$

$$\frac{AF}{FP} - \frac{N^{\frac{1}{3}} \cdot \left(N^{\frac{1}{3}} + 1\right)}{N - 1} = 0$$



121193R2

Unit.

$AB := 1$

Given.

$Y := 4$

$X := 20$

Descriptions.

$AL := \frac{X}{Y}$ $BL := AL - AB$ $BK := \frac{BL}{2}$ $AE := (AL)^{\frac{1}{3}}$ $AJ := AL^{\frac{2}{3}}$

$BE := AE - AB$ $BJ := AJ - AB$ $JL := BL - BJ$ $EJ := AJ - AE$ $FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $BF := BL - FL$ $FP := \sqrt{BF \cdot FL}$ $KR := BK$ $KL := BK$

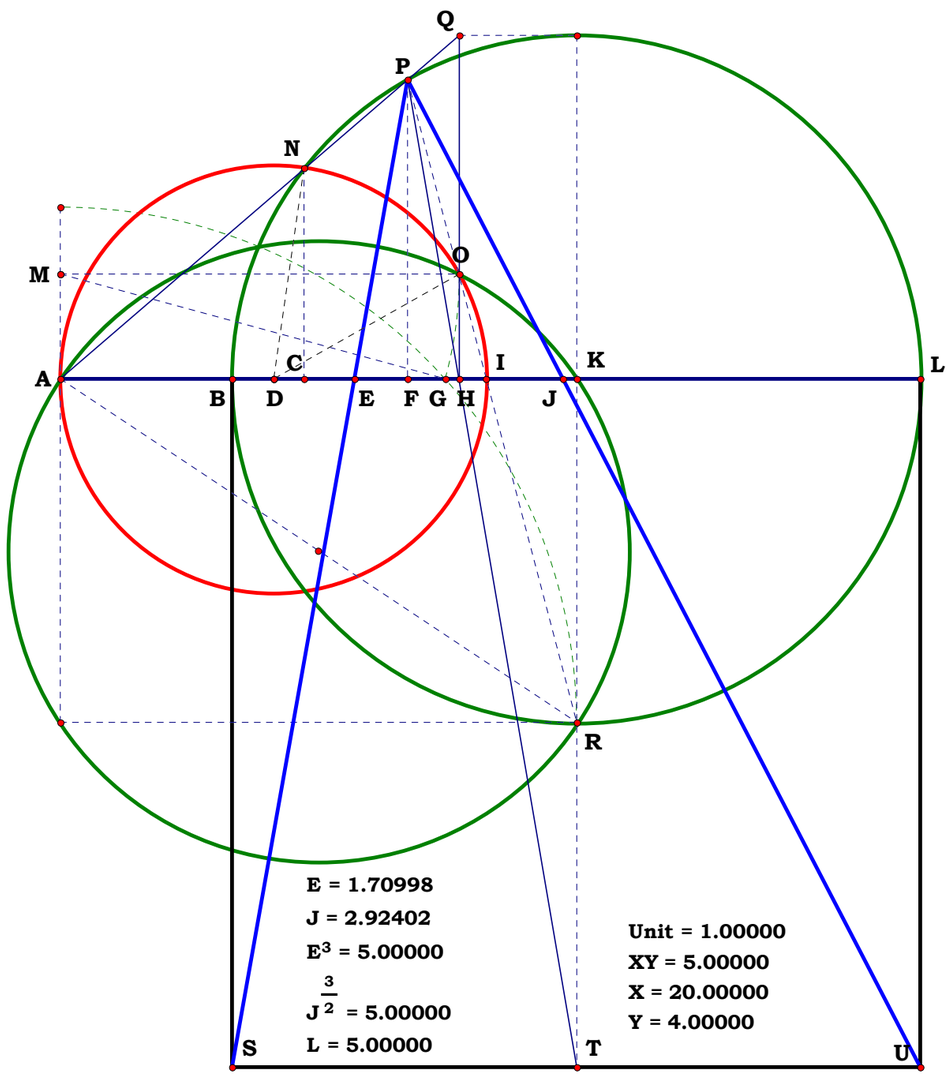
$FK := FL - KL$ $IK := \frac{FK \cdot KR}{KR + FP}$ $AK := BK + AB$ $AI := AK - IK$ $AD := \frac{AI}{2}$

$KT := BL$ $FH := \frac{FK \cdot FP}{KT + FP}$ $AF := BF + AB$ $AH := AF + FH$ $HI := AI - AH$

$HO := \sqrt{AH \cdot HI}$ $DN := AD$ $KN := BK$ $DK := AK - AD$ $CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $CI := AI - AC$ $CN := \sqrt{AC \cdot CI}$ $\frac{KR}{IK} - \frac{HO}{HI} = 0$ $\frac{AF}{FP} - \frac{AC}{CN} = 0$

The structure in red appears to be a constant.



Definitions.

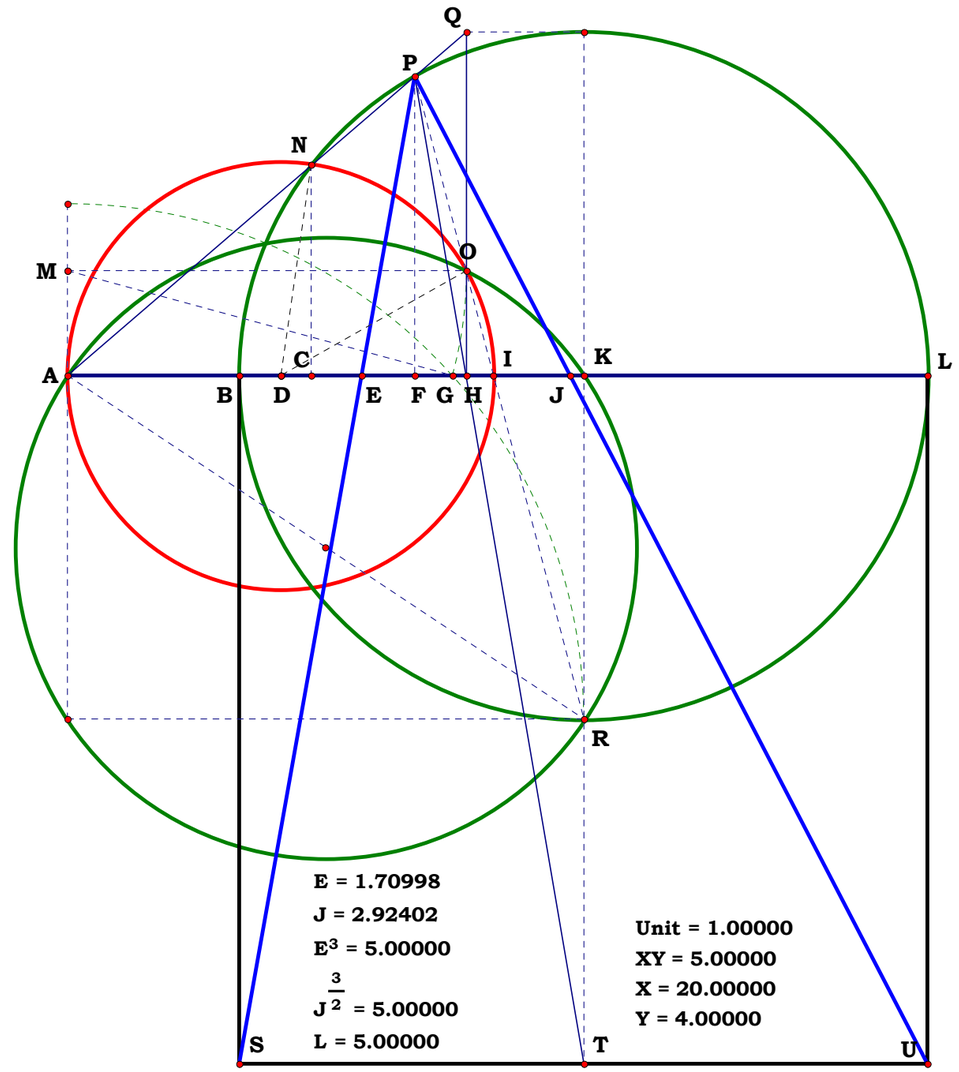
$$AL - \frac{X}{Y} = 0 \qquad BL - \frac{X-Y}{Y} = 0 \qquad BK - \frac{X-Y}{2\cdot Y} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{1}{3}} = 0 \qquad AJ - \left(\frac{X}{Y}\right)^{\frac{2}{3}} = 0$$

$$BE - \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0 \qquad BJ - \left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} - 1\right] = 0 \qquad JL - \left[\frac{X}{Y} - \left(\frac{X}{Y}\right)^{\frac{2}{3}}\right] = 0 \qquad EJ - \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0$$

$$FJ - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FL - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad BF - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FP - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

$$FK - \left[\frac{X-Y}{2\cdot Y} - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}\right] = 0 \qquad IK - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]\cdot(X-Y)}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AK - \frac{X+Y}{2\cdot Y} = 0$$

$$AI - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AD - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}{2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad FH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 2} = 0 \qquad AF - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

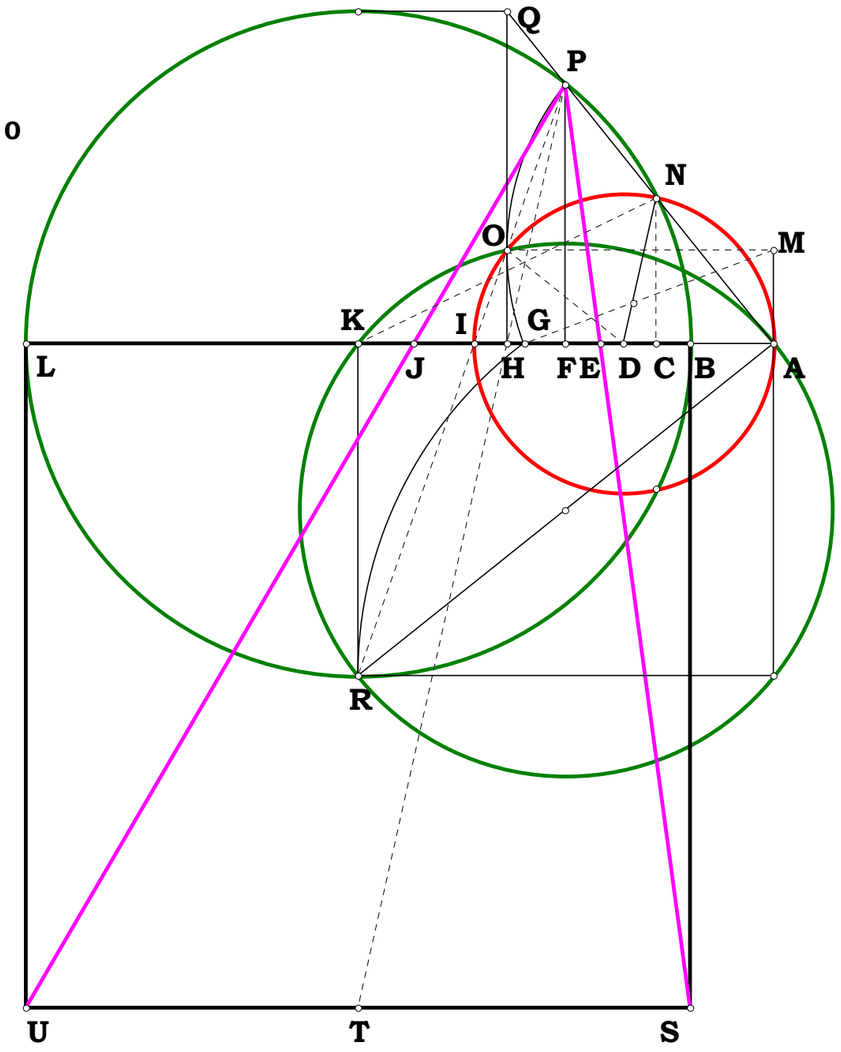


$$AH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2} = 0 \qquad HI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} + 2} = 0 \qquad HO - \frac{\sqrt{\left(\frac{X}{Y}\right)^{\frac{2}{3}} - \frac{2 \cdot X}{Y} + \frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}}}{2} = 0$$

$$DK - \frac{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad CK - -\frac{(X - Y) \cdot \left[Y - X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}\right]}{2 \cdot Y^2 \cdot \left[\frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y} + 1\right]} = 0 \qquad AC - \frac{X \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]}{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + \left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]^2}{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CN - \frac{(X - Y) \cdot \left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}} = 0 \qquad \frac{KR}{IK} - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1}{\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{X - Y} = 0$$



121193R3

Unit.

$BL := 1$

Given.

$Y := 5$

$X := 10$

Descriptions.

$AB := 1$ $\quad AL := \frac{X}{Y}$ $\quad BK := \frac{BL}{2}$ $\quad AE := \left(AB^2 \cdot AL\right)^{\frac{1}{3}}$ $\quad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BE := AE - AB$ $\quad BJ := AJ - AB$ $\quad JL := BL - BJ$ $\quad EJ := AJ - AE$ $\quad FJ := \frac{JL \cdot EJ}{JL + BE}$

$FL := JL + FJ$ $\quad BF := BL - FL$ $\quad FP := \sqrt{BF \cdot FL}$ $\quad KR := BK$ $\quad KL := BK$

$FK := FL - KL$ $\quad IK := \frac{FK \cdot KR}{KR + FP}$ $\quad AK := BK + AB$ $\quad AI := AK - IK$ $\quad AD := \frac{AI}{2}$

$KT := BL$ $\quad FH := \frac{FK \cdot FP}{KT + FP}$ $\quad AF := BF + AB$ $\quad AH := AF + FH$ $\quad HI := AI - AH$

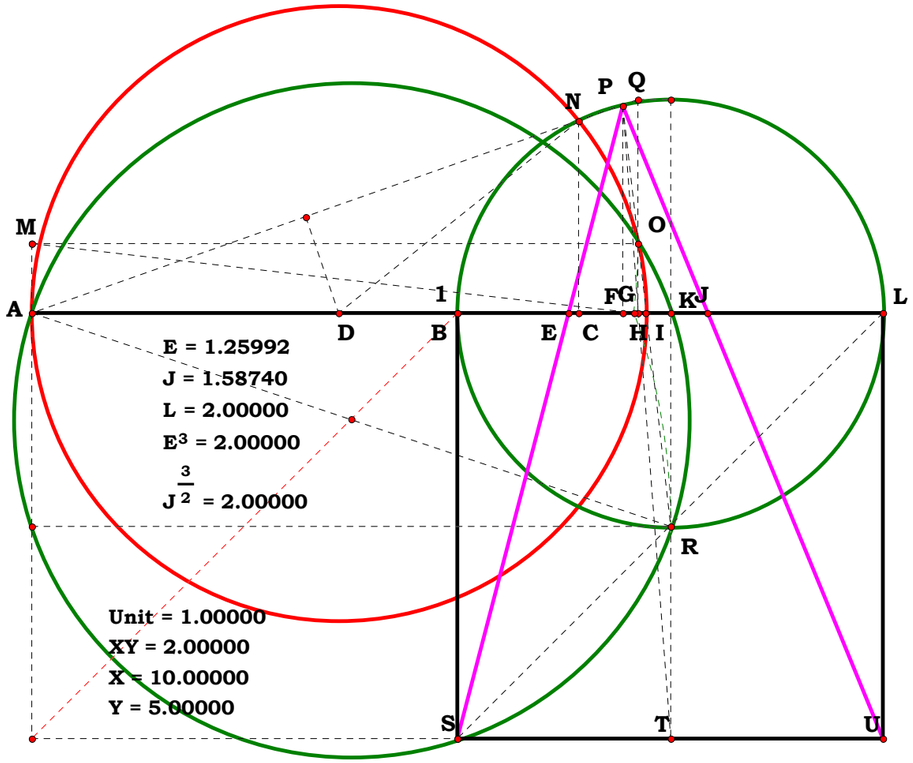
$HO := \sqrt{AH \cdot HI}$ $\quad DN := AD$ $\quad KN := BK$ $\quad DK := AK - AD$ $\quad CK := \frac{KN^2 + DK^2 - DN^2}{2 \cdot DK}$

$AC := AK - CK$ $\quad CI := AI - AC$ $\quad CN := \sqrt{AC \cdot CI}$ $\quad \frac{KR}{IK} - \frac{HO}{HI} = 0$ $\quad \frac{AF}{FP} - \frac{AC}{CN} = 0$

$AE = 1.259921$ $\quad AE^3 = 2$

$AJ = 1.587401$ $\quad AJ^{\frac{3}{2}} = 2$

The structure in red appears to be a constant.



Definitions.

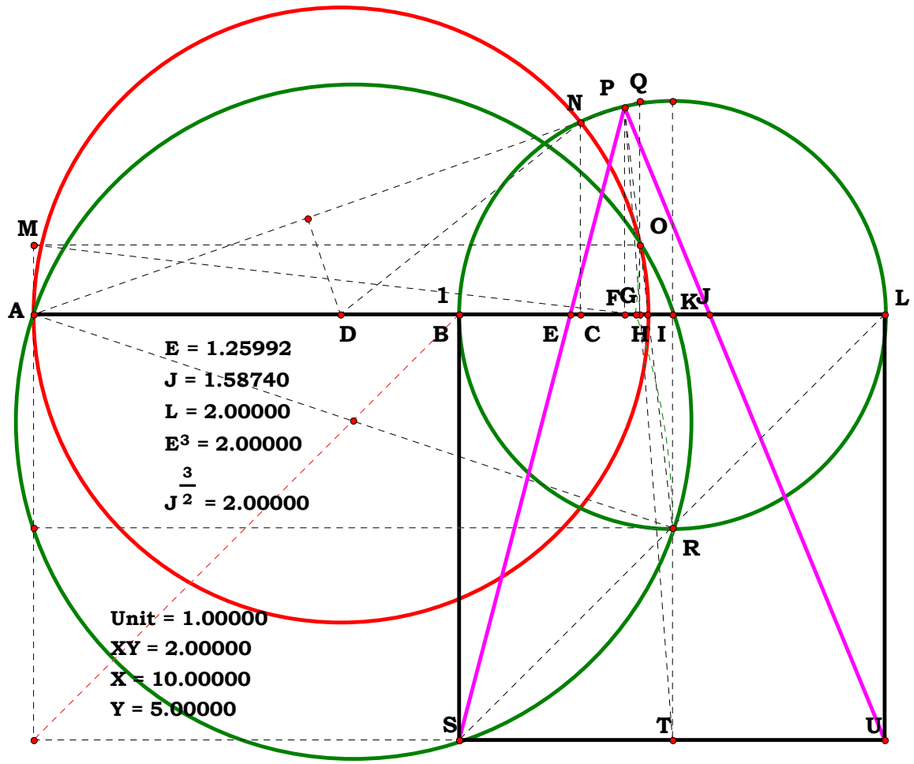
$$AL - \frac{X}{Y} = 0 \qquad BL - \frac{X-Y}{Y} = 0 \qquad BK - \frac{X-Y}{2\cdot Y} = 0 \qquad AE - \left(\frac{X}{Y}\right)^{\frac{1}{3}} = 0 \qquad AJ - \left(\frac{X}{Y}\right)^{\frac{2}{3}} = 0$$

$$BE - \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0 \qquad BJ - \left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} - 1\right] = 0 \qquad JL - \left[\frac{X}{Y} - \left(\frac{X}{Y}\right)^{\frac{2}{3}}\right] = 0 \qquad EJ - \left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right] = 0$$

$$FJ - \frac{X\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FL - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad BF - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0 \qquad FP - \frac{(X-Y)\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

$$FK - \left[\frac{X-Y}{2\cdot Y} - \frac{X-Y}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}\right] = 0 \qquad IK - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]\cdot(X-Y)}{2\cdot Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AK - \frac{X+Y}{2\cdot Y} = 0$$

$$AI - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad AD - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}}\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]}{2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad FH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2\cdot\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]\cdot\left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 2} = 0 \qquad AF - \frac{X+Y\cdot\left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y\cdot\left[\left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]} = 0$$

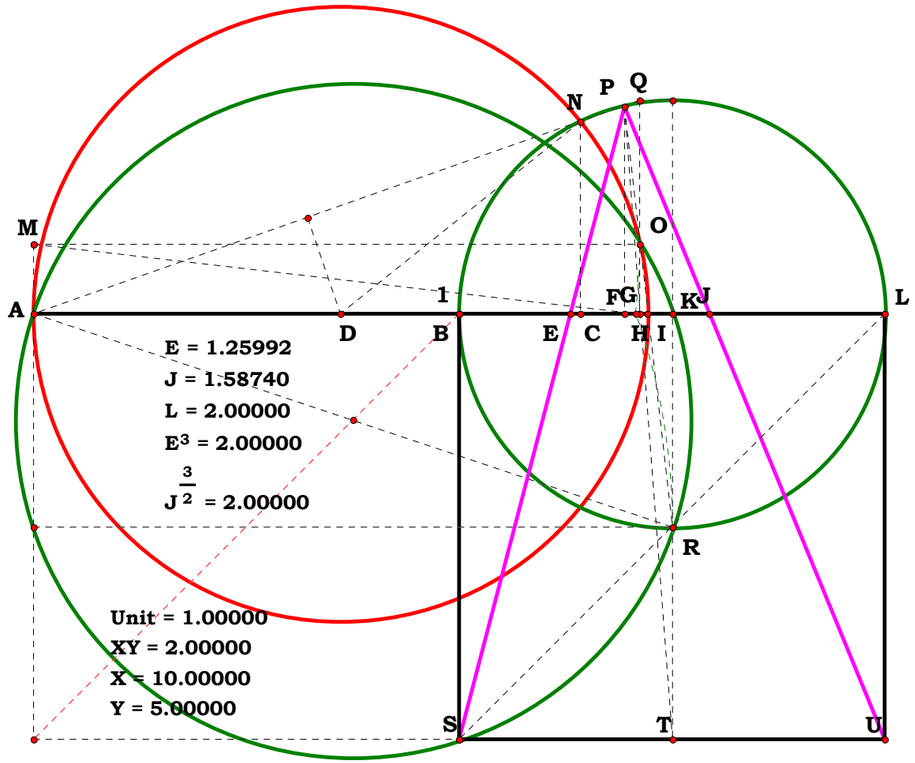


$$AH - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2} = 0 \qquad HI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} + 2} = 0 \qquad HO - \frac{\sqrt{\left(\frac{X}{Y}\right)^{\frac{2}{3}} - \frac{2 \cdot X}{Y} + \frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y}}}{2} = 0$$

$$DK - \frac{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{2 \cdot Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]} = 0 \qquad CK - -\frac{(X - Y) \cdot \left[Y - X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}\right]}{2 \cdot Y^2 \cdot \left[\frac{X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{Y} + 1\right]} = 0 \qquad AC - \frac{X \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right]}{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CI - \frac{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1\right]^2 \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}} \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + \left(\frac{X}{Y}\right)^{\frac{2}{3}} + 1\right]^2}{\left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left[\left(\frac{X}{Y}\right)^{\frac{4}{3}} + 1\right]} = 0$$

$$CN - \frac{(X - Y) \cdot \left(\frac{X}{Y}\right)^{\frac{2}{3}}}{Y + X \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}} = 0 \qquad \frac{KR}{IK} - \frac{\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1}{\left(\frac{X}{Y}\right)^{\frac{1}{3}} - 1} = 0 \qquad \frac{AF}{FP} - \frac{Y \cdot \left[\left(\frac{X}{Y}\right)^{\frac{1}{3}} + 1\right] \cdot \left(\frac{X}{Y}\right)^{\frac{1}{3}}}{X - Y} = 0$$



Given.

$N_1 := 5$

$N_2 := 3$

121293A1R

Descriptions.

$AF := N_1$ $\quad BE := N_2$

$AD := \frac{AF}{2}$ $\quad BD := \frac{BE}{2}$ $\quad AB := AD - BD$

$AE := BE + AB$ $\quad AC := \frac{AE}{2}$ $\quad CG := AC$

$CD := AD - AC$ $\quad GH := 2 \cdot \sqrt{CG^2 - CD^2}$

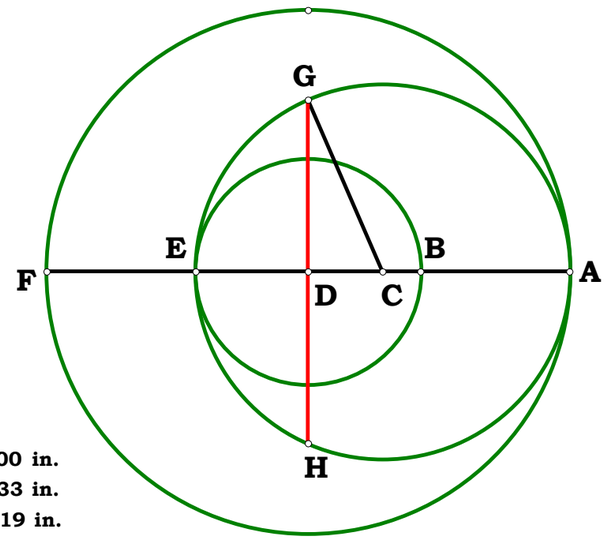
Definitions.

$GH - \sqrt{AF \cdot BE} = 0$

$GH - \sqrt{N_1 \cdot N_2} = 0$

Common Segment Common Midpoint

Square root by common segment common midpoint.
Given AFand BE is GH their root?



AF = 3.45000 in.
BE = 1.48333 in.
GH = 2.26219 in.
$\sqrt{AF \cdot BE}$ - GH = 0.00000 in.

121293A2R

Unit.

$AF := 1$

Given.

$Y := 20$

$X := 9$

Descriptions.

$BE := \frac{X}{Y}$ $AD := \frac{AF}{2}$ $BD := \frac{BE}{2}$

$AB := AD - BD$ $AE := BE + AB$

$AC := \frac{AE}{2}$ $CG := AC$ $CD := AD - AC$

$GH := 2 \cdot \sqrt{CG^2 - CD^2}$ $GH - \sqrt{AF \cdot BE} = 0$

Definitions.

$BE - \frac{X}{Y} = 0$ $AD - \frac{1}{2} = 0$ $BD - \frac{X}{2 \cdot Y} = 0$

$AB - \left(\frac{Y - X}{2 \cdot Y}\right) = 0$ $AE - \left(\frac{X + Y}{2 \cdot Y}\right) = 0$

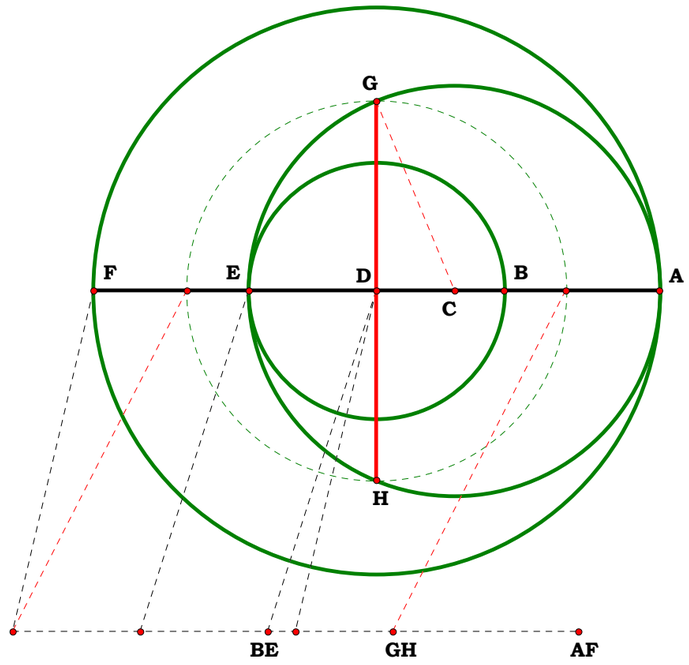
$AC - \frac{X + Y}{4 \cdot Y} = 0$ $CG - \frac{X + Y}{4 \cdot Y} = 0$

$CD - \frac{Y - X}{4 \cdot Y} = 0$ $GH - \frac{\sqrt{X}}{\sqrt{Y}} = 0$

$GH - \sqrt{\frac{X}{Y}} = 0$

Common Segment Common Midpoint

Square root by common segment common midpoint.
Given AFand BE is GH their root?



Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000

AF = 1.00000
BE = 0.45000
GH = 0.67082
$\sqrt{BE}$ = 0.67082

121293B1R

Given.

$N_1 := 1$

$N_2 := 4$

$N_3 := 3$

Descriptions.

$AF := N_1 \qquad DF := \frac{AF}{N_2} \qquad AD := AF - DF$

$DE := \frac{DF}{N_3} \qquad AE := AD + DE \qquad AB := \frac{AE}{2}$

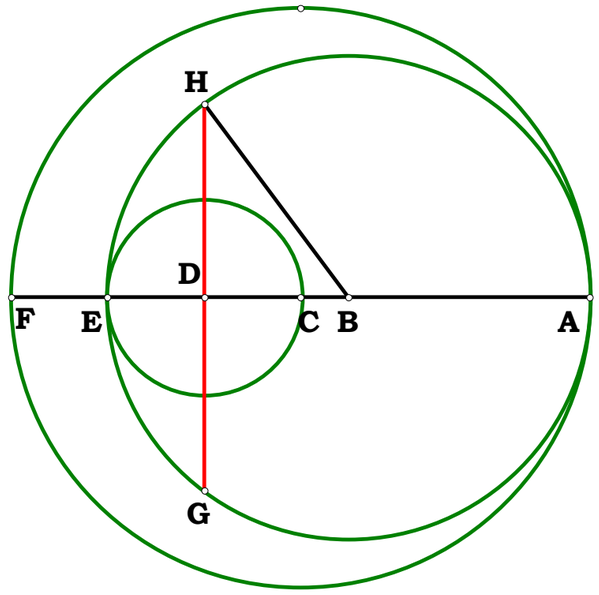
$BD := AD - AB \qquad BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$

Definitions.

$$GH - 2 \cdot \frac{N_1 \cdot \sqrt{N_2 - 1}}{N_2 \cdot \sqrt{N_3}} = 0$$

Generalize The Previous Square Root Figure



AF = 3.81667 in.

DF = 1.27449 in.

DE = 0.64116 in.

$\frac{AF}{DF} = 2.99466$

$\frac{DF}{DE} = 1.98779$

$\frac{2 \cdot AF \cdot \sqrt{\frac{AF}{DF} - 1}}{\frac{AF}{DF} \cdot \sqrt{\frac{DF}{DE}}} = 2.55338 \text{ in.}$

HG = 2.55338 in.

$\frac{2 \cdot AF \cdot \sqrt{\frac{AF}{DF} - 1}}{\frac{AF}{DF} \cdot \sqrt{\frac{DF}{DE}}} - HG = 0.00000 \text{ in.}$

121293B2R

Descriptions.

$AF := 1$

Given.

$Y := 20$

$X := 8$

$W := 9$

$V := 3$

$DF := \frac{X}{Y}$ $AD := AF - DF$

$DE := DF - \frac{DF \cdot V}{W}$ $AE := AD + DE$

$AB := \frac{AE}{2}$ $BD := AD - AB$ $BH := AB$

$GH := 2 \cdot \sqrt{(BH)^2 - (BD)^2}$ $GH = 0.8$

Definitions.

$DF - \frac{X}{Y} = 0$ $AD - \frac{Y - X}{Y} = 0$ $DE - \frac{X \cdot (W - V)}{W \cdot Y}$

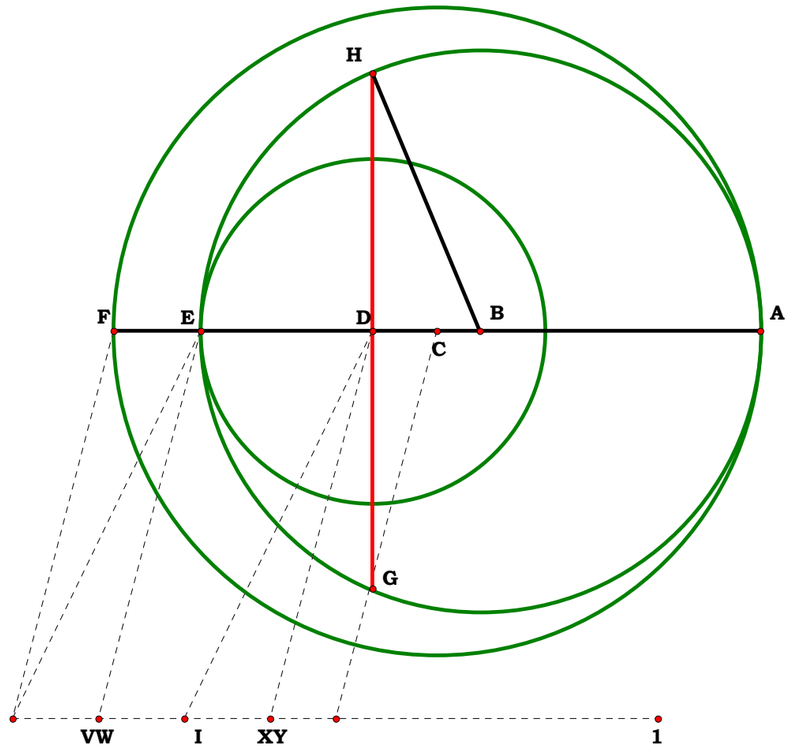
$AE - \frac{W \cdot Y - V \cdot X}{W \cdot Y} = 0$ $AB - \frac{W \cdot Y - V \cdot X}{2 \cdot W \cdot Y} = 0$

$BD - \frac{V \cdot X - 2 \cdot W \cdot X + W \cdot Y}{2 \cdot W \cdot Y} = 0$

$BH - \frac{W \cdot Y - V \cdot X}{2 \cdot W \cdot Y} = 0$ $GH = 0.8$

$GH - 2 \cdot \frac{\sqrt{X \cdot (V - W) \cdot (X - Y)}}{\sqrt{W} \cdot Y} = 0$

Generalize The Previous Square Root Figure



Unit = 1.00000

XY = 0.40000

X = 8.00000

Y = 20.00000

VW = 0.33333

V = 3.00000

W = 9.00000

$\frac{Y}{X} = 2.50000$

$\frac{W}{V} = 3.00000$

$\frac{GH}{AF} = 0.80000$

$\frac{2 \cdot \sqrt{X \cdot (V-W) \cdot (X-Y)}}{\sqrt{W} \cdot Y} - \frac{GH}{AF} = 0.00000$

Unit.

$AR := 1$ **Using 120493**

Given.

$\Delta := 5 \qquad \delta := 2 .. \Delta + 1$

121693AR

Descriptions.

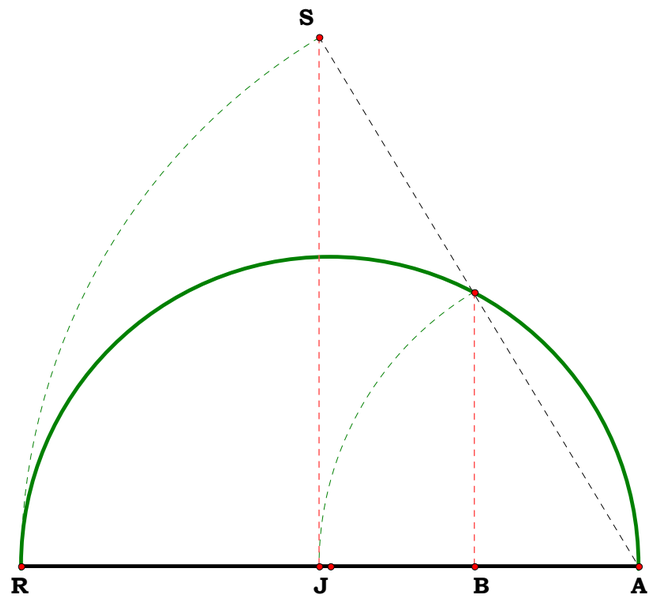
$$AB_\delta := \frac{AR}{\delta}$$

$$AJ_\delta := \sqrt{AB_\delta \cdot AR} \qquad JR_\delta := AR - AJ_\delta$$

$$JW_\delta := \sqrt{AJ_\delta \cdot JR_\delta} \qquad AW_\delta := \sqrt{(AJ_\delta)^2 + (JW_\delta)^2}$$

AB_δ =	AJ_δ =
0.5	0.707107
0.333333	0.57735
0.25	0.5
0.2	0.447214
0.166667	0.408248

The figure presents me with a progression. What is it's formula? It turns out not only that I can do square roots, but any two-prime root. It took me 48 pages of sieve work to realize what I was looking at.



Euclidean Exponential Series

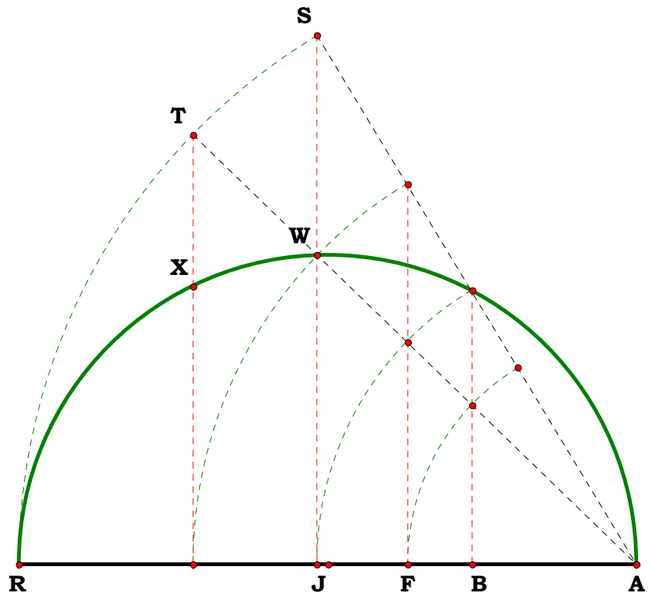
$$AT := AR \qquad AN_\delta := AW_\delta \qquad AF_\delta := \frac{(AJ_\delta)^2}{AW_\delta}$$

$$NR_\delta := AR - AN_\delta \qquad NX_\delta := \sqrt{AN_\delta \cdot NR_\delta}$$

$$AX_\delta := \sqrt{(AN_\delta)^2 + (NX_\delta)^2}$$

Definitions.

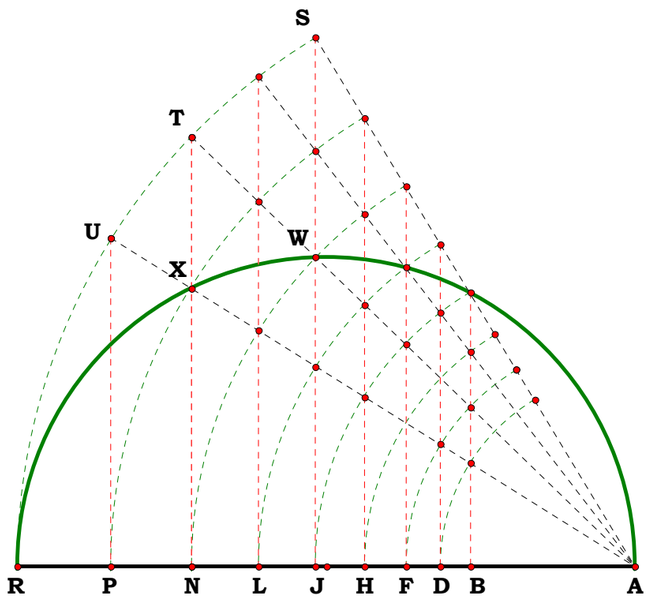
AB_δ =	AF_δ =	AJ_δ =	AN_δ =
0.5	0.594604	0.707107	0.840896
0.333333	0.438691	0.57735	0.759836
0.25	0.353553	0.5	0.707107
0.2	0.29907	0.447214	0.66874
0.166667	0.260847	0.408248	0.638943



What I find interesting, foremost, is the implication for number theory-- We learn exponential notation prior to any naturally occurring exponential examples- Here is one example. The figure removes the conception that exponentiation is purely a notational device and raises it's rank to that of a realistic abstraction.

$$AU := AR \qquad AP_\delta := AX_\delta \qquad AL_\delta := \frac{(AN_\delta)^2}{AX_\delta} \qquad AH_\delta := \frac{(AJ_\delta)^2}{AL_\delta} \qquad AD_\delta := \frac{(AF_\delta)^2}{AH_\delta}$$

$$PR_\delta := AR - AP_\delta \qquad PY_\delta := \sqrt{AP_\delta \cdot PR_\delta} \qquad AY_\delta := \sqrt{(AP_\delta)^2 + (PY_\delta)^2}$$

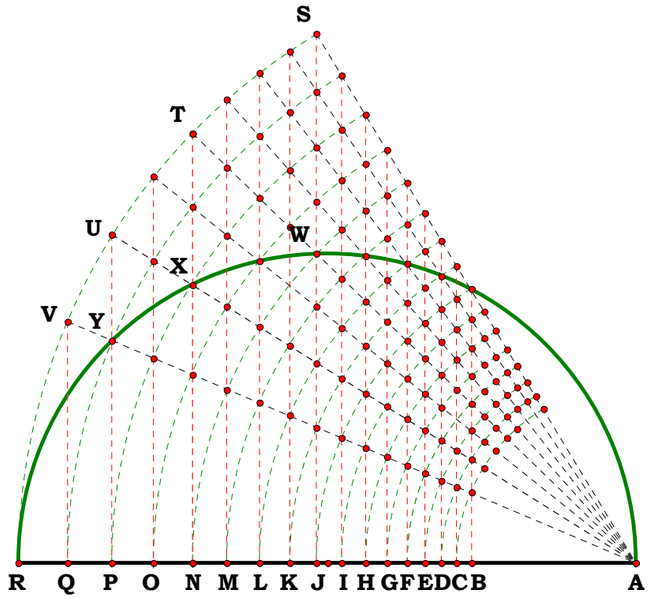


AB_δ =	AD_δ =	AF_δ =	AH_δ =	AJ_δ =	AL_δ =	AN_δ =	AP_δ =
0.5	0.545254	0.594604	0.64842	0.707107	0.771105	0.840896	0.917004
0.333333	0.382401	0.438691	0.503268	0.57735	0.662338	0.759836	0.871686
0.25	0.297302	0.353553	0.420448	0.5	0.594604	0.707107	0.840896
0.2	0.244569	0.29907	0.365716	0.447214	0.546873	0.66874	0.817765
0.166667	0.208506	0.260847	0.326329	0.408248	0.510732	0.638943	0.799339

$$AV := AR \qquad AQ_\delta := AY_\delta \qquad AO_\delta := \frac{(AP_\delta)^2}{AY_\delta} \qquad AM_\delta := \frac{(AN_\delta)^2}{AO_\delta}$$

$$AK_\delta := \frac{(AL_\delta)^2}{AM_\delta} \qquad AI_\delta := \frac{(AJ_\delta)^2}{AK_\delta} \qquad AG_\delta := \frac{(AH_\delta)^2}{AI_\delta}$$

$$AE_\delta := \frac{(AF_\delta)^2}{AG_\delta} \qquad AC_\delta := \frac{(AD_\delta)^2}{AE_\delta}$$



AB_δ =	AC_δ =	AD_δ =	AE_δ =	AF_δ =
0.5	0.522137	0.545254	0.569394	0.594604
0.333333	0.357025	0.382401	0.40958	0.438691
0.25	0.272627	0.297302	0.32421	0.353553
0.2	0.221165	0.244569	0.27045	0.29907
0.166667	0.186416	0.208506	0.233213	0.260847

AG_δ =
0.620929
0.469872
0.385553
0.330718
0.291757

AH_δ =
0.64842
0.503268
0.420448
0.365716
0.326329

AI_δ =
0.677128
0.539038
0.458502
0.404417
0.364998

AJ_δ =
0.707107
0.57735
0.5
0.447214
0.408248

AK_δ =
0.738413
0.618386
0.545254
0.494539
0.456624

AL_δ =
0.771105
0.662338
0.594604
0.546873
0.510732

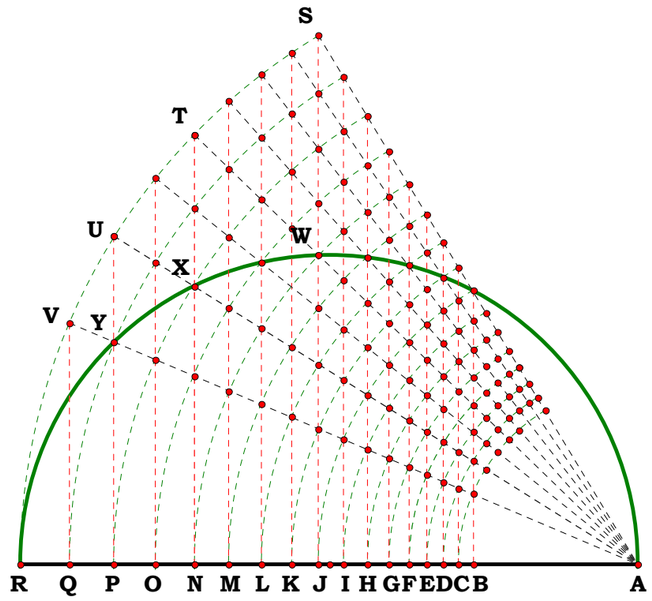
AM_δ =
0.805245
0.709414
0.64842
0.604744
0.571252

AN_δ =
0.840896
0.759836
0.707107
0.66874
0.638943

AO_δ =
0.878126
0.813841
0.771105
0.739508
0.714655

AP_δ =
0.917004
0.871686
0.840896
0.817765
0.799339

AQ_δ =
0.957603
0.933641
0.917004
0.904304
0.894058



Values found by the investigator of 12_14_93

AQ_δ =
0.957603
0.933641
0.917004
0.904304
0.894058

$(AB_\delta)^{\frac{1}{16}}$ =
0.957603
0.933641
0.917004
0.904304
0.894058

AP_δ =
0.917004
0.871686
0.840896
0.817765
0.799339

$(AB_\delta)^{\frac{1}{8}}$ =
0.917004
0.871686
0.840896
0.817765
0.799339

AN_δ =
0.840896
0.759836
0.707107
0.66874
0.638943

$(AB_\delta)^{\frac{1}{4}}$ =
0.840896
0.759836
0.707107
0.66874
0.638943

AL_δ =
0.771105
0.662338
0.594604
0.546873
0.510732

$(AB_\delta)^{\frac{3}{8}}$ =
0.771105
0.662338
0.594604
0.546873
0.510732

AI_δ =
0.677128
0.539038
0.458502
0.404417
0.364998

$(AB_\delta)^{\frac{9}{16}}$ =
0.677128
0.539038
0.458502
0.404417
0.364998

AH_δ =
0.64842
0.503268
0.420448
0.365716
0.326329

$(AB_\delta)^{\frac{5}{8}}$ =
0.64842
0.503268
0.420448
0.365716
0.326329

AG_δ =
0.620929
0.469872
0.385553
0.330718
0.291757

$(AB_\delta)^{\frac{11}{16}}$ =
0.620929
0.469872
0.385553
0.330718
0.291757

AF_δ =
0.594604
0.438691
0.353553
0.29907
0.260847

$(AB_\delta)^{\frac{3}{4}}$ =
0.594604
0.438691
0.353553
0.29907
0.260847

$(AB_\delta)^{\frac{13}{16}}$

$AE_\delta =$

0.569394
0.40958
0.32421
0.27045
0.233213

$(AB_\delta)^{\frac{13}{16}} =$

0.569394
0.40958
0.32421
0.27045
0.233213

$AD_\delta =$

0.545254
0.382401
0.297302
0.244569
0.208506

$(AB_\delta)^{\frac{7}{8}} =$

0.545254
0.382401
0.297302
0.244569
0.208506

$AC_\delta =$

0.522137
0.357025
0.272627
0.221165
0.186416

$(AB_\delta)^{\frac{15}{16}} =$

0.522137
0.357025
0.272627
0.221165
0.186416

$AQ_\delta =$

0.957603
0.933641
0.917004
0.904304
0.894058

$(AB_\delta)^{\frac{1}{16}} =$

0.957603
0.933641
0.917004
0.904304
0.894058

$AP_\delta =$

0.917004
0.871686
0.840896
0.817765
0.799339

$(AB_\delta)^{\frac{2}{16}} =$

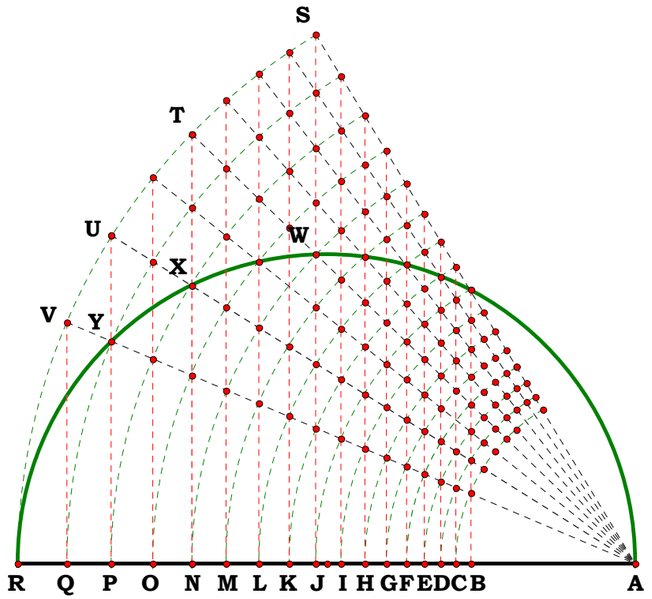
0.917004
0.871686
0.840896
0.817765
0.799339

$AO_\delta =$

0.878126
0.813841
0.771105
0.739508
0.714655

$(AB_\delta)^{\frac{3}{16}} =$

0.878126
0.813841
0.771105
0.739508
0.714655



$AN_\delta =$

0.840896
0.759836
0.707107
0.66874
0.638943

$(AB_\delta)^{\frac{4}{16}} =$

0.840896
0.759836
0.707107
0.66874
0.638943

$AM_\delta =$

0.805245
0.709414
0.64842
0.604744
0.571252

$(AB_\delta)^{\frac{5}{16}} =$

0.805245
0.709414
0.64842
0.604744
0.571252

$AL_\delta =$

0.771105
0.662338
0.594604
0.546873
0.510732

$(AB_\delta)^{\frac{6}{16}} =$

0.771105
0.662338
0.594604
0.546873
0.510732

$AK_\delta =$

0.738413
0.618386
0.545254
0.494539
0.456624

$(AB_\delta)^{\frac{7}{16}} =$

0.738413
0.618386
0.545254
0.494539
0.456624

$AJ_\delta =$

0.707107
0.57735
0.5
0.447214
0.408248

$(AB_\delta)^{\frac{8}{16}} =$

0.707107
0.57735
0.5
0.447214
0.408248

$AI_\delta =$

0.677128
0.539038
0.458502
0.404417
0.364998

$(AB_\delta)^{\frac{9}{16}} =$

0.677128
0.539038
0.458502
0.404417
0.364998

$AH_\delta =$

0.64842
0.503268
0.420448
0.365716
0.326329

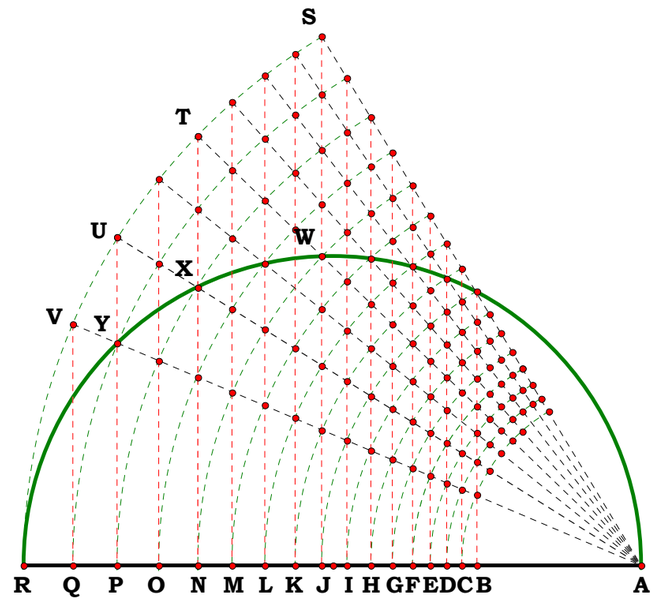
$(AB_\delta)^{\frac{10}{16}} =$

0.64842
0.503268
0.420448
0.365716
0.326329

AG_δ =	$(AB_\delta)^{\frac{11}{16}}$ =	AF_δ =	$(AB_\delta)^{\frac{12}{16}}$ =
0.620929	0.620929	0.594604	0.594604
0.469872	0.469872	0.438691	0.438691
0.385553	0.385553	0.353553	0.353553
0.330718	0.330718	0.29907	0.29907
0.291757	0.291757	0.260847	0.260847

AE_δ =	$(AB_\delta)^{\frac{13}{16}}$ =	AD_δ =	$(AB_\delta)^{\frac{14}{16}}$ =
0.569394	0.569394	0.545254	0.545254
0.40958	0.40958	0.382401	0.382401
0.32421	0.32421	0.297302	0.297302
0.27045	0.27045	0.244569	0.244569
0.233213	0.233213	0.208506	0.208506

AC_δ =	$\left[(AB_\delta)^{\frac{15}{16}}\right]$ =	AB_δ =	$\left[(AB_\delta)^{\frac{16}{16}}\right]$ =
0.522137	0.522137	0.5	0.5
0.357025	0.357025	0.333333	0.333333
0.272627	0.272627	0.25	0.25
0.221165	0.221165	0.2	0.2
0.186416	0.186416	0.166667	0.166667



Resultant Equation

$$\left(A^{\delta} \cdot B^{DIV-\delta}\right)^{\frac{1}{DIV}}$$

or

$$\left(A^{DIV-\delta} \cdot B^{\delta}\right)^{\frac{1}{DIV}}$$

depending on direction of transcription.

Given.

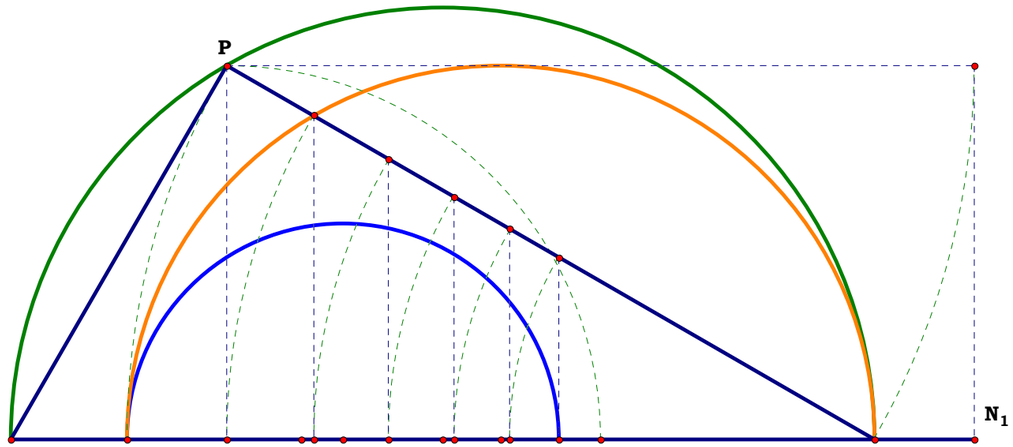
$DIV \equiv 7$

$\Delta := DIV$

$\delta := 1 .. \Delta$

121693BR

Exponential Series



Descriptions.

$AP := 10 \qquad AH := AP \qquad PF_1 := 5 \qquad AF_1 := \sqrt{AP^2 - (PF_1)^2}$

$AF_2 := \frac{AF_1 \cdot AF_1}{AP} \qquad AF_{\delta+1} := \frac{AF_1 \cdot AF_\delta}{AP}$

$\frac{PF_1^{\,2}}{AF_2} = 3.333333 \qquad \frac{PF_1^{\,2}}{\left(\frac{10}{3}\right)} = 7.5 \qquad \frac{AP}{PF_1} = 2$

$\sqrt{AF_\Delta \cdot AH} = 6.044456$

$AF_\Delta = 3.653545 \qquad \frac{AH}{AF_\Delta} = 2.737068$

Definitions.

$\left[(AF_\Delta)^{\Delta-\delta} \cdot (AH)^\delta\right]^{\frac{1}{\Delta}} = \quad if(\Delta - \delta, AF_{\Delta-\delta}, 0) = \quad \Delta - \delta =$

4.21875	4.21875	6
4.871393	4.871393	5
5.625	5.625	4
6.495191	6.495191	3
7.5	7.5	2
8.660254	8.660254	1
10	0	0

J H F_1 1 F_2 F_3 F_4 F_5 F_6 N_2 A

$F_1 1 = 1.25400$ cm

$AP = 12.54000$ cm

$AH = 12.54000$ cm

$AF_1 = 10.85996$ cm

$AF_2 = 9.40500$ cm

$AF_3 = 8.14497$ cm

$AF_4 = 7.05375$ cm

$AF_5 = 6.10873$ cm

$AF_6 = 5.29031$ cm

$\frac{AH}{F_1 1} = 10.00000 \qquad \frac{AF_3}{F_1 1} = 6.49519$

$\frac{AP}{F_1 1} = 10.00000 \qquad \frac{AF_4}{F_1 1} = 5.62500$

$\frac{AF_1}{F_1 1} = 8.66025 \qquad \frac{AF_5}{F_1 1} = 4.87139$

$\frac{AF_2}{F_1 1} = 7.50000 \qquad \frac{AF_6}{F_1 1} = 4.21875$

Unit.

$L := 1$

Given.

$Y := 20$

$X := 14$

$DIV := 10$

$\Delta := DIV$

$\delta := 1 .. \Delta$

121693CR

Descriptions.

Every number is expressible as a ratio.

$OJ := \frac{X}{Y}$ $OK := \sqrt{OJ}$ $OJ_1 := OK^2$

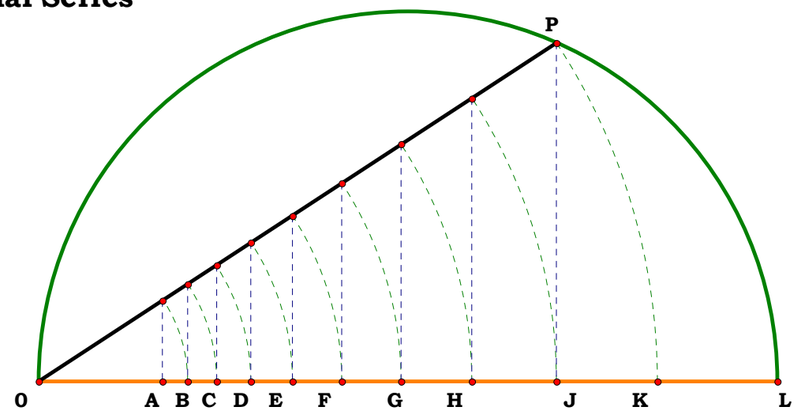
$OJ_2 := \frac{OJ^2}{OK}$ $OJ_{\delta+1} := \frac{OJ_1 \cdot OJ_\delta}{OK}$

$\left[\left(OJ_\Delta\right)^{\Delta-\delta} \cdot (OK)^\delta\right]^{\frac{1}{\Delta}} = \text{if}\left(\Delta-\delta, OJ_{\Delta-\delta}, 0\right) = \Delta-\delta = \text{if}\left[\Delta-\delta, \left[(OJ)_{\Delta-\delta}\right]^{\frac{1}{[\Delta-(\delta-1)]}}, 0\right] =$

0.16807	0.16807	9	0.83666
0.200882	0.200882	8	0.83666
0.2401	0.2401	7	0.83666
0.286974	0.286974	6	0.83666
0.343	0.343	5	0.83666
0.409963	0.409963	4	0.83666
0.49	0.49	3	0.83666
0.585662	0.585662	2	0.83666
0.7	0.7	1	0.83666
0.83666	0	0	0

Definitions.

Exponential Series



A = 0.16807
B = 0.20088
C = 0.24010
D = 0.28697
E = 0.34300
F = 0.40996
G = 0.49000
H = 0.58566
J = 0.70000
K = 0.83666
L = 1.00000
Unit = 1.00000
XY = 0.70000
X = 14.00000
Y = 20.00000

$A^{\frac{1}{10}}\text{-}K = 0.00000$
$A^{\frac{2}{10}}\text{-}J = 0.00000$
$A^{\frac{3}{10}}\text{-}H = 0.00000$
$A^{\frac{4}{10}}\text{-}G = 0.00000$
$A^{\frac{5}{10}}\text{-}F = 0.00000$
$A^{\frac{6}{10}}\text{-}E = 0.00000$
$A^{\frac{7}{10}}\text{-}D = 0.00000$
$A^{\frac{8}{10}}\text{-}C = 0.00000$
$A^{\frac{9}{10}}\text{-}B = 0.00000$

$\frac{X}{Y} = 0.70000$
$A^{\frac{1}{5}} = 0.70000$
$B^{\frac{2}{9}} = 0.70000$
$C^{\frac{3}{12}} = 0.70000$
$D^{\frac{4}{14}} = 0.70000$
$F^{\frac{6}{15}} = 0.70000$
$G^{\frac{7}{14}} = 0.70000$
$H^{\frac{8}{12}} = 0.70000$
$K^2 = 0.70000$

$\sqrt{\frac{X}{Y}} = 0.83666$
$J^{\frac{1}{2}}\text{-}K = 0.00000$
$H^{\frac{1}{3}}\text{-}K = 0.00000$
$G^{\frac{1}{4}}\text{-}K = 0.00000$
$F^{\frac{1}{5}}\text{-}K = 0.00000$
$E^{\frac{1}{6}}\text{-}K = 0.00000$
$D^{\frac{1}{7}} = 0.83666$
$C^{\frac{1}{8}}\text{-}K = 0.00000$
$B^{\frac{1}{9}}\text{-}K = 0.00000$
$A^{\frac{1}{10}}\text{-}K = 0.00000$

$K^2\text{-}J = 0.00000$
$K^3\text{-}H = 0.00000$
$K^4\text{-}G = 0.00000$
$K^5\text{-}F = 0.00000$
$K^6\text{-}E = 0.00000$
$K^7\text{-}D = 0.00000$
$K^8\text{-}C = 0.00000$
$K^9\text{-}B = 0.00000$
$K^{10}\text{-}A = 0.00000$

121693DR

Unit.

$L := 1$

Given.

$Y := 20$

$X := 14$

$DIV := 10$

$\Delta := DIV$

$\delta := 1 .. \Delta$

Exponential Series

Descriptions.

The first was X/Y, and this is Y/X.

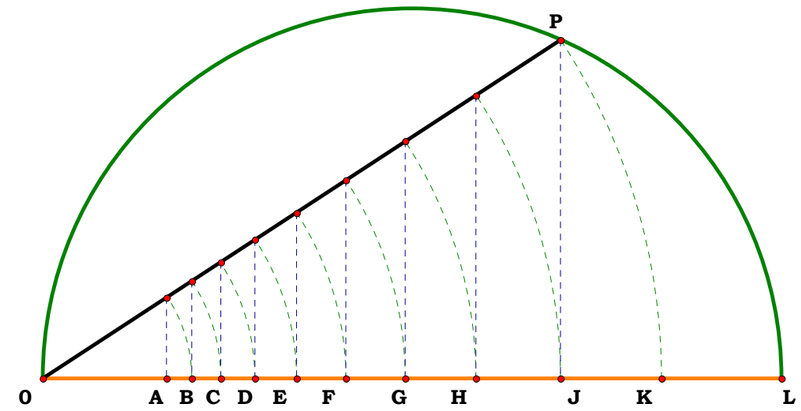
$OJ := \frac{Y}{X}$ $OK := \sqrt{OJ}$ $OJ_1 := OK^2$

$OJ_2 := \frac{OJ^2}{OK}$ $OJ_{\delta+1} := \frac{OJ_1 \cdot OJ_\delta}{OK}$ $\sqrt{\frac{Y}{X}} = 1.195229$

$\left[\left(OJ_\Delta\right)^{\Delta-\delta} \cdot (OK)^\delta\right]^{\frac{1}{\Delta}} = \text{if}\left(\Delta - \delta, OJ_{\Delta-\delta}, 0\right) = \Delta - \delta + 1 = \left(\sqrt{\frac{Y}{X}}\right)^{\Delta-\delta+1} = \delta = \left(\sqrt{\frac{Y}{X}}\right)^{\delta} =$

5.949902	5.949902	10	5.949902	1	1.195229
4.978045	4.978045	9	4.978045	2	1.428571
4.164931	4.164931	8	4.164931	3	1.707469
3.484632	3.484632	7	3.484632	4	2.040816
2.915452	2.915452	6	2.915452	5	2.439242
2.439242	2.439242	5	2.439242	6	2.915452
2.040816	2.040816	4	2.040816	7	3.484632
1.707469	1.707469	3	1.707469	8	4.164931
1.428571	1.428571	2	1.428571	9	4.978045
1.195229	0	1	1.195229	10	5.949902

Definitions.



A = 0.16807
B = 0.20088
C = 0.24010
D = 0.28697
E = 0.34300
F = 0.40996
G = 0.49000
H = 0.58566
J = 0.70000
K = 0.83666
L = 1.00000
Unit = 1.00000
XY = 0.70000
X = 14.00000
Y = 20.00000

$A^{\frac{1}{10}}\text{-}K = 0.00000$
$A^{\frac{2}{10}}\text{-}J = 0.00000$
$A^{\frac{3}{10}}\text{-}H = 0.00000$
$A^{\frac{4}{10}}\text{-}G = 0.00000$
$A^{\frac{5}{10}}\text{-}F = 0.00000$
$A^{\frac{6}{10}}\text{-}E = 0.00000$
$A^{\frac{7}{10}}\text{-}D = 0.00000$
$A^{\frac{8}{10}}\text{-}C = 0.00000$
$A^{\frac{9}{10}}\text{-}B = 0.00000$

$\frac{X}{Y} = 0.70000$
$A^{\frac{1}{5}} = 0.70000$
$B^{\frac{2}{9}} = 0.70000$
$C^{\frac{3}{12}} = 0.70000$
$D^{\frac{4}{14}} = 0.70000$
$F^{\frac{6}{15}} = 0.70000$
$G^{\frac{7}{14}} = 0.70000$
$H^{\frac{8}{12}} = 0.70000$
$K^2 = 0.70000$

$\sqrt{\frac{X}{Y}} = 0.83666$
$J^{\frac{1}{2}}\text{-}K = 0.00000$
$H^{\frac{1}{3}}\text{-}K = 0.00000$
$G^{\frac{1}{4}}\text{-}K = 0.00000$
$F^{\frac{1}{5}}\text{-}K = 0.00000$
$E^{\frac{1}{6}}\text{-}K = 0.00000$
$D^{\frac{1}{7}} = 0.83666$
$C^{\frac{1}{8}}\text{-}K = 0.00000$
$B^{\frac{1}{9}}\text{-}K = 0.00000$
$A^{\frac{1}{10}}\text{-}K = 0.00000$

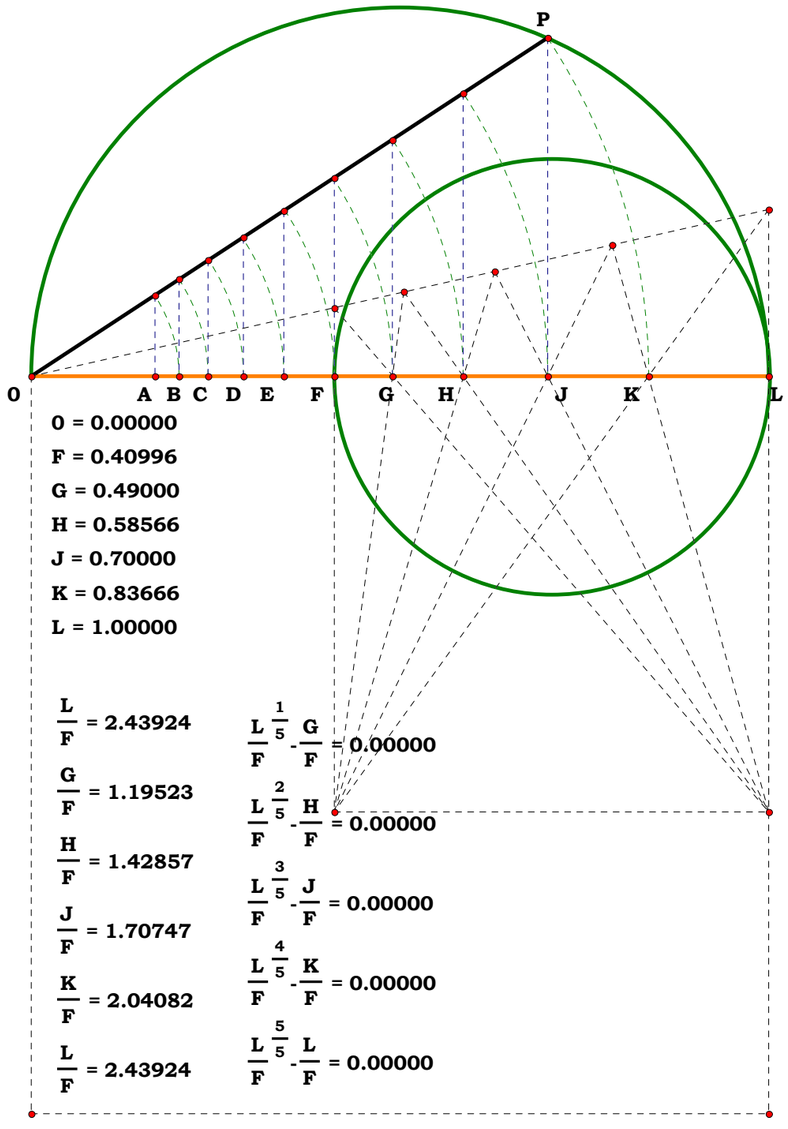
$K^2\text{-}J = 0.00000$
$K^3\text{-}H = 0.00000$
$K^4\text{-}G = 0.00000$
$K^5\text{-}F = 0.00000$
$K^6\text{-}E = 0.00000$
$K^7\text{-}D = 0.00000$
$K^8\text{-}C = 0.00000$
$K^9\text{-}B = 0.00000$
$K^{10}\text{-}A = 0.00000$
$\frac{L}{K} = 1.19523$

And if given any member of a series, save the square root, we have from 042906

Let us plead the 5th.

So, if one is clever enough, which I am not, one can actually do any root seriesl from the fact that any number is expressible as a ratio, and triangles just love ratios.

At one time I was seriously considering changing the name of the Delian Quest to One Circle, One Square, and One Line, but it did not stick.



$\sqrt{\frac{X}{Y}} = 0.83666$

$J^{\frac{1}{2}} - K = 0.00000$

$H^{\frac{1}{3}} - K = 0.00000$

$G^{\frac{1}{4}} - K = 0.00000$

$F^{\frac{1}{5}} - K = 0.00000$

$E^{\frac{1}{6}} - K = 0.00000$

$D^{\frac{1}{7}} = 0.83666$

$C^{\frac{1}{8}} - K = 0.00000$

$B^{\frac{1}{9}} - K = 0.00000$

$A^{\frac{1}{10}} - K = 0.00000$

$K^2 - J = 0.00000$

$K^3 - H = 0.00000$

$K^4 - G = 0.00000$

$K^5 - F = 0.00000$

$K^6 - E = 0.00000$

$K^7 - D = 0.00000$

$K^8 - C = 0.00000$

$K^9 - B = 0.00000$

$K^{10} - A = 0.00000$

$\frac{L}{K} = 1.19523$

040694

Unit.

Given.

$N_1 := 3 \qquad AB := N_1$

$N_2 := 4 \qquad BC := N_2$

$N_3 := 5 \qquad AC := N_3$

Inscribing A Circle In A Given Triangle

Given three sides of a triangle, what is the length of the inscribed radius?

Descriptions.

$AK := AC \qquad BD := BC \qquad AF := \frac{AC^2 + AB^2 - BC^2}{2 \cdot AB} \qquad FK := AK - AF \qquad CF := \sqrt{AC^2 - AF^2}$

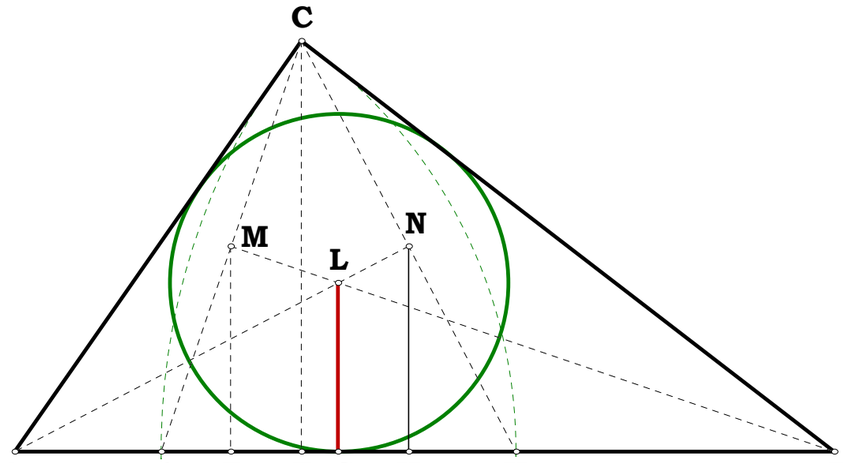
$CK := \sqrt{FK^2 + CF^2} \qquad AN := \frac{CF \cdot AK}{CK} \qquad AH := \frac{CF \cdot AN}{CK} \qquad HN := \frac{FK \cdot AN}{CK} \qquad BF := AB - AF$

$DF := BD - BF \qquad CD := \sqrt{CF^2 + DF^2} \qquad BM := \frac{CF \cdot BD}{CD} \qquad BE := \frac{CF \cdot BM}{CD} \qquad GL := \frac{HN \cdot AB}{AH + BE}$

Definitions.

$S_1 := \begin{pmatrix} AB \\ BC \\ AC \end{pmatrix} \qquad S_2 := \begin{pmatrix} BC \\ AC \\ AB \end{pmatrix} \qquad S_3 := \begin{pmatrix} AC \\ AB \\ BC \end{pmatrix} \qquad \delta := 0 .. 2$

$Radius_\delta := \frac{\sqrt{-S1_\delta + S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta - S2_\delta + S3_\delta} \cdot \sqrt{S1_\delta + S2_\delta - S3_\delta}}{2 \cdot \sqrt{S1_\delta + S2_\delta + S3_\delta}} \qquad Radius = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$



$AK - N_3 = 0 \qquad BD - N_2 = 0 \qquad AF - \frac{N_1^2 - N_2^2 + N_3^2}{2 \cdot N_1} = 0 \qquad FK - \frac{(N_2 - N_1 + N_3) \cdot (N_1 + N_2 - N_3)}{2 \cdot N_1} = 0 \qquad CF - \frac{\sqrt{(N_1 + N_2 + N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_1 + N_2 - N_3) \cdot (N_2 - N_1 + N_3)}}{2 \cdot N_1} = 0$

$CK - \frac{\sqrt{N_3 \cdot (N_1 + N_2 - N_3) \cdot (N_2 - N_1 + N_3)}}{\sqrt{N_1}} = 0 \qquad AN - \frac{N_3 \cdot \sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot \sqrt{N_1} \cdot \sqrt{N_3 \cdot (N_1 + N_2 - N_3) \cdot (N_2 - N_1 + N_3)}} = 0 \qquad AH - \frac{(N_1 - N_2 + N_3) \cdot (N_1 + N_2 + N_3)}{4 \cdot N_1} = 0$

$HN - \frac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{4 \cdot N_1} = 0 \qquad BF - \frac{N_1^2 + N_2^2 - N_3^2}{2 \cdot N_1} = 0 \qquad DF - \frac{(N_2 - N_1 + N_3) \cdot (N_1 - N_2 + N_3)}{2 \cdot N_1} = 0$

$CD - \frac{\sqrt{N_2 \cdot (N_2 - N_1 + N_3) \cdot (N_1 - N_2 + N_3)}}{\sqrt{N_1}} = 0 \qquad BM - \frac{N_2 \cdot \sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot \sqrt{N_1} \cdot \sqrt{N_2 \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3)}} = 0 \qquad BE - \frac{(N_1 + N_2 - N_3) \cdot (N_1 + N_2 + N_3)}{4 \cdot N_1} = 0$

$GL - \frac{\sqrt{(N_1 + N_2 - N_3) \cdot (N_1 - N_2 + N_3) \cdot (N_2 - N_1 + N_3) \cdot (N_1 + N_2 + N_3)}}{2 \cdot (N_1 + N_2 + N_3)} = 0$

Imagine that, another equation for the radius along with every other definition!

DE = 0.58830

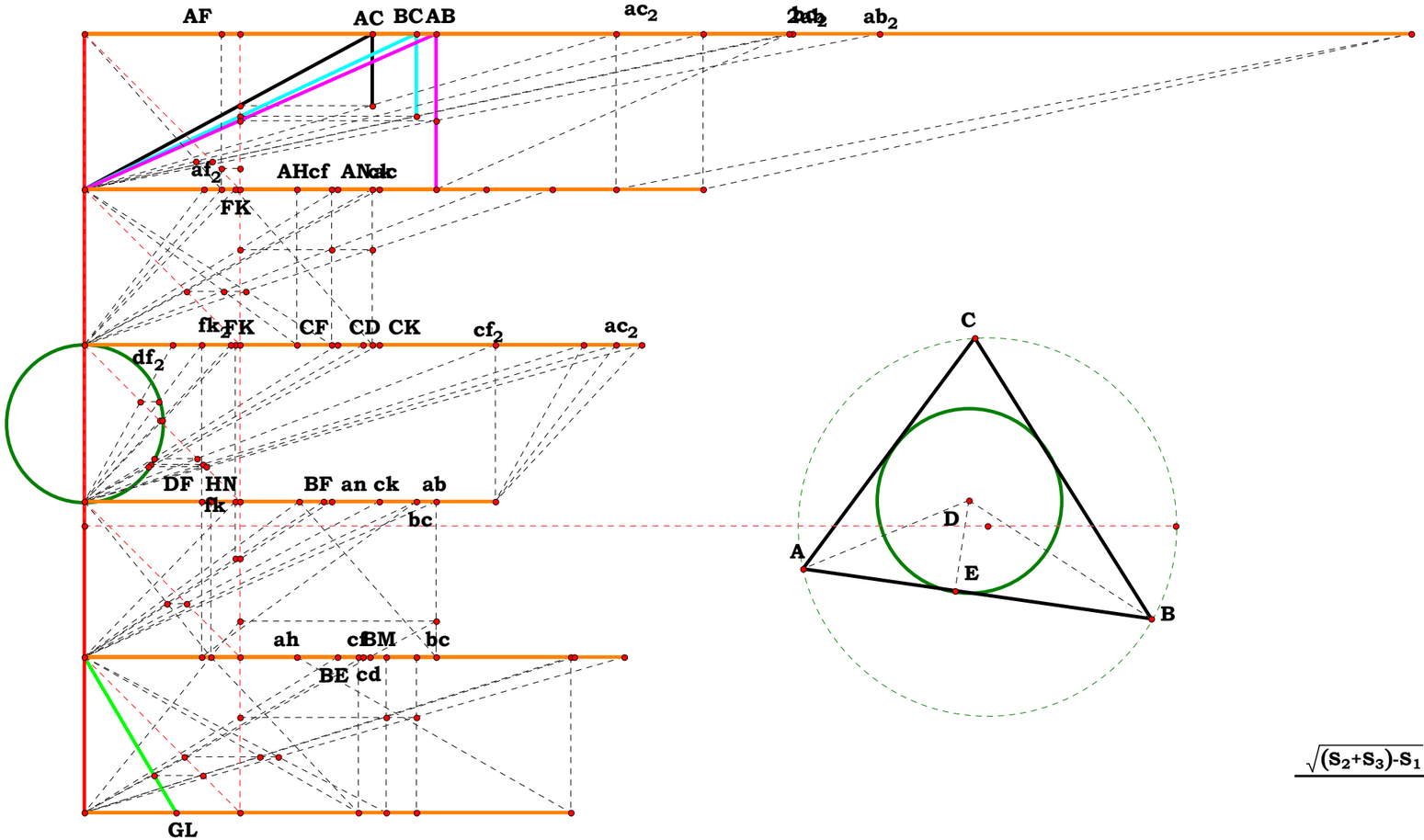
GL = 0.58830

DE-GL = 0.00000

AB = 2.25959

BC = 2.13235

AC = 1.84623



$S_1 = 2.25959$

$S_2 = 2.13235$

$S_3 = 1.84623$

$$\frac{\sqrt{(S_2+S_3)-S_1}\cdot\sqrt{(S_1+S_3)-S_2}\cdot\sqrt{(S_1+S_2)-S_3}}{2\cdot\sqrt{S_1+S_2+S_3}}-GL = 0.00000$$

Unit.

$AB := 1$

Given.

$N := 5$

042194

Descriptions.

$AL := AB \cdot N \qquad AF := \sqrt{AB \cdot AL}$

$AC := \left(AB^2 \cdot AL\right)^{\frac{1}{3}} \qquad AJ := \left(AB \cdot AL^2\right)^{\frac{1}{3}}$

$BL := AL - AB \qquad BP := BL \qquad LR := BL \qquad FL := AL - AF$

$BC := AC - AB \qquad BJ := AJ - AB \qquad JL := BL - BJ$

$BF := AF - AB \qquad FJ := AJ - AF \qquad CF := AF - AC$

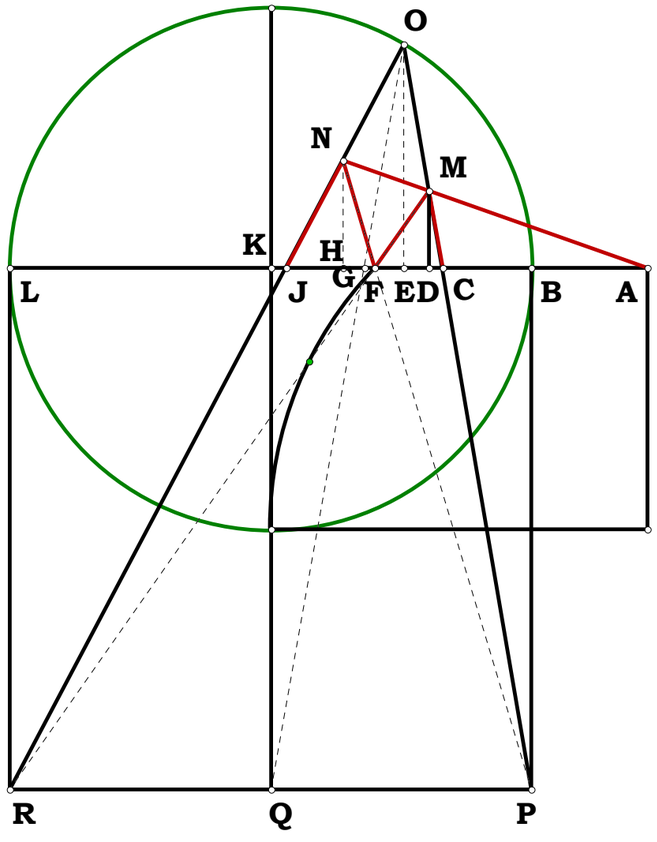
$FG := \frac{BF \cdot FJ}{BF + JL} \qquad GN := \frac{BP \cdot FG}{BF} \qquad CD := \frac{BC \cdot CF}{BC + FL}$

$DM := \frac{BP \cdot CD}{BC} \qquad AD := AC + CD \qquad AG := AF + FG$

$\frac{AG}{GN} - \frac{AD}{DM} = 0$

The Cradle

Are A, M, N colinear?



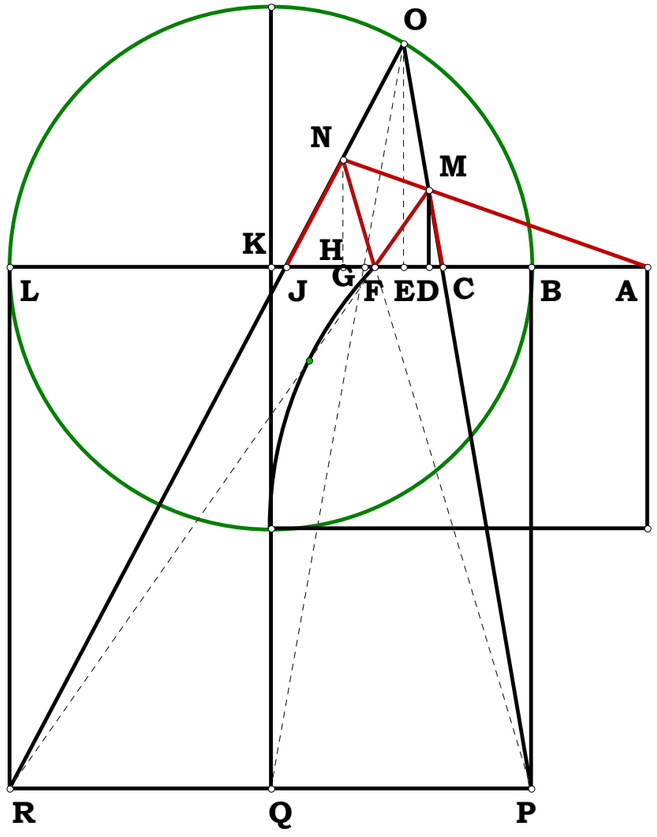
Definitions.

$AL - N = 0$ $\quad AF - \sqrt{N} = 0$ $\quad AC - N^{\frac{1}{3}} = 0$

$AJ - N^{\frac{2}{3}} = 0$ $\quad BL - (N - 1) = 0$ $\quad FL - \left(N - \sqrt{N}\right) = 0$

$BC - \left(N^{\frac{1}{3}} - 1\right) = 0$ $\quad BJ - \left(N^{\frac{2}{3}} - 1\right) = 0$ $\quad JL - \left(N - N^{\frac{2}{3}}\right) = 0$ $\quad BF - \left(\sqrt{N} - 1\right) = 0$

$FJ - \left(N^{\frac{2}{3}} - \sqrt{N}\right) = 0$ $\quad CF - \left[\sqrt{N} - (N)^{\frac{1}{3}}\right] = 0$ $\quad FG - \dfrac{\sqrt{N} \cdot \left(\sqrt{N} - 1\right)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$



$GN - \dfrac{\sqrt{N} \cdot (N - 1)}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$ $\quad CD - \dfrac{N^{\frac{2}{3}} - N^{\frac{1}{3}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$

$DM - \dfrac{N^{\frac{4}{3}} - N^{\frac{1}{3}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$ $\quad AD - \dfrac{N + \sqrt{N} + N^{\frac{2}{3}} + N^{\frac{5}{6}} + N^{\frac{7}{6}}}{\sqrt{N} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$

$AG - \dfrac{N + N^{\frac{2}{3}} + N^{\frac{4}{3}} + N^{\frac{5}{6}} + N^{\frac{7}{6}}}{N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}} + 1} = 0$ $\quad \dfrac{AG}{GN} - \dfrac{\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}}}{N - 1} = 0$ $\quad \dfrac{AD}{DM} - \dfrac{\sqrt{N} + N^{\frac{1}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{6}} + N^{\frac{5}{6}}}{N - 1} = 0$

042694

Given.

$N_1 := 3$

$N_2 := 2$

$N_3 := 7$

Descriptions.

$R_1 := \sqrt{N_1^2}$ $R_2 := \sqrt{N_2^2}$

What is the length of the AO, O being the similarity point?

$AC := R_1$ $BD := R_2$ $AB := N_3$

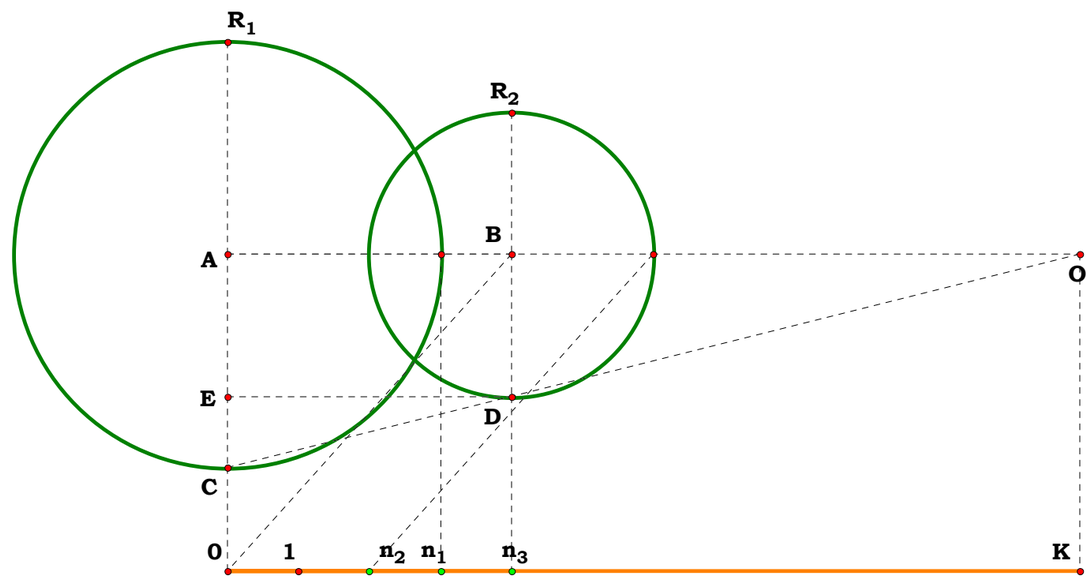
$DE := AB$ $AE := BD$ $CE := AC - AE$

$AO := \frac{DE \cdot AC}{CE}$ $AO = 21$

$\frac{N_3 \cdot R_1}{R_1 - R_2} = 21$

$AO - \frac{N_3 \cdot R_1}{R_1 - R_2} = 0$

Tangents and Similarity Points.



What is the length of the tangent GO?

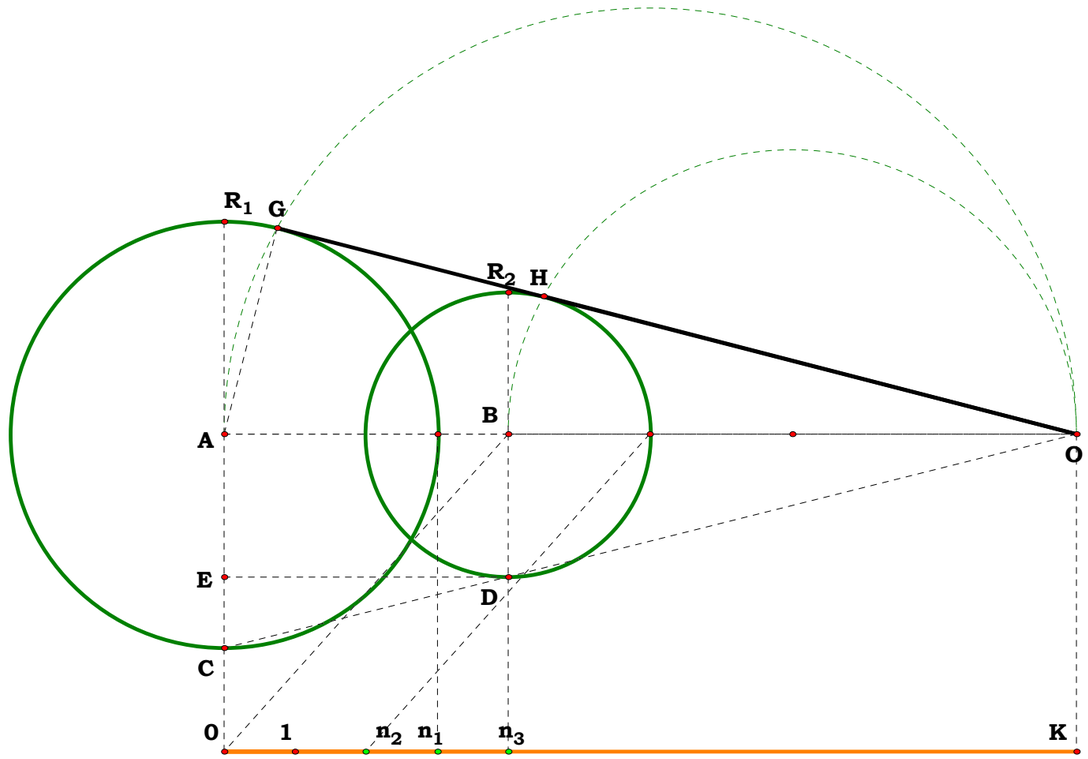
$$AG := R_1 \qquad GO := \sqrt{AO^2 - AG^2}$$

$$GO - \frac{R_1 \cdot \sqrt{N_3^2 - R_1^2 + 2 \cdot R_1 \cdot R_2 - R_2^2}}{\sqrt{(R_1 - R_2)^2}} = 0$$

What is the length of the tangent HO?

$$BH := R_2 \qquad HO := \sqrt{\left(\frac{N_3 \cdot R_1}{R_1 - R_2} - N_3\right)^2 - R_2^2}$$

$$HO - \frac{R_2 \cdot \sqrt{N_3^2 - R_1^2 + 2 \cdot R_1 \cdot R_2 - R_2^2}}{\sqrt{(R_1 - R_2)^2}} = 0$$



What is the length of line tangent to tangent of these circles?

$$GH := \frac{GO \cdot AB}{AO} \qquad GH - \sqrt{N_3^{\,2} - R_1^{\,2} + 2 \cdot R_1 \cdot R_2 - R_2^{\,2}} = 0$$

What are the names of the tangents AP and BP to the similarity point P?

$$AP := \frac{N_3 \cdot R_1}{R_1 + R_2} \qquad BP := N_3 - \frac{N_3 \cdot R_1}{R_1 + R_2}$$

What is JP?

$$JP := \sqrt{AP^2 - R_1^{\,2}} \qquad JP - \frac{R_1 \cdot \sqrt{N_3^{\,2} - R_1^{\,2} - 2 \cdot R_1 \cdot R_2 - R_2^{\,2}}}{R_1 + R_2} = 0$$

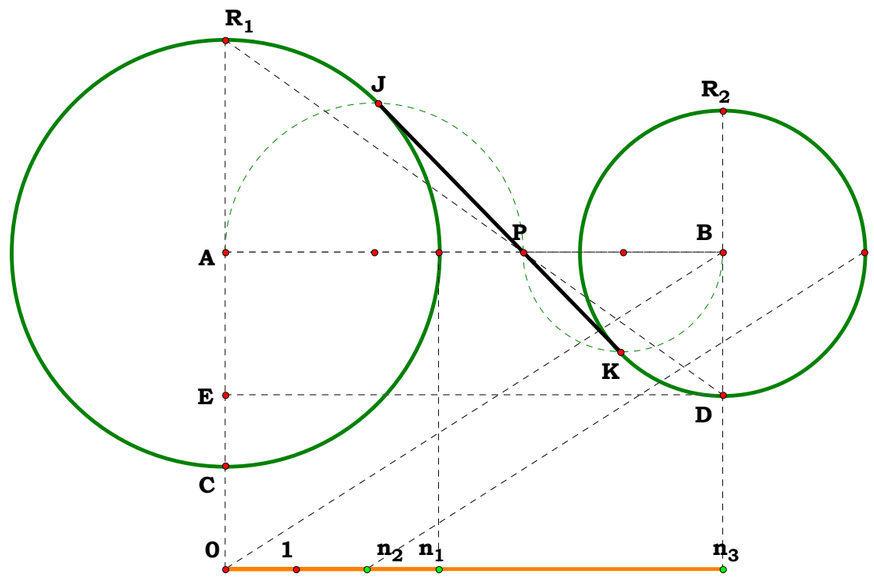
What is KP?

$$BP - \frac{N_3 \cdot R_2}{R_1 + R_2} = 0$$

$$KP := \frac{JP \cdot BP}{AP} \qquad KP - \frac{R_2 \cdot \sqrt{N_3^{\,2} - R_1^{\,2} - 2 \cdot R_1 \cdot R_2 - R_2^{\,2}}}{R_1 + R_2} = 0$$

What is JK?

$$JK := \frac{JP \cdot AB}{AP} \qquad JK - \sqrt{N_3^{\,2} - R_1^{\,2} - 2 \cdot R_1 \cdot R_2 - R_2^{\,2}} = 0$$



042794

Given.

$N_1 := 3$

$N_2 := 2$

$N_3 := 6$

Descriptions.

$R_1 := \sqrt{N_1^2}$ $R_2 := \sqrt{N_2^2}$ $AB := N_3$

$AC := \frac{R_1^2}{AB}$ $BD := \frac{R_2^2}{AB}$ $CD := AB - (AC + BD)$

$CE := \frac{CD}{2}$ $AE := AC + CE$ $BE := AB - AE$

$$AE - \frac{N_3^2 + R_1^2 - R_2^2}{2 \cdot N_3} = 0$$

$$BE - \frac{N_3^2 - R_1^2 + R_2^2}{2 \cdot N_3} = 0$$

Definitions.

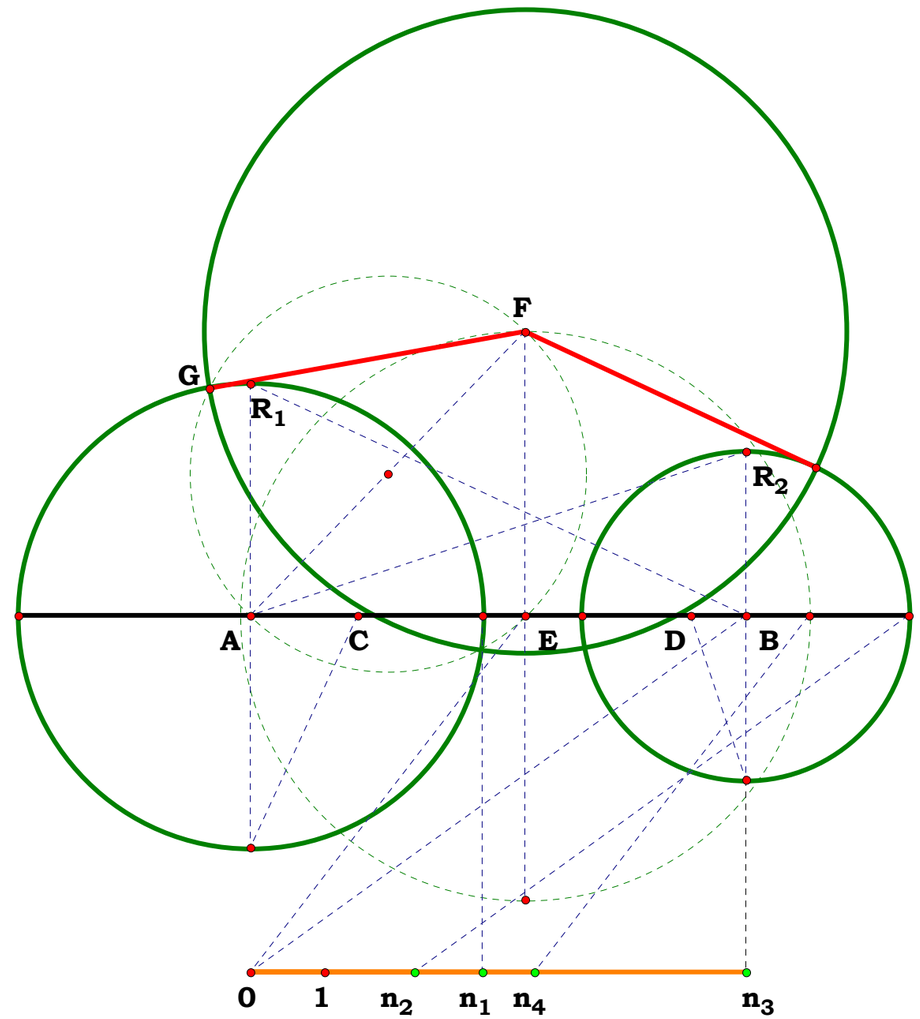
If these equations look familiar, see 01/08/93 The perpendicular of a Triangle would be on the powerline.

$N_4 := .698$ $AF := \sqrt{AE^2 + N_4^2}$ $AG := R_1$ $FG := \sqrt{AF^2 - R_1^2}$

$$FG - \frac{\sqrt{N_3^2 \cdot \left(N_3^2 + 4 \cdot N_4^2 - 2 \cdot R_1^2 - 2 \cdot R_2^2\right) + \left(R_1 - R_2\right)^2 \cdot \left(R_1 + R_2\right)^2}}{2 \cdot \sqrt{N_3^2}} = 0$$

The Chordal or Power Line of two Circles

The solution for finding the chordal as outlined in *100 Great Problems of Elementary Mathematics Their History and Solution* by Heinrich Dörrie did not lend itself to an ordered process, so I took a couple of minuets (Bach) and developed my own method.
One of the attributes of the powerline is that a circle drawn with a center on the powerline which cuts one perpendicularly will cut the other as such.
Given two circles find their chordal or power line given just their radius and difference between their centers. Then, pick a spot on the power line and write the equation for the tangent circle.



042894

Given.

$N_1 := 3$ $D_1 := 7.81447$

$N_2 := 1$ $D_2 := 6.96686$

$N_3 := 2$ $D_3 := 5.33279$

Given three circles find their power point and if it is at all possible, cut all three perpendicularly. Demonstrate an Algebraic name for the power point and the length of the resultant tangent.

Descriptions.

$R_1 := \sqrt{N_1^2}$ $R_2 := \sqrt{N_2^2}$ $R_3 := \sqrt{N_3^2}$ $AC := D_1$ $AE := D_2$ $CE := D_3$

$$AG := \frac{\sqrt{\left(R_1^2 + D_1^2 - R_2^2\right)^2}}{2 \cdot D_1} \qquad AH := \frac{\sqrt{\left(R_1^2 + D_2^2 - R_3^2\right)^2}}{2 \cdot D_2} \qquad AM := \frac{\sqrt{\left(D_2^2 + D_1^2 - D_3^2\right)^2}}{2 \cdot D_1}$$

$EM := \sqrt{AE^2 - AM^2}$ $AK := \frac{AE \cdot AH}{AM}$ $GK := AK - AG$ $GJ := \frac{AM \cdot GK}{EM}$

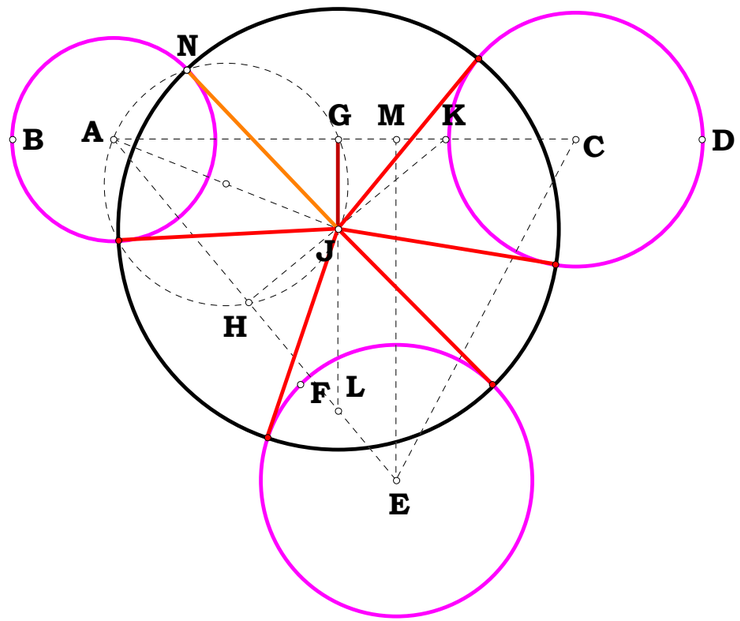
$AJ := \sqrt{AG^2 + GJ^2}$ $AN := R_1$ $JN := \sqrt{AJ^2 - AN^2}$

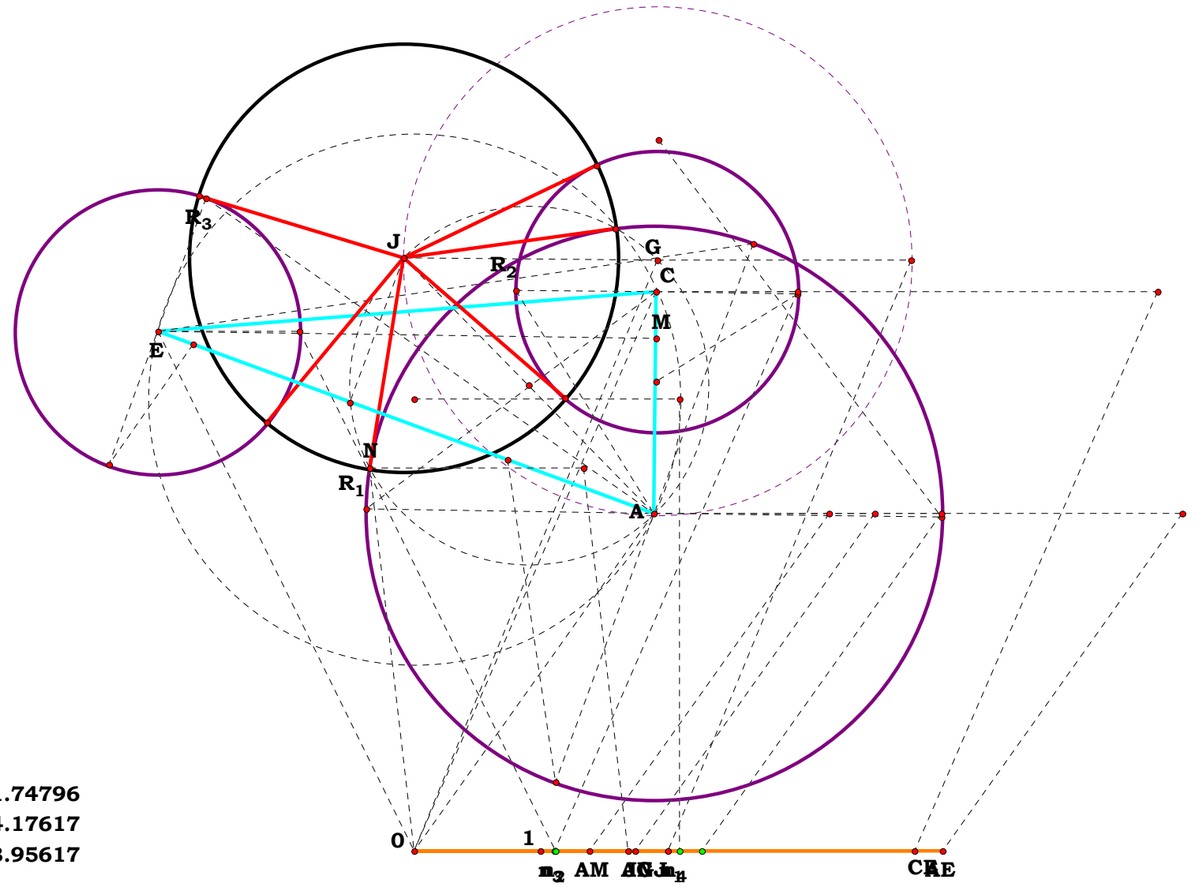
Definitions.

$$GJ - \frac{2 \cdot D_1^2 \cdot \sqrt{\left(D_2^2 + R_1^2 - R_3^2\right)^2} - \sqrt{\left(D_1^2 + D_2^2 - D_3^2\right)^2} \cdot \sqrt{\left(D_1^2 + R_1^2 - R_2^2\right)^2}}{2 \cdot \sqrt{D_1^2} \cdot \sqrt{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_2 - D_1 + D_3\right) \cdot \left(D_1 + D_2 + D_3\right)}} = 0$$

$$JN - \sqrt{\frac{\begin{aligned} &D_1^2 \cdot R_3^2 \cdot \left(R_1^2 + R_2^2 - R_3^2 - D_1^2\right) + D_1^2 \cdot D_2^2 \cdot \left(R_2^2 + R_3^2 - D_3^2\right) \ \ldots \\ &+ D_2^2 \cdot R_1^2 \cdot \left(D_3^2 + R_2^2 - R_3^2\right) + D_1^2 \cdot D_3^2 \cdot \left(R_1^2 + R_3^2\right) + D_3^2 \cdot R_1^2 \cdot \left(R_2^2 + R_3^2 - D_3^2 - R_1^2\right) \ \ldots \\ &+ D_2^2 \cdot R_2^2 \cdot \left(R_3^2 - D_2^2 - R_2^2\right) + D_3^2 \cdot R_2^2 \cdot \left(D_2 - R_3\right) \cdot \left(D_2 + R_3\right) - D_1^2 \cdot R_1^2 \cdot R_2^2 \end{aligned}}{\left(D_1 + D_2 - D_3\right) \cdot \left(D_1 + D_2 + D_3\right) \cdot \left(D_1 - D_2 + D_3\right) \cdot \left(D_1 - D_2 - D_3\right)}} = 0$$

Power Point





$N_1 = 2.27017$ $R_1 = 2.27017$ $AC = 1.74796$ $D_1 = 1.74796$

$N_2 = 1.11001$ $R_2 = 1.11001$ $AE = 4.17617$ $D_2 = 4.17617$

$N_3 = 1.12235$ $R_3 = 1.12235$ $CE = 3.95617$ $D_3 = 3.95617$

$N_4 = 2.09689$ $AM = 1.38576$

$GJ = 2.00571$

$JN = 1.68883$

Animate Points

$$\frac{\sqrt{((D_1^2+D_2^2)-D_3^2)^2}}{2\cdot D_1}\text{-AM} = 0.00000$$

$$\frac{2\cdot D_1^2\cdot\sqrt{((D_2^2+R_1^2)-R_3^2)^2}-\sqrt{((D_1^2+D_2^2)-D_3^2)^2}\cdot\sqrt{((D_1^2+R_1^2)-R_2^2)^2}}{2\cdot\sqrt{D_1^2}\cdot\sqrt{((D_1+D_2)-D_3)\cdot((D_1-D_2)+D_3)\cdot((D_2-D_1)+D_3)\cdot(D_1+D_2+D_3)}}\text{-GJ} = 0.00000$$

$$\sqrt{\frac{(D_1^2.R_3^2.((R_1^2+R_2^2)-R_3^2-D_1^2)+D_1^2.D_2^2.((R_2^2+R_3^2)-D_3^2))+(D_2^2.R_1^2.((D_3^2+R_2^2)-R_3^2)+D_1^2.D_3^2.(R_1^2+R_3^2)+D_3^2.R_1^2.((R_2^2+R_3^2)-D_3^2-R_1^2))+((D_2^2.R_2^2.(R_3^2-D_2^2-R_2^2)+D_3^2.R_2^2.(D_2-R_3)\cdot(D_2+R_3))-D_1^2.R_1^2.R_2^2)}{(((D_1+D_2)-D_3)\cdot(D_1+D_2+D_3)\cdot((D_1-D_2)+D_3)\cdot(D_1-D_2-D_3))}}\text{-JN} = 0.00000$$

042994

Given.

$N_1 := 2$ $\quad AF := N_1$

$N_2 := 3$ $\quad BE := N_2$

$N_3 := 5$ $\quad AB := N_3$

Descriptions.

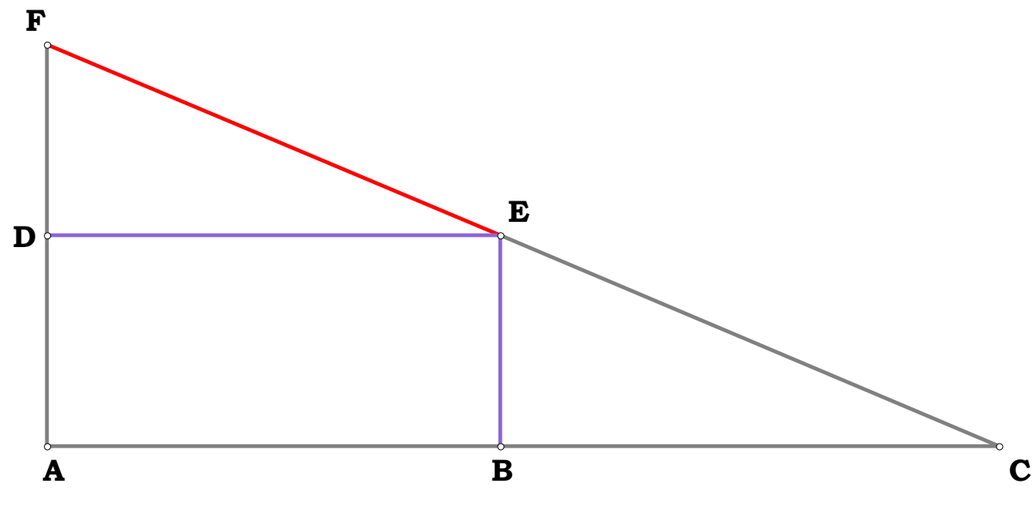
$AD := BE \qquad DE := AB$

$DF := AF - AD \qquad EF := \left(DF^2 + DE^2\right)^{\frac{1}{2}}$

Definitions.

$EF - \sqrt{N_1{}^2 - 2 \cdot N_1 \cdot N_2 + N_2{}^2 + N_3{}^2} = 0$

Given AB, AF, BE, what is EF?



Given.

$N_1 := 5 \qquad AB := N_1$

$N_2 := 2 \qquad BC := N_2$

043094

Descriptions.

$$AC := \sqrt{AB^2 + BC^2}$$

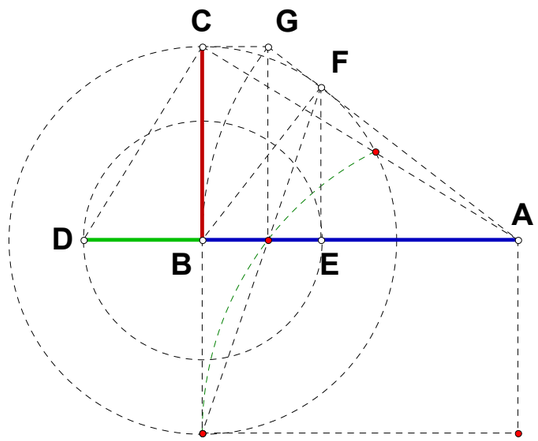
$$CD := \frac{BC \cdot AC}{AB}$$

$$BD := \sqrt{CD^2 - BC^2}$$

Definitions.

$$\frac{N_2^2}{N_1} - BD = 0$$

Division N^2



Given.

$R_1 := 3$ $DE := R_1$

$R_2 := 2$ $BC := R_2$

Two Circles And A Parallel

Given the radius of two tangent circles find the radius of the third that is tangent to the two circles and tangent to the parallel opposite AP which is tangent to the larger circle.

050194

Descriptions.

$CN := BC$ $EQ := DE$ $CD := BC$ $CE := CD + DE$ $ES := CN$ $NS := CE$

$SQ := EQ - ES$ $AE := \frac{NS \cdot EQ}{SQ}$ $AD := AE - DE$ $EP := DE$ $AP := \sqrt{AE^2 - EP^2}$

$DO := \frac{EP \cdot AD}{AP}$ $DL := \frac{DO \cdot DE}{CD}$ $AC := AD - CD$ $AM := \frac{AP \cdot AC}{AE}$ $AO := \frac{AE \cdot AD}{AP}$

$MO := AO - AM$ $MR := \frac{AD \cdot MO}{AO}$ $RO := \frac{DO \cdot MR}{AD}$ $DR := DO - RO$ $LR := DR + DL$

$ML := \sqrt{MR^2 + LR^2}$ $DK := \frac{MR \cdot DL}{LR}$ $CK := DK - CD$ $CH := \frac{LR \cdot CK}{ML}$ $CM := BC$

$MH := \sqrt{CM^2 - CH^2}$ $MG := 2 \cdot MH$ $GL := ML - MG$ $GJ := \frac{CM \cdot GL}{MG}$

Definitions.

$R_3 := \frac{R_1^2}{4 \cdot R_2}$ $GJ - R_3 = 0$

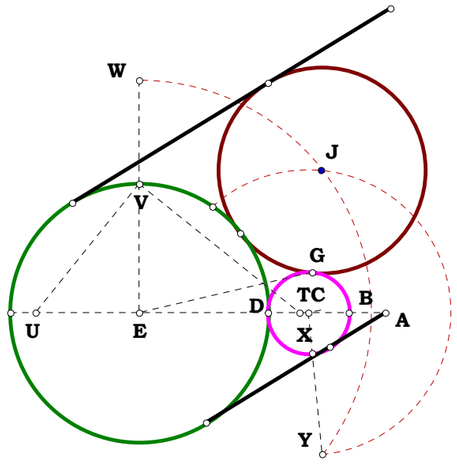
The Algebraic name for GJ suggests a simpler method of construction.

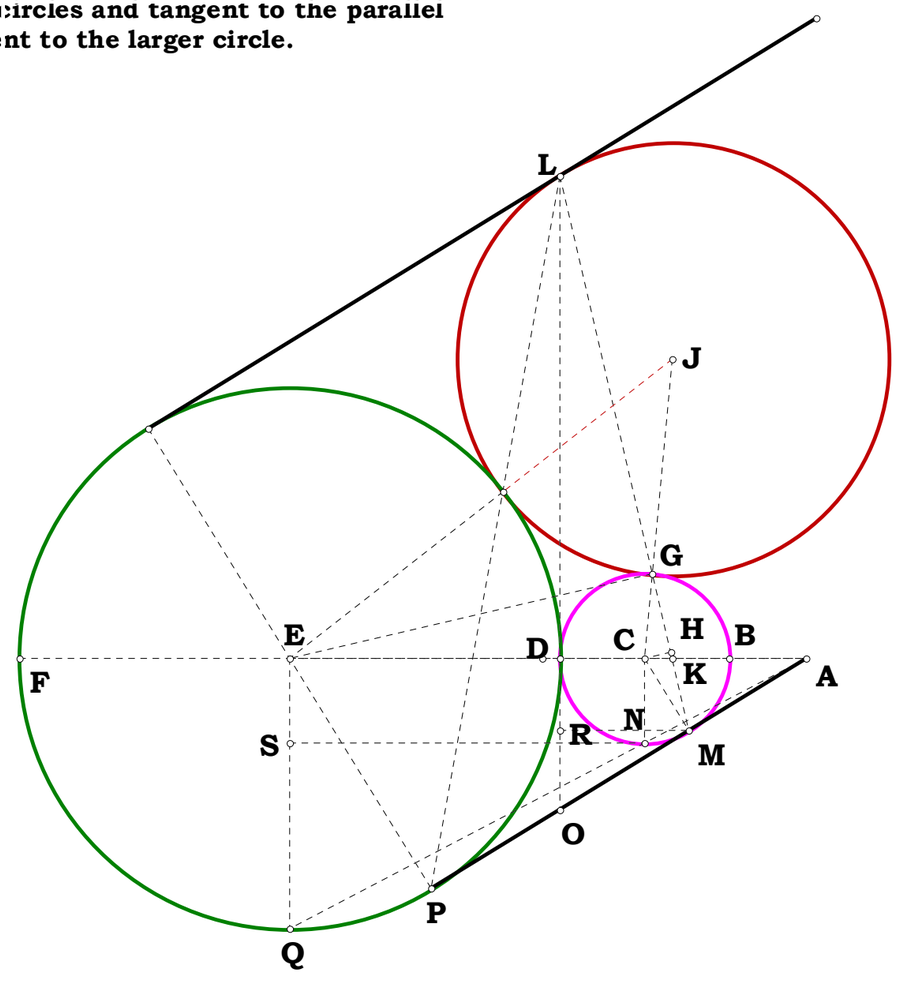
$ET := 4 \cdot BC$ $EV := DE$ $EU := \frac{DE^2}{ET}$

$VW := EU$ $XY := EU$ $EV := DE$ $CX := BC$

$EW := EV + VW$ $CY := CX + XY$

$GJ_2 := EU$ $GJ - GJ_2 = 0$





050494

Descriptions.

Given.

$R_1 := 1.49167$ $FK := R_1$

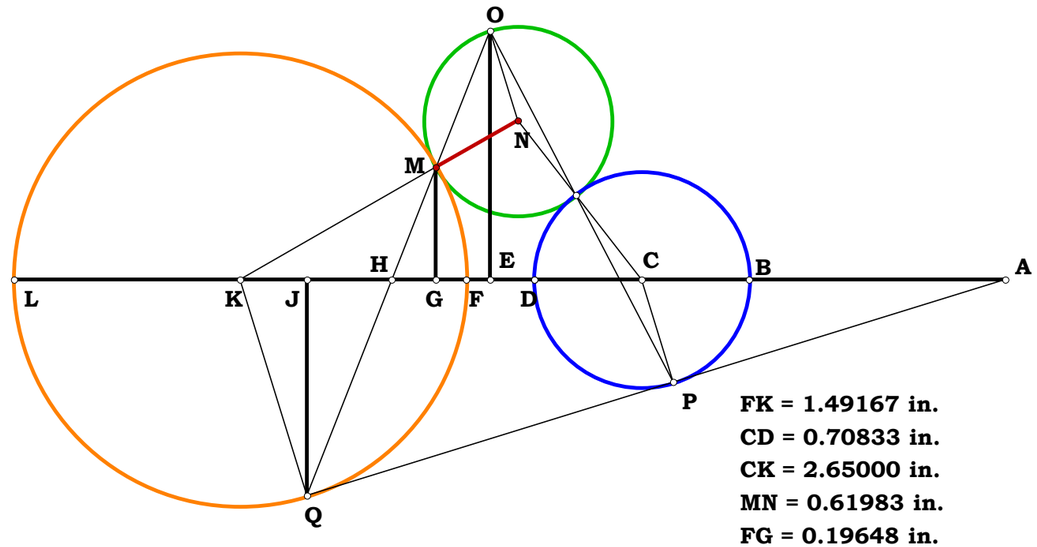
$R_2 := .70833$ $BC := R_2$

$D := 2.65$ $CK := D$

$N := 15.18411$

Two Circles And A Tangent

Given two circles, the difference between them and a point on one expressed as a ratio of the diameter, find the circle tangent to both at that point.



FK = 1.49167 in.
CD = 0.70833 in.
CK = 2.65000 in.
MN = 0.61983 in.
FG = 0.19648 in.
$\frac{2 \cdot FK}{FG} = 15.18411$

$FL := 2 \cdot FK$ $AK := \frac{D \cdot R_1}{R_1 - R_2}$ $EK := \frac{R_1^2 + D^2 - R_2^2}{2 \cdot D}$

$AQ := R_1 \cdot \frac{\sqrt{(R_1 - R_2 + D) \cdot (-R_1 + R_2 + D)}}{R_1 - R_2}$ $FG := \frac{FL}{N}$

$GL := FL - FG$ $GM := \sqrt{FG \cdot GL}$ $AJ := \frac{AQ \cdot AQ}{AK}$ $AF := AK - FK$

$FJ := AJ - AF$ $JL := FL - FJ$ $JQ := \sqrt{FJ \cdot JL}$ $GJ := FJ - FG$

$QM := \sqrt{(JQ + GM)^2 + GJ^2}$ $GH := \frac{GJ \cdot GM}{JQ + GM}$ $HM := \frac{QM \cdot GM}{JQ + GM}$

$EF := EK - FK$ $EH := EF + FG + GH$ $HO := \frac{HM \cdot EH}{GH}$ $MO := HO - HM$

$KM := FK$ $MN := \frac{KM \cdot MO}{QM}$ $MN = 0.619833$

Definitions.

$$MN - \frac{(4 \cdot R_1 \cdot D) - N \cdot (R_2 + D - R_1) \cdot (R_2 + R_1 - D)}{2N \cdot (R_2 + D - R_1) - 4 \cdot D} = 0$$

Unit

$FH := 1$

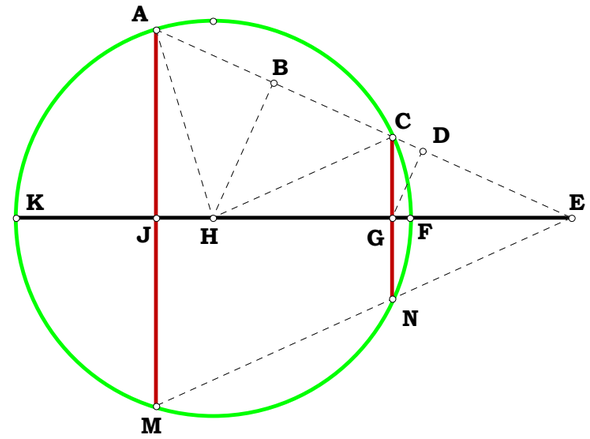
Given.

$N := 2.423$

050694

A Ratio In Trisection

What is AJ to CG?



Descriptions.

$CE := FH \qquad CG := \frac{FH}{N} \qquad EG := \sqrt{CE^2 - CG^2} \qquad CD := \frac{CG^2}{CE}$

$DG := \sqrt{CG^2 - CD^2} \qquad EH := 2 \cdot EG \qquad BH := \frac{DG \cdot EH}{EG} \qquad CH := FH$

$BC := \sqrt{CH^2 - BH^2} \qquad AC := 2 \cdot BC \qquad AE := AC + CE \qquad AJ := \frac{CG \cdot AE}{CE}$

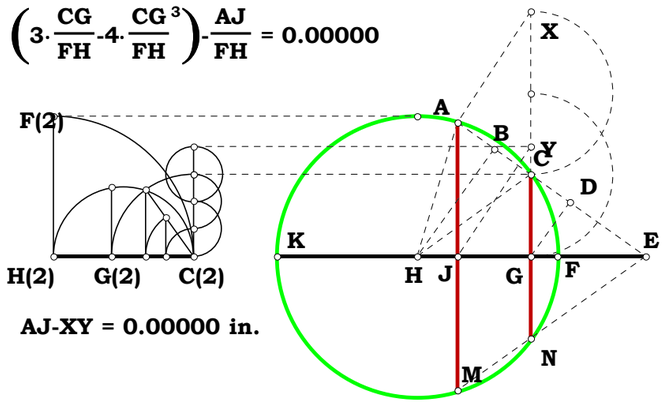
$3 \cdot CG - \frac{4 \cdot CG^3}{CE^2} - AJ = 0 \qquad 3 \cdot CG - 4 \cdot CG^3 - AJ = 0 \qquad AJ - \frac{\left(3 \cdot N^2 - 4\right)}{N^3} = 0$

Definitions.

$AJ - \left(\frac{3}{N} - \frac{4}{N^3}\right) = 0$

The resultant equation suggests this construction.

$\left(3 \cdot \frac{CG}{FH} - 4 \cdot \frac{CG^3}{FH}\right) - \frac{AJ}{FH} = 0.00000$



AJ-XY = 0.00000 in.

Unit.

$AE := 1$

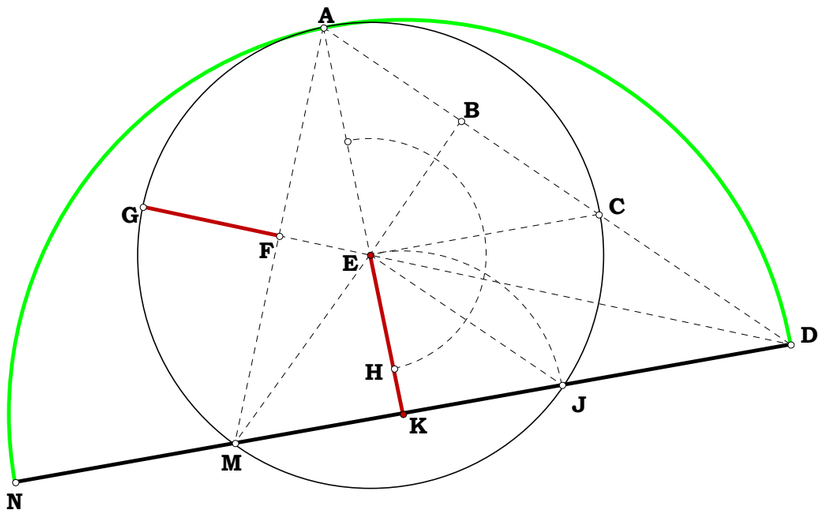
Given.

$N := 2$

050794

A Trisection Ratio

In trisection, what is the ratio of FG/EK?



Descriptions.

$$EH := \frac{AE}{2} \qquad HK := AE \cdot N \qquad AK := AE + EH + HK$$

$$EJ := AE \qquad EK := EH + HK \qquad AD := \frac{EJ \cdot AK}{EK} \qquad CD := AE$$

$$AC := AD - CD \qquad BC := \frac{AC}{2} \qquad CE := AE \qquad BE := \sqrt{CE^2 - BC^2}$$

$$BD := CD + BC \qquad DE := \sqrt{BD^2 + BE^2} \qquad DF := \frac{BD \cdot AD}{DE}$$

$$EG := AE \qquad DG := DE + EG \qquad FG := DG - DF$$

Definitions.

$$FG - \left[1 + \frac{(N+1)\cdot(2\cdot N-1)}{(2N+1)\cdot\sqrt{(N+1)\cdot(2N+1)}}\right] = 0 \qquad EK - \frac{1}{2} - N = 0$$

$$\frac{FG}{EK} - 2\cdot\frac{\left[2\cdot N\cdot\sqrt{(N+1)\cdot(2\cdot N+1)} + \sqrt{(N+1)\cdot(2\cdot N+1)} + (N+1)\cdot(2\cdot N-1)\right]}{\left[(2\cdot N+1)^2\cdot\sqrt{(N+1)\cdot(2\cdot N+1)}\right]} = 0$$

Tangent Diameter and Circles

Unit

$CF := 1$

Given.

$N_1 := 3.78824$

$N_2 := 2$

051694A

Descriptions.

$$CE := \frac{CF}{2} \qquad CD := \frac{CE}{N_1} \qquad DE := CE - CD \qquad EJ := CE \cdot N_2$$

$$DJ := \sqrt{DE^2 + EJ^2} \qquad JG := \frac{EJ^2}{DJ} \qquad BE := CE$$

$$EG := \sqrt{EJ^2 - JG^2} \qquad BG := \sqrt{BE^2 - EG^2} \qquad BJ := BG + JG$$

$$JK := CE \qquad BD := BJ - DJ \qquad DH := \frac{JK \cdot BD}{BJ}$$

Definitions.

$$CD - \frac{1}{2 \cdot N_1} = 0 \quad DE - \frac{(N_1 - 1)}{2 \cdot N_1} = 0$$

$$EJ - \frac{N_2}{2} = 0 \qquad DJ - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}}{2 \cdot N_1} = 0$$

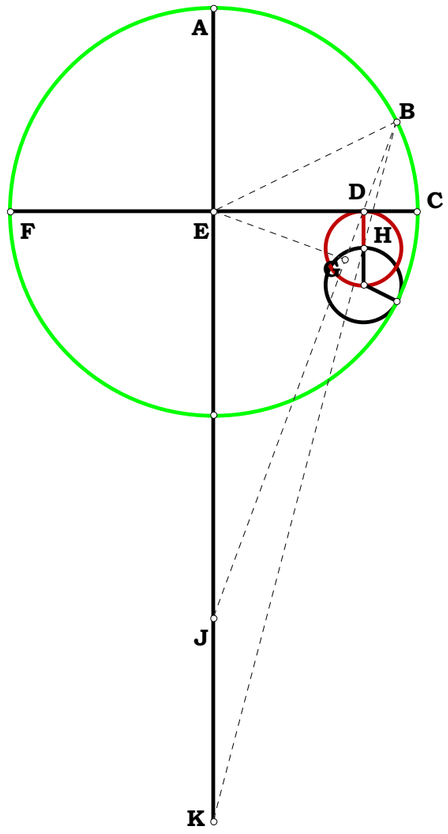
$$EG - \frac{(N_1 - 1) \cdot N_2}{2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0 \qquad JG - \frac{N_1 \cdot N_2^2}{2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0$$

$$BG - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2}}{2 \cdot \sqrt{\left(N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2\right)}} = 0 \qquad BJ - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1}{2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2}} = 0$$

$$BD - \frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{N_1^2 + 2 \cdot N_1 \cdot N_2^2 - 2 \cdot N_1 - N_2^2 + 1} - 1}{2 \cdot \sqrt{N_1^2 - 2 \cdot N_1 + 1 + N_2^2 \cdot N_1^2} \cdot N_1} = 0$$

$$DH - \frac{\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} \cdot N_1 - N_1^2 + 2 \cdot N_1 - 1}{2 \cdot N_1 \cdot \left(\sqrt{N_1^2 - 2 \cdot N_1 + 1 + 2 \cdot N_2^2 \cdot N_1 - N_2^2} + N_2^2 \cdot N_1\right)} = 0$$

Choose a point along CF and the number of circles tanget to it and to the circumscribing circle and place them in the downright position.



CF = 2.68333 in.

DH = 0.24196 in.

CE = 1.34167 in.

CE = 1.34167 in.

CD = 0.35417 in.

$\frac{CE}{CD} = 3.78824$

051694B

Unit.

$AF := 1$

Given.

$N_1 := 2$

$N_2 := 2$

Tangent Diameter and Circles

Choose a point along DF and the number of circles tanget to it and to the circumscribing circle and place them in the upright position.

Descriptions.

$DF := AF \quad DE := \frac{DF}{N_1} \quad AJ := AF \cdot N_2$

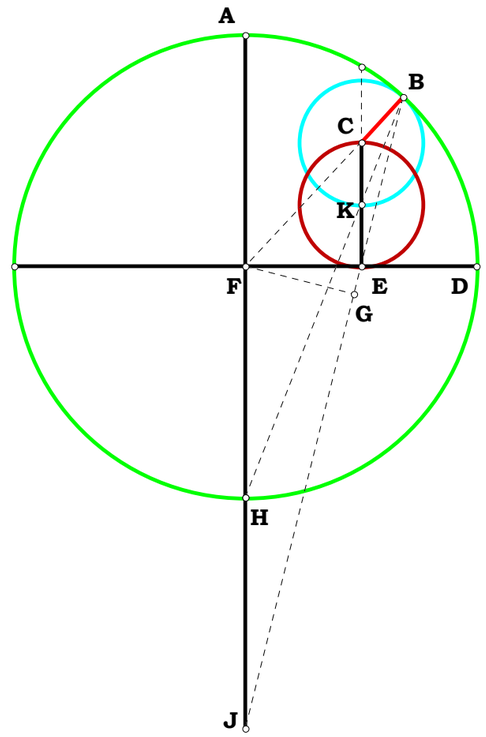
$HJ := AF \quad EF := DF - DE \quad FJ := AJ - AF$

$EJ := \sqrt{EF^2 + FJ^2} \quad EG := \frac{EF^2}{EJ} \quad BF := AF$

$FG := \sqrt{EF^2 - EG^2} \quad BG := \sqrt{BF^2 - FG^2} \quad BE := BG - EG$

$BJ := BE + EJ \quad KE := \frac{HJ \cdot BE}{BJ} \quad BC := BF - \sqrt{EF^2 + \left[\left(\frac{AJ - AF}{AF}\right) \cdot KE\right]^2}$

$BC - KE = 0 \quad KE = 0.375$



Definitions.

$$\frac{2 \cdot N_1 - N_1^2 + N_1 \cdot \sqrt{(2 \cdot N_1 - 1) \cdot N_2^2 + (2 - 4 \cdot N_1) \cdot N_2 + N_1^2} - 1}{N_1 \cdot \left[N_1 + N_1 \cdot N_2^2 - 2 \cdot N_1 \cdot N_2 + \sqrt{(2 \cdot N_1 - 1) \cdot N_2^2 + (2 - 4 \cdot N_1) \cdot N_2 + N_1^2}\right]} - BC = 0$$

102794

Unit.

$AB := 1$

Given.

$N := 5$

Descriptions.

$AG := AB \cdot N \quad BG := AG - AB \quad BF := \frac{BG}{2}$

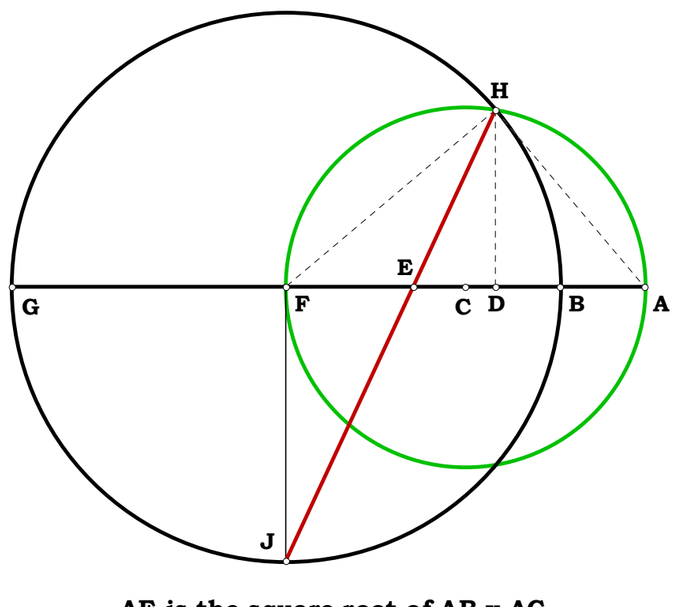
$AF := AB + BF \quad FH := BF \quad DF := \frac{FH^2}{AF} \quad AD := AF - DF$

$BD := AD - AB \quad DG := BG - BD \quad FJ := BF$

$DH := \sqrt{BD \cdot DG} \quad DE := \frac{DF \cdot DH}{DH + FJ} \quad AE := AB + BD + DE$

$\sqrt{AB \cdot AG} - AE = 0 \quad AE = 2.236068$

Trivial Method Square Root



AE is the square root of AB x AG.

Definitions.

The following is as far as Mathcad 15 will get you, the rest you have to do by hand. As I noted elsware, Mathcad is not the sharpest tool in the shed, I hope!

$$AE - \frac{\sqrt{(N+1)^2} \cdot \left[\sqrt{N \cdot (N-1)^2} \cdot \sqrt{(N+1)^2} - 2 \cdot N + 2 \cdot N^2\right]}{(N+1) \cdot \left[N \cdot \sqrt{(N+1)^2} - \sqrt{(N+1)^2} + 2 \cdot \sqrt{N \cdot (N-1)^2}\right]} = 0$$

$AE - \sqrt{N} = 0$

Unit.

$AB := 1$

Given.

$N := 5$

102894

Descriptions.

$AH := AB \cdot N \quad BH := AH - AB$

$BG := \frac{BH}{2} \quad GK := BG \quad AG := AB + BG$

$DG := \frac{GK^2}{AG} \quad AD := AG - DG \quad AL := BG$

$GL := \sqrt{AL^2 + AG^2} \quad BD := BG - DG \quad DH := BH - BD$

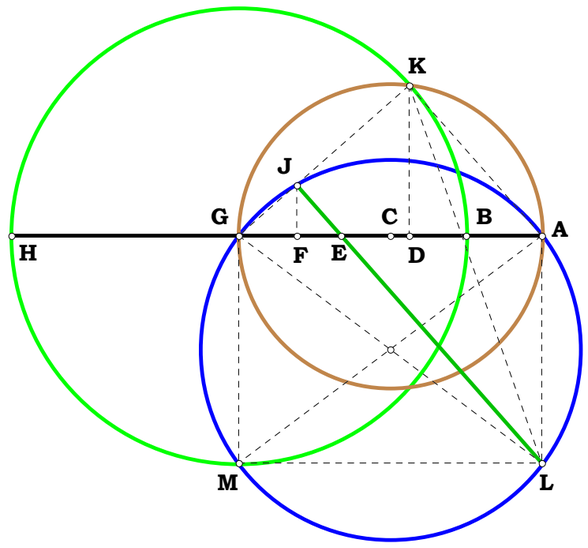
$DK := \sqrt{BD \cdot DH} \quad KL := \sqrt{AD^2 + (AL + DK)^2}$

$GJ := \frac{GL^2 + GK^2 - KL^2}{2 \cdot GK} \quad FG := \frac{DG \cdot GJ}{GK}$

$AF := AG - FG \quad FJ := \frac{DK \cdot GJ}{GK} \quad EF := \frac{AF \cdot FJ}{FJ + AL}$

$AE := AF - EF \quad \sqrt{AB \cdot AH} - AE = 0 \quad AE = 2.236068$

Trivial Method Square Root



$S_1 := GK \quad S_2 := GL \quad S_3 := KL$

$GJ - \frac{S_2^2 + S_1^2 - S_3^2}{2 \cdot S_1} = 0$

AE is the square root of AB x AH.

Definitions.

Another fine example of Mathcad's inability to function rationally. Even to get the reduction this far required too much manual labor, but if one wants to continue, then one will get to the simple result.

$$AE - \frac{(N-1)^2 \cdot (N+1) \cdot \sqrt{N \cdot (N-1)^2} + 2 \cdot N \cdot (N-1) \cdot (N+1) \cdot \sqrt{(N+1)^2}}{\left(N^3 - 3 \cdot N^2 + 3 \cdot N - 1\right) \cdot \sqrt{(N+1)^2} + \sqrt{N \cdot (N-1)^2} \cdot \left(2 \cdot N^2 + 4 \cdot N + 2\right)} = 0$$

$AE - \sqrt{N} = 0$

Unit.

$AB := 1$

Given.

$N := 22$

103194

Descriptions.

$AF := N \quad BF := AF - AB \quad AD := \frac{AF}{2}$

$AJ := AF \quad FK := AF \quad BD := AD - AB \quad BJ := \sqrt{AB^2 + AJ^2}$

$BG := \frac{AB \cdot BD}{BJ} \quad DH := AD \quad DG := \frac{AJ \cdot BD}{BJ} \quad GH := \sqrt{DH^2 - DG^2}$

$HJ := BJ + BG + GH \quad BC := \frac{AB \cdot (BG + GH)}{BJ} \quad AC := AB + BC$

$CF := AF - AC \quad CH := \sqrt{AC \cdot CF} \quad CE := \frac{CF \cdot CH}{(CH + FK)}$

$EF := CF - CE \quad BE := BC + CE \quad DF := BF - BE \quad CD := AD - AC$

Definitions.

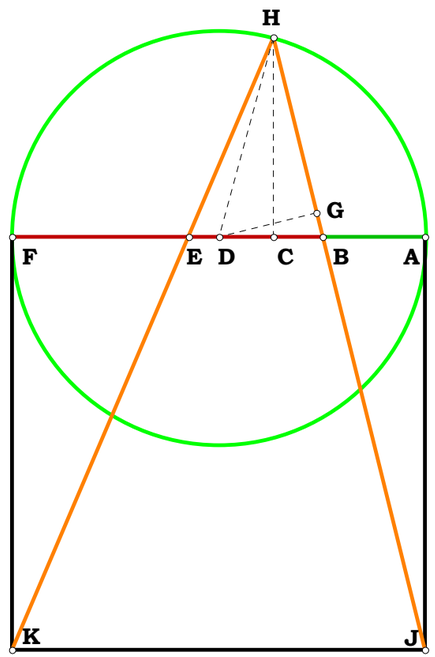
I cannot get Mathcad to transform one into the other.

$BE^2 - EF = 0 \quad \sqrt{EF} - BE = 0$

$BE - \frac{N - 2 + N \cdot \sqrt{4 \cdot N - 3}}{2 \cdot N + 1 + \sqrt{4 \cdot N - 3}} = 0$

$EF - \frac{2 \cdot N^2 - \sqrt{4 \cdot N - 3} - 2 \cdot N + 1}{2 \cdot N + \sqrt{4 \cdot N - 3} + 1} = 0$

Square Root of a Segment



Given a unit divide a segment into N and its square. Let AB be the unit and BF the segment then BE is N and EF its square.

110194a

Given.

$N_1 := 4.55192$

$N_2 := 3.86362$

Given BG and BC find AB, AG, such that $(AB^2 \cdot AG)^{1/3} = BC$.
For obvious reasons, BC between BG. It seems this is the first time I drew the figure in Sketchpad, all my other graphics came from TommyCad in the early 90's,

I am going to be presenting two identical write-ups of the figure, the only difference will be in N_2 in order to show a problem with so called mathematicians today.

Descriptions.

$BG := N_1 \quad BC := N_1 - N_2 \quad BN := BG \quad BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN} \quad FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN} \quad GM := DL \quad BD := BC + CD$

$DG := BG - BD \quad LM := DG \quad GO := BG \quad MO := GO + GM \quad EG := \frac{LM \cdot GO}{MO}$

$CG := BG - BC \quad CE := CG - EG \quad CJ := BC \quad CK := CE \quad IJ := BC$

$JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2} \quad BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

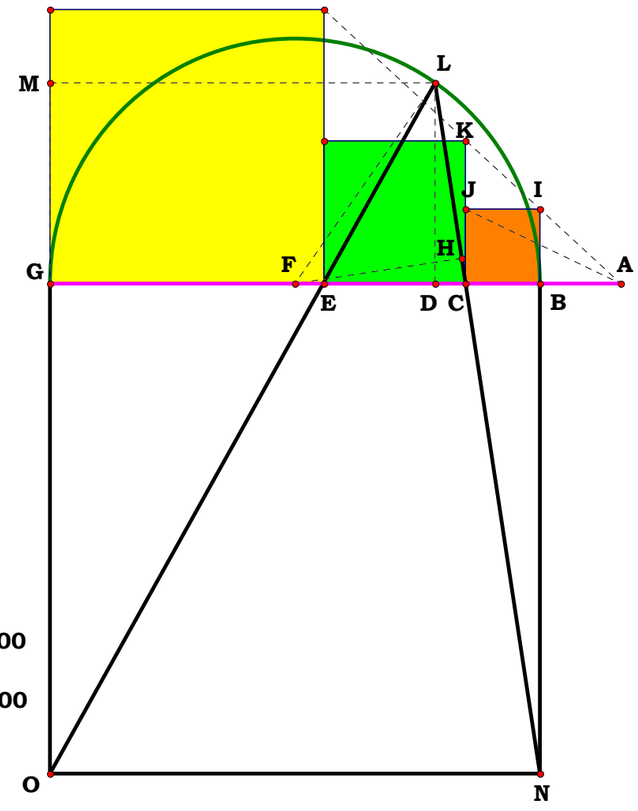
$AE := AB + BC + CE \quad AC := AB + BC \quad AB = 0.746998 \quad AG = 5.298918$

$AE = 2.757812$

Definitions.

$\left(AB^2 \cdot AG\right)^{\frac{1}{3}} - AC = 0 \qquad \left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 0$

$N_1 = 4.55192$
$N_2 = 3.86362$
$AB = 0.74699$
$BF = 2.27596$
$AG = 5.29891$
$AC = 1.43528$
$AE = 2.75779$
$(AB^2 \cdot AG)^{\frac{1}{3}} = 1.43528$
$(AB \cdot AG^2)^{\frac{1}{3}} = 2.75779$
$(AB^2 \cdot AG)^{\frac{1}{3}} - AC = 0.00000$
$(AB \cdot AG^2)^{\frac{1}{3}} - AE = 0.00000$



Here, everything looks as it should and we might get comfortable, but let us just move C.

$EG = 2.541106 \quad CE = 1.322514 \quad BC = 0.6883 \quad \frac{EG}{CE} - \frac{CE}{BC} = 0$

Given any point on a segment, cut the segment into duplicate ratios with that point.

110194b

Given.

$N_1 := 4.55192$

$N_2 := 2.63187$

Given AG - AB = BG and $(AB^2 \cdot AG)^{1/3}$ - AB = BC, find AB, AG, and $(AB \cdot AG^2)^{1/3}$.
For obvious reasons, BC between BG. It seems this is the first time I drew the figure in Sketchpad, all my other graphics came from TommyCad in the early 90's,

I am going to be presenting two identical write-ups of the figure, the only difference will be in N_2 in order to show a problem with so called mathematicians today.

Descriptions.

$BG := N_1 \quad BC := N_1 - N_2 \quad BN := BG \quad BF := \frac{BG}{2} \quad FL := BF \quad CF := BF - BC$

$CN := \sqrt{BC^2 + BN^2} \quad CH := \frac{BC \cdot CF}{CN} \quad FH := \frac{BN \cdot CF}{CN} \quad HL := \sqrt{FL^2 - FH^2}$

$CL := CH + HL \quad CD := \frac{BC \cdot CL}{CN} \quad DL := \frac{BN \cdot CL}{CN} \quad GM := DL \quad BD := BC + CD$

$DG := BG - BD \quad LM := DG \quad GO := BG \quad MO := GO + GM \quad EG := \frac{LM \cdot GO}{MO}$

$CG := BG - BC \quad CE := CG - EG \quad CJ := BC \quad CK := CE \quad IJ := BC$

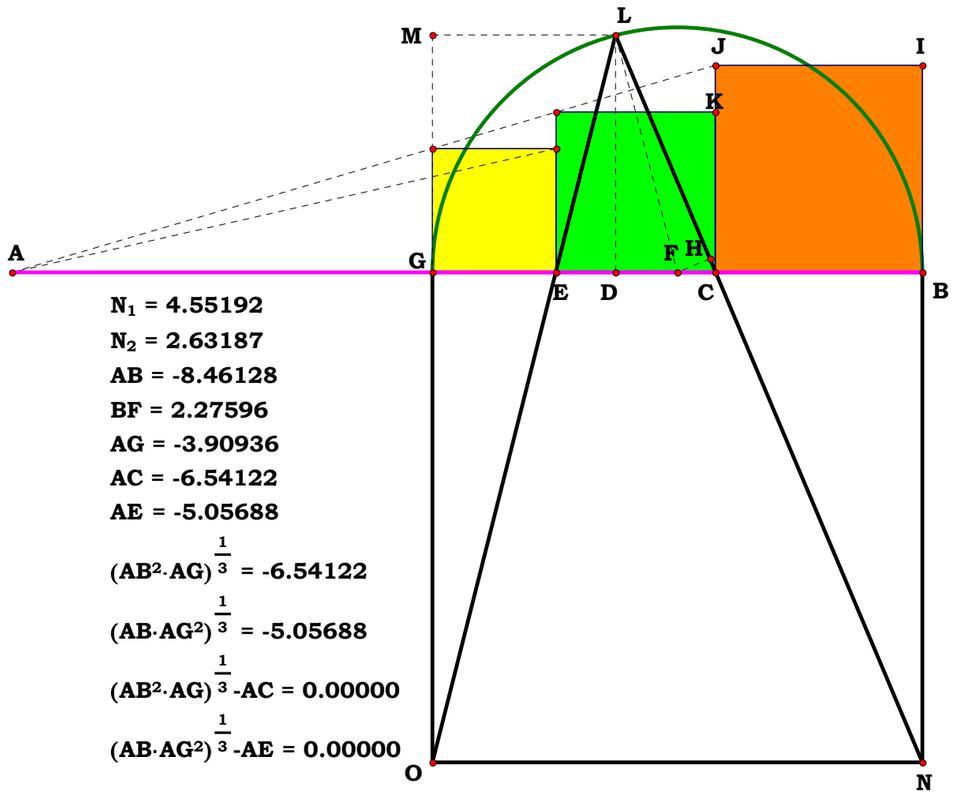
$JK := CK - CJ \quad IK := \sqrt{IJ^2 + JK^2} \quad BI := BC \quad AB := \frac{IJ \cdot BI}{JK} \quad AG := AB + BG$

$AE := AB + BC + CE \quad AC := AB + BC \quad AB = -8.461312 \quad AG = -3.909392$

$AE = -5.056911$

Definitions.

$\left(AB^2 \cdot AG\right)^{\frac{1}{3}} = 3.270631 + 5.664899i \quad \left(AB \cdot AG^2\right)^{\frac{1}{3}} - AE = 7.585367 + 4.379414i$



N_1 = 4.55192
N_2 = 2.63187
AB = -8.46128
BF = 2.27596
AG = -3.90936
AC = -6.54122
AE = -5.05688
$(AB^2 \cdot AG)^{\frac{1}{3}}$ = -6.54122
$(AB \cdot AG^2)^{\frac{1}{3}}$ = -5.05688
$(AB^2 \cdot AG)^{\frac{1}{3}}$ -AC = 0.00000
$(AB \cdot AG^2)^{\frac{1}{3}}$ -AE = 0.00000

For those who believe that math is perfect, that mathematicians are literate, here we have an example of simple geometry and a child's drawing program showing the illiteracy of one of the top math programs world-wide. How is it possible for two identical equations with identical inputs produce two different resullts? How is it possible not to know what your own grammar means? Do the equation in Windows Calculator, it will even get it right. Why can't a so called high end math program?

122494

Unit.

$AB := 1$

Given.

$N := 5$

Descriptions.

$AJ := AB \cdot N$ $\quad AF := \sqrt{AB \cdot AJ}$ $\quad BJ := AJ - AB$ $\quad BG := \frac{BJ}{2}$

$AG := AB + BG$ $\quad GS := BG$ $\quad DG := \frac{GS^2}{AG}$ $\quad FG := AG - AF$

$BD := BG - DG$ $\quad DJ := BJ - BD$ $\quad DS := \sqrt{BD \cdot DJ}$

$FK := \frac{DS \cdot FG}{DG}$ $\quad BF := AF - AB$ $\quad BK := \sqrt{BF^2 + FK^2}$

$FI := \frac{DJ \cdot FK}{DS}$ $\quad BI := FI + BF$ $\quad BP := \frac{BK \cdot BJ}{BI}$

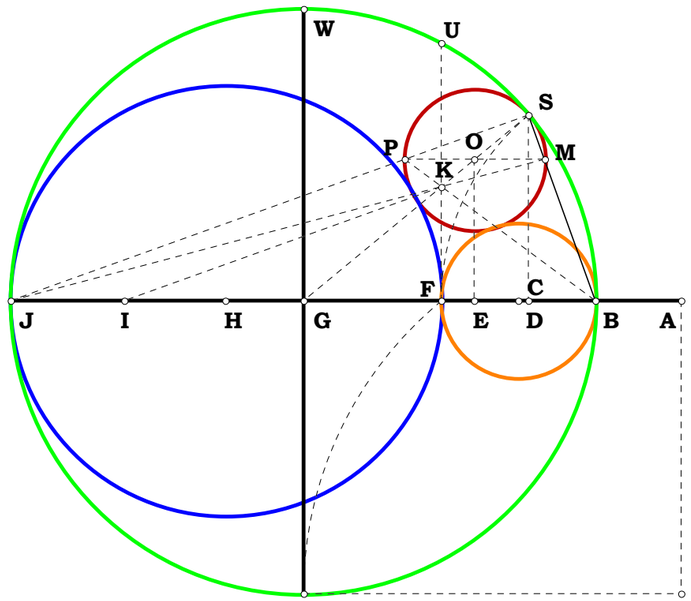
$KP := BP - BK$ $\quad MP := \frac{BJ \cdot KP}{BK}$ $\quad OS := \frac{MP}{2}$

Definitions.

$DG - \frac{(N-1)^2}{2 \cdot (N+1)} = 0$ $\quad OS - \frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(N + \sqrt{N} + 1\right)} = 0$

Power Line At Square Root

In this square root figure, what is the Algebraic definition of the tangent circle OS?



AB = 1.43933 cm
AJ = 11.23950 cm
OS = 1.18009 cm

$\frac{AB}{AB} = 1.00000$ $\quad AB = 1.00000$

$\frac{AJ}{AB} = 7.80882$ $\quad N = 7.80882$

$\frac{OS}{AB} = 0.81989$ $\quad OS = 0.81989$

$\frac{\sqrt{N} \cdot (N-1)}{2 \cdot \left(\sqrt{N} + N + 1\right)} - OS = 0.00000$

122595

Descriptions.

Unit.

$AB := 1$

Given.

$N := 5$

$\Delta := 2$

$\delta := 1 .. \Delta$

Two Prime Exponential Series Developed Through The Powerline Progression

$AO := AB \cdot N \quad AG := \sqrt{AB \cdot AO} \quad BO := AO - AB \quad BJ := \frac{BO}{2} \quad JZ := BJ \quad JV := BJ \quad JO := BJ$

$BG_1 := AG - AB \quad GO_1 := BO - BG_1 \quad GW_1 := \sqrt{BG_1 \cdot GO_1} \quad GJ_1 := BJ - BG_1 \quad GH_1 := \frac{GJ_1 \cdot GW_1}{JZ + GW_1}$

$$\begin{pmatrix} BG_{\delta+1} \\ GO_{\delta+1} \\ GW_{\delta+1} \\ GJ_{\delta+1} \\ GH_{\delta+1} \end{pmatrix} := \begin{bmatrix} BG_\delta + GH_\delta \\ BO - (BG_\delta + GH_\delta) \\ \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]} \\ BJ - (BG_\delta + GH_\delta) \\ \frac{[BJ - (BG_\delta + GH_\delta)] \cdot \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}}{JZ + \sqrt{(BG_\delta + GH_\delta) \cdot [BO - (BG_\delta + GH_\delta)]}} \end{bmatrix}$$

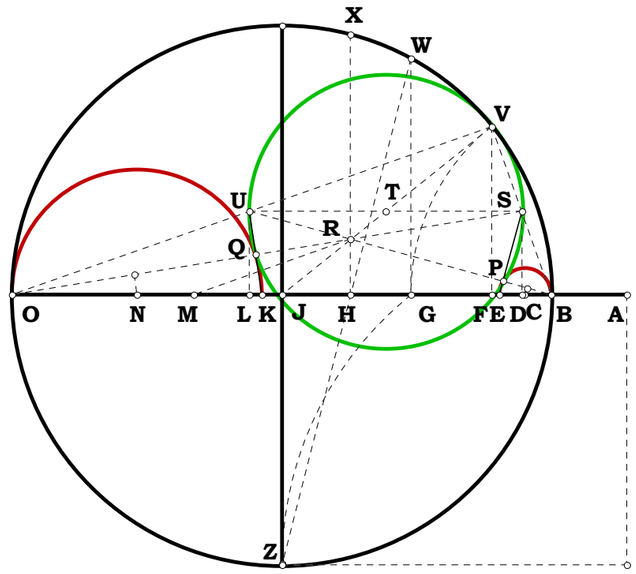
$HJ := BJ - BG_\Delta \quad FJ := \frac{(N-1)^2}{2 \cdot (N+1)} \quad BF := BJ - FJ \quad FO := FJ + JO \quad FV := \sqrt{BF \cdot FO} \quad HR := \frac{FV \cdot HJ}{FJ}$

$BH := BJ - HJ \quad BR := \sqrt{HR^2 + BH^2} \quad HM := \frac{FO \cdot HR}{FV} \quad BU := \frac{BR \cdot BO}{BH + HM} \quad RU := BU - BR \quad SU := \frac{BO \cdot RU}{BR}$

$TV := \frac{SU}{2} \quad PU := \frac{BH \cdot SU}{BR} \quad BP := BU - PU \quad BE := \frac{BR \cdot BP}{BH} \quad AE := AB + BE$

Definitions.

$N^{\frac{1}{2^\Delta}} - AE = 0$



Descriptions.

$$LU := \frac{HR \cdot BU}{BR} \qquad BL := \frac{BH \cdot BU}{BR} \qquad HO := JO + HJ$$

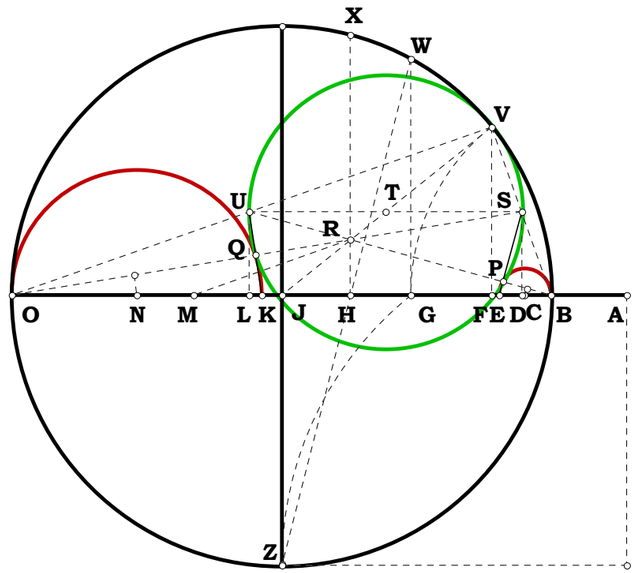
$$OR := \sqrt{HR^2 + HO^2} \qquad DS := LU \qquad OS := \frac{OR \cdot DS}{HR}$$

$$DO := \frac{HO \cdot DS}{HR} \qquad QS := \frac{DO \cdot SU}{OS} \qquad OQ := OS - QS$$

$$KO := \frac{OS \cdot OQ}{DO} \qquad AK := AO - KO$$

Definitions.

$$N^{\frac{2^{\Delta}-1}{2^{\Delta}}} - AK = 0$$



Unit.

$AB := 1$

Given.

$N := 5$

122694

Descriptions.

$AF := AB \cdot N \quad BF := AF - AB$

$BE := \frac{BF}{2} \quad EF := BE \quad EK := BE$

$AD := \sqrt{AB \cdot AF} \quad CE := \frac{(N-1)^2}{2 \cdot (N+1)} \quad CF := CE + EF$

$DF := AF - AD \quad BC := BF - CF \quad CH := \sqrt{BC \cdot CF}$

$DG_1 := \frac{CH \cdot DF}{CF} \qquad DG_1 = 1.236068$

$BD := AD - AB \quad DG_2 := \frac{EK \cdot BD}{BE} \quad DG_1 - DG_2 = 0$

Definitions.

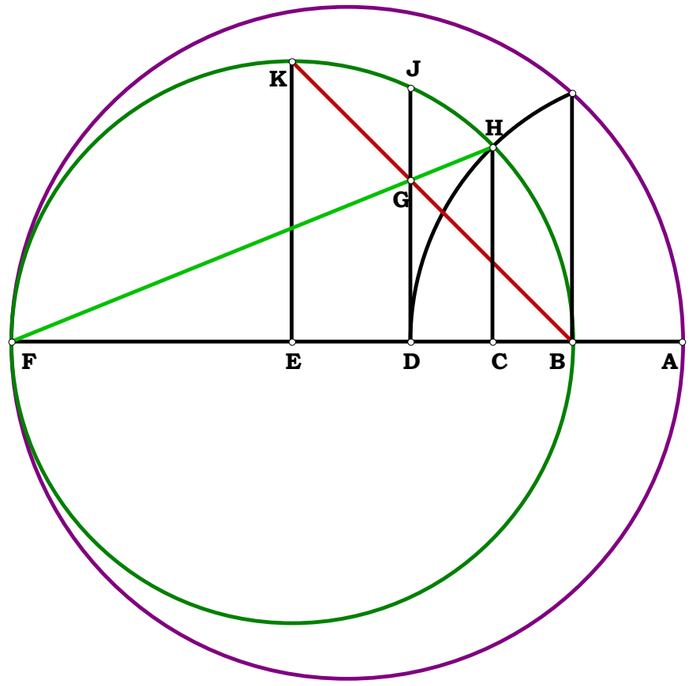
This might give you something to think about.

$$DG_1 - \frac{(N+1) \cdot \sqrt{N \cdot (N-1)^2} \cdot \left(N - \sqrt{N}\right)}{N \cdot (N-1) \cdot \sqrt{(N+1)^2}} = 0$$

$$DG_2 - \left(\sqrt{N} - 1\right) = 0$$

Is Point G on DJ?

Is G, the intersection of FH and BK, on DJ?



Unit.

$AC := 1$

Given.

$N := 5$

010695

Descriptions.

$AJ := AC \cdot N \quad CJ := AJ - AC \quad AE := \sqrt{AC \cdot AJ} \quad CE := AE - AC$

$EJ := CJ - CE \quad EN := \sqrt{CE \cdot EJ} \quad BM := EN \quad HO := EN$

$MN := EN \quad NO := 1BE := EN \quad EH := EN \quad BJ := BE + EJ$

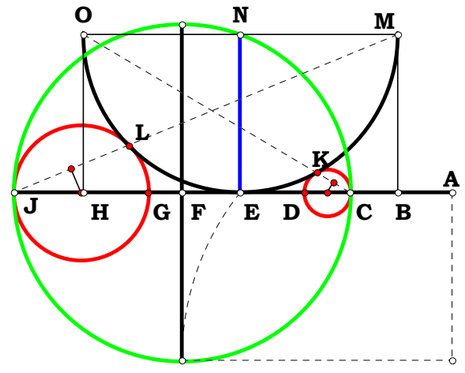
$MJ := \sqrt{BJ^2 + BM^2} \quad MO := MN + NO \quad ML := \frac{BJ \cdot MO}{MJ}$

$JL := MJ - ML \quad GJ := \frac{MJ \cdot JL}{BJ} \quad AG := AJ - GJ \quad \left(AC \cdot AJ^3\right)^{\frac{1}{4}} - AG = 0$

$CH := CE + EH \quad CO := \sqrt{CH^2 + HO^2} \quad KO := \frac{CH \cdot MO}{CO} \quad CK := CO - KO \quad CD := \frac{CO \cdot CK}{CH}$

$AD := AC + CD \quad \left(AC^3 \cdot AJ\right)^{\frac{1}{4}} - AD = 0$

Alternate Method Quad Roots



Definitions.

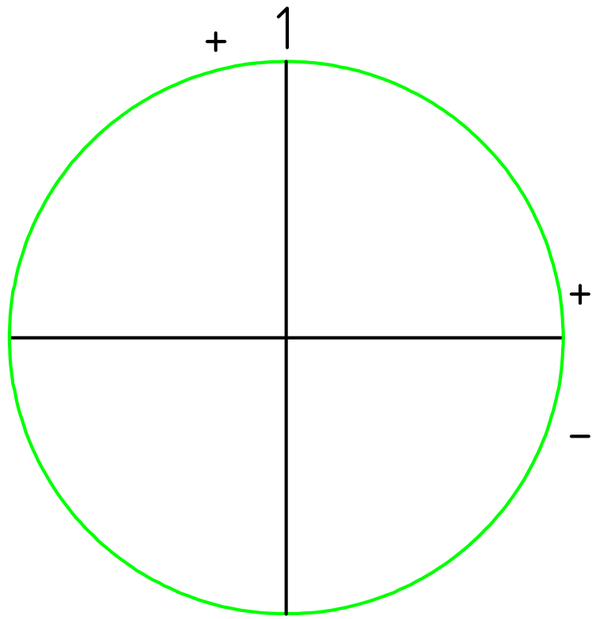
$N^{\frac{3}{4}} - AG = 0 \qquad N^{\frac{1}{4}} - AD = 0$

011295

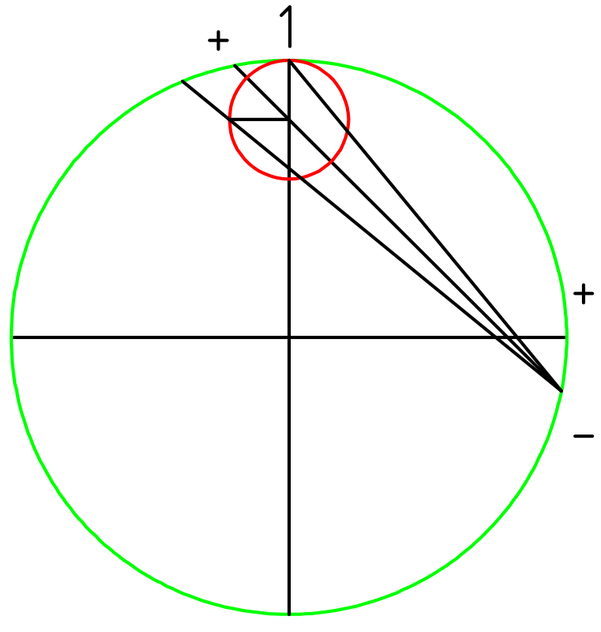
Archamedian Trisection Revisited.

I am curious as to why the Archamedian trisection is still taught the way it was given so long ago and the idea from which it derives is not in any of my books.

I am going to use a circle to demonstrate some principles concerning angles. To help log the results in mathematical terms, I will divide the circle into quadrants and label them either plus or minus. The top is labeled one.



The circle is divided into 1/8 segments of 90 degrees for each quadrant. I start at the top. Since the angle at the perimeter is 1/2 of the angle at the center, when I start the figure I use twice the arc segment for the angle I am working with. I have marked some quadrants with plus and minus and have found that for the figure, I would say that I have 1 + 1/8 - 1/8. From this I have] $\frac{90}{8} = 11.25$ my starting angle of the si will be 90 degrees.



$$90 + \frac{90}{8} - \frac{90}{8} = 90 \qquad \left(1 + \frac{1}{8} - \frac{1}{8}\right) \cdot 90 = 90$$

$$B := 1 + \frac{1}{8} - \frac{1}{8} \qquad B = 1$$

$$\frac{B \cdot 4}{4} \cdot 90 = 90 \qquad \frac{B \cdot 3}{4} \cdot 90 = 67.5$$

$$\frac{B \cdot 2}{4} \cdot 90 = 45 \qquad \frac{B}{4} \cdot 90 = 22.5$$

$$8 + 1 - 1 = 8 \qquad 8 \cdot 11.25 = 90$$

$$8 + 1 - 1 - 2 = 6 \qquad 6 \cdot 11.25 = 67.5$$

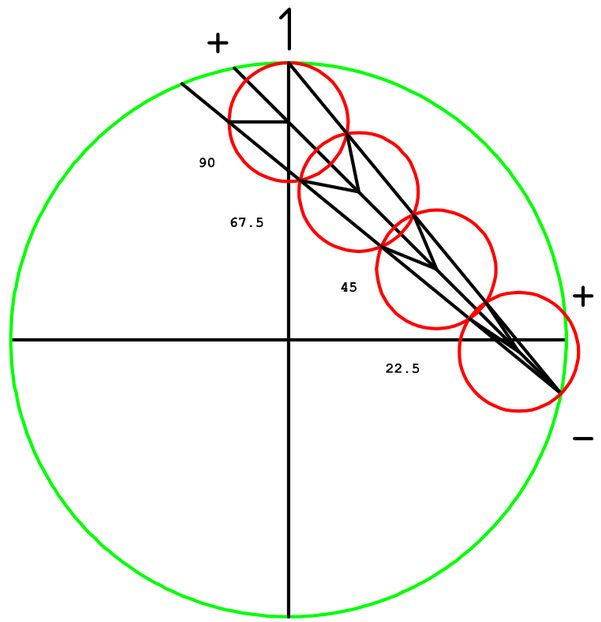
$$8 + 1 - 1 - 2 - 2 = 4 \qquad 4 \cdot 11.25 = 45$$

$$8 + 1 - 1 - 2 - 2 - 2 = 2 \qquad 2 \cdot 11.25 = 22.5$$

$$8 + 1 - 1 - 2 - 2 - 2 - 2 = 0$$

$$\mathbf{mod}(8 + 1 - 1, 2) = 0$$

I have added another plus to a quadrant at the bottom of the figure.



$$B := 1 + \frac{1}{8} + \frac{1}{8} - \frac{1}{8} \qquad B = 1.125 \qquad \frac{9}{8} = 1.125$$

$$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25 \qquad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$$

$$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25 \qquad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$$

$$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$$

$$8 + 1 + 1 - 1 = 9 \qquad 9 \cdot 11.25 = 101.25$$

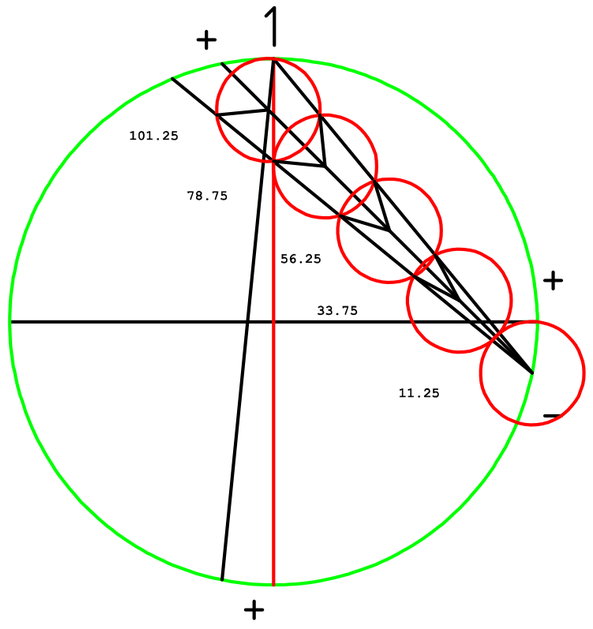
$$8 + 1 + 1 - 1 - 2 = 7 \qquad 7 \cdot 11.25 = 78.75$$

$$8 + 1 + 1 - 1 - 2 - 2 = 5 \qquad 5 \cdot 11.25 = 56.25$$

$$8 + 1 + 1 - 1 - 2 - 2 - 2 = 3 \qquad 3 \cdot 11.25 = 33.75$$

$$8 + 1 + 1 - 1 - 2 - 2 - 2 - 2 = 1 \qquad 1 \cdot 11.25 = 11.25$$

$$\mathbf{mod}(8 + 1 + 1 - 1, 2) = 1$$



$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{1}{8}$ $\quad B = 1.125 \quad \frac{9}{8} = 1.125$

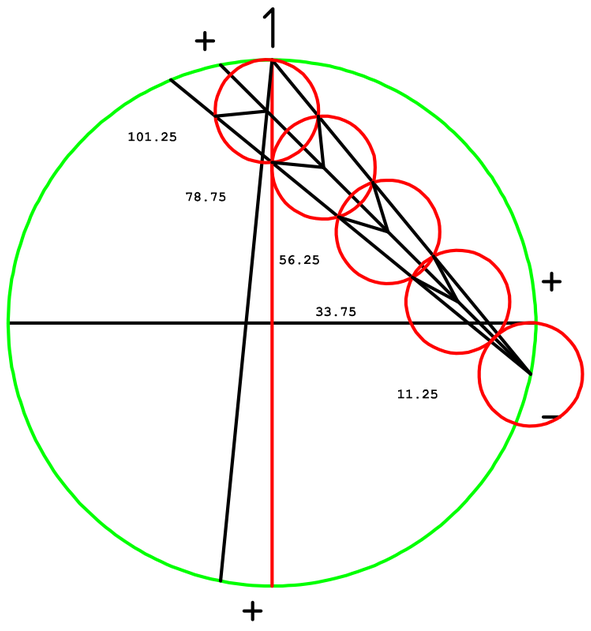
$\frac{B \cdot 4.5}{4.5} \cdot 90 = 101.25$ $\quad \frac{B \cdot 3.5}{4.5} \cdot 90 = 78.75$

$\frac{B \cdot 2.5}{4.5} \cdot 90 = 56.25$ $\quad \frac{B \cdot 1.5}{4.5} \cdot 90 = 33.75$

$\frac{B \cdot .5}{4.5} \cdot 90 = 11.25$

$8 + 1 = 9$	$9 \cdot 11.25 = 101.25$
$8 + 1 - (1 \cdot 2) = 7$	$7 \cdot 11.25 = 78.75$
$8 + 1 - (2 \cdot 2) = 5$	$5 \cdot 11.25 = 56.25$
$8 + 1 - (3 \cdot 2) = 3$	$3 \cdot 11.25 = 33.75$
$8 + 1 - (4 \cdot 2) = 1$	$1 \cdot 11.25 = 11.25$

$\mathrm{mod}(8 + 1, 2) = 1$



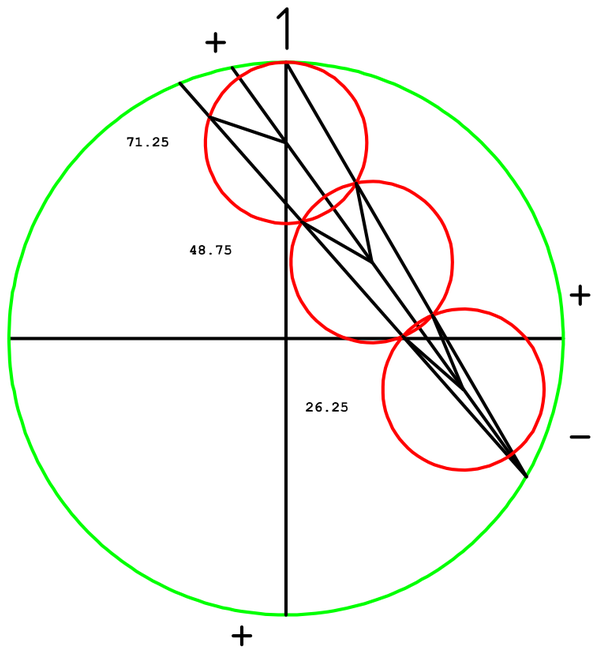
$B := 1 + \frac{3}{24} - \frac{8}{24} = 0.791667$ $\quad \frac{19}{24} = 0.791667$

$\frac{B \cdot 3.1666}{3.1666} \cdot 90 = 71.25$ $\quad \frac{B \cdot 2.1666}{3.1666} \cdot 90 = 48.749526$

$\frac{B \cdot 1.16666}{3.16666} \cdot 90 = 26.249905$ $\quad \frac{B \cdot .166666}{3.166666} \cdot 90 = 3.749986$

$(24 + 3) - 8 = 19$	$19 \cdot 3.75 = 71.25$
$(24 + 3) - 8 - (1 \cdot 6) = 13$	$13 \cdot 3.75 = 48.75$
$(24 + 3) - 8 - (2 \cdot 6) = 7$	$7 \cdot 3.75 = 26.25$
$(24 + 3) - 8 - (3 \cdot 6) = 1$	$1 \cdot 3.75 = 3.75$

$\mathrm{mod}(24 + 3 - 8, 2) = 1$



$$B := 1 + \frac{1}{8} - \frac{1}{8} + \frac{10}{8} \qquad B = 2.25 \qquad \frac{18}{8} = 2.25$$

$$\frac{B \cdot 9}{9} \cdot 90 = 202.5 \qquad \frac{B \cdot 8}{9} \cdot 90 = 180$$

$$\frac{B \cdot 7}{9} \cdot 90 = 157.5 \qquad \frac{B \cdot .6}{9} \cdot 90 = 13.5$$

$8 + 1 - 1 + 10 = 18 \qquad 18 \cdot 11.25 = 202.5$

$8 + 1 - 1 + 10 - (2 \cdot 1) = 16 \quad 16 \cdot 11.25 = 180$

$8 + 1 - 1 + 10 - (2 \cdot 2) = 14 \quad 14 \cdot 11.25 = 157.5$

$8 + 1 - 1 + 10 - (2 \cdot 3) = 12 \quad 12 \cdot 11.25 = 135$

$8 + 1 - 1 + 10 - (2 \cdot 4) = 10 \quad 10 \cdot 11.25 = 112.5$

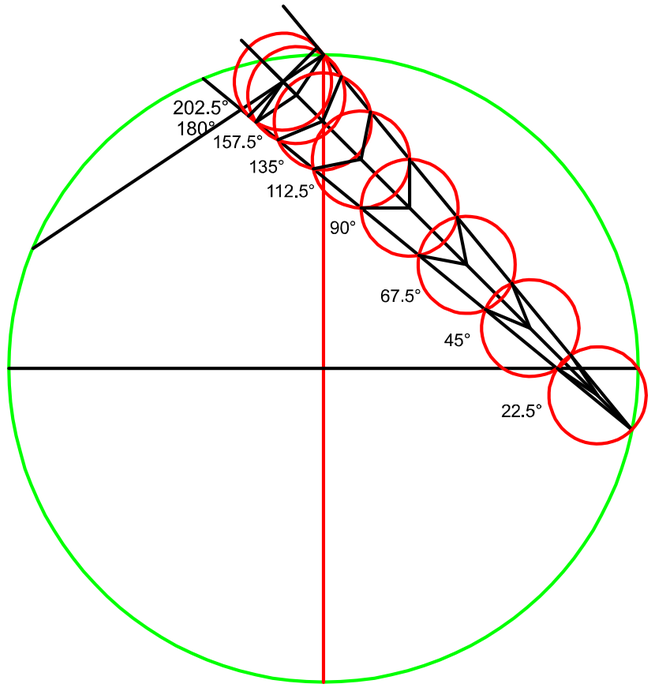
$8 + 1 - 1 + 10 - (2 \cdot 5) = 8 \quad 8 \cdot 11.25 = 90$

$8 + 1 - 1 + 10 - (2 \cdot 6) = 6 \quad 6 \cdot 11.25 = 67.5$

$8 + 1 - 1 + 10 - (2 \cdot 7) = 4 \quad 4 \cdot 11.25 = 45$

$8 + 1 - 1 + 10 - (2 \cdot 8) = 2 \qquad 2 \cdot 11.25 = 22.5$

$$\mathbf{mod}[(8 + 1 - 1) + 10, 2] = 0$$



$$B := 1 + \frac{1}{7} - \frac{2}{7} \qquad B = 0.857143 \qquad \frac{6}{7} = 0.857143$$

$$\frac{B \cdot 6}{6} \cdot 90 = 77.142857 \qquad \frac{B \cdot 4}{6} \cdot 90 = 51.428571$$

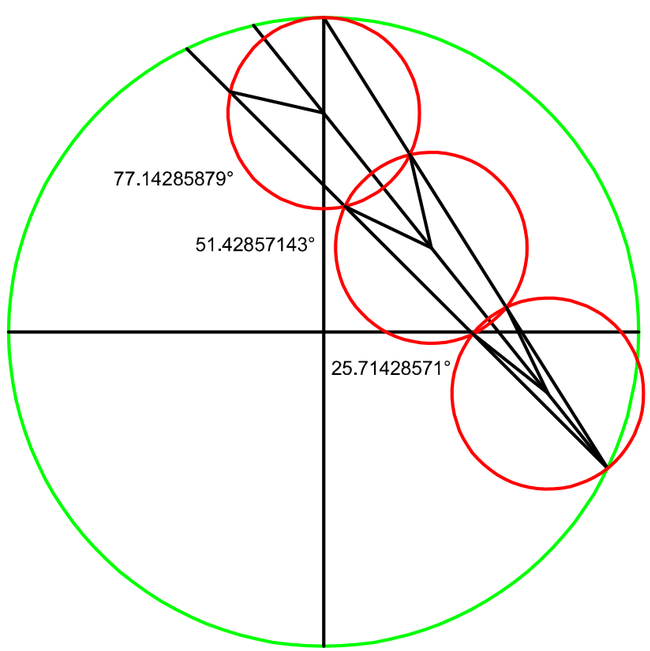
$$\frac{B \cdot 2}{6} \cdot 90 = 25.714286 \qquad c := \frac{90}{7}$$

$7 + 1 - (1 \cdot 2) = 6 \qquad 6 \cdot c = 77.142857$

$7 + 1 - (2 \cdot 2) = 4 \qquad 4 \cdot c = 51.428571$

$7 + 1 - (3 \cdot 2) = 2 \qquad 2 \cdot c = 25.714286$

$\mathbf{mod}(7 + 1 - 2, 2) = 0$



$B := 1 + \frac{1}{7} - \frac{1}{7}$ $B = 1$ $\frac{7}{7} = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\frac{B \cdot 5}{7} \cdot 90 = 64.285714$

$\frac{B \cdot 3}{7} \cdot 90 = 38.571429$ $\frac{B \cdot 1}{7} \cdot 90 = 12.857143$

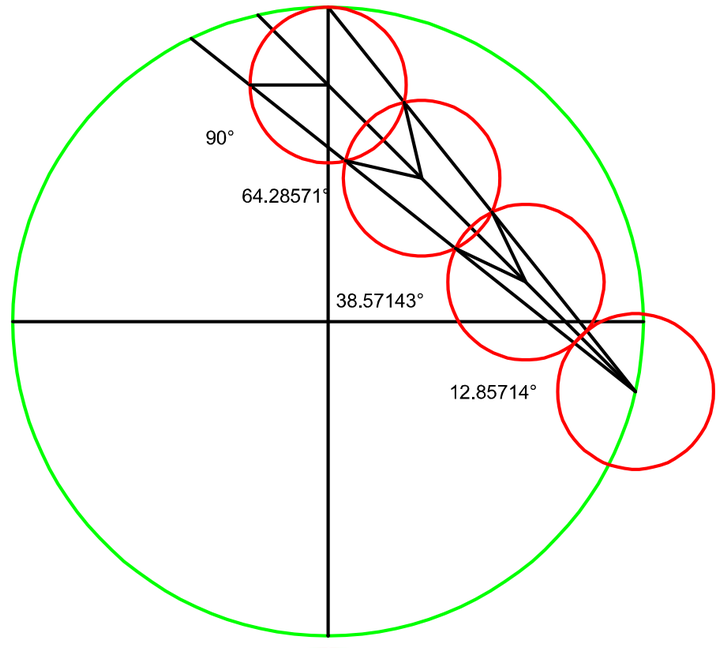
$7 + 1 - 1 = 7$ $7 \cdot c = 90$

$7 + 1 - 1 - (1 \cdot 2) = 5$ $5 \cdot c = 64.285714$

$7 + 1 - 1 - (2 \cdot 2) = 3$ $3 \cdot c = 38.571429$

$7 + 1 - 1 - (3 \cdot 2) = 1$ $1 \cdot c = 12.857143$

$\mathbf{mod}(7 + 1 - 1, 2) = 1$



$B := 1 + \frac{8}{56} - \frac{7}{56}$ $B = 1.017857$

$\frac{B \cdot 57}{57} \cdot 90 = 91.607143$ $\frac{B \cdot 41}{57} \cdot 90 = 65.892857$

$\frac{B \cdot 25}{57} \cdot 90 = 40.178571$ $c := \frac{90}{56}$

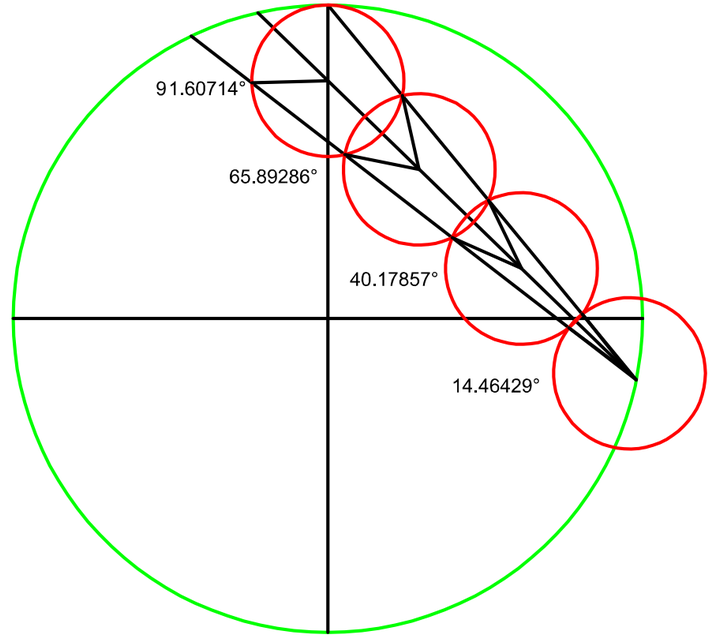
$56 + 8 - 7 = 57$ $57 \cdot c = 91.607143$

$56 + 8 - 7 - (1 \cdot 16) = 41$ $41 \cdot c = 65.892857$

$56 + 8 - 7 - (2 \cdot 16) = 25$ $25 \cdot c = 40.178571$

$56 + 8 - 7 - (3 \cdot 16) = 9$ $9 \cdot c = 14.464286$

$\mathbf{mod}(56 + 8 - 7, 16) = 9$



$B := 1 + \frac{1}{7} - \frac{1}{7}$ $\quad B = 1$

$\frac{B \cdot 7}{7} \cdot 90 = 90$ $\quad \frac{B \cdot 5}{7} \cdot 90 = 64.285714$

$\frac{B \cdot 3}{7} \cdot 90 = 38.571429$ $\quad \frac{B \cdot 1}{7} \cdot 90 = 12.857143$

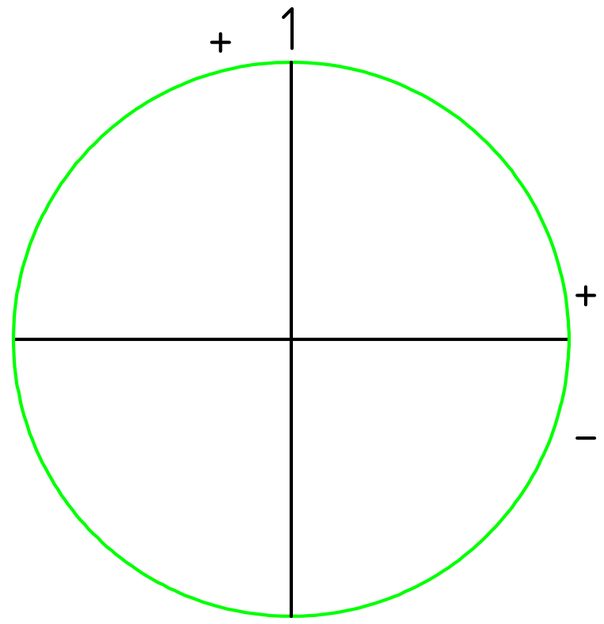
$7 + 1 - 1 = 7$

$7 + 1 - 1 - (1 \cdot 2) = 5$

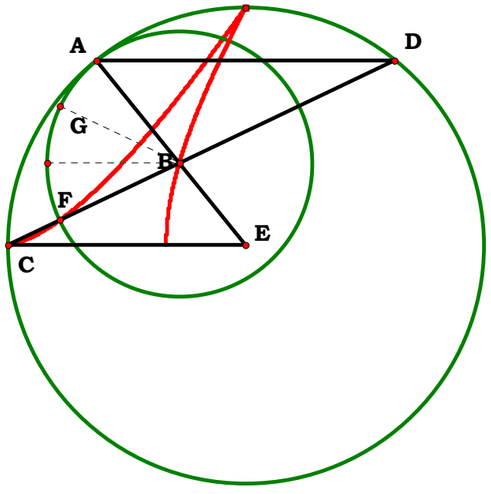
$7 + 1 - 1 - (2 \cdot 2) = 3$

$7 + 1 - 1 - 2 - 2 - 2 = 1$

$\mathrm{mod}(7 + 1 - 1, 2) = 1$



Work in progress.



$m\angle ABF = 76.48479°$

$m\angle ABG = 25.49493°$

$\frac{m\angle ABF}{m\angle ABG} = 3.00000$

$m\angle ABC = 76.48479°$

$m\angle ADC = 25.49493°$

$\frac{m\angle ABC}{m\angle ADC} = 3.00000$

$m\angle BCE = 25.49493°$

040195

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 5$

$N_2 := 3$

$\delta := 0 .. 3$

Exponential Series-Roots and Powers

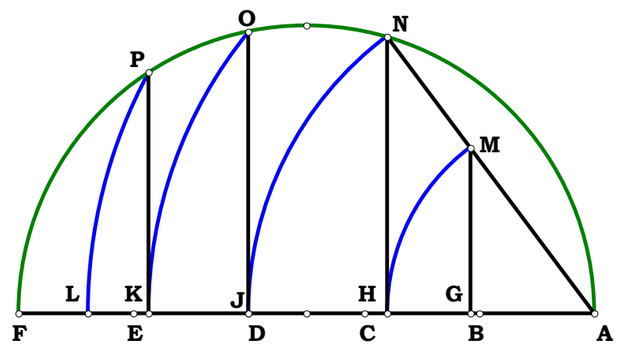
I remember a dream was it? That exponential notation is not demonstrable in Geometric Grammar, that it is a pure conceptual abstract.

$$AF := AB \cdot N_1 \quad AJ_0 := AB \cdot N_2 \quad AN := AJ_0 \quad AJ_1 := \frac{AN^2}{AF}$$

$$AJ_{\delta+1} := \frac{AJ_\delta \cdot AN}{AF} \quad AD_0 := AJ_0 \quad DF_0 := AF - AD_0$$

$$DO_0 := \sqrt{AD_0 \cdot DF_0} \quad AO_0 := \sqrt{(DO_0)^2 + (AD_0)^2}$$

$$\begin{pmatrix} AD_{\delta+1} \\ DF_{\delta+1} \\ DO_{\delta+1} \\ AO_{\delta+1} \end{pmatrix} := \begin{bmatrix} AO_\delta \\ AF - AO_\delta \\ \sqrt{AO_\delta \cdot (AF - AO_\delta)} \\ \sqrt{AO_\delta \cdot (AF - AO_\delta) + (AO_\delta)^2} \end{bmatrix}$$



Definitions.

$$\sum_\delta \left[\frac{AF}{AJ_\delta} - \left(\frac{N_1}{N_2} \right)^{\delta+1} \right] = 0 \quad \sum_\delta \left[\frac{AF}{AD_\delta} - \left(\frac{N_1}{N_2} \right)^{\frac{1}{2^\delta}} \right] = 0$$

$AE := 5.20700$

$Unit := AE$

Given.

$AB := 1.11300 \quad N_1 := AB$

$AC := 2.02712 \quad N_2 := AC$

091395a

Given AE, AB, AC what is GH?

Descriptions.

$BC := (AC - AB) \qquad GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2}$

$GK = 4.27657 \qquad GH = 2.138285$

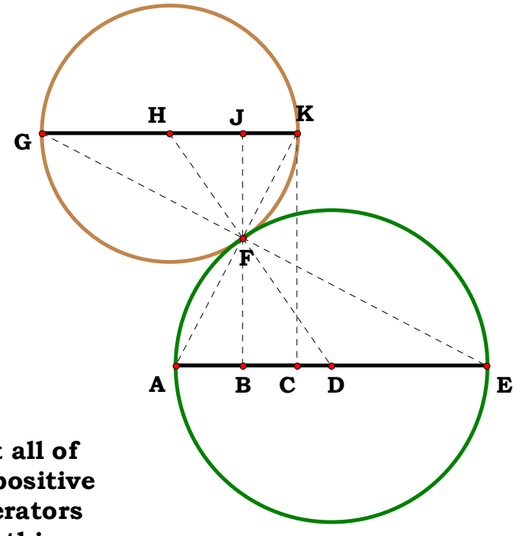
Definitions.

When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_2 - N_1}{\sqrt{(N_2 - N_1)^2}} = 1$$

$$GH - \left[\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot \frac{N_2 - N_1}{\sqrt{(N_2 - N_1)^2}}\right] = 0 \qquad \text{Combinaing we get:} \qquad GH - \frac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} = 0$$

A Study In Placement



$AE = 5.20700$ cm

$AB = 1.11300$ cm

$AC = 2.02712$ cm

$CK = 3.88781$ cm

$EF = 4.61708$ cm

$GK = 4.27653$ cm

$GH = 2.13827$ cm

$BC = 0.91412$ cm

$Unit = 5.20700$ cm

$N_1 = 1.11300$ cm

$N_2 = 2.02712$ cm

$AC - AB = 0.91412$ cm

$\frac{AE}{AB} - \frac{GK}{BC} = 0.00000$

$\frac{AE \cdot BC}{AB} - GK = 0.00000$ cm

$\frac{(AC-AB)}{\sqrt{(AC-AB)^2}} = 1.00000$

Is GK outside side AE? = 1.00000

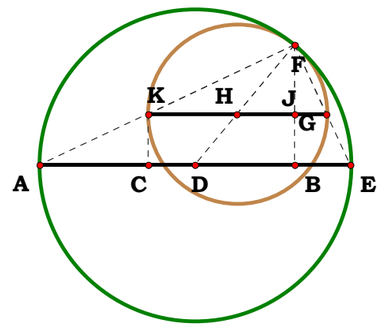
X = 1.00000

$\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} = 2.13827$ cm

$\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot X - GH = 0.00000$ cm

$\frac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} - GH = 0.00000$ cm

$\frac{Unit \cdot (N_2 - N_1)}{2 \cdot N_1} \cdot \frac{(AC-AB)}{\sqrt{(AC-AB)^2}} - GH = 0.00000$ cm



$\frac{Unit \cdot \sqrt{(N_1 - N_2)^2}}{2 \cdot N_1} - GH = 0.00000$ cm

$AE := 5.20700$

$Unit := AE$

Given.

$EF := 4.58113 \quad N_1 := EF$

$AC := 1.96362 \quad N_2 := AC$

091395b

Given AE, EF AC what is GH?

Descriptions.

$AB := AE - \frac{EF^2}{AE} \qquad BC := (AC - AB) \qquad GK := \frac{AE \cdot BC}{AB}$

$GH := \frac{GK}{2} \qquad GK = 3.48358 \qquad GH = 1.74179$

Definitions.

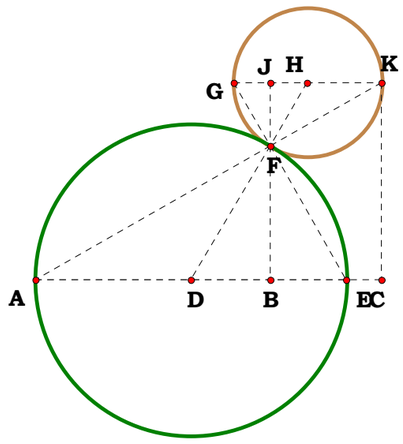
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_1^2 - Unit^2 + N_2 \cdot Unit}{\sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}} = 1$$

$$GH - \left[\frac{\left(N_2 \cdot Unit^2 - Unit^3 + Unit \cdot N_1^2\right)}{2 \cdot Unit^2 - 2 \cdot N_1^2} \cdot \frac{N_1^2 - Unit^2 + N_2 \cdot Unit}{\sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}}\right] = 0$$

$$GH - \frac{Unit \cdot \sqrt{\left(N_1^2 - Unit^2 + N_2 \cdot Unit\right)^2}}{2 \cdot \left(Unit^2 - N_1^2\right)} = 0$$

A Study In Placement



$AE = 5.20700$ cm

$AB = 3.92817$ cm

$AC = 5.79479$ cm

$CK = 3.30635$ cm

$EF = 2.58048$ cm

$GK = 2.47430$ cm

$GH = 1.23715$ cm

$BC = 1.86662$ cm

$\frac{Unit \cdot \sqrt{((N_1^2 - Unit^2) + N_2 \cdot Unit)^2}}{2 \cdot (Unit^2 - N_1^2)} - GH = 0.00000$ cm

$Unit = 5.20700$ cm

$N_1 = 2.58048$ cm

$N_2 = 5.79479$ cm

$\frac{AE}{AB} - \frac{GK}{BC} = 0.00000$

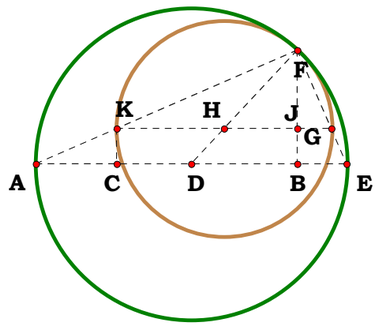
$\frac{AE \cdot BC}{AB} - GK = 0.00000$ cm

$AC - AB = 1.86662$ cm

$\frac{(AC - AB)}{\sqrt{(AC - AB)^2}} = 1.00000$

Is GK outside AE? = 1.00000

X = 1.00000



$\frac{Unit \cdot \sqrt{((N_1^2 - Unit^2) + N_2 \cdot Unit)^2}}{2 \cdot (Unit^2 - N_1^2)} - GH = 0.00000$ cm

$AE := 5.20700$

Unit := AE

Given.

$CK := 2.82631$ $N_1 := CK$

091395c $AC := 4.41895$ $N_2 := AC$

Given AE, CK, AC what is GH?

Descriptions.

$$CM := \frac{CK^2}{AC} \qquad AB := \frac{AC \cdot AE}{AC + CM} \qquad BC := AC - AB$$

$$GK := \frac{AE \cdot BC}{AB} \qquad GH := \frac{GK}{2} \qquad GK = 1.019626 \qquad GH = 0.509813$$

Definitions.

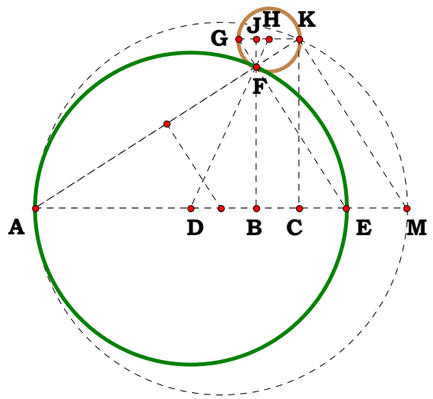
When selecting a unit, one has to make sure that one draws the figure so that all of the givens are in terms of the unit. These terms will not be in reference to a positive or negative unit, as there is no such thing. Then one can write the logical operators which answer questions in terms of the resulting operations with the unit. In this example, we can put the remaining question as: ***Is the Brown Circle outside the Green one?*** If it is not we have to change the sign of the equation. One can look at logical operators which are 1 and - 1 as is such and such and is not such and such or as simple assertion and denial. Therefore, this one logical operator is universal for every question.

$$\frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2} \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2} \cdot \left(N_1^2 + N_2^2\right)} = 1$$

$$GH - \left[\frac{N_1^2 + N_2^2 - Unit \cdot N_2}{2 \cdot N_2} \cdot \frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2} \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2} \cdot \left(N_1^2 + N_2^2\right)}\right] = 0$$

$$GH - \frac{\left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2}{2 \cdot \sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2}} = 0$$

A Study In Placement



AE = 5.20700 cm
AB = 3.69534 cm
AC = 4.41895 cm
CK = 2.82631 cm
EF = 2.80557 cm
GK = 1.01963 cm
GH = 0.50981 cm
BC = 0.72362 cm
CM = 1.80767 cm

Unit = 5.20700 cm
N_1 = 2.82631 cm
N_2 = 4.41895 cm
AC-AB = 0.72362 cm

$\frac{AE}{AB} - \frac{GK}{BC}$ = 0.00000

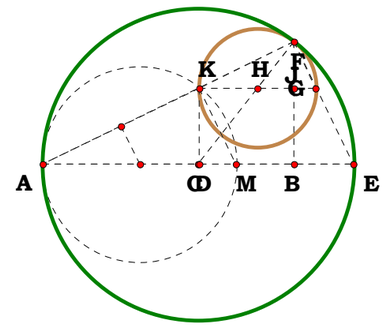
$\frac{AE \cdot BC}{AB}$ - GK = 0.00000 cm

$\frac{(AC-AB)}{\sqrt{(AC-AB)^2}}$ = 1.00000

Is GK outside AE? = 1.00000

X = 1.00000

$$\frac{((N_1^2+N_2^2)-Unit \cdot N_2)^2}{2 \cdot \sqrt{N_2^2 \cdot ((N_1^2+N_2^2)-Unit \cdot N_2)^2}} - GH = 0.00000 \text{ cm}$$



$$\frac{((N_1^2+N_2^2)-Unit \cdot N_2)^2}{2 \cdot \sqrt{N_2^2 \cdot ((N_1^2+N_2^2)-Unit \cdot N_2)^2}} - GH = 0.00000 \text{ cm}$$

Combinaing we get: $\frac{N_2 \cdot \sqrt{\left(N_1^2 + N_2^2\right)^2} \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)}{\sqrt{N_2^2 \cdot \left(N_1^2 + N_2^2 - Unit \cdot N_2\right)^2} \cdot \left(N_1^2 + N_2^2\right)}$

And since MC cannot figure this out, we finish it by hand.

101495

Unit.

$AB := 1$

Given.

$N_1 := 5$

$N_2 := 4$

Descriptions.

$AF := N_1 \quad AK := N_2 \quad BF := AF - AB$

$BO := \frac{BF}{2} \quad AO := AB + BO \quad KM := AO \quad AM := \sqrt{AK^2 + KM^2}$

$DJ := BO \quad AJ := AM \quad AD := \sqrt{AJ^2 - DJ^2}$

$CK := AD \quad AC := \sqrt{CK^2 - AK^2}$

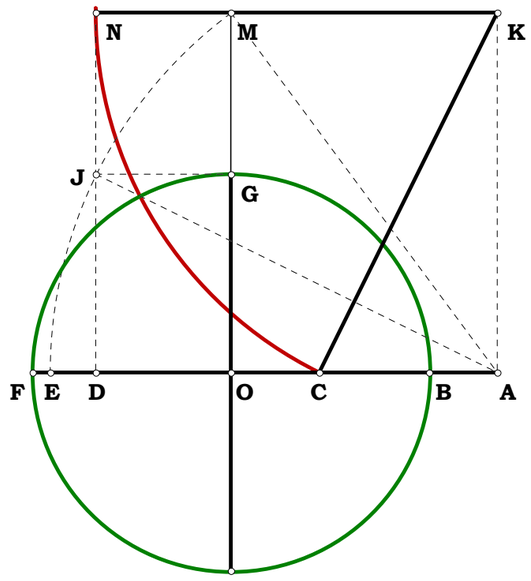
$\sqrt{AB \cdot AF} - AC = 0$

Definitions.

$AC - \sqrt{N_1} = 0$

Alternate Method Square Root

For any AK is AC the root of AB x AF?



Unit.

$AE := 1$

Given.

$N_1 := 2$

102095

Descriptions.

$$AB := \frac{AE}{N_1} \qquad BE := AE - AB \qquad BF := \sqrt{AB \cdot BE} \qquad AF := \sqrt{AB^2 + BF^2}$$

$$AC := AF \qquad CE := AE - AC \qquad EG := CE \qquad DE := \frac{EG^2}{AE}$$

$$BD := AE - (AB + DE) \qquad \frac{BD^2}{4 \cdot (AB \cdot DE)} = 1 \qquad AD := AE - DE \qquad DG := \sqrt{AD \cdot DE}$$

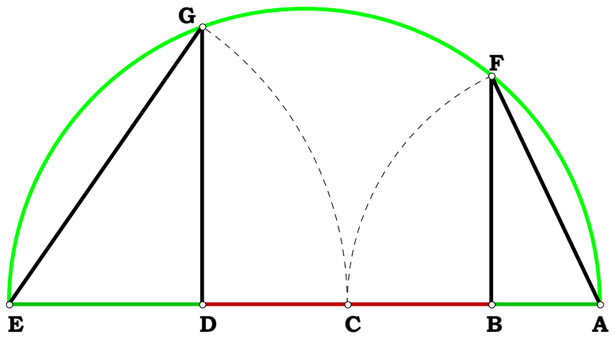
Definitions.

$$\frac{1}{N_1} - AB = 0 \qquad 1 - \frac{1}{N_1} - BE = 0 \qquad \sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} - BF = 0 \qquad \sqrt{\frac{N_1}{N_1^2}} - AF = 0$$

$$1 - \sqrt{\frac{N_1}{N_1^2}} - CE = 0 \qquad 1 - 2 \cdot \sqrt{\frac{1}{N_1}} + \frac{1}{N_1} - DE = 0 \qquad 2 \cdot \sqrt{\frac{1}{N_1}} - \frac{2}{N_1} - BD = 0$$

$$2 \cdot \sqrt{\frac{1}{N_1}} - \frac{1}{N_1} - AD = 0 \qquad \frac{\sqrt{4 \cdot \sqrt{N_1} - 5 \cdot N_1 + 2 \cdot N_1^{\frac{3}{2}} - 1}}{N_1} - DG = 0$$

Four Times The Square



110195

Unit.

$AG := 1$

Given.

$N_1 := 5$

$N_2 := 2$

Descriptions.

$AE := \frac{AG}{2}$ $EG := AE$ $EF := \frac{AG}{2 \cdot N_1}$ $AF := AE + EF$

$FG := EG - EF$ $FN := \sqrt{AF \cdot FG}$ $GN := \sqrt{FN^2 + FG^2}$ $GK := GN$

$EK := \sqrt{GK^2 - EG^2}$ $EO := \frac{EG \cdot EF}{EK}$ $OK := EO + EK$ $DE := \frac{AE}{N_2}$

$DO := \sqrt{DE^2 + EO^2}$ $DJ := OK - DO$ $CD := \frac{DE \cdot DJ}{DO}$ $CE := CD + DE$

$CJ := \frac{EO \cdot DJ}{DO}$ $AC := AE - CE$ $AJ := \sqrt{AC^2 + CJ^2}$ $AL := AJ$ $AB := \frac{AL^2}{AG}$

$CG := AG - AC$ $GJ := \sqrt{CG^2 + CJ^2}$ $GM := GJ$ $DG := \frac{GM^2}{AG}$

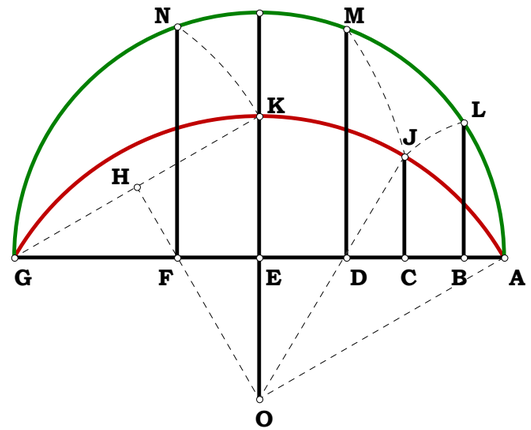
$BD := AG - (AB + DG)$

Definitions.

$$\frac{N_1 - 1}{2} - \frac{\sqrt{AB \cdot DG}}{BD} = 0$$

A Modification Of The Square Root Figure, Gemini Roots

On a given segment and from any point on that segment construct a square and a segment that will divide that square by (N-1)/2 times.



110595

Unit.

$AG := 1$

Given.

$N_1 := 3$

$N_2 := 5$

Descriptions.

$AF := \frac{AG}{2}$ $AR := AF$ $FQ := AF$ $FG := AF$

$AL := \frac{AR}{N_1}$ $IM := \frac{AR}{N_2}$ $AK := \frac{AL \cdot IM}{AR}$ $DO := IM$

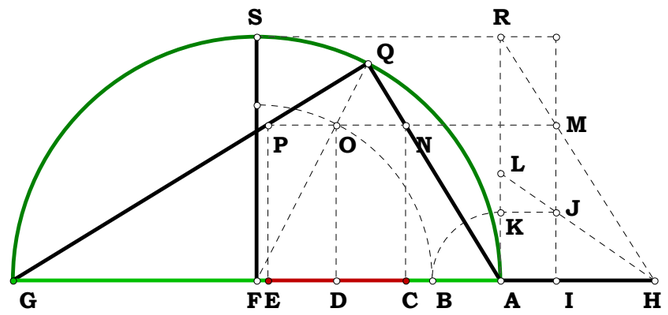
$AB := AK$ $BF := AF - AB$ $FO := BF$ $OQ := FQ - FO$

$NP := \frac{(AG - 2 \cdot AB) \cdot OQ}{FO}$ $NP - 2 \cdot AK = 0$ $CD := AK$

$DE := AK$ $DF := \sqrt{FO^2 - DO^2}$ $AD := AF - DF$

$AC := AD - CD$ $EG := FG + DF - DE$ $CE := NP$

Alternate Method Gemini Roots



Definitions.

$\frac{N_1}{2} - \frac{\sqrt{AC \cdot EG}}{CE} = 0$

Unit.

$AH := 1$

Given.

$N_1 := 5$

120195

Descriptions.

$$AE := \frac{AH}{2} \qquad EH := AE \quad EP := AE \qquad AP := \sqrt{2 \cdot AE^2}$$

$$AB := \frac{AE}{N_1} \qquad CE := AB \quad CH := EH + CE \qquad CL := \sqrt{2 \cdot CE^2}$$

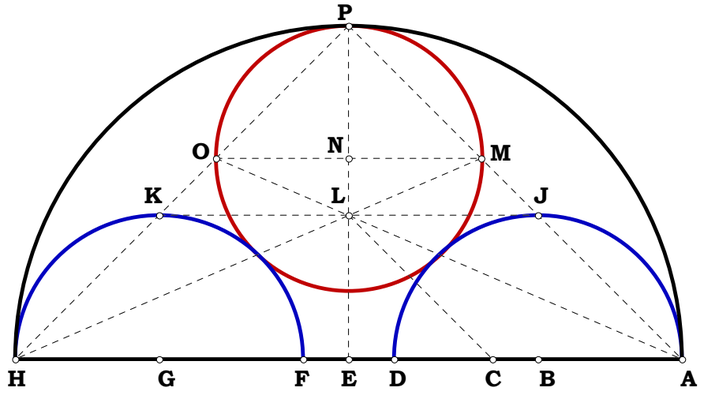
$$AM := \frac{CL \cdot AH}{CH} \qquad MP := AP - AM \qquad NP := \frac{EP \cdot MP}{AP}$$

Definitions.

$$\frac{1}{2} \cdot \frac{(N_1 - 1)}{(N_1 + 1)} - NP = 0 \qquad \frac{(N_1 - 1)}{(N_1 + 1)} - 2 \cdot NP = 0$$

Method For Equals

Given AB find NP.



Unit.

Given.

$\delta := 0 .. 2$

120795

Scholia: ***100 Great Problems of Elementary Mathematics*** **H. Dorrie Problem 27 Euler's Line**

Given three sides of a triangle, determine the length of the Euler line. Work the drawing from each of the sides.

Descriptions.

$$AC := \begin{pmatrix} Side\_1 \\ Side\_2 \\ Side\_3 \end{pmatrix} \quad BC := \begin{pmatrix} Side\_2 \\ Side\_3 \\ Side\_1 \end{pmatrix} \quad AB := \begin{pmatrix} Side\_3 \\ Side\_1 \\ Side\_2 \end{pmatrix}$$

$$TRIANGLE := (Side\_1 + Side\_2 > Side\_3) \cdot (Side\_1 + Side\_3 > Side\_2) \cdot (Side\_2 + Side\_3 > Side\_1)$$

$$AE_\delta := \frac{AB_\delta}{2} \quad Ak_\delta := AC_\delta \quad Bl_\delta := BC_\delta \quad Ai_\delta := \frac{(Ak_\delta)^2}{AB_\delta} \quad Bh_\delta := \frac{(Bl_\delta)^2}{AB_\delta}$$

$$Ah_\delta := AB_\delta - Bh_\delta \quad hi_\delta := Ah_\delta - Ai_\delta \quad Aj_\delta := Ai_\delta + \frac{hi_\delta}{2} \quad Cj_\delta := \sqrt{(AC_\delta)^2 - (Aj_\delta)^2}$$

$$BE_\delta := AE_\delta \quad Bj_\delta := AB_\delta - Aj_\delta \quad Bg_\delta := \frac{BC_\delta}{2} \quad Bf_\delta := \frac{BC_\delta \cdot BE_\delta}{Bj_\delta} \quad fg_\delta := Bf_\delta - Bg_\delta$$

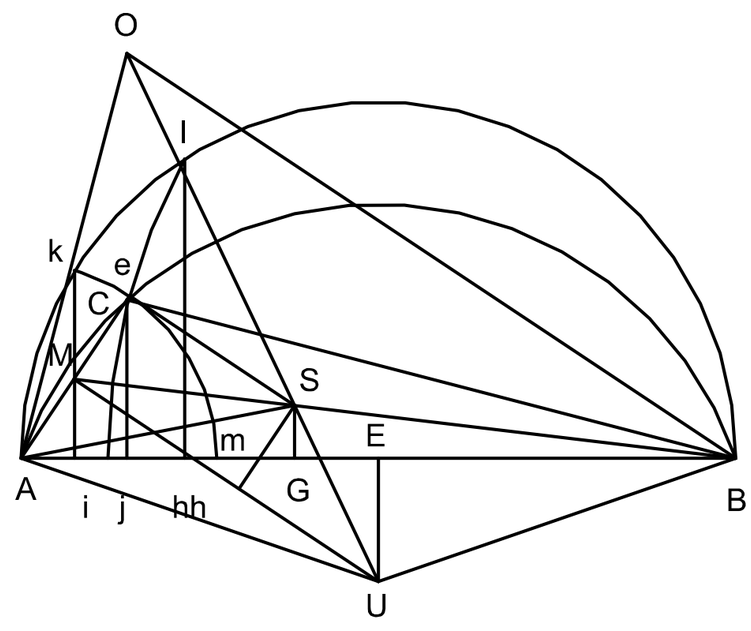
$$Ug_\delta := if\left(Cj_\delta, \frac{Bj_\delta \cdot fg_\delta}{Cj_\delta}, 0\right) \quad BU_\delta := if\left[Ug_\delta, \sqrt{(Ug_\delta)^2 + (Bg_\delta)^2}, \infty\right] \quad AM_\delta := \frac{AC_\delta}{2}$$

$$AGG_\delta := \frac{Aj_\delta \cdot AM_\delta}{AC_\delta} \quad BGG_\delta := AB_\delta - AGG_\delta \quad GGM_\delta := \sqrt{(AM_\delta)^2 - (AGG_\delta)^2}$$

$$BM_\delta := \sqrt{(GGM_\delta)^2 + (BGG_\delta)^2} \quad BS_\delta := \frac{2 \cdot BM_\delta}{3} \quad BG_\delta := \frac{BGG_\delta \cdot BS_\delta}{BM_\delta} \quad GS_\delta := \frac{GGM_\delta \cdot BS_\delta}{BM_\delta}$$

$$AG_\delta := AB_\delta - BG_\delta \quad AS_\delta := \sqrt{(AG_\delta)^2 + (GS_\delta)^2} \quad MS_\delta := BM_\delta - BS_\delta \quad AU_\delta := BU_\delta$$

$$MU_\delta := \sqrt{(AU_\delta)^2 - (AM_\delta)^2} \quad Ae_\delta := \frac{1}{2} \cdot \frac{(AS_\delta)^2}{AM_\delta} + \frac{1}{2} \cdot AM_\delta - \frac{1}{2} \cdot \frac{(MS_\delta)^2}{AM_\delta}$$



The following sequence is from 06_07_93.MCD. In that paper I treated it as a problem between two triangles, which it is. One can restate it as finding the fourth side of a quadrilateral by pulling up a line.

$eM_\delta := Ae_\delta - AM_\delta$ $\quad Sm_\delta := eM_\delta$ $\quad Se_\delta := \sqrt{(AS_\delta)^2 - (Ae_\delta)^2}$ $\quad Mm_\delta := Se_\delta$

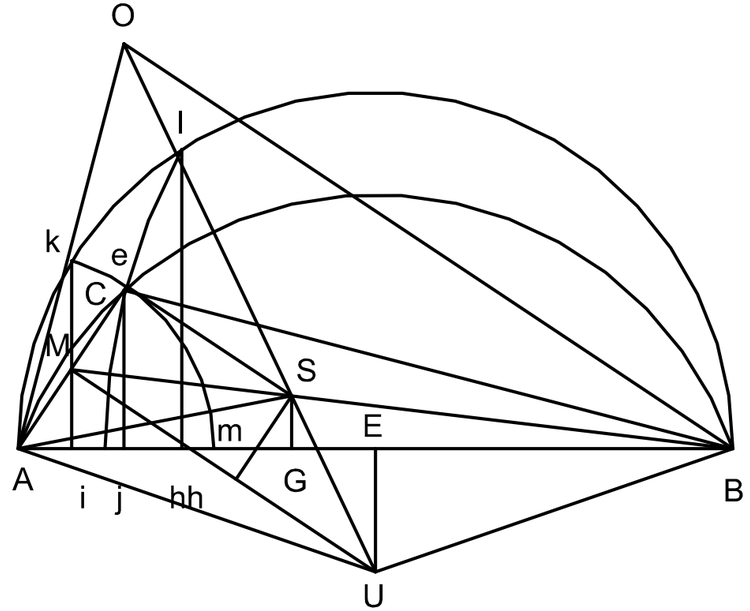
$Um_\delta := \text{if}\left[AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}\,,\ MU_\delta - Mm_\delta\,,\ MU_\delta + Mm_\delta\right]$ $\quad SU_\delta := \sqrt{(Um_\delta)^2 + (Sm_\delta)^2}$ $\quad UO_\delta := 3 \cdot SU_\delta$

Due to the way in which certain lines lay, the above switch was needed.

Definitions.

Is this a TRIANGLE = 1 ?

Side_1 ≡ 21 **Side_2 ≡ 14.4** **Side_3 ≡ 7.75**



$AC_\delta < \sqrt{(BC_\delta)^2 + (AB_\delta)^2}$ =

0
1
1

SU_δ =	UO_δ =	AU_δ =
14.15083	42.452489	16.702084
14.15083	42.452489	16.702084
14.15083	42.452489	16.702084

121695

Descriptions.

$AD := n \cdot a \qquad BE := \sqrt{a^2 + b^2} \qquad BC := \frac{a^2}{BE}$

$CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$FG := \frac{AD}{2} \qquad CG := \sqrt{FG^2 - CF^2}$

$AG := FG \qquad AC := AG + CG$

$BG := CG - BC \qquad DG := FG$

$BD := DG - BG \qquad AB := AG + BG$

$AE := AB + BE \qquad DE := BE - BD$

$DH := \frac{b^2}{DE} \qquad DI := AE \qquad HI := DI - DH$

$z := AE \qquad z = 12.621556$

$c := DE \qquad c = 0.621556$

$d := HI \qquad d = 6.18609$

$z^2 - \left(n \cdot a \cdot z + b^2 + c \cdot d\right) = 0$

Given.

$n \equiv 3$

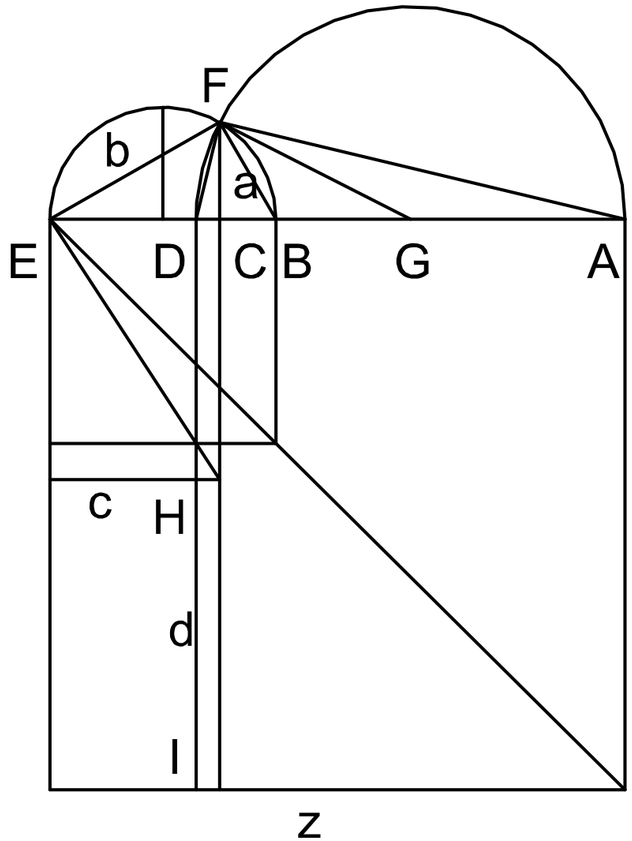
$a \equiv 4$

$b \equiv 2$

Place values here :

Descartes gives a figure for solving $z^2 = az + b^2$ which should have been stated as $z^2 = 2az + b^2$, generalize the figure. Descartes' figure was given only when n = 2. In this form, it is an equation of two givens and one unknown, generalized it as three givens and solves for three unknowns. Attempt to eliminate two of the unknowns as a function of the three givens.

Given a, n and b for the equation $z^2 =$ $n\,az + b^2 + cd$ find z, c, and d.



Expressing c and d in terms of the givens does not really look esthetically pleasing.

$$d - 2\cdot a\cdot\frac{b^2}{\sqrt{a^2+b^2}}\cdot\frac{\left(2\cdot a-\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{-2\cdot b+n\cdot\sqrt{a^2+b^2}}\cdot\sqrt{2\cdot b+n\cdot\sqrt{a^2+b^2}}\right)}=0$$

$$c-\frac{-1}{2}\cdot\frac{\left(-2\cdot b^2+n\cdot a\cdot\sqrt{a^2+b^2}-a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\sqrt{a^2+b^2}}=0$$

The symbolic processor could not reduce cd directly so I had to do it in terms of the equation by resolving *z*.

$$z-\frac{1}{2}\cdot\frac{\left(n\cdot a\cdot\sqrt{a^2+b^2}+a\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}+2\cdot b^2\right)}{\sqrt{a^2+b^2}}=0$$

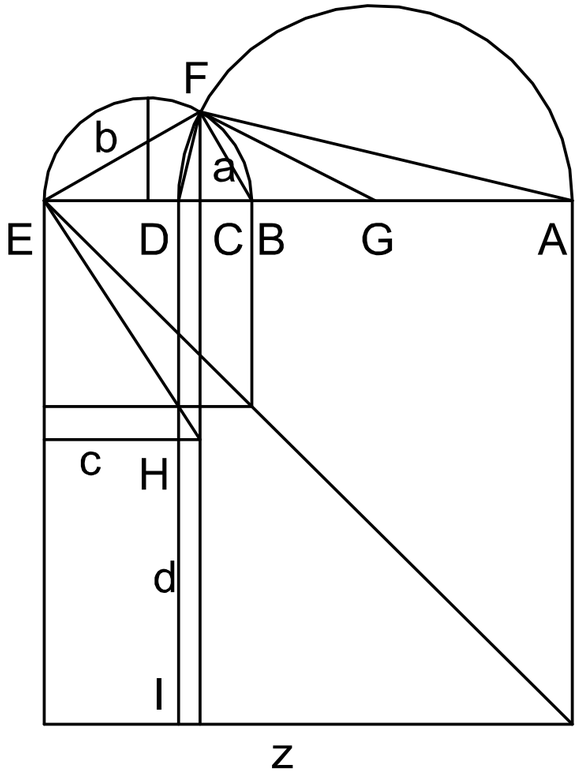
$$z^2-\left(n\cdot a\cdot z+b^2+c\cdot d\right)=0 \qquad p:=-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}$$

$$(c\cdot d)-p=0$$

$$z^2-\left[n\cdot a\cdot z+b^2+\left[-a\cdot b^2\cdot\frac{\left(2\cdot a-\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}\right)}{\left(a^2+b^2\right)}\right]\right]=0$$

Solve for *z* below.

$$\begin{pmatrix}\frac{1}{2}\cdot n\cdot a+\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}}\\ \frac{1}{2}\cdot n\cdot a-\frac{1}{2}\cdot\frac{\sqrt{n^2\cdot a^4+n^2\cdot a^2\cdot b^2-4\cdot a^2\cdot b^2+4\cdot b^4+4\cdot a\cdot b^2\cdot\sqrt{n^2\cdot a^2+n^2\cdot b^2-4\cdot b^2}}}{\sqrt{a^2+b^2}}\end{pmatrix}$$



121895

Descriptions.

Scholia: The Geometry of Rene Descartes with a facsimile of the first edition. Translated by D. Eugene and M. Latham

$z^2 := az - b^2$

The problem is given for the solution of z when a and b are given. Working the problem backward, $z^2 + b^2 := az$ **one can see constants in the figure for solving when only a and b are given.**

$b := 2.12 \qquad z := 1.41 \qquad c := \frac{b^2}{z}$

Finding a is just a matter of expressing b in terms of cz, and a becomes z + c.

$a := z + c$

We find that this c has another relation to z, for it holds a proportion to it in the given equation.

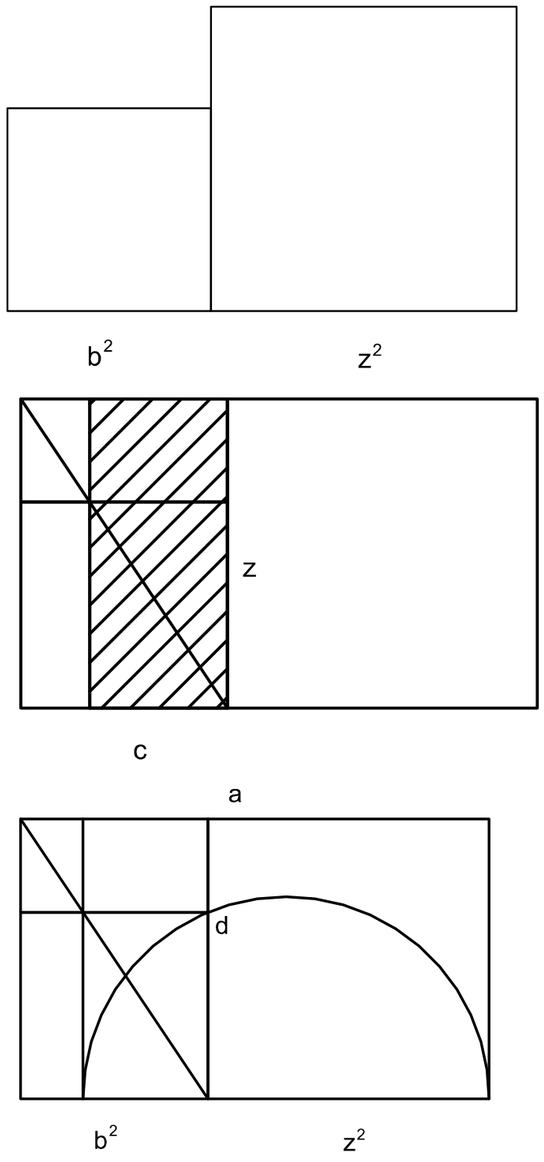
$\left(c^2 + b^2\right) - a \cdot c = 1.776357 \times 10^{-15}$

$\left(c^2 + b^2\right) - [(z + c) \cdot c] = 1.776357 \times 10^{-15}$

$\left(c^2 + b^2\right) - (c \cdot z) - c^2 = 0$

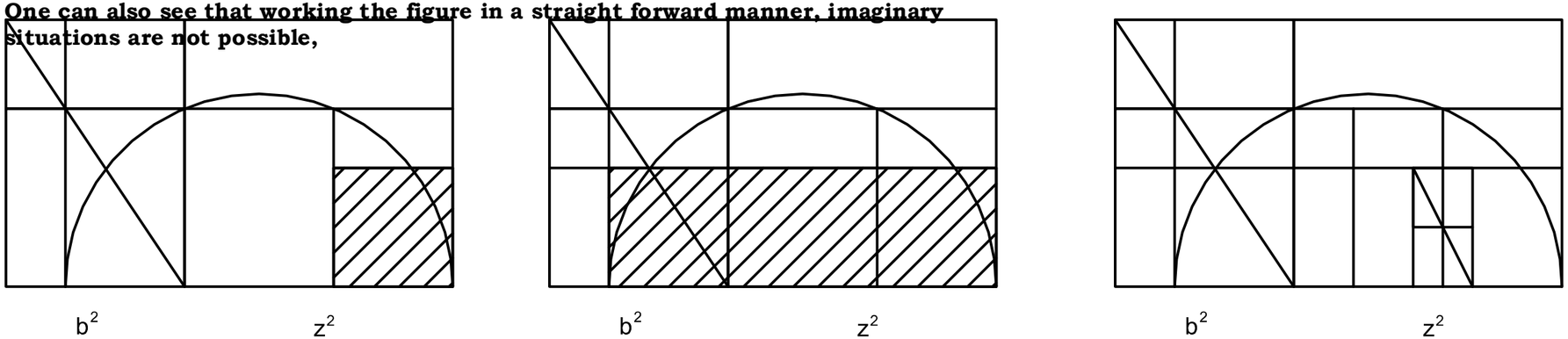
$\left(z^2 + b^2\right) - (z + c) \cdot z = 0$

$\left(z^2 + b^2\right) - (c \cdot z) - z^2 = 0$

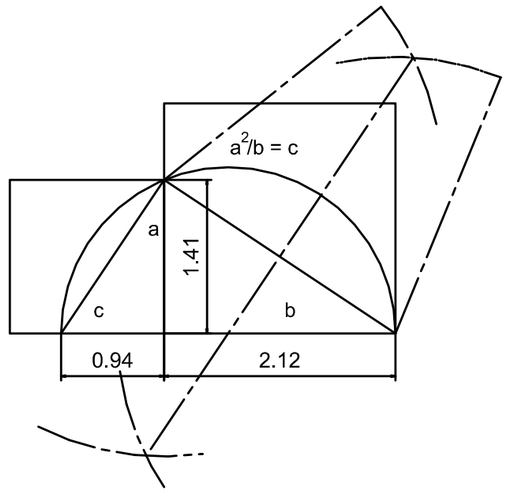


Descartes and other mathematicians speak as if we have two different values for z, however, I see quite plainly that we have a z and a c that was found. The unique name of the symbols in context are thus preserved.

One can also see that working the figure in a straight forward manner, imaginary situations are not possible,



The primitive form of construction is very suggestive of Descartes process. If however we use Pythagorean division, the figure may not be so apparent. One should note that contrary to his statement on page 4, one cannot divide a line by a line. In geometry one can divide a square by a line. This is the law of tautology, which holds for all mathematics. It will also be noted that Descartes claims to be working with a quadratic equation, and his solution is indeed for one, however he states it as a quadratic deficient by a rectangle, deficient in fact by az. The authors of the notes make the same mistake.



Unit.

$EF := 1$

Given.

$N := 2$

122095

Descriptions.

$EJ := EF \cdot N \quad AE := \frac{EJ^2}{EF} \quad AF := AE + EF$

$AB := \frac{AF}{2} \quad BF := AB \quad BE := BF - EF \quad BH := BE$

$BJ := \sqrt{EJ^2 + BE^2} \quad BD := \frac{BE \cdot BH}{BJ} \quad BG := BD$

$BC := \frac{BE \cdot BG}{BJ} \quad GH := BH - BG \quad DE := BE - BD$

$HO := DE \quad EG_1 := \sqrt{BE^2 - BG^2} \quad GJ := BJ - BG$

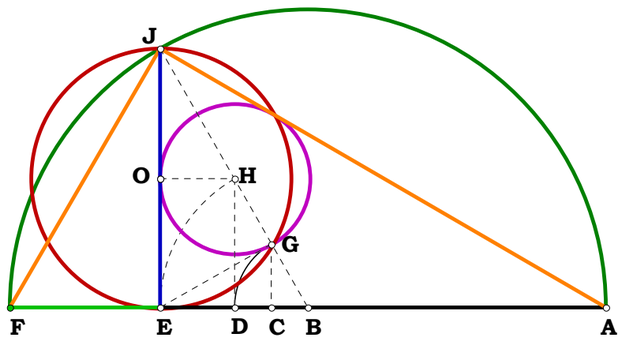
$EG_2 := \sqrt{EJ^2 - GJ^2} \quad \frac{GH}{HO} = 1 \quad \frac{EG_1}{EG_2} = 1$

$\left(BC^2 \cdot BF\right)^{\frac{1}{3}} - BD = 0 \quad \left(BC \cdot BF^2\right)^{\frac{1}{3}} - BE = 0$

Definitions.

$BC - \frac{(N+1)^3 \cdot (N-1)^3}{2 \cdot \left(N^2 + 1\right)^2} = 0$

Just For Fun



Unit.

$AB := 1$

Given.

$N := 7$

122195

Descriptions.

$AD := AB \cdot N \quad AC := \sqrt{AB \cdot AD} \quad BD := AD - AB \quad CD := AD - AC$

$BC := BD - CD \quad CF := \sqrt{BC \cdot CD} \quad CF := \sqrt{BC \cdot CD} \quad DF := \sqrt{CF^2 + CD^2}$

$DK := \frac{DF \cdot BD}{CD} \quad FK := \frac{DK \cdot BC}{BD} \quad HK := \frac{FK \cdot FK}{DK} \quad JK := \frac{HK \cdot HK}{FK}$

Definitions.

$\frac{DK}{FK} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \quad \frac{DK}{HK} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \quad \frac{DK}{JK} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

A more civil figure.

$AB := 1 \quad AF := N \quad BF := AF - AB \quad AE := \sqrt{AB \cdot AF} \quad BE := AE - AB \quad BH := BE$

$BD := \frac{BE \cdot BH}{BF} \quad BG := BD \quad BC := \frac{BD \cdot BG}{BE}$

Definitions.

$\frac{BF}{BE} - \frac{(N-1)}{\left(\sqrt{N}-1\right)} = 0 \quad \frac{BF}{BD} - \frac{N^2 - 2 \cdot N + 1}{N - 2 \cdot \sqrt{N} + 1} = 0 \quad \frac{BF}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\frac{3}{2}} - 3 \cdot N + 3 \cdot \sqrt{N} - 1} = 0$

Another figure.

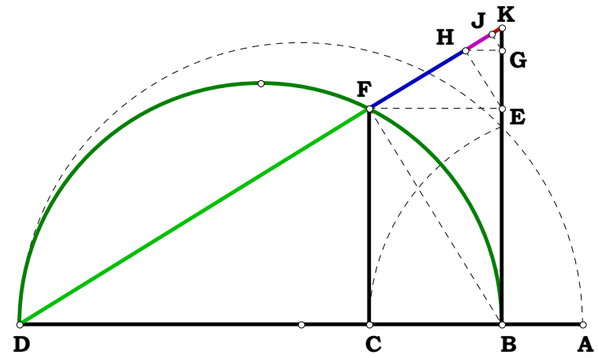
$AB := 1 \quad AG := N \quad BG := AG - AB \quad AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}}$

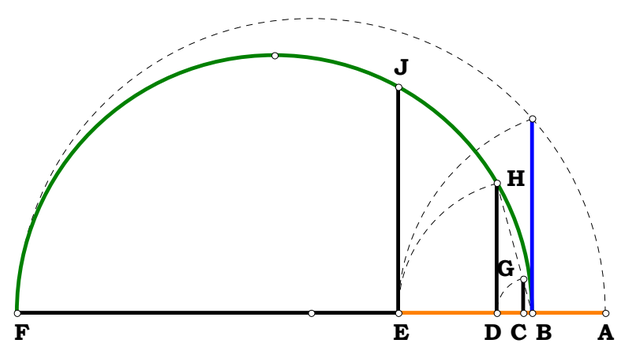
$BF := AF - AB \quad BJ := BF \quad BE := \frac{BJ \cdot BF}{BG} \quad BH := BE$

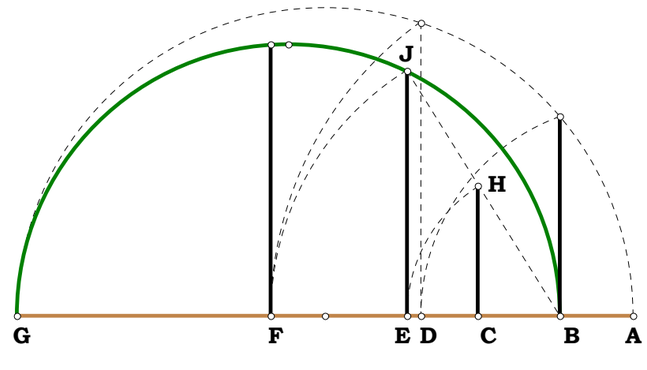
$BC := \frac{BH \cdot BE}{BF}$

$\frac{BG}{BF} - \frac{N-1}{N^{\frac{3}{4}} - 1} = 0 \quad \frac{BG}{BE} - \frac{N^2 - 2 \cdot N + 1}{N^{\left(\frac{3}{2}\right)} - 2 \cdot N^{\left(\frac{3}{4}\right)} + 1} = 0 \quad \frac{BG}{BC} - \frac{N^3 - 3 \cdot N^2 + 3 \cdot N - 1}{N^{\left(\frac{9}{4}\right)} - 3 \cdot N^{\left(\frac{3}{2}\right)} + 3 \cdot N^{\left(\frac{3}{4}\right)} - 1} = 0$

Pascal's Triangle With Exponential Division







122995

Descriptions.

Unit.

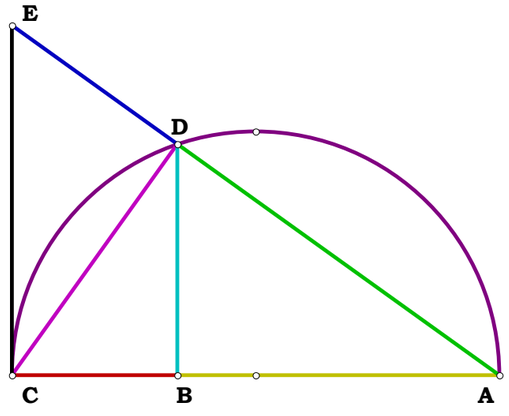
Given.

Definitions.

$N_1 := 4$

$N_2 := 9$

Given AC and CE find BC.



$AC := N_1 \quad CE := N_2 \quad AE := \sqrt{AC^2 + CE^2} \quad AD := \frac{AC \cdot AC}{AE}$

$AB := \frac{AD^2}{AC} \quad BC := AC - AB \quad BD := \sqrt{AB \cdot BC}$

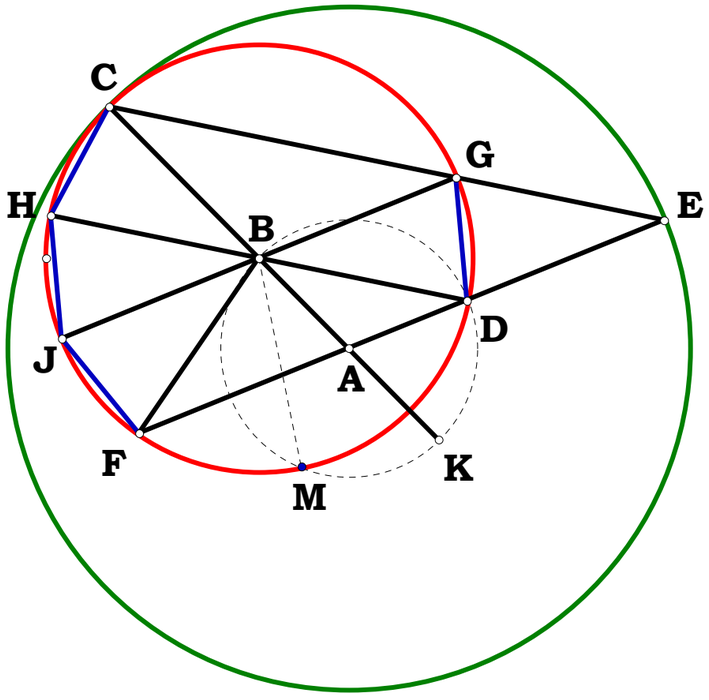
$CD := \sqrt{BC^2 + BD^2} \quad DE := AE - AD$

$BC - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} = 0 \quad BD - \frac{N_1^2 \cdot N_2}{N_1^2 + N_2^2} = 0 \quad AB - \left(N_1 - \frac{N_1 \cdot N_2^2}{N_1^2 + N_2^2} \right) = 0$

$AD - \frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} = 0 \quad DE - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

010496

The Archamedian Paper Trisector- Without the Numbers.



Given any circle AB.

Given any circle BC such that BC ≤ 2AB.

Construct AE such that AE = AC.

As AC = AB + BC and AD = AB so too DE = BC.

Construct DH parallel to BD. Construct CE.

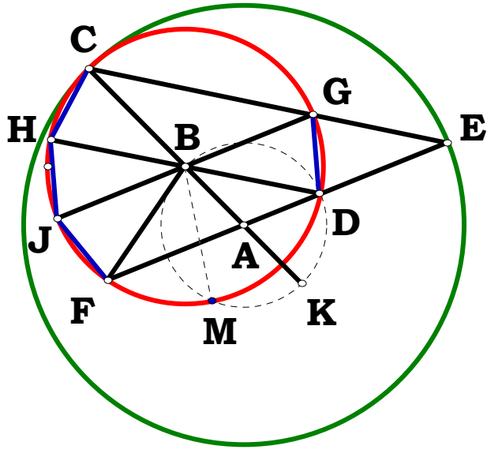
As AB = AD and AC = AE, Δ ABD is proportional to Δ ACE, therefore CE is parallel to BD.

From here one can take two paths.

Construct GJ parallel to EF.

As CE is parallel to DH, DG = CH.

As GJ is parallel to EF, DG = FJ.



As ∠ HBJ is opposite and equal to ∠ GBD, DG = HJ, therefore ∠ DG is $\frac{1}{3}$ CF.

As CE is parallel to DH and EF is parallel to GJ, EGBD is equilateral.

By construction DK = KM.

As DH is parallel to CE, CH = DG.

As DK is equal and opposite CH, MK + DK + DG is $\frac{1}{3}$ DG.

Unit.

$AD := 2$

Given.

$N := AD$

010796

A rusty Compass construction for the duplication of the cube.

Descriptions.

$AB := \frac{AD}{2}$ $AG := \sqrt{2 \cdot AB^2}$ $AF := \frac{AG}{9} \cdot 8$

$AC := AF$ $AC = 1.257079$

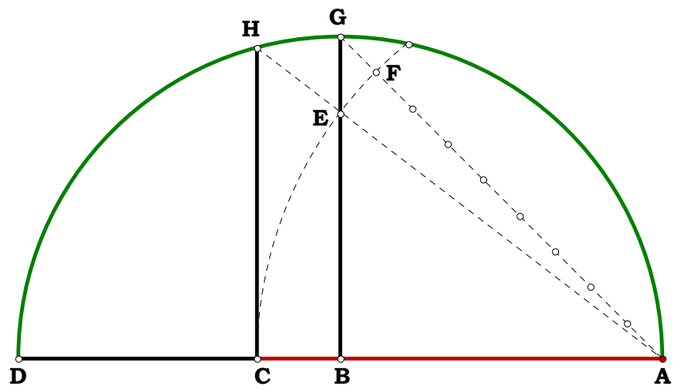
$\left(AB^2 \cdot AD\right)^{\frac{1}{3}} = 1.259921$ $\frac{\left(AB^2 \cdot AD\right)^{\frac{1}{3}}}{AC} = 1.002261$

I remember a rusty construction for squaring a circle off the base of a right triangle. Here is one more rust pile construction.

Definitions.

$\frac{2^{\frac{1}{3}}}{2} \cdot \left(N^3\right)^{\frac{1}{3}}$ $\frac{9 \cdot 2^{\frac{5}{6}} \cdot \left(N^3\right)^{\frac{1}{3}}}{16 \cdot \sqrt{N^2}}$

Rusty Cubes



010896a

Given.

$R_1 := 2$

$R_2 := 5$

$D := 4$

Descriptions.

$DE := R_1 \quad KM := R_2 \quad EK := D \quad DM := DE + EK + KM$

$EF := DE \quad JK := KM \quad FJ := EK - (EF + AD := DM$

$BM := DM \quad DF := DE + EF \quad JM := JK + KM$

$FG := \frac{DF \cdot FJ}{DF + JM} \quad GJ := FJ - FG \quad DI := \frac{DM}{2}$

$DG := DF + FG \quad GI := DI - DG \quad CI := DI \quad \underline{GN} := \frac{AD \cdot FG}{DF}$

$GH := \frac{GI \cdot GN}{CI + GN} \quad DH := DF + FG + GH \quad DH = 1.375$

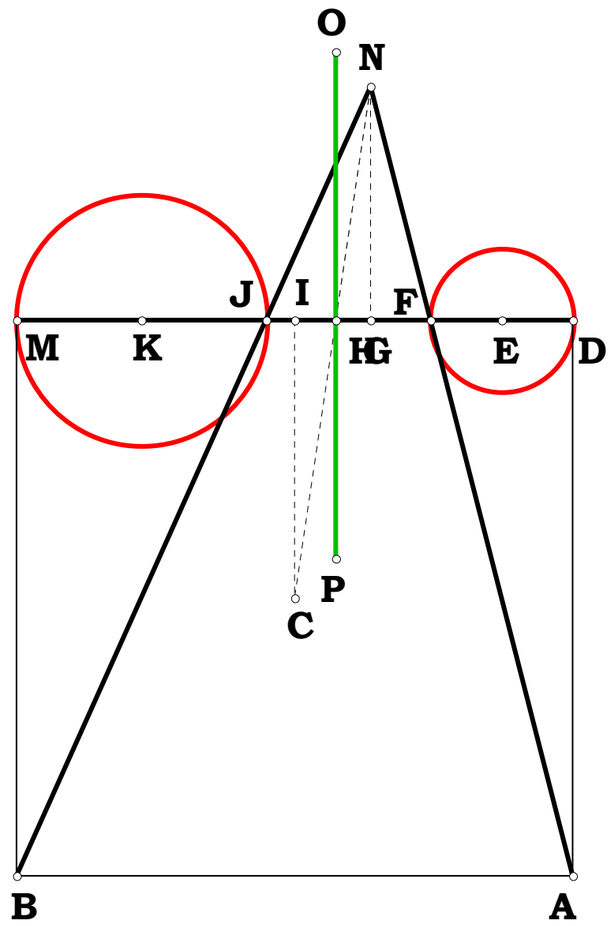
$HM := DM - DH$

Definitions.

$$DH - \frac{(R_1 + R_2 + D) \cdot (R_1 - R_2 + D)}{2 \cdot D} = 0$$

$$HM - \frac{(R_2 + R_1 + D) \cdot (R_2 - R_1 + D)}{2 \cdot D} = 0$$

Alternate Method Power Line



Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

010896

Descriptions.

$BG := AG - AB$

$BO := \frac{BG}{2} \quad AD := \sqrt{AB \cdot AG} \quad AC := \left(AB^3 \cdot AG\right)^{\frac{1}{4}}$

$AF := \left(AB \cdot AG^3\right)^{\frac{1}{4}} \quad BC := AC - AB \quad FG := AG - AF$

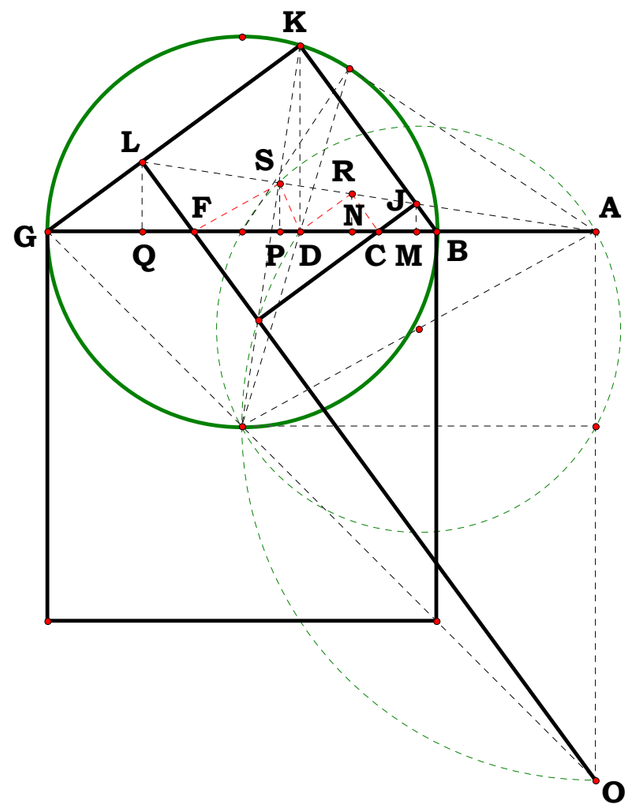
$DG := AG - AD \quad BD := AD - AB \quad DK := \sqrt{BD \cdot DG}$

$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG} \quad FQ := \frac{BD \cdot FG}{BG}$

$\frac{GL}{BJ} = 5 \quad \frac{AG}{AB} = 5$

The figure cuts the base line in quad roots. As such it would be another trivial method for doing quad roots, however I am interested in some of the ratio's of the figure instead.



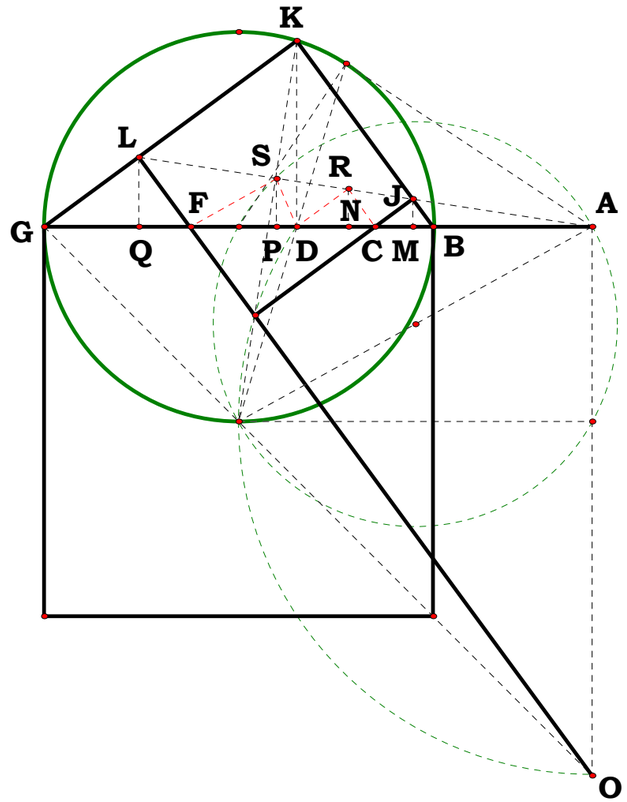
Definitions.

$$\frac{GK}{GL} - \left(\frac{N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}}}{N^{\frac{3}{4}}} \right) = 0 \qquad \frac{BK}{BJ} - \left(N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} \right) = 0$$

$$CD := AD - AC \quad DF := AF - AD \quad BM := \frac{BD \cdot BC}{BG} \quad CN := \frac{BD \cdot CD}{BG} \quad DP := \frac{BD \cdot DF}{BG}$$

$$\frac{BG}{BM} - \left(N^{\frac{5}{4}} + N + N^{\frac{3}{4}} + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} \right) = 0$$

$$\frac{BG}{CN} - \left(N + N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} \right) = 0$$

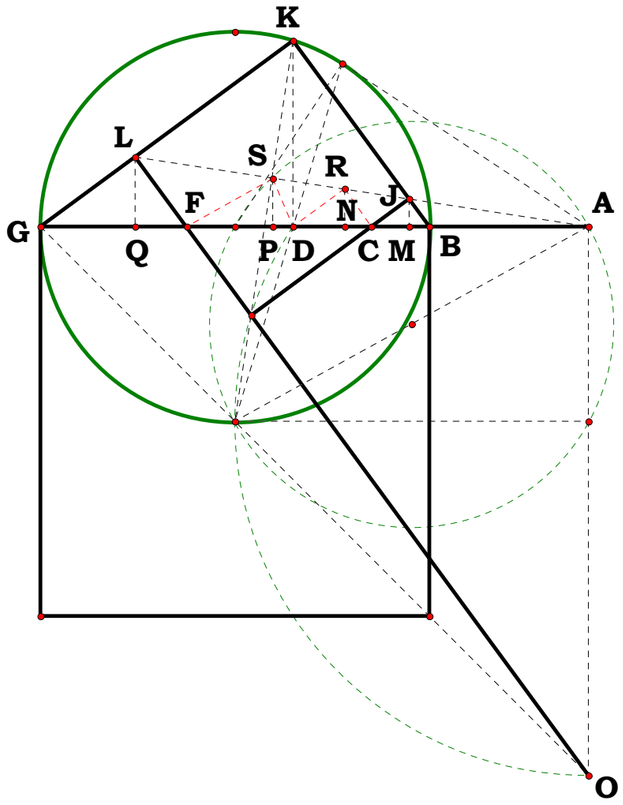


$$\frac{BG}{DP} - \left(N^{\frac{3}{4}} + N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} + N^{\frac{-2}{4}} \right) = 0$$

$$\frac{BG}{FQ} - \left(N^{\frac{2}{4}} + N^{\frac{1}{4}} + N^{\frac{0}{4}} + N^{\frac{0}{4}} + N^{\frac{-1}{4}} + N^{\frac{-1}{4}} + N^{\frac{-2}{4}} + N^{\frac{-3}{4}} \right) = 0$$

$$\frac{AG}{BM} - \left(\frac{N^{\frac{3}{2}} + N}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0 \qquad \frac{AG}{CN} - \left(\frac{N^{\frac{5}{4}} + N^{\frac{3}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0$$

$$\frac{AG}{DP} - \left(\frac{N + N^{\frac{2}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0 \qquad \frac{AG}{FQ} - \left(\frac{N^{\frac{3}{4}} + N^{\frac{1}{4}}}{N^{\frac{1}{4}} - N^{\frac{0}{4}}} \right) = 0$$



011396

Descriptions.

Unit.

$AD := 1$

Given.

$N_1 := 5$

$N_2 := 7$

$AB := \frac{AD}{N_1}$ $BD := AD - AB$ $BG := \sqrt{AB \cdot BD}$ $BE := \frac{BG}{N_2}$

$BC := \frac{BD \cdot BE}{BG}$ $AE := \sqrt{AB^2 + BE^2}$ $AC := AB + BC$ $AF := \frac{AE \cdot AD}{AC}$

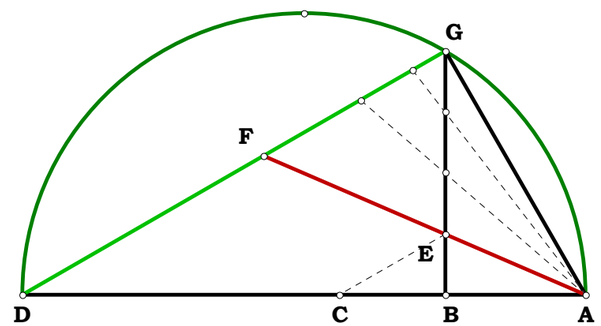
$EF := AF - AE$ $\frac{AF}{EF} - \frac{N_1 \cdot N_2}{(N_1 - 1) \cdot (N_2 - 1)} = 0$ $\Delta := 2 .. N_1$ $\delta := 2 .. N_2$

Definitions.

$SeriesAF_{\Delta, \delta} := \frac{\Delta \cdot \delta}{(\Delta - 1) \cdot (\delta - 1)}$

$$SeriesAF = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2.666667 & 2.5 & 2.4 & 2.333333 \\ 0 & 0 & 3 & 2.25 & 2 & 1.875 & 1.8 & 1.75 \\ 0 & 0 & 2.666667 & 2 & 1.777778 & 1.666667 & 1.6 & 1.555556 \\ 0 & 0 & 2.5 & 1.875 & 1.666667 & 1.5625 & 1.5 & 1.458333 \end{pmatrix}$$

Pyramid of Ratios VI, Moving the Point



$DG := \sqrt{BD^2 + BG^2}$

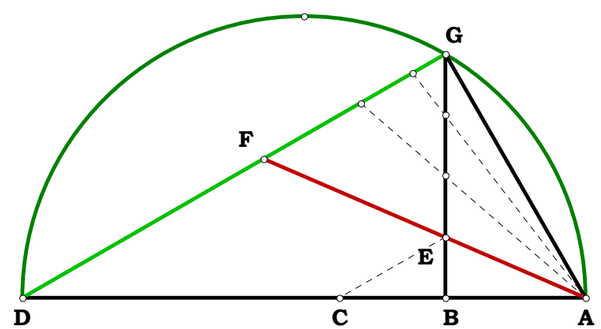
$CE := \sqrt{BC^2 + BE^2}$

$DF := \frac{CE \cdot AD}{AC}$

$GF := DG - DF$

$\frac{DG}{GF} - \frac{(N_2 + N_1 - 1)}{(N_2 - 1)} = 0$ $SeriesDG_{\Delta, \delta} := \frac{(\delta + \Delta - 1)}{(\delta - 1)}$

$$SeriesDG = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1.666667 & 1.5 & 1.4 & 1.333333 \\ 0 & 0 & 4 & 2.5 & 2 & 1.75 & 1.6 & 1.5 \\ 0 & 0 & 5 & 3 & 2.333333 & 2 & 1.8 & 1.666667 \\ 0 & 0 & 6 & 3.5 & 2.666667 & 2.25 & 2 & 1.833333 \end{pmatrix}$$



011696

Unit.

$AB := 1$

Given.

$N := 10 \qquad AG := N$

Descriptions.

$BG := AG - AB \qquad BO := \frac{BG}{2}$

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}} \qquad BC := AC - AB$

$AF := \left(AB \cdot AG^2\right)^{\frac{1}{3}} \qquad BF := AF - AB \qquad FG := BG - BF$

$HJ := \frac{BC \cdot BG}{BC + FG} \qquad BD := HJ \qquad DG := BG - BD$

$DJ := \sqrt{BD \cdot DG} \qquad GJ := \sqrt{DJ^2 + DG^2} \qquad BJ := \sqrt{DJ^2 + BD^2}$

$GN := \frac{GJ \cdot FG}{BG} \qquad BM := \frac{BJ \cdot BC}{BG}$

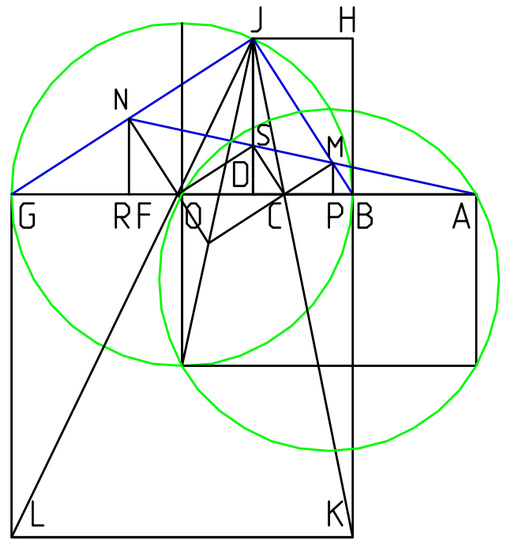
$\frac{AG}{AB} = 10 \qquad \frac{GN}{BM} = 10$

Definitions.

$\left(\frac{AB}{AG}\right)^{\frac{2}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} = 1.679602 \qquad \frac{GJ}{GN} = 1.679602 \qquad \frac{N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}}}{N^{\frac{2}{3}}} = 1.679602$

$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 7.796024 \qquad \frac{BJ}{BM} = 7.796024 \qquad N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 7.796024$

The figure cuts the base in Cube Roots and provides some interesting ratios.



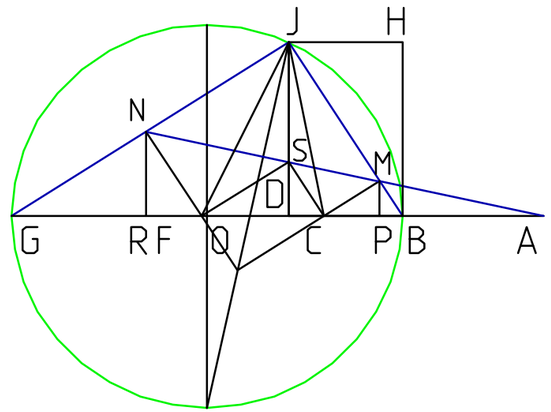
Descriptions.

$CF := AF - AC$ $\quad BP := \frac{BD \cdot BC}{BG}$ $\quad CD := \frac{BD \cdot CF}{BG}$ $\quad FR := \frac{BD \cdot FG}{BG}$

Definitions.

$$\left(\frac{AG}{AB}\right)^{\frac{4}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} = 43.981959$$

$$N^{\frac{4}{3}} + N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} = 43.981959 \qquad \frac{BG}{BP} = 43.981959$$



$$\left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{3}{3}} + \left(\frac{AB}{AG}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} = 20.414617$$

$$N^{\frac{3}{3}} + N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} = 20.414617 \qquad \frac{BG}{CD} = 20.414617$$

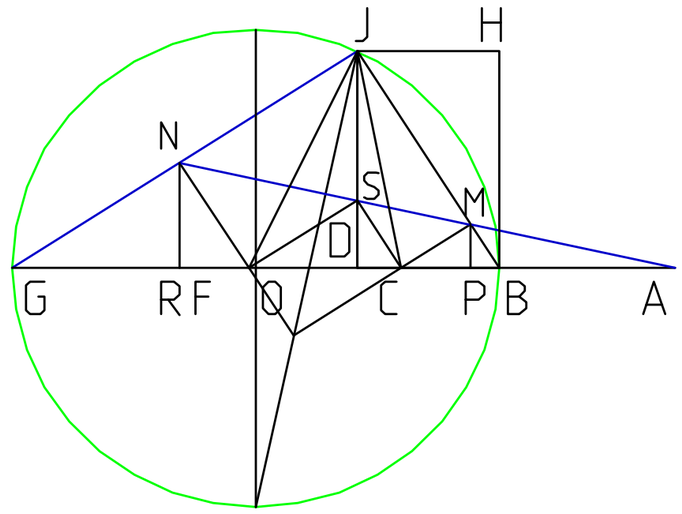
$$\left(\frac{AG}{AB}\right)^{\frac{2}{3}} + \left(\frac{AG}{AB}\right)^{\frac{1}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AG}{AB}\right)^{\frac{0}{3}} + \left(\frac{AB}{AG}\right)^{\frac{1}{3}} + \left(\frac{AB}{AG}\right)^{\frac{2}{3}} = 9.475626$$

$$N^{\frac{2}{3}} + N^{\frac{1}{3}} + N^{\frac{0}{3}} + N^{\frac{0}{3}} + \frac{1}{N^{\frac{1}{3}}} + \frac{1}{N^{\frac{2}{3}}} = 9.475626 \qquad \frac{BG}{FR} = 9.475626$$

$$\frac{AG^{\frac{5}{3}} + AB^{\frac{2}{3}} \cdot AG}{AG^{\frac{1}{3}} \cdot AB^{\frac{4}{3}} - AB^{\frac{5}{3}}} = 48.868844 \qquad \frac{AG}{BP} = 48.868844 \qquad \frac{N^{\frac{5}{3}} + N}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 48.868844$$

$$\frac{AG^{\frac{4}{3}} + AB^{\frac{2}{3}} \cdot AG^{\frac{2}{3}}}{AG^{\frac{1}{3}} \cdot AB - AB^{\frac{4}{3}}} = 22.682908 \qquad \frac{AG}{CD} = 22.682908 \qquad \frac{N^{\frac{4}{3}} + N^{\frac{2}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 22.682908$$

$$\frac{AG + AB^{\frac{2}{3}} \cdot AG^{\frac{1}{3}}}{AG^{\frac{1}{3}} \cdot AB^{\frac{2}{3}} - AB} = 10.528473 \qquad \frac{AG}{FR} = 10.528473 \qquad \frac{N + N^{\frac{1}{3}}}{N^{\frac{1}{3}} - N^{\frac{0}{3}}} = 10.528473$$



011796A

Unit.

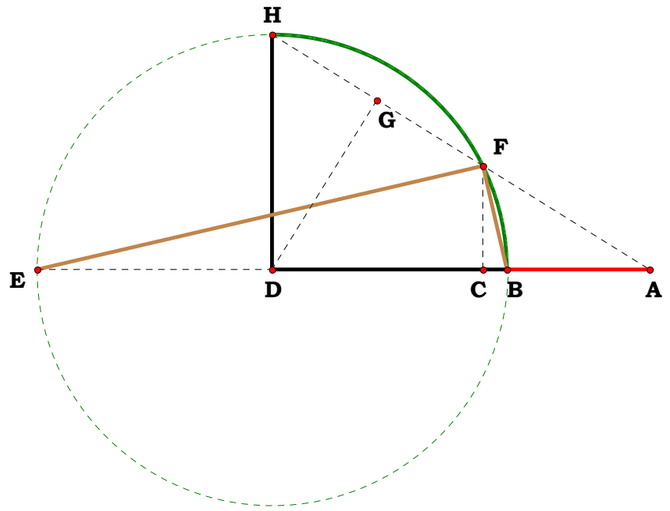
$AB := 1$

Given.

$N := 3$

Given AE and AB on AE, place a right triangle on BE as base such that the opposite sides are in the ratio of AB to AE.

Right Triangle In A Given Ratio



Descriptions.

$AE := AB \cdot N \quad BE := AE - AB \quad BD := \frac{BE}{2} \quad DF := BD \quad DE := BD$

$AD := AB + BD \quad DH := BD \quad AH := \sqrt{AD^2 + DH^2} \quad AG := \frac{AD \cdot AD}{AH}$

$GH := AH - AG \quad FG := GH \quad AF := AH - (FG + GH)$

$CD := \frac{AD^2 - AF^2 + DF^2}{2 \cdot AD} \quad BC := BD - CD \quad CE := CD + DE$

$CF := \sqrt{BC \cdot CE} \quad BF := \sqrt{BC^2 + CF^2} \quad EF := \sqrt{CE^2 + CF^2}$

$AC := AD - CD \quad AC = 1.2$

Definitions.

$\frac{AE}{AB} - \frac{EF}{BF} = 0 \quad AC - \frac{N^2 + N}{N^2 + 1} = 0 \quad BF - \frac{N - 1}{\sqrt{N^2 + 1}} = 0 \quad EF - \frac{N^2 - N}{\sqrt{N^2 + 1}} = 0$

$S_1 := AD \quad S_2 := AF \quad S_3 := DF$

$\frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} - CD = 0$

011796B

Unit.

$AB := 1$

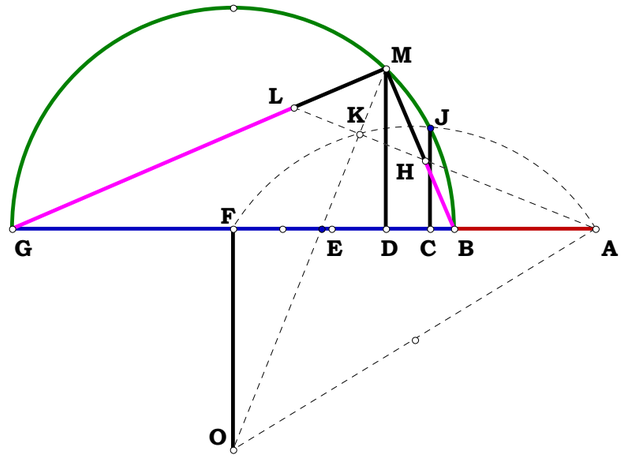
Given.

$N_1 := 5$

$N_2 := 9$

Divide The Sides Of A Triangle In A Given Ratio

Given AG and AB on AG and a right triangle on BG divide the sides of the triangle such that a section on one side is to the other as AB is to AG.



Descriptions.

$AG := AB \cdot N_1 \quad BG := AG - AB \quad BF := \frac{BG}{2} \quad AC := \frac{N_1^2 + N_1}{N_1^2 + 1}$

$AF := AB + BF \quad CF := AF - AC \quad BC := AC - AB \quad BD := \frac{CF}{N_2} + BC$

$DF := BF - BD \quad DG := BG - BD \quad DM := \sqrt{BD \cdot DG} \quad FO := BF$

$EF := \frac{DF \cdot FO}{FO + DM} \quad AE := AF - EF \quad EO := \sqrt{EF^2 + FO^2}$

$MO := \frac{EO \cdot (DM + FO)}{FO} \quad EK := \frac{EF \cdot AE}{EO} \quad KM := MO - (EK + EO)$

$HM := \sqrt{2 \cdot KM^2} \quad BM := \sqrt{BD^2 + DM^2} \quad BH := BM - HM$

$GM := \sqrt{DG^2 + DM^2} \quad LM := HM \quad GL := GM - LM \quad \frac{AG}{AB} - \frac{GL}{BH} = 0$

Definitions.

$AG - N_1 = 0 \quad BG - (N_1 - 1) = 0 \quad BF - \frac{N_1 - 1}{2} = 0 \quad AC - \frac{N_1^2 + N_1}{N_1^2 + 1} = 0 \quad FO - \frac{N_1 - 1}{2} = 0 \quad AF - \frac{N_1 + 1}{2} = 0$

$CF - \frac{(N_1 + 1) \cdot (N_1 - 1)^2}{2 \cdot (N_1^2 + 1)} = 0 \quad BC - \frac{N_1 - 1}{N_1^2 + 1} = 0 \quad BD - \frac{(N_1 - 1) \cdot (N_1^2 + 2 \cdot N_2 - 1)}{2 \cdot N_2 \cdot (N_1^2 + 1)} = 0 \quad DF - \frac{(N_1 + 1) \cdot (N_1 - 1)^2 \cdot (N_2 - 1)}{2 \cdot N_2 \cdot (N_1^2 + 1)} = 0 \quad DG - \frac{(N_1 - 1) \cdot (2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1)}{2 \cdot N_2 \cdot (N_1^2 + 1)} = 0$

$DM - \frac{(N_1 - 1) \cdot \sqrt{(N_1^2 + 2 \cdot N_2 - 1) \cdot (2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1)}}{2 \cdot N_2 \cdot (N_1^2 + 1)} = 0 \quad EF - \frac{(N_1 - 1)^2 \cdot (N_1 + 1) \cdot (N_2 - 1)}{2 \cdot \left[N_2 + N_1^2 \cdot N_2 + \sqrt{(N_1^2 + 2 \cdot N_2 - 1) \cdot (2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1)}\right]} = 0$

$AE - \frac{(N_1 + 1) \cdot \left[N_1^2 - 2 \cdot N_1 + \sqrt{(2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1) \cdot (N_1^2 + 2 \cdot N_2 - 1)} + 2 \cdot N_1 \cdot N_2 + 1\right]}{2 \cdot \left[N_2 + N_1^2 \cdot N_2 + \sqrt{(2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1) \cdot (N_1^2 + 2 \cdot N_2 - 1)}\right]} = 0$

As Mathcad has started misbehaving, it is time to simply put an etc., here.

Unit.

$BH := 1$

Given.

$N := 5$

012196

Descriptions.

$BP := BH \quad HQ := BH \quad BG := \frac{BH}{2} \quad GO := BG \quad GN := BG$

$NO := BH \quad GH := BG \quad BE := \frac{BG}{N} \quad EG := BG - BE$

$EO := \sqrt{EG^2 + GO^2} \quad MO := \frac{GO \cdot NO}{EO} \quad EM := MO - EO$

$EL := \frac{EM}{2} \quad LK := EL \quad LO := EO + EL \quad LJ := \frac{LK^2}{LO}$

$EJ := EL - LJ \quad AE := \frac{EO \cdot EJ}{EG} \quad AH := AE + EG + GH$

$AB := AH - BH \quad DE := \frac{EG \cdot EM}{EO} \quad DM := \frac{GO \cdot EM}{EO}$

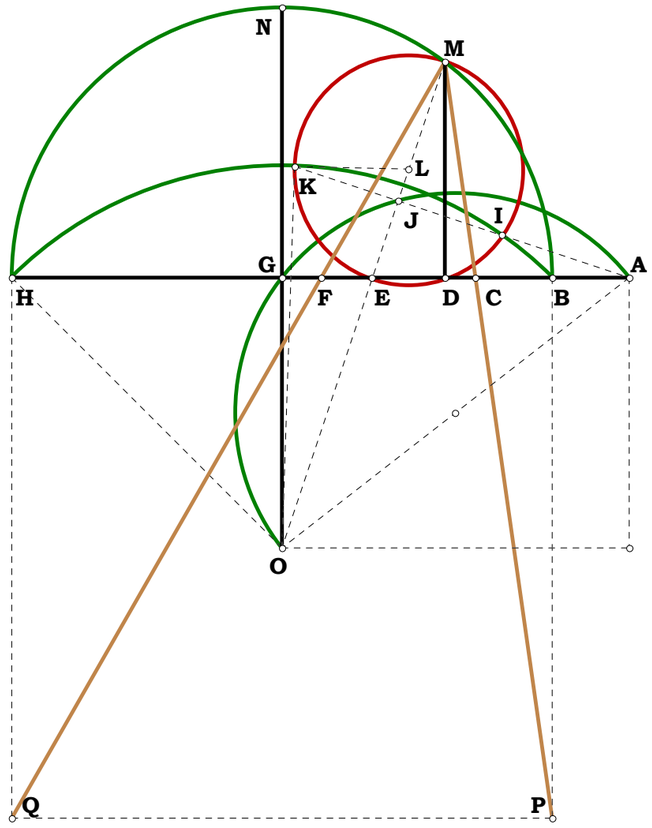
$BD := BG - (EG + DE) \quad BC := \frac{BD \cdot BP}{BP + DM} \quad DH := BH - BD$

$DF := \frac{DH \cdot DM}{DM + HQ} \quad AC := AB + BC \quad AF := AB + BD + DF$

Definitions.

$(AB^2 \cdot AH)^{\frac{1}{3}} - AC = 0 \quad (AB \cdot AH^2)^{\frac{1}{3}} - AF = 0$

More On Cube Roots



Unit.

$AF := 1$

Given.

$N_1 := 6$

$N_2 := 3$

012296

Descriptions.

$AL := AF \cdot N_1$ $FL := AL - AF$ $FJ := \frac{FL}{2}$

$AM := \sqrt{AF \cdot AL}$ $AJ := AF + FJ$ $AG := \frac{AM^2}{AJ}$

$GL := AL - AG$ $GK := \frac{GL}{N_2}$ $FG := AG - AF$

$FK := GK + FG$ $KL := FL - FK$ $EK := \sqrt{FK \cdot KL}$

$AK := FK + AF$ $AE := \sqrt{AK^2 + EK^2}$ $AD := \frac{AK \cdot AJ}{AE}$

$DE := AE - AD$ $BD := DE$ $AB := AE - 2 \cdot BD$

$\sqrt{AB \cdot AE} - AM = 0$

Definitions.

$AL - N_1 = 0$ $FL - (N_1 - 1) = 0$ $FJ - \frac{N_1 - 1}{2} = 0$ $AM - \sqrt{N_1} = 0$

$AJ - \frac{N_1 + 1}{2} = 0$ $AG - \frac{2 \cdot N_1}{N_1 + 1} = 0$ $GL - \frac{N_1 \cdot (N_1 - 1)}{N_1 + 1} = 0$ $GK - \frac{N_1 \cdot (N_1 - 1)}{N_2 \cdot (N_1 + 1)} = 0$

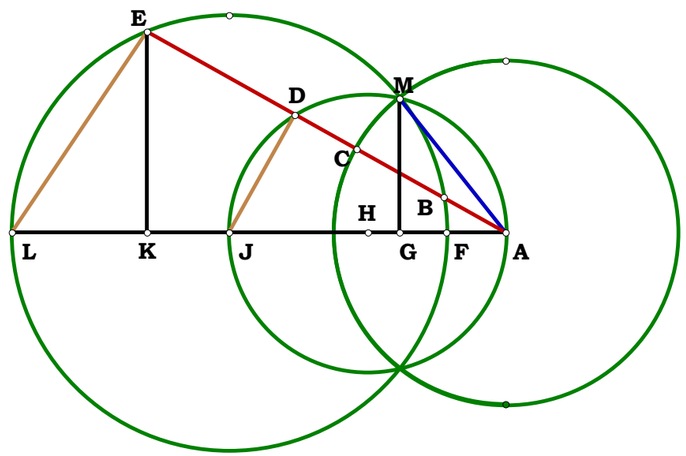
$FG - \frac{N_1 - 1}{N_1 + 1} = 0$ $FK - \frac{(N_1 - 1) \cdot (N_1 + N_2)}{N_2 \cdot (N_1 + 1)} = 0$ $KL - \frac{N_1 \cdot (N_2 - 1) \cdot (N_1 - 1)}{N_2 \cdot (N_1 + 1)} = 0$

$EK - \frac{(N_1 - 1) \cdot \sqrt{N_1 \cdot (N_2 - 1) \cdot (N_1 + N_2)}}{N_2 \cdot (N_1 + 1)} = 0$ $AK - \frac{N_1 \cdot (N_1 + 2 \cdot N_2 - 1)}{N_2 \cdot (N_1 + 1)} = 0$ $AE - \frac{\sqrt{N_1 \cdot (N_1 + N_2 - 1)}}{\sqrt{N_2}} = 0$

$AD - \frac{N_1 \cdot (N_1 + 2 \cdot N_2 - 1)}{2 \cdot \sqrt{N_2} \cdot \sqrt{N_1 \cdot (N_1 + N_2 - 1)}} = 0$ $DE - \frac{N_1 \cdot (N_1 - 1)}{2 \cdot \sqrt{N_2} \cdot \sqrt{N_1^2 - N_1 + N_1 \cdot N_2}} = 0$ $AB - \frac{N_1 \cdot \sqrt{N_2}}{\sqrt{N_1^2 - N_1 + N_1 \cdot N_2}} = 0$

For any E between M and L, AM is the square root of AB x AE.

Trivial Method Square Root



Unit.

$AB := 1$

Given.

$N := 5$

The tangent from any point outside a circle is equal to the square root of the difference to the circle and that difference plus the diameter of the circle.

012496

Descriptions.

$AE := N \quad BE := AE - AB \quad BD := \frac{BE}{2} \quad AD := AB + BD$

$DM := BD \quad DH := BD \quad CD := \frac{DH \cdot DM}{AD} \quad BC := BD - CD$

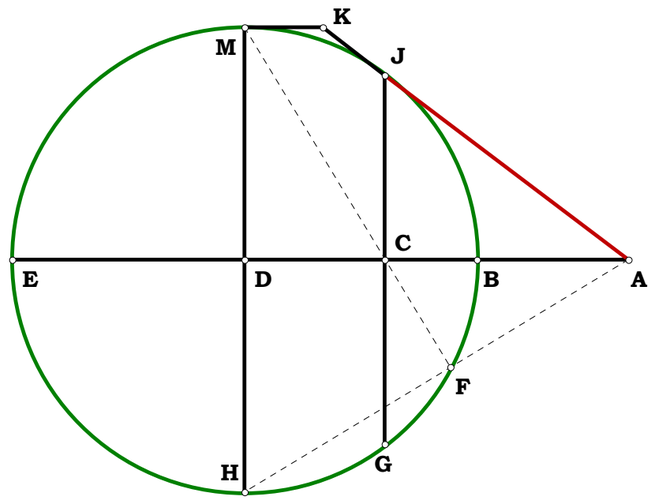
$CE := BE - BC \quad CJ := \sqrt{BC \cdot CE} \quad AC := AB + BC$

$AJ := \sqrt{(AB + BC)^2 + CJ^2}$

Definitions.

$AJ - \sqrt{N} = 0 \quad 2 \cdot \frac{N}{(1 + N)} - AC = 0$

Tangent



$AE - N = 0 \quad BE - (N - 1) = 0 \quad BD - \frac{N - 1}{2} = 0 \quad AD - \left(\frac{N + 1}{2}\right)$

$DM - \frac{N - 1}{2} = 0 \quad DH - \frac{N - 1}{2} = 0 \quad CD - \frac{(N - 1)^2}{2 \cdot (N + 1)} \quad BC - \frac{N - 1}{N + 1} = 0$

$CE - \frac{N \cdot (N - 1)}{N + 1} = 0 \quad CJ - \frac{\sqrt{N} \cdot (N - 1)}{(N + 1)} = 0 \quad AC - \frac{2 \cdot N}{N + 1} = 0 \quad AJ - \sqrt{N} = 0$

Unit.

$BE := 1$

Given.

$N := 9$

012596

Descriptions.

$$BD := \frac{BE}{2} \quad DK := BDJ := BD \quad JK := BE \quad DE := BD$$

$$BC := \frac{BD}{N} \quad CD := BD - BC \quad CK := \sqrt{CD^2 + DK^2}$$

$$HK := \frac{DK \cdot JK}{CK} \quad CH := HK - CK \quad CF := \frac{CH}{2}$$

$$FK := CK + CF \quad GK := \frac{DK \cdot FK}{CK} \quad FG := \frac{CD \cdot FK}{CK}$$

$$GJ := JK - GK \quad AD := \frac{GJ \cdot DK}{FG} \quad AE := AD + DE$$

$$AB := AE - BE$$

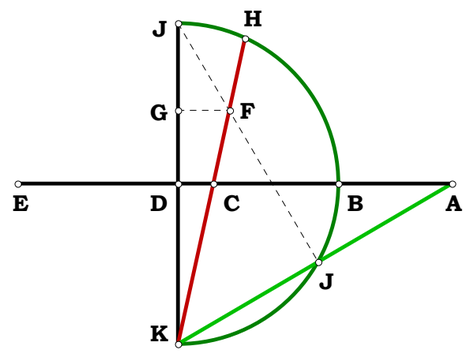
Definitions.

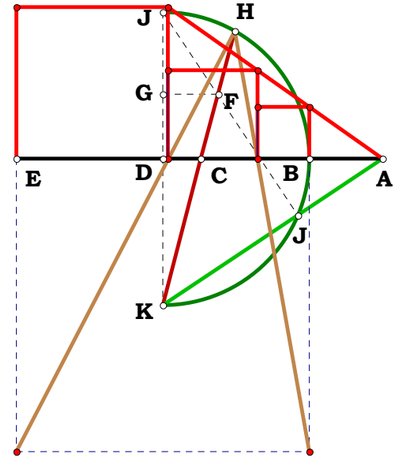
$$AE - \frac{(2 \cdot N - 1)^3}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$AB - \frac{1}{2 \cdot (N - 1) \cdot \left(4 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

Given a point on BD (a point on the cubes powerline), project to the point of cubic similarity.

On Cubes





012996

Unit.

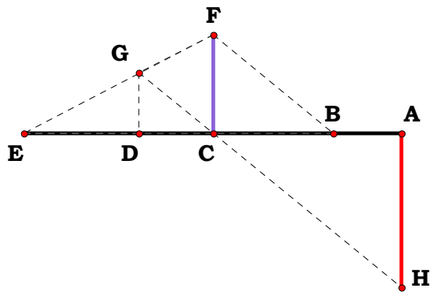
$AE := 1$

Given.

$N_1 := 3$

$N_2 := 9$

Linear division $\frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)}$



Descriptions.

$AH := AE \cdot N_1 \quad AC := \frac{AE}{2} \quad CF := AE \cdot N_2 \quad BC := \frac{AC \cdot CF}{AH}$

$CE := AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{AH \cdot CD}{AC}$

Definitions.

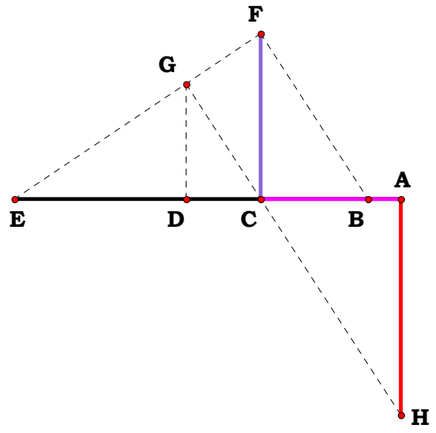
$DE - \frac{N_1}{2 \cdot (N_1 + N_2)} = 0 \quad AD - \frac{N_1 + 2 \cdot N_2}{2 \cdot (N_1 + N_2)} = 0 \quad CD - \frac{N_2}{2 \cdot (N_1 + N_2)} = 0 \quad DG - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$

Linear division $\frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)}$

Given.

$N_1 := 4 \quad N_3 := 2$

$N_2 := 4$



Descriptions.

$AH := AE \cdot N_1 \quad AC := \frac{AE}{N_2} \quad CF := AE \cdot N_3 \quad BC := \frac{AC \cdot CF}{AH}$

$CE := AE - AC \quad BE := CE + BC \quad CD := \frac{BC \cdot CE}{BE} \quad DE := CE - CD$

$AD := AC + CD \quad DG := \frac{CF \cdot CE}{BE}$

Definitions.

$DE - AE \cdot \frac{N_1 \cdot (N_2 - 1)^2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0 \quad AD - AE \cdot \frac{N_1 \cdot N_2 - N_1 + N_3 \cdot N_2}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

$DG - AE \cdot \frac{N_1 \cdot N_3 \cdot (N_2 - 1)}{N_1 \cdot N_2 - N_1 + N_3} = 0 \quad CD - AE \cdot \frac{N_3 \cdot (N_2 - 1)}{N_2 \cdot (N_1 \cdot N_2 - N_1 + N_3)} = 0$

013196

On Gemini Roots

Unit.

$BC := 1$

Given.

$N_1 := 5$

$N_2 := 3$

Descriptions.

$BJ := N_1$ $CJ := BJ - BC$ $CI := \frac{CJ}{2}$ $IJ := CI$ $BF := \sqrt{BC \cdot BJ}$ $AB := BF$

$AF := AB + BF$ $CF := BF - BC$ $FJ := CJ - CF$ $FO := \sqrt{CF \cdot FJ}$ $CR := CJ \cdot N_2$

$HS := CR$ $FI := FJ - IJ$ $FG := \frac{FI \cdot FO}{FO + HS}$ $AG := AB + BF + FG$

$OS := \sqrt{(HS + FO)^2 + FI^2}$ $GO := \frac{OS \cdot FO}{HS + FO}$ $AJ := AF + FJ$ $GL := \frac{HS \cdot GO}{OS}$

$FU := \frac{AG \cdot FO}{GL}$ $AH := \frac{FU \cdot AJ}{FU + FJ}$ $DK := \frac{FO \cdot (AF - CF)}{FU - CF}$ $AD := \frac{AG \cdot DK}{GL}$ $AC := AF - CF$

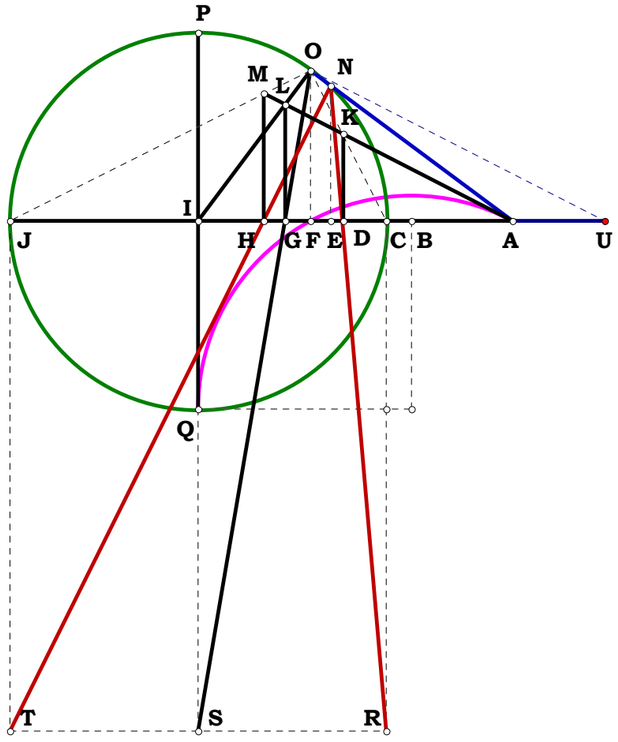
$CD := AD - AC$ $CH := AH - AC$ $DH := CH - CD$ $HJ := CJ - CH$ $EN := \frac{CR \cdot DH}{CD + HJ}$

$CE := \frac{CD \cdot (CR + EN)}{CR}$ $AE := AC + CE$ $BD := BC + CD$ $BH := BC + CH$

$\frac{AF}{FO} - \frac{AE}{EN} = 0$ $\sqrt{BC \cdot BJ} - \sqrt{BD \cdot BH} = 0$

Definitions.

Hitting AO from any RT while maintaining Gemini Roots.



Given.

$N_1 := 3.467$

020296

$N_2 := 1.728$

Descriptions.

$BE := N_1 \quad BD := \frac{BE}{2} \quad CH := BD \quad BD := \frac{BE}{2}$

$CD := BD - \frac{BD}{N_2} \quad DH := \sqrt{CD^2 + CH^2} \quad DF := \frac{DH}{2}$

$AD := \frac{DH \cdot DF}{CD} \quad AB := AD - BD \quad BC := BD - CD$

$\sqrt{(AB + BE) \cdot AB} - (AB + BC) = 0$

Definitions.

$BE - N_1 = 0 \quad BD - \frac{N_1}{2} = 0 \quad CH - \frac{N_1}{2} = 0 \quad BD - \frac{N_1}{2} = 0$

$CD - \frac{N_1 \cdot (N_2 - 1)}{2 \cdot N_2} = 0 \quad DH - \frac{N_1 \cdot \sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{2 \cdot N_2} = 0$

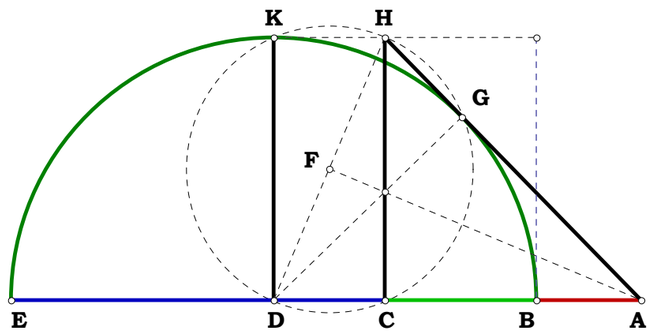
$DF - \frac{N_1 \cdot \sqrt{2 \cdot N_2^2 - 2 \cdot N_2 + 1}}{4 \cdot N_2} = 0 \quad AD - \frac{N_1 \cdot (2 \cdot N_2^2 - 2 \cdot N_2 + 1)}{4 \cdot N_2 \cdot (N_2 - 1)} = 0$

$AB - \frac{N_1}{4 \cdot N_2 \cdot (N_2 - 1)} = 0 \quad BC - \frac{N_1}{2 \cdot N_2} = 0$

$\sqrt{(AB + BE) \cdot AB} - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{4 \cdot N_2 \cdot (N_2 - 1)} = 0 \quad AB + BC - \frac{N_1 \cdot (2 \cdot N_2 - 1)}{4 \cdot N_2 \cdot (N_2 - 1)}$

Find A Segment

Find segment AB.



Given BE and BC such that
$\sqrt{(AB + BE) \cdot AB}$ **= AB + BC, find AB.**

Unit.

Given.

$N := 2$

$\Delta := 40 \quad \delta := 0 .. \Delta$

Or, the 17 decimal place rustic solution.

021496

Descriptions.

$CI := 1 \quad CG := \frac{CI}{2} \quad GI := CG \quad BC := 1$

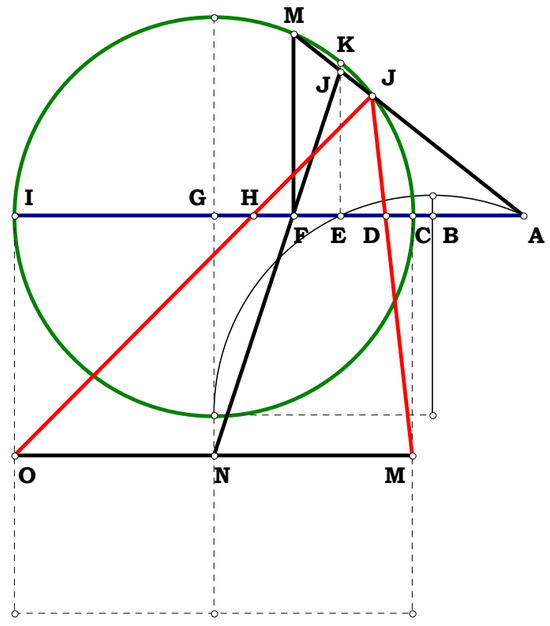
$BI := BC + CI \quad BE := \sqrt{BC \cdot BI} \quad CE := BE - BC$

$EI := CI - CE \quad EK := \sqrt{CE \cdot EI} \quad EG := CG - CE$

$AE := \frac{EK^2}{EG} \quad AC := AE - CE \quad AG := AC + CG$

$GN := CG \cdot N \quad IO := GN \quad CM := GN$

Use iteration to find any root pair for BE.
Remember that when N is set to 2, we have cube roots.



$$\begin{pmatrix} EP_0 \\ FG_0 \\ AF_0 \\ FI_0 \\ CF_0 \\ FL_0 \end{pmatrix} := \begin{bmatrix} EK \\ \frac{EG \cdot GN}{GN + EK} \\ AG - \frac{EG \cdot GN}{GN + EK} \\ GI + \frac{EG \cdot GN}{GN + EK} \\ \left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC \\ \sqrt{\left[\left[AG - \left(\frac{EG \cdot GN}{GN + EK}\right)\right] - AC\right] \cdot \left[GI + \left(\frac{EG \cdot GN}{GN + EK}\right)\right]} \end{bmatrix}$$

$$\begin{pmatrix} EP_{\delta+1} \\ FG_{\delta+1} \\ AF_{\delta+1} \\ FI_{\delta+1} \\ CF_{\delta+1} \\ FL_{\delta+1} \end{pmatrix} := \begin{pmatrix} \frac{FL_\delta \cdot AE}{AF_\delta} \\ \frac{EG \cdot GN}{GN + EP_\delta} \\ AG - FG_\delta \\ GI + FG_\delta \\ AF_\delta - AC \\ \sqrt{CF_\delta \cdot FI_\delta} \end{pmatrix}$$

$AK := \sqrt{AE^2 + EK^2}$ $AL := \sqrt{(AF_\Delta)^2 + (FL_\Delta)^2}$ $AJ := \frac{AK^2}{AL}$ $AQ := \frac{AF_\Delta \cdot AJ}{AL}$ $CQ := AQ - AC$

$IQ := CI - CQ$ $JQ := \sqrt{CQ \cdot IQ}$ $CD := \frac{CQ \cdot CM}{CM + JQ}$ $HI := \frac{IQ \cdot IO}{IO + JQ}$ $DH := CI - (CD + HI)$

$BD := BC + CD$ $BH := BC + CD + DH$

$\frac{DH}{\sqrt{CD \cdot HI}} = 1$ $BE - \sqrt{BD \cdot BH} = 0$

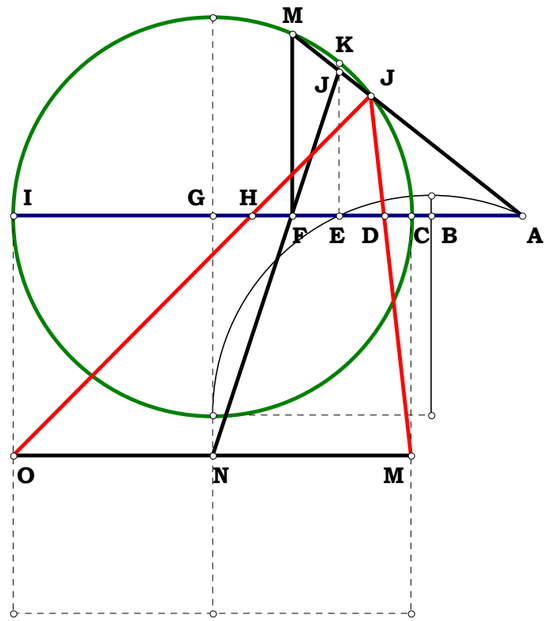
The next two equations are for the Delian Problem only.
Resolution set to max of the program.

$(BC^2 \cdot BI)^{\frac{1}{3}} - BD = 0$ $(BC \cdot BI^2)^{\frac{1}{3}} - BH = 0$

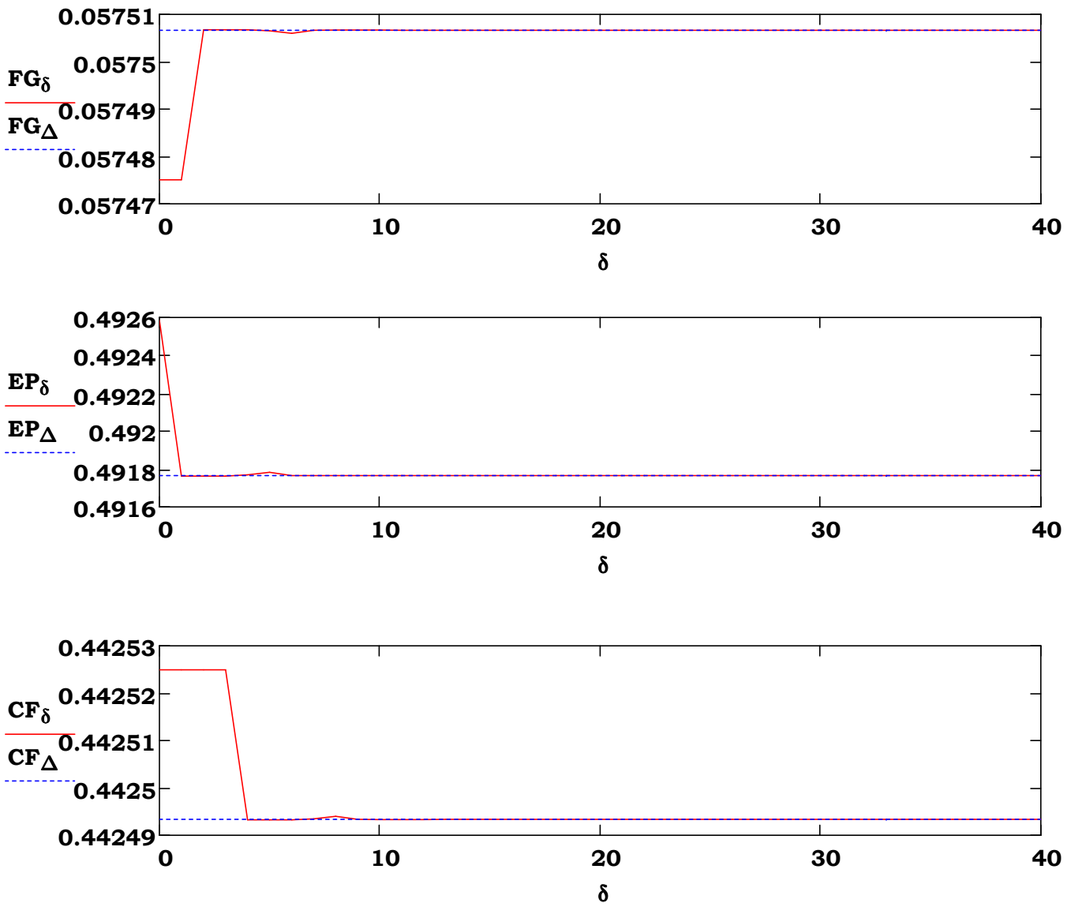
$BD = 1.2599210498948732$ $2^{\frac{1}{3}} = 1.2599210498948732$

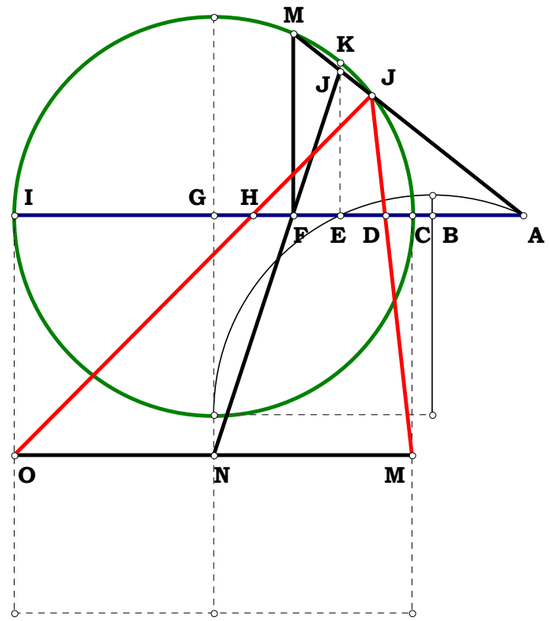
17 decimal places. Good to the limits of the program.

$BH = 1.5874010519681994$ $4^{\frac{1}{3}} = 1.5874010519681994$



A graphics peek.





The problem as stated was to duplicate the altar of Apollo. The solution demanded was material and not theoretical, and therefore has been solved by a finite number of steps. The next step is to solve it theoretically. That solution alone will conquer contiguous domains and will satisfy the purist. The solution is only good to material differences, on the atomic level, so to speak.

041496

Unit.

$CM := 1$

Given.

$N_1 := 7$

$N_2 := 3$

Descriptions.

$$CK := \frac{CM}{2} \quad CE := \frac{CM}{N_1} \quad LM := \frac{CM}{N_2} \quad EL := CM - (CE + LM)$$

$$BL := \frac{EL \cdot LM}{LM - CE} \quad BM := BL + LM \quad BC := BM - CM \quad BK := \frac{CM}{2} + BC$$

$$R_1 := LM \quad R_2 := CE \quad D := EL \quad KS := CK \quad EH := \frac{\left(R_2^{\,2} + D^2 - R_1^{\,2}\right)}{2 \cdot D}$$

$$FK := \frac{KS^2}{BK} \quad CF := CK - FK \quad FM := CM - CF \quad FS := \sqrt{CF \cdot FM}$$

$$HK := CK - (CE + EH) \quad CH := CK - HK \quad HN := \frac{FS \cdot HK}{FK} \quad AF := \frac{CH \cdot FS}{HN}$$

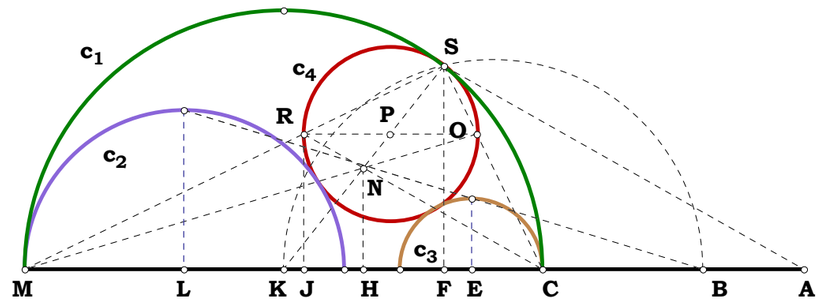
$$JR := \frac{FS \cdot CM}{AF + FM} \quad RO := \frac{CM \cdot (FS - JR)}{FS} \quad PS := \frac{RO}{2}$$

Definitions.

$$PS - \frac{\left(N_2 - 2\right) \cdot \left(N_1 - 2\right)}{2 \cdot \left(N_1 \cdot N_2 - 4\right)} = 0$$

Given c_1, c_2, c_3, find c_4. I had this sketched out in 95, but if I put it there I would have had a lot of document links to redo in "The Quest."

Method for Unequals



Unit.

$AB := 1$

Given.

$N_1 := 4$

041596

Descriptions.

On Gemini Roots

$BE := N_1 \qquad BD := \frac{BE}{2}$

$AE := AB + BE \qquad AC := \sqrt{AB \cdot AE} \qquad BC := AC - AB \qquad CE := BE - BC \qquad CF := \sqrt{BC \cdot CE}$

$CD := BD - BC \qquad CG := \frac{CF^2}{CD} \qquad BG := CG - BC \qquad EG := BG + BE \qquad CH := \frac{1}{2} \cdot CF$

$DH := \sqrt{CH^2 + CD^2} \qquad DI := \frac{1}{2} \cdot DH \qquad DL := \frac{CD \cdot DI}{DH} \qquad BL := BD - DL \qquad EL := BE - BL$

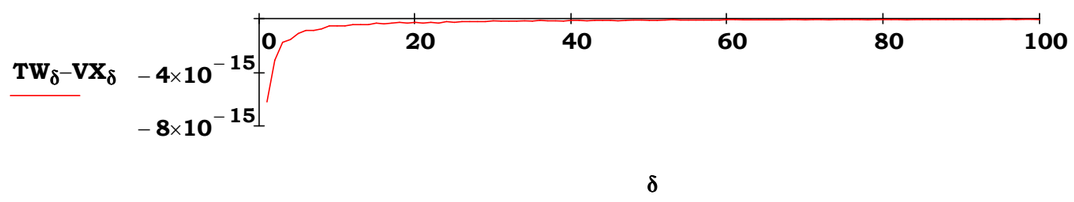
$JL := \sqrt{BL \cdot EL} \qquad GL := BL + BG \qquad GJ := \sqrt{JL^2 + GL^2} \qquad GK := \frac{BG \cdot EG}{GJ} \qquad GM := \frac{GL \cdot GK}{GJ}$

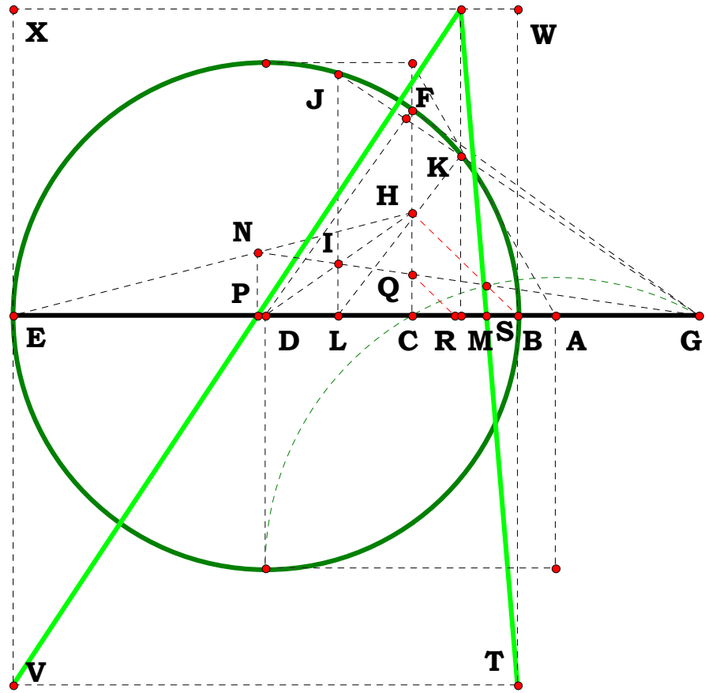
$BM := GM - BG \qquad EM := BE - BM \qquad IL := \sqrt{DI^2 - DL^2} \qquad CO := \frac{GL \cdot CH}{IL} \qquad NP := \frac{CH \cdot EG}{(CO + CE)}$

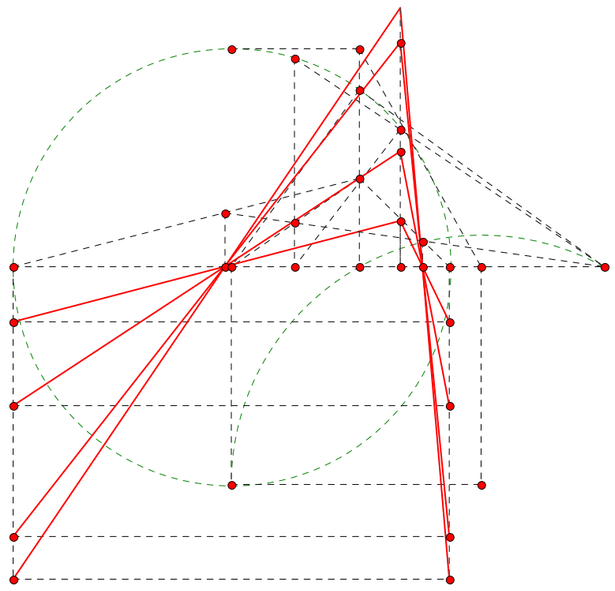
$EP := \frac{CE \cdot NP}{CH} \qquad CQ := \frac{IL \cdot CG}{GL} \qquad CR := \frac{BC \cdot CQ}{CH} \qquad GR := CG - CR \qquad BS := \frac{CR \cdot BG}{GR}$

Definitions.

$\delta := 1 .. 100 \qquad E_\delta := \frac{BE}{\delta} \qquad BT_\delta := E_\delta \qquad EV_\delta := E_\delta \qquad TW_\delta := \frac{BT_\delta \cdot BM}{BS} \qquad VX_\delta := \frac{EV_\delta \cdot EM}{EP}$







041696

Descriptions.

$AF := \frac{AJ}{2}$ $HJ := \frac{AJ}{N_1}$ $NO := \frac{AJ}{N_2}$ $HM := HJ$ $MO := NO$

$HO := HM + MO$ $FO := AF - NO$ $AH := AJ - HJ$ $FH := AH - AF$

$EH := \frac{HO^2 + FH^2 - FO^2}{2 \cdot FH}$ $EO := \sqrt{HO^2 - EH^2}$ $OP := NO$

$EG := OP$ $AE := AH - EH$ $AG := AE + EG$ $GP := EO$

$AP := \sqrt{AG^2 + GP^2}$ $PL := \frac{AG \cdot (NO + OP)}{AP}$ $AL := AP - PL$ $AB := \frac{AP \cdot AL}{2 \cdot AG}$

Definitions.

$$AB - \frac{N_2 \cdot N_1 - 2 \cdot N_2 - 2 \cdot N_1}{2 \cdot (N_2 \cdot N_1 - 2 \cdot N_2 - 4)} = 0$$

$S_1 := FH$ $S_2 := FO$ $S_3 := HO$

$$\frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1} - EH = 0$$

Unit.

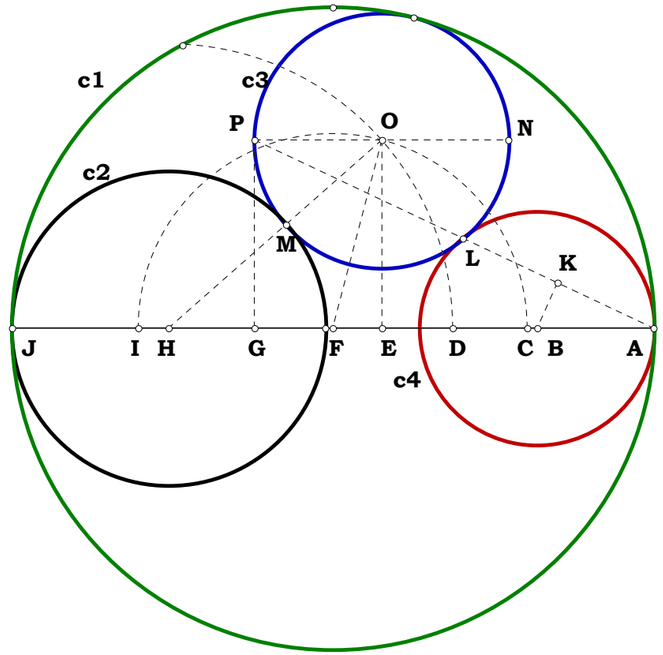
$AJ := 1$

Given.

$N_1 := 4$

$N_2 := 16$

Given Three Radii



Given c1, c2 and c3 find c4 such that AB is collinear with c1 and c2.

Unit.

Given.

$R_1 := 3.72533$ $D_1 := 2.44091$

$R_2 := 2.00643$ $D_2 := 4.29291$

041796

Descriptions.

$AG := R_1$ $AJ := 2 \cdot R_1$ $CF := R_2$ $CG := D_1$ $BG := \frac{\left(R_1^2 + D_1^2 - R_2^2\right)}{2 \cdot D_1}$

$BC := CG - BG$ $FG := CG - CF$ $AF := AG - FG$ $AD := \frac{AF}{2}$ $AB := AG - BG$

$BJ := AJ - AB$ $BH := D_2$ $AH := BH + AB$ $GH := AH - AG$ $HR := \sqrt{AG^2 - GH^2}$

$AP := HR$ $PR := BH$ $PS := \frac{GH \cdot PR}{HR}$ $BS := AP + PS$ $RS := \sqrt{PR^2 + PS^2}$ $NS := RS$

$CN := CF$ $CS := \sqrt{NS^2 + CN^2}$ $CK := \frac{CN^2}{CS}$ $SK := CS - CK$ $KN := \sqrt{CN^2 - CK^2}$

$KM := \frac{BC \cdot KN}{BS}$ $SM := SK + KM$ $SL := \frac{BS \cdot SM}{CS}$ $BL := BS - SL$ $EN := BL$

$CE := \sqrt{CN^2 - EN^2}$ $HT := \frac{CE \cdot HR}{EN}$ $GT := HT - GH$ $GO := \frac{AG \cdot CG}{GT}$ $OR := AG - GO$

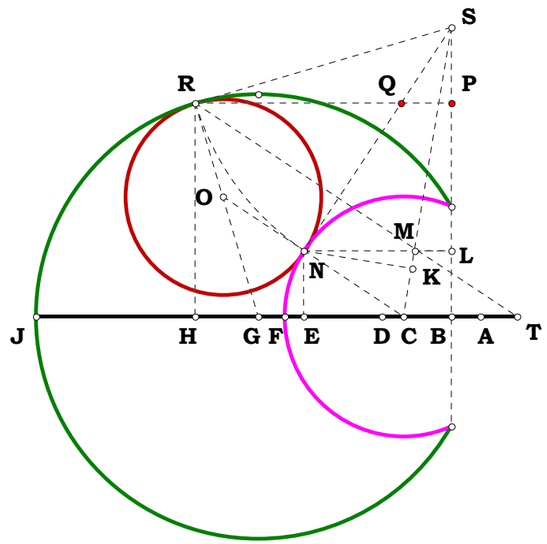
$OR = 1.631533$

Definitions.

$AG - R_1 = 0$ $AJ - 2 \cdot R_1 = 0$ $CF - R_2 = 0$ $CG - D_1 = 0$ $BG - \frac{\left(R_1^2 + D_1^2 - R_2^2\right)}{2 \cdot D_1} = 0$

$BC - \frac{D_1^2 - R_1^2 + R_2^2}{2 \cdot D_1} = 0$ $FG - \left(D_1 - R_2\right) = 0$ $AF - \left[R_1 - \left(D_1 - R_2\right)\right] = 0$ $AD - \frac{R_1 - \left(D_1 - R_2\right)}{2} = 0$ $AB - \frac{\left(D_1 - R_1 + R_2\right) \cdot \left(R_1 - D_1 + R_2\right)}{2 \cdot D_1} = 0$

A Circle In A Crescent



AG = 3.72533 cm
AJ = 7.45067 cm
CF = 2.00643 cm
CG = 2.44091 cm
BH = 4.29291 cm
OR = 1.63154 cm

$$BJ - \frac{\left(D_1 + R_1 - R_2\right)\cdot\left(D_1 + R_1 + R_2\right)}{2\cdot D_1} = 0 \qquad BH - D_2 = 0 \qquad AH - \frac{2\cdot D_1\cdot\left(D_2 + R_1\right) - {D_1}^2 + {R_2}^2 - {R_1}^2}{2\cdot D_1} = 0 \qquad GH - \frac{2\cdot D_1\cdot D_2 - \left({D_1}^2 + {R_1}^2 - {R_2}^2\right)}{2\cdot D_1} = 0$$

$$HR - \frac{\sqrt{\left(2\cdot D_1\cdot R_1 - {D_1}^2 + 2\cdot D_2\cdot D_1 - {R_1}^2 + {R_2}^2\right)\cdot\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)}}{2\cdot D_1} = 0 \qquad PR - D_2 = 0$$

$$AP - \frac{\sqrt{\left(2\cdot D_1\cdot R_1 - {D_1}^2 + 2\cdot D_2\cdot D_1 - {R_1}^2 + {R_2}^2\right)\cdot\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)}}{2\cdot D_1} = 0$$

$$PS - \frac{D_2\cdot\left(2\cdot D_2\cdot D_1 - {D_1}^2 - {R_1}^2 + {R_2}^2\right)}{\sqrt{\left(2\cdot D_1\cdot R_1 - {D_1}^2 + 2\cdot D_2\cdot D_1 - {R_1}^2 + {R_2}^2\right)\cdot\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)}} = 0$$

$$BS - \left[\frac{\left[\left(D_1 + R_1 - R_2\right)\cdot\left(D_1 - R_1 + R_2\right)\cdot\left(R_1 - D_1 + R_2\right)\cdot\left(D_1 + R_1 + R_2\right) + D_2\cdot\left(2\cdot {D_1}^3 + 2\cdot D_1\cdot {R_1}^2 - 2\cdot D_1\cdot {R_2}^2\right)\right]}{2\cdot D_1\cdot\sqrt{\left[-\left({D_1}^2 - 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)\cdot\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)\right]}}\right] = 0$$

$$RS - \frac{2\cdot D_1\cdot D_2\cdot R_1}{\sqrt{\left(2\cdot D_1\cdot R_1 - {D_1}^2 + 2\cdot D_2\cdot D_1 - {R_1}^2 + {R_2}^2\right)\cdot\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)}} = 0$$

$$NS - \frac{2\cdot D_1\cdot D_2\cdot R_1}{\sqrt{\left(2\cdot D_1\cdot R_1 - {D_1}^2 + 2\cdot D_2\cdot D_1 - {R_1}^2 + {R_2}^2\right)\cdot\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)}} = 0 \qquad CN - R_2 = 0$$

$$CS - \frac{\sqrt{\left({D_1}^2\cdot R_2 - 2\cdot D_2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1\cdot R_2 - {R_1}^2\cdot R_2 - 2\cdot R_1\cdot {R_2}^2 - {R_2}^3\right)\cdot\left({D_1}^2\cdot R_2 + 2\cdot D_2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1\cdot R_2 - {R_1}^2\cdot R_2 + 2\cdot R_1\cdot {R_2}^2 - {R_2}^3\right)}}{\sqrt{\left({D_1}^2 + 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)\cdot\left({D_1}^2 - 2\cdot D_1\cdot R_1 - 2\cdot D_2\cdot D_1 + {R_1}^2 - {R_2}^2\right)}} = 0$$

$$CK - \frac{R_2^2 \cdot \sqrt{-\left(2 \cdot D_1 \cdot R_1 - D_1^2 + 2 \cdot D_2 \cdot D_1 - R_1^2 + R_2^2\right) \cdot \left(D_1^2 + 2 \cdot D_1 \cdot R_1 - 2 \cdot D_2 \cdot D_1 + R_1^2 - R_2^2\right)}}{\sqrt{\left(2 \cdot D_2 \cdot D_1 \cdot R_2 - 2 \cdot D_2 \cdot D_1 \cdot R_1 - D_1^2 \cdot R_2 + R_1^2 \cdot R_2 - 2 \cdot R_1 \cdot R_2^2 + R_2^3\right) \cdot \left(2 \cdot D_2 \cdot D_1 \cdot R_1 - D_1^2 \cdot R_2 + 2 \cdot D_2 \cdot D_1 \cdot R_2 + R_1^2 \cdot R_2 + 2 \cdot R_1 \cdot R_2^2 + R_2^3\right)}} = 0$$

$$SK - \frac{4 \cdot D_1^2 \cdot D_2^2 \cdot R_1^2}{\sqrt{\begin{pmatrix} D_1^2 - 2 \cdot D_1 \cdot R_1 & \dots \\ + & -2 \cdot D_2 \cdot D_1 + R_1^2 - R_2^2 \end{pmatrix} \cdot \begin{pmatrix} D_1^2 \cdot R_2 - 2 \cdot D_2 \cdot D_1 \cdot R_1 & \dots \\ + & -2 \cdot D_2 \cdot D_1 \cdot R_2 - R_1^2 \cdot R_2 - 2 \cdot R_1 \cdot R_2^2 - R_2^3 \end{pmatrix} \cdot \begin{pmatrix} D_1^2 \cdot R_2 + 2 \cdot D_2 \cdot D_1 \cdot R_1 & \dots \\ + & -2 \cdot D_2 \cdot D_1 \cdot R_2 - R_1^2 \cdot R_2 + 2 \cdot R_1 \cdot R_2^2 - R_2^3 \end{pmatrix} \cdot \begin{pmatrix} D_1^2 + 2 \cdot D_1 \cdot R_1 & \dots \\ + & -2 \cdot D_2 \cdot D_1 + R_1^2 - R_2^2 \end{pmatrix}}} = 0$$

$$KM - \frac{BC \cdot KN}{BS} = 0 \qquad SM - (SK + KM) = 0 \qquad SL - \frac{BS \cdot SM}{CS} = 0 \qquad BL - (BS - SL) = 0 \qquad EN - BL = 0$$

$$CE - \sqrt{CN^2 - EN^2} = 0 \qquad HT - \frac{CE \cdot HR}{EN} = 0 \qquad GT - (HT - GH) = 0 \qquad GO - \frac{AG \cdot CG}{GT} = 0 \qquad OR - (AG - GO) = 0$$

Unit.

Given.

$R_1 := 1.62500$ $D_1 := 1.32541$

$R_2 := .75196$ $D_2 := 2.25410$

041796b

Descriptions.

$AF := R_1$ $AK := 2 \cdot R_1$ $FP := AF$ $FK := AF$ $DH := R_2$ $DF := D_1$

$CD := \frac{DH^2 + DF^2 - AF^2}{2 \cdot DF}$ $FH := DF - DH$ $AH := AF - FH$ $AE := \frac{AH}{2}$

$EH := AE$ $AD := DF - AF$ $DE := AE - AD$ $DN := DH$ $AC := CD - AD$

$CK := AK - AC$ $CJ := D_2$ $CF := CK - FK$ $CF := DF - CD$ $FJ := CJ - CF$

$JP := \sqrt{FP^2 - FJ^2}$ $FS := \frac{FP \cdot CF}{FJ}$ $PS := FS + FP$ $QS := \frac{FP \cdot PS}{JP}$ $PQ := \frac{FJ \cdot QS}{FP}$

$CS := \frac{JP \cdot CF}{FJ}$ $CQ := QS - CS$ $DQ := \sqrt{CD^2 + CQ^2}$ $DL := \frac{DH^2}{DQ}$ $LN := \sqrt{DN^2 - DL^2}$

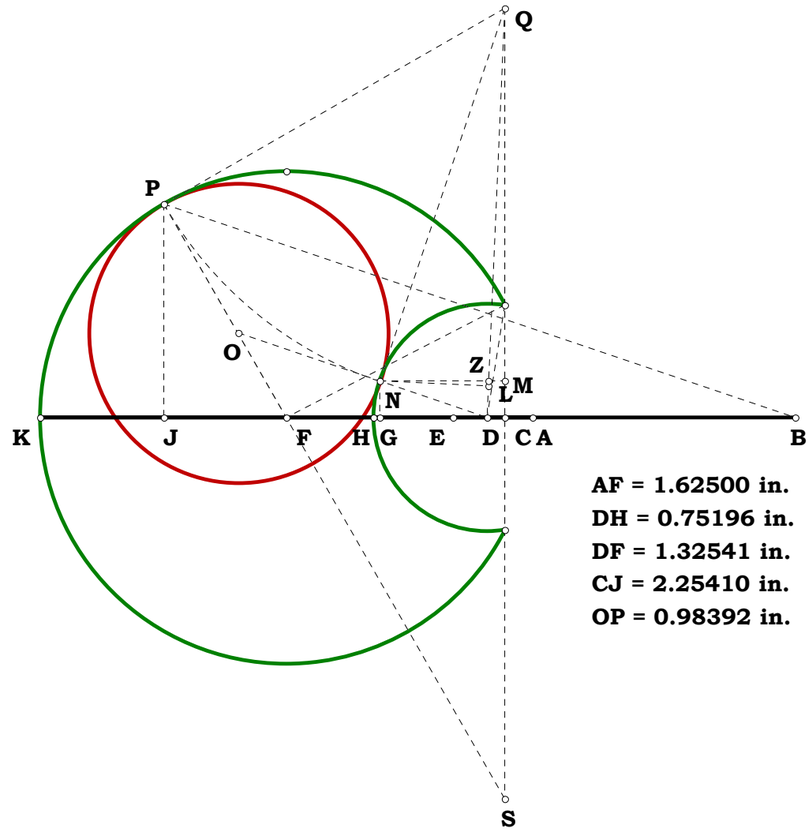
$LZ := \frac{CD \cdot LN}{CQ}$ $QZ := DQ - DL + LZ$ $MQ := \frac{CQ \cdot QZ}{DQ}$ $CM := CQ - MQ$ $GN := CM$

$DG := \sqrt{DN^2 - GN^2}$ $BJ := \frac{DG \cdot JP}{GN}$ $BF := BJ - FJ$ $FO := \frac{FP \cdot DF}{BF}$ $OP := FP - FO$

$OP = 0.983917$

Definitions.

A Circle In A Crescent



AF = 1.62500 in.
DH = 0.75196 in.
DF = 1.32541 in.
CJ = 2.25410 in.
OP = 0.98392 in.

042296a

Unit.

Given.

$N_1 := 7.87400$ $D_1 := .26381$

$D_2 := .74461$

Descriptions.

$MT := N_1$ $AB := \frac{MT}{2}$ $BM := AB$ $BE := AB$ $BL := AB$ $MF := D_1 \cdot N_1$

$BF := N_1 - MF - AB$ $EF := \sqrt{\sqrt{\left(BE^2 - BF^2\right)^2}}$ $EI := 2 \cdot EF$ $EG := EI \cdot D_2$

$FG := EG - EF$ $BG := \sqrt{BF^2 + FG^2} \cdot \frac{FG}{\sqrt{FG^2}}$ $GL := BL - BG$ $DG := GL$

$GH := \frac{FG \cdot GL}{BG}$ $HL := \sqrt{GL^2 - GH^2}$ $EH := EG + GH$ $EL := \sqrt{EH^2 + HL^2}$

$JL := \frac{EL}{2}$ $BJ := \sqrt{BL^2 - JL^2}$ $LN := \frac{BL \cdot JL}{BJ}$ $GN := \sqrt{LN^2 + GL^2}$ $JN := \sqrt{LN^2 - JL^2}$

$EJ := JL$ $EN := \sqrt{JN^2 + EJ^2}$ $GO := \frac{GN^2 + EG^2 - EN^2}{2 \cdot EG}$ $NO := \sqrt{GN^2 - GO^2}$ $NR := \frac{NO^2}{GN}$

$GS := \frac{DG^2}{GN}$ $RS := GN - (NR + GS)$ $DT := RS$ $DS := \sqrt{DG^2 - GS^2}$ $RT := DS$

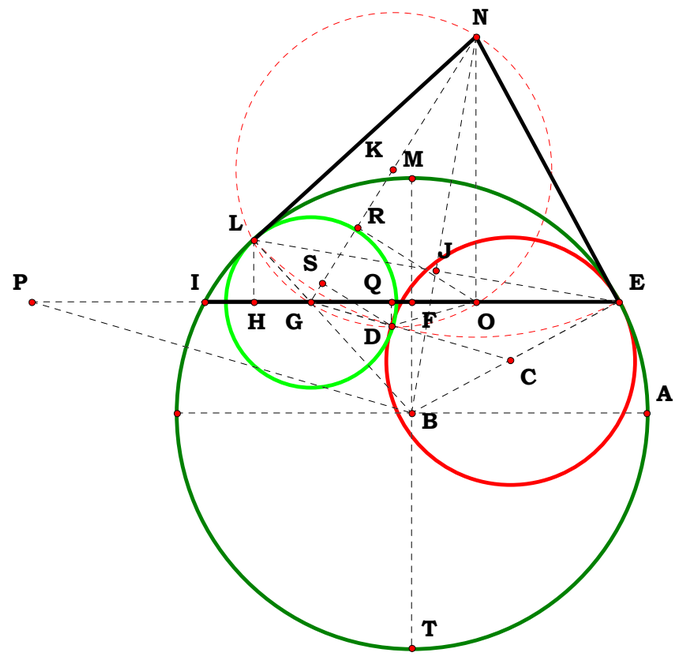
$OR := \sqrt{NO^2 - NR^2}$ $OT := OR - RT$ $DO := \sqrt{DT^2 + OT^2}$ $OQ := \frac{DO^2 + GO^2 - DG^2}{2 \cdot GO}$

$GQ := GO - OQ$ $DQ := \sqrt{DO^2 - OQ^2}$ $FP := \frac{GQ \cdot BF}{DQ}$ $CE := \sqrt{\left(\frac{BE \cdot EG}{FP + EF}\right)^2}$

$CE = 2.066678$

Definitions.

Given BF as a ratio to BM and EG as a ratio to EI, what is CE?



N_1 = 7.87400 cm

D_1 = 0.26381

D_2 = 0.74461

CE = 2.06670 cm

Unit.

Given.

$N_1 := 9.27617 \quad D_1 := .812152$

$N_2 := 2.67694 \quad D_2 := 3.19428$

042296B

Descriptions.

$AC := N_1 \quad AB := \frac{AC}{2} \quad BC := AB \quad BD := N_2 \quad Aa := N_1 \cdot D_1 \quad Ba := Aa - AB$

$CD := \sqrt{BD^2 + BC^2} \quad EF := \frac{CD \cdot \left(\sqrt{CD^2 - Ba^2} - BD\right)}{\sqrt{CD^2 - Ba^2}} \quad DE := CD - EF$

$BE := \sqrt{DE^2 - BD^2} \quad AE := AB + BE \quad GH := \frac{AE - EF}{D_2} \quad EG := EF + GH$

$DG := CD - GH \quad Db := \frac{DE^2 + DG^2 - EG^2}{2 \cdot DG} \quad Eb := \sqrt{DE^2 - Db^2} \quad Ec := \frac{DE^2}{Eb}$

$Dc := \frac{Db \cdot DE}{Eb} \quad Dd := \frac{DE^2}{BD} \quad Ed := \frac{BE \cdot Dd}{DE} \quad Ee := \frac{DE \cdot Ed}{Dc} \quad Ef := \frac{Ee \cdot DE}{DE + Ee}$

$Eg := \frac{Ec \cdot Ef}{DE} \quad bg := Eb - Eg \quad BM := \frac{bg \cdot BD}{Db} \quad DM := \sqrt{BD^2 + BM^2}$

$Bk := \frac{BM \cdot CD}{DM} \quad HM := CD - DM \quad Hk := \frac{BD \cdot HM}{DM} \quad Mk := \frac{BM \cdot Hk}{BD} \quad Ik := \frac{Hk^2}{Mk}$

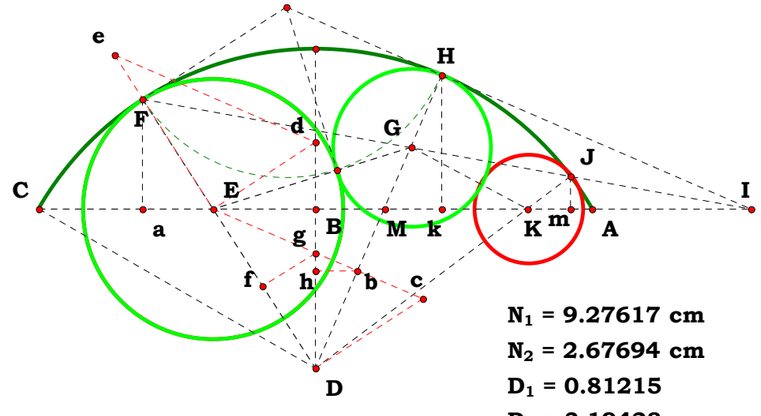
$HI := \sqrt{Hk^2 + Ik^2} \quad Ea := \frac{BE \cdot EF}{DE} \quad Ba := BE + Ea \quad Ia := Ik + Ba + Bk$

$Fa := \frac{BD \cdot EF}{DE} \quad FI := \sqrt{Ia^2 + Fa^2} \quad JI := \frac{HI^2}{FI} \quad Jm := \frac{Fa \cdot JI}{FI} \quad JK := \frac{CD \cdot Jm}{BD + Jm}$

$JK = 0.901404$

Definitions.

Place EF and GH and find JK.



N_1 = 9.27617 cm

N_2 = 2.67694 cm

D_1 = 0.81215

D_2 = 3.19428

JK = 0.90141 cm

Unit.

$AB := 1$

Given.

$N_1 := 3$

042396

Descriptions.

$AH := AB \cdot N_1$ $BH := AH - AB$ $BG := \frac{BH}{2}$ $BN := BG$ $GO := BG$ $HP := BG$

$GM := BG$ $GH := BG$ $AG := AH - GH$ $AM := \sqrt{GM^2 + AG^2}$ $AL := \frac{AG^2}{AM}$

$LM := AM - AL$ $JL := LM$ $AJ := AM - (JL + LM)$ $AD := \frac{AG \cdot AJ}{AM}$

$BD := AD - AB$ $DH := BH - BD$ $DJ := \sqrt{BD \cdot DH}$ $BC := \frac{BD \cdot BN}{BN + DJ}$

$DF := \frac{DH \cdot DJ}{BN + DJ}$ $CD := BD - BC$ $CF := CD + DF$ $CE := \frac{CF}{2}$ $BE := BC + CE$

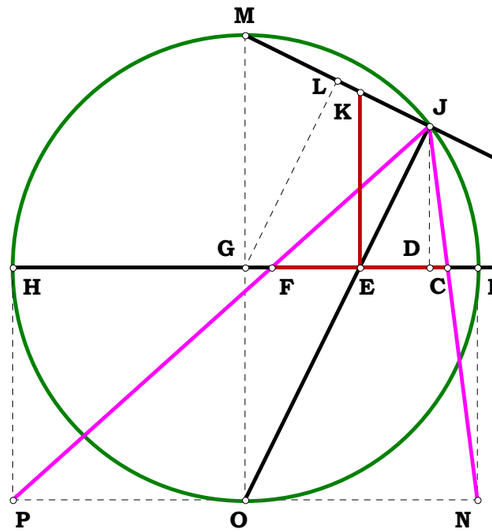
$AE := AB + BE$ $EK := \frac{GM \cdot AE}{AG}$ $EK - CF = 0$ $EK = 0.75$

Definitions.

$$EK - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{(N_1 + 1)^2} = 0$$

$$CF - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{(N_1 + 1)^2} = 0$$

Is CF always equal to EK?



AB = 1.57415 in.

AH = 4.64082 in.

EK = 1.16000 in.

CF = 1.16000 in.

$\frac{AB}{AB} = 1.00000$

$\frac{AH}{AB} = 2.94814$

$\frac{EK}{AB} = 0.73691$

$\frac{CF}{AB} = 0.73691$

Unit = 1.00000

$N_1 = 2.94814$

$$\frac{2 \cdot N_1 \cdot (N_1 - Unit)}{(N_1 + Unit)^2} = 0.73691$$

Sometimes the process of naming becomes very complex and it is impossible given the working constraints to put everything in terms of the givens, actually, they often do not illicit any recognition in the mind as to their truth, so one has to write all the steps down anyway, but sometimes one would like to show all the definitions in a given step by step process.

$$BH - (N_1 - 1) = 0 \qquad BG - \frac{N_1 - 1}{2} = 0 \qquad AG - \left(\frac{N_1 + 1}{2}\right) = 0$$

$$AM - \frac{1}{2} \cdot \sqrt{2 \cdot N_1^{\,2} + 2} = 0 \qquad AL - \frac{1}{2} \cdot \frac{(N_1 + 1)^2}{\sqrt{2 \cdot N_1^{\,2} + 2}} = 0 \qquad LM - \frac{1}{2} \cdot \frac{(N_1 - 1)^2}{\sqrt{2 \cdot N_1^{\,2} + 2}} = 0$$

$$AJ - \frac{2}{\sqrt{2 \cdot N_1^{\,2} + 2}} \cdot N_1 = 0 \qquad AD - (N_1 + 1) \cdot \frac{N_1}{(N_1^{\,2} + 1)} = 0 \qquad BD - \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} = 0$$

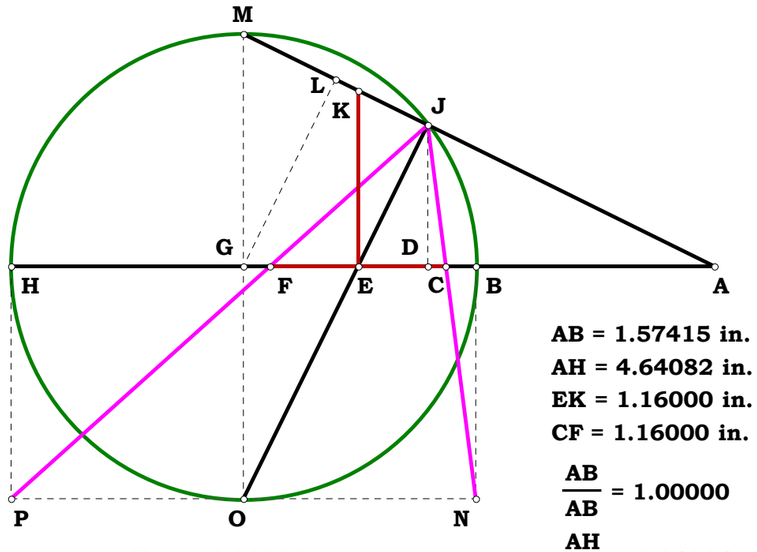
$$DH - N_1^{\,2} \cdot \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} = 0 \qquad DJ - \frac{(N_1 - 1)}{(N_1^{\,2} + 1)} \cdot N_1 = 0 \qquad BC - \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0$$

$$DF - 2 \cdot N_1^{\,3} \cdot \frac{(N_1 - 1)}{\left[(N_1 + 1)^2 \cdot (N_1^{\,2} + 1)\right]} = 0 \qquad CD - 2 \cdot (N_1 - 1) \cdot \frac{N_1}{\left[(N_1^{\,2} + 1) \cdot (N_1 + 1)^2\right]} = 0$$

$$CF - 2 \cdot N_1 \cdot \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad CE - N_1 \cdot \frac{(N_1 - 1)}{(N_1 + 1)^2} = 0 \qquad BE - \frac{(N_1 - 1)}{(N_1 + 1)} = 0$$

$$AE - 2 \cdot \frac{N_1}{(N_1 + 1)} = 0 \qquad EK - 2 \cdot (N_1 - 1) \cdot \frac{N_1}{(N_1 + 1)^2}$$

$$EK - CF = 0 \qquad EK = 0.75$$



AB = 1.57415 in.
AH = 4.64082 in.
EK = 1.16000 in.
CF = 1.16000 in.

Unit = 1.00000
N_1 = 2.94814
$\frac{2 \cdot N_1 \cdot (N_1\text{-Unit})}{(N_1\text{+Unit})^2}$ = 0.73691

$\frac{AB}{AB}$ = 1.00000
$\frac{AH}{AB}$ = 2.94814
$\frac{EK}{AB}$ = 0.73691
$\frac{CF}{AB}$ = 0.73691

Unit.

$AB := 1$

Given.

$N_1 := .36802$

042496

Descriptions.

$AF := AB$ $\quad AD := \frac{AF}{2}$ $\quad AC := N_1$ $\quad DO := \frac{AB}{2}$ $\quad OR := AF$ $\quad CD := AD - AC$

$CO := \sqrt{CD^2 + DO^2}$ $\quad PO := \frac{DO \cdot OR}{CO}$ $\quad CP := PO - CO$ $\quad CK := \frac{DO \cdot CP}{PO}$

$JK := CK$ $\quad KO := \sqrt{CD^2 + (DO + CK)^2}$ $\quad JO := \sqrt{KO^2 - JK^2}$ $\quad KS := \frac{JK^2}{KO}$

$SO := KO - KS$ $\quad JS := \frac{JK \cdot SO}{JO}$ $\quad ST := \frac{CD \cdot SO}{DO + CK}$ $\quad JT := JS + ST$

$TO := \frac{KO \cdot ST}{CD}$ $\quad TU := \frac{CD \cdot JT}{KO}$ $\quad DU := TO - (DO + TU)$

$CV := DU$ $\quad CQ := 2 \cdot CK$ $\quad QV := CQ - CV$ $\quad BH := \frac{CK \cdot CV}{QV}$

$CK = 0.232581$ $\quad BH = 0.082585$

Definitions.

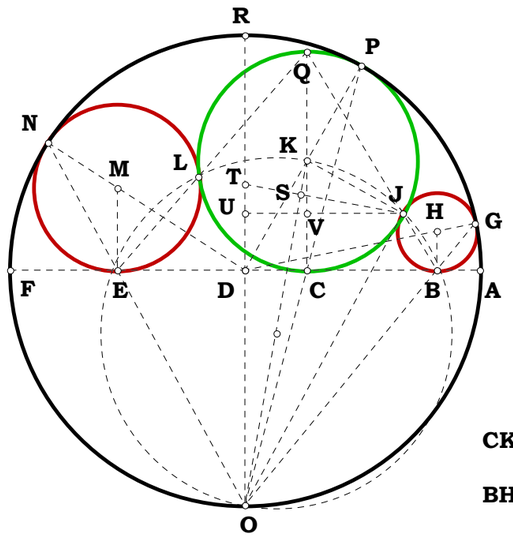
$$CK - \left(N_1 - N_1^2\right) = 0$$

$$BH - \frac{N_1 \cdot \left(N_1 - 1\right) \cdot \left(2 \cdot N_1 - 2 \cdot N_1^2 - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} - 2\right)}{2 \cdot N_1 + 6 \cdot N_1^2 - 16 \cdot N_1^3 + 8 \cdot N_1^4 - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} + 2} = 0$$

If one takes the time to work MC by hand we get; $\quad BH - \frac{N_1 \cdot \left(1 - N_1\right)}{\left[\sqrt{2} - \left(2N_1 - 1\right)\right]^2} = 0$

Three Circles.

Given AC, find CK and BH.



AF = 7.87400 cm
AC = 2.89777 cm
CK = 1.83134 cm
BH = 0.65027 cm

$\frac{AF}{AF} = 1.00000$ $\quad AF = 1.00000$

$\frac{AC}{AF} = 0.36802$ $\quad N_1 = 0.36802$

$\frac{CK}{AF} = 0.23258$ $\quad CK = 0.23258$

$\frac{BH}{AF} = 0.08258$ $\quad BH = 0.08258$

$CK\text{-}N_1\text{-}N_1^2 = 0.00000$

$BH\text{-}\frac{N_1 \cdot (1\text{-}N_1)}{(\sqrt{2}\text{-}2 \cdot N_1\text{-}1)^2} = 0.00000$

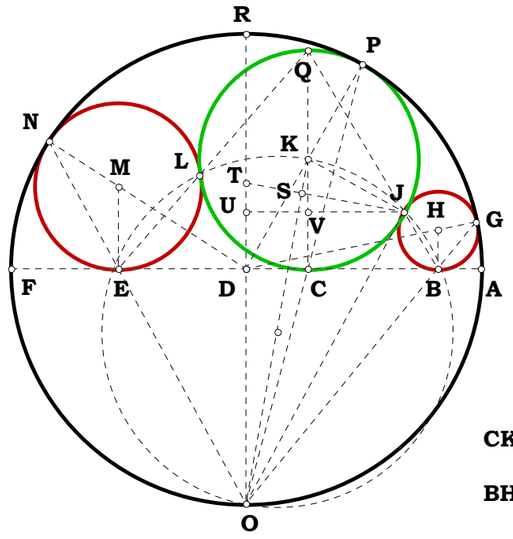
Some Algebraic Names, or Definitions.

$$AD - \frac{1}{2} = 0 \qquad AC - N_1 = 0 \qquad CD - \frac{(1 - 2 \cdot N_1)}{2} = 0 \qquad CO - \frac{\sqrt{2 \cdot N_1^2 - 2 \cdot N_1 + 1}}{\sqrt{2}} = 0$$

$$PO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0 \qquad CP - \frac{\sqrt{2} \cdot N_1 \cdot (1 - N_1)}{\sqrt{2 \cdot N_1^2 - 2 \cdot N_1 + 1}} = 0 \qquad CK - \left(N_1 - N_1^2\right) = 0$$

$$KO - \frac{\sqrt{\left(2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1\right)}}{\sqrt{2}} = 0 \qquad KS - \frac{\sqrt{2} \cdot N_1^2 \cdot (N_1 - 1)^2}{\sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0$$

$$SO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0 \qquad JS - \frac{N_1 \cdot (1 - N_1)}{\sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0$$



AF = 7.87400 cm
AC = 2.89777 cm
CK = 1.83134 cm
BH = 0.65027 cm

$\frac{AF}{AF}$ = 1.00000 $\frac{AC}{AF}$ = 0.36802 $\frac{CK}{AF}$ = 0.23258 $\frac{BH}{AF}$ = 0.08258

AF = 1.00000
N_1 = 0.36802
CK = 0.23258
BH = 0.08258

$CK\text{-}N_1\text{-}N_1^2$ = 0.00000

$BH\text{-}\frac{N_1 \cdot (1\text{-}N_1)}{(\sqrt{2}\text{-}2 \cdot N_1\text{-}1)^2}$ = 0.00000

$$ST - \frac{\sqrt{2} \cdot (1 - 2 \cdot N_1)}{2 \cdot (2 \cdot N_1 - 2 \cdot N_1^2 + 1) \cdot \sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0 \qquad JO - \frac{1}{\sqrt{2}} = 0$$

$$JT - \frac{N_1 + N_1^2 - 4 \cdot N_1^3 + 2 \cdot N_1^4 - \sqrt{2} \cdot N_1 + \frac{\sqrt{2}}{2}}{(2 \cdot N_1 - 2 \cdot N_1^2 + 1) \cdot \sqrt{2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1}} = 0 \qquad TO - \frac{1}{2 \cdot N_1 - 2 \cdot N_1^2 + 1} = 0$$

$$TU - \left[\frac{1}{2 \cdot N_1 - 2 \cdot N_1^2 + 1} - \frac{(3 \cdot \sqrt{2} - 2) \cdot N_1^2 - 2 \cdot \sqrt{2} \cdot N_1^3 + (2 - \sqrt{2}) \cdot N_1 + 1}{2 \cdot (2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1)}\right] = 0 \qquad DU - \left[\frac{(3 \cdot \sqrt{2} - 2) \cdot N_1^2 - 2 \cdot \sqrt{2} \cdot N_1^3 + (2 - \sqrt{2}) \cdot N_1 + 1}{2 \cdot (2 \cdot N_1^4 - 4 \cdot N_1^3 + 2 \cdot N_1^2 + 1)} - \frac{1}{2}\right] = 0$$

$$CQ - 2 \cdot \left(N_1 - N_1^2\right) = 0 \qquad QV - \frac{N_1 \cdot (N_1 - 1) \cdot \left(2 \cdot N_1 + 6 \cdot N_1^2 - 16 \cdot N_1^3 + 8 \cdot N_1^4 - 2 \cdot \sqrt{2} \cdot N_1 + \sqrt{2} + 2\right)}{2 \cdot \left(4 \cdot N_1^3 - 2 \cdot N_1^4 - 2 \cdot N_1^2 - 1\right)} = 0$$

Unit.

$AC := 1$

Given.

$N_1 := 2.817$ $AF := N_1$

$N_2 := 3$ $FK := N_2$

One Over N + One

Descriptions.

042596

$CF := AF - AC$ $CE := \frac{CF}{2}$ $AE := AC + CE$

$EJ := \frac{FK \cdot AE}{AF}$ $DL := FK$ $EF := CE$ $DF := \frac{EF \cdot DL}{EJ}$

$CG := \frac{FK \cdot AC}{AF}$ $CD := CF - DF$ $DH := CG$

$HL := DL - DH$ $BC := \frac{CD \cdot CG}{HL}$

Definitions.

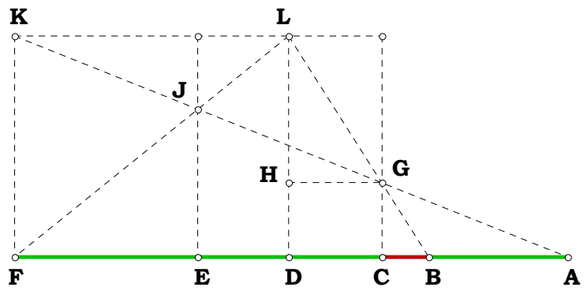
$CF - (N_1 - 1) = 0$ $CE - \frac{1}{2} \cdot (N_1 - 1) = 0$

$AE - \frac{1}{2} \cdot (1 + N_1) = 0$

$EJ - \frac{N_2 \cdot (N_1 + 1)}{2 \cdot N_1} = 0$ $DF - \frac{(N_1 - 1)}{(1 + N_1)} \cdot N_1 = 0$

$CG - \frac{N_2}{N_1} = 0$ $CD - \frac{(N_1 - 1)}{(1 + N_1)} = 0$

$HL - N_2 \cdot \frac{(N_1 - 1)}{N_1} = 0$ $\frac{1}{N_1 + 1} - BC = 0$



$N_1 = 2.96971$

$N_2 = 1.18938$

Construct 1/(N+1)

In this construction, N_2 has to be something, it can be anything and will not change the results BC.

$AC = 1.22908$ in.
$AF = 3.65000$ in.
$CF = 2.42092$ in.
$CE = 1.21046$ in.
$AE = 2.43954$ in.
$FK = 1.46184$ in.
$EJ = 0.97705$ in.
$DL = 1.46184$ in.
$EF = 1.21046$ in.
$DF = 1.81107$ in.
$CG = 0.49225$ in.
$CD = 0.60985$ in.
$DH = 0.49225$ in.
$HL = 0.96959$ in.
$BC = 0.30961$ in.

$\frac{AC}{AC} = 1.00000$

$\frac{AF}{AC} - N_1 = 0.00000$

$\frac{FK}{AC} - N_2 = 0.00000$

$\frac{CF}{AC} - N_1 - 1 = 0.00000$

$\frac{CE}{AC} - \frac{N_1 - 1}{2} = 0.00000$

$\frac{AE}{AC} - \frac{N_1 + 1}{2} = 0.00000$

$\frac{FK}{AC} - N_2 = 0.00000$

$\frac{EJ}{AC} - \frac{N_2 \cdot (N_1 + 1)}{2 \cdot N_1} = 0.00000$

$\frac{DL}{AC} - N_2 = 0.00000$

$\frac{EF}{AC} - \frac{N_1 - 1}{2} = 0.00000$

$\frac{DF}{AC} - \frac{N_1 \cdot (N_1 - 1)}{N_1 + 1} = 0.00000$

$\frac{CG}{AC} - \frac{N_2}{N_1} = 0.00000$

$\frac{CD}{AC} - \frac{N_1 - 1}{N_1 + 1} = 0.00000$

$\frac{DH}{AC} - \frac{N_2}{N_1} = 0.00000$

$\frac{HL}{AC} - \frac{N_2 \cdot (N_1 - 1)}{N_1} = 0.00000$

$\frac{BC}{AC} - \frac{1}{N_1 + 1} = 0.00000$

Unit.

Given.

$N_1 := .64966$ $\quad BC := N_1$

$N_2 := 7.51417$ $\quad CI := N_2$

$N_3 := 1.92131$ $\quad BM := N_3$

04_26_96

Descriptions.

$CG := \frac{CI}{2}$ $\quad BI := BC + CI$ $\quad BE := \sqrt{BC \cdot BI}$ $\quad EM := \sqrt{BM^2 + BE^2}$

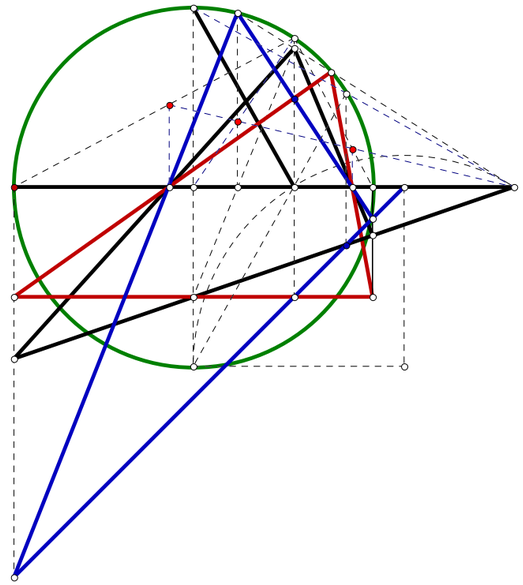
$BD := EM - BM$ $\quad BH := BM + EM$ $\quad GN := CG$ $\quad CE := BE - BC$

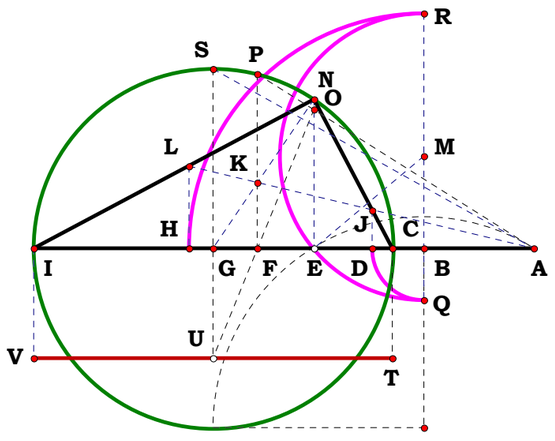
$EI := CI - CE$ $\quad EN := \sqrt{CE \cdot EI}$ $\quad EH := BH - BE$ $\quad EG := EI - CG$

$AE := \frac{EN^2}{EG}$ $\quad HI := EI - EH$ $\quad HL := \frac{EN \cdot HI}{EI}$ $\quad AG := AE + EG$

Three Bases.

Three peaks, three predictable bases that intersect our hypotenuse through the Gemini roots.





$$BD - \left(\sqrt{N_1^2 + N_1 \cdot N_2 + N_3^2} - N_3\right) = 0$$

$$BH - \left(N_3 + \sqrt{N_1^2 + N_1 \cdot N_2 + N_3^2}\right) = 0$$

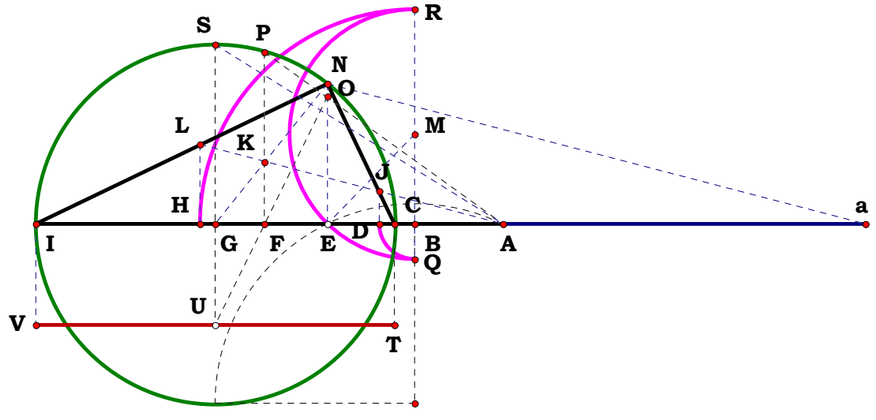
$$AH := AE + EH \qquad Ea := \frac{AH \cdot EN}{HL}$$

$$FG := \frac{EG \cdot AG}{(Ea + EG)} \qquad CF := CG - FG$$

$$FI := CG + FG \qquad FP := \sqrt{CF \cdot FI}$$

$$AF := AG - FG \qquad EO := \frac{FP \cdot AE}{AF}$$

$$EF := AF - AE \qquad GU := \frac{EO \cdot FG}{EF}$$



$$AC := AE - CE \qquad AI := AC + CI \qquad AP := \sqrt{AF^2 + FP^2}$$

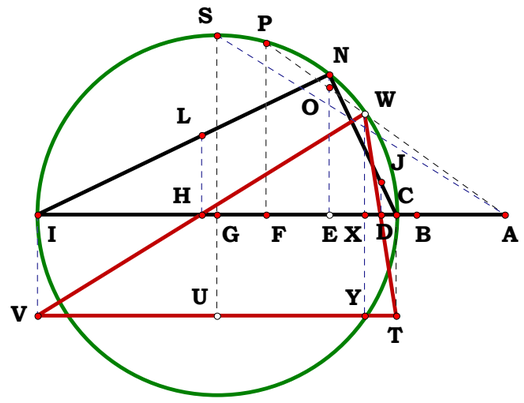
$$AW := \frac{AC \cdot AI}{AP} \qquad AX := \frac{AF \cdot AW}{AP} \qquad CX := AX - AC$$

$$XI := CI - CX \qquad WX := \sqrt{CX \cdot XI} \qquad XG := CG - CX$$

$$YU := XG \qquad UV := CG \qquad YV := YU + UV \qquad XH := \frac{YV \cdot WX}{WX + GU}$$

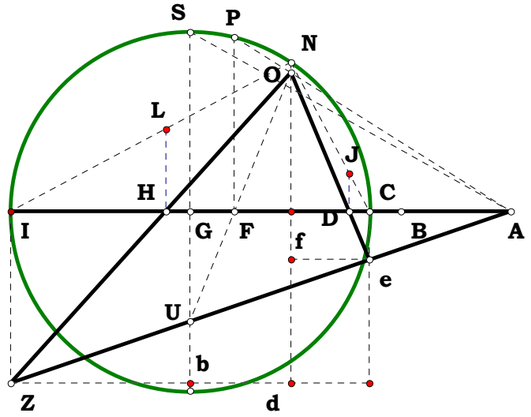
$$CH := AH - AC \qquad \frac{CH}{XH + CX} = 1 \qquad CD := BD - BC$$

$$DX := \frac{CX \cdot WX}{WX + GU} \qquad \frac{CD}{CX - DX} = 1$$



$$IZ := \frac{GU \cdot AI}{AG} \qquad Ed := IZ \quad \frac{\frac{EI \cdot EO}{EO+Ed}}{EH} = 1$$

$$Ce := \frac{GU \cdot AC}{AG} \qquad Ef := Ce \quad \frac{CD}{\frac{CE \cdot Ce}{EO+Ef}} = 1$$

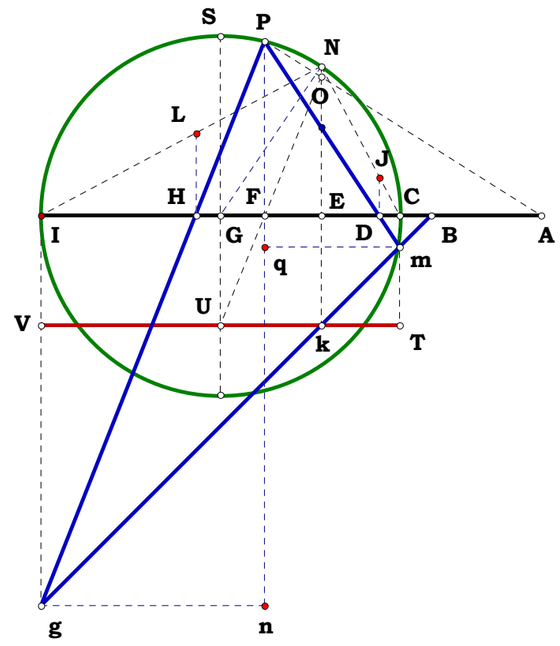


$$Ek := GU \qquad Ig := \frac{Ek \cdot BI}{BE} \qquad Cm := \frac{Ek \cdot BC}{BE}$$

$$Fn := Ig\,gn := FI \qquad FH := \frac{gn \cdot FP}{FP + Fn}$$

$$\frac{FH}{AH - AF} = 1 \qquad DF := \frac{CF \cdot FP}{FP + Cm}$$

$$\frac{CD}{CF - DF} = 1$$



042796

Unit.

$CF := 1$

Given.

$N_1 := 4$

$N_2 := 9$

Descriptions.

$CE := \frac{CF}{2}$ $CD := \frac{CE}{N_1}$ $FK := N_2$ $DM := FK$ $EL := FK$

$DF := CF - CD$ $EF := \frac{CF}{2}$ $EJ := \frac{DM \cdot EF}{DF}$ $JL := EL - EJ$

$KL := EF$ $AF := \frac{KL \cdot FK}{JL}$ $AC := AF - CF$ $CG := \frac{FK \cdot AC}{AF}$

$DH := CG$ $HM := DM - DH$ $BC := \frac{CD \cdot DH}{HM}$ $BF := BC + CF$

$BD := BC + CD$ $AB := AF - BF$

$AB^2 - BC \cdot BF = 0$ $\sqrt{BC \cdot BF} - BD = 0$ $BD - (CD + BC) = 0$

$AB - BD = 0$ $\sqrt{BC \cdot BF} - AB = 0$ $AB = 0.145833$

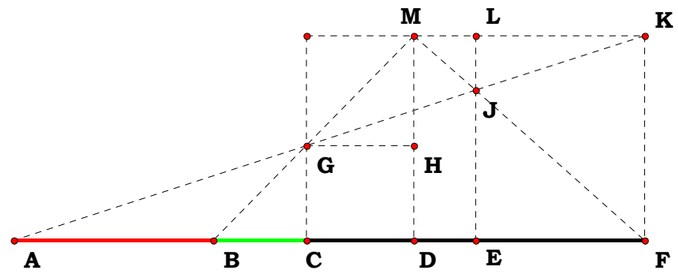
One may notice that can be any value at all, it just has to be some value. Every use of symbols require prue intelligble concepts which are perceptible in the grammar, but not all people can see them intelligibly.

A Root Figure

The difference between the perceptible and the intelligible.

CD + BC is the square root of $\sqrt{BC \cdot BF}$. What is BC?

When I originally did this story, I said to myself, well I can name that tune in 2 variables, which it should be, however, it is one of those things one says that they will do later as it has little to do with the main goal of the Delian Quest. Sometimes later means later in the day, sometimes later means decades away, as in this case. What are revisions for? One should also be aware, while learning, that one is not going to have the experience required to do a story justice until they mature, thus waiting is certainly an option.



CF = 5.67267 cm $\frac{CF}{CF} = 1.00000$

CE = 2.83633 cm $\frac{CE}{CD} = 1.57637$

CD = 1.79928 cm $\frac{AB}{CF} = 0.59234$

AB = 3.36014 cm $\frac{BC}{CF} = 0.27515$

BC = 1.56086 cm $\frac{BF}{CF} = 1.27515$

BF = 7.23353 cm

$N_1 = 1.57637$

AB = 0.59234

BC = 0.27515

BF = 1.27515

$\frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.59234$ $AB - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$\frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.27515$ $BC - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$\frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 1.27515$ $BF - \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0.00000$

$AB - \sqrt{BC \cdot BF} = 0.00000$

$AB - \sqrt{BC \cdot BF} = 0.00000$ cm

This equation is in the universal and any relatative difference can use it.

This equation is particular. It is from these particular examples, perceptible examples, that one acquires what Plato called, the similie in multis, or the Universal, or again, the intelligible. One aims for three things then, the correct range for a varaible built in to the equation, the answer expressed only in the givens, and the equation be free from any particular perceptible as grammar is always applicable universally. Not all my examples fill that bill, but then this is my storybook, my rather odd novel.

Definitions.

$$CF - 1 = 0 \qquad CE - \frac{1}{2} = 0 \qquad CD - \frac{1}{2 \cdot N_1} = 0 \qquad FK - N_2 = 0 \qquad DM - N_2 = 0 \qquad EL - N_2 = 0 \qquad DF - \frac{2 \cdot N_1 - 1}{2 \cdot N_1} = 0 \qquad EF - \frac{1}{2} = 0 \qquad EJ - \frac{N_1 \cdot N_2}{2 \cdot N_1 - 1} = 0$$

$$JL - \frac{N_2 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0 \qquad KL - \frac{1}{2} = 0 \qquad AF - \frac{2 \cdot N_1 - 1}{2 \cdot (N_1 - 1)} = 0 \qquad AC - \frac{1}{2 \cdot (N_1 - 1)} = 0 \qquad CG - \frac{N_2}{2 \cdot N_1 - 1} = 0 \qquad DH := \frac{N_2}{2 \cdot N_1 - 1} \qquad HM - \frac{2 \cdot N_2 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0$$

$$BC - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad BF - \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad BD - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad AB - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0$$

$$\left[\frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)}\right]^2 - \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} \cdot \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad \sqrt{\frac{1}{4 \cdot N_1 \cdot (N_1 - 1)} \cdot \frac{(2 \cdot N_1 - 1)^2}{4 \cdot N_1 \cdot (N_1 - 1)}} - \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} = 0 \qquad \frac{2 \cdot N_1 - 1}{4 \cdot N_1 \cdot (N_1 - 1)} - \left[\frac{1}{2 \cdot N_1} + \frac{1}{4 \cdot N_1 \cdot (N_1 - 1)}\right] = 0$$

$$AB - BD = 0 \qquad \sqrt{BC \cdot BF} - AB = 0 \qquad AB = 0.145833$$

Unit.

Given.

$N_1 := 2$

$N_2 := 6$

Given one root in a prime root series, determine the rest of the series using the straight edge method. The other method uses a compass and is shown at the end of document.

042896

I had finally decided to write this up in Oct. of 94, and being punctual, here it is.

Process Summary will use a 5th root series for an example.

Descriptions.

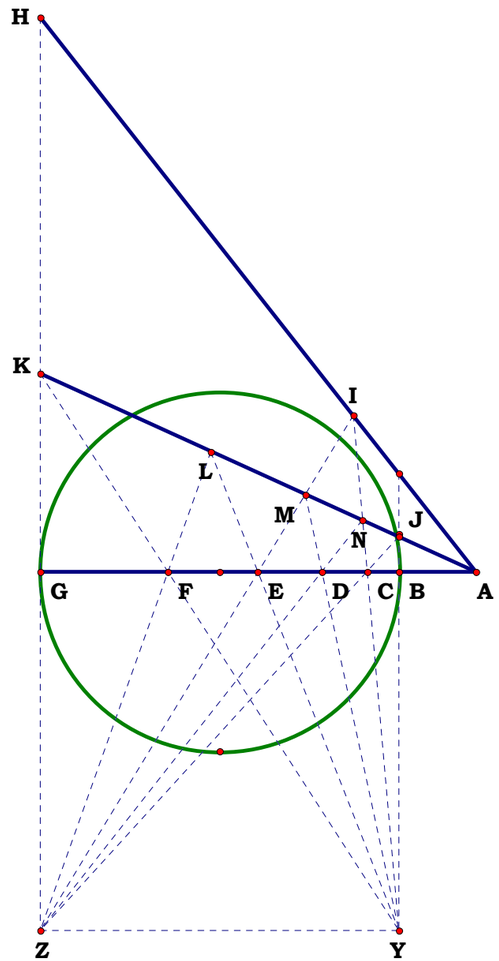
$$AB := N_1 \qquad AG := N_2 \qquad AE := \left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}$$

$$BG := AG - AB \qquad GZ := BG \qquad YZ := BG$$

$$BY := BG \qquad BE := AE - AB \qquad EG := BG - BE$$

Definitions.

$$GH := \frac{BY \cdot EG}{BE}$$



AB = 1.29117 cm
AG = 7.30394 cm
AC = 1.82599 cm
AD = 2.58233 cm
AE = 3.65197 cm
AF = 5.16467 cm

$(AB^4 \cdot AG^1)^{\frac{1}{5}} - AC = 0.00000$

$(AB^3 \cdot AG^2)^{\frac{1}{5}} - AD = 0.00000$

$(AB^2 \cdot AG^3)^{\frac{1}{5}} - AE = 0.00000$

$(AB^1 \cdot AG^4)^{\frac{1}{5}} - AF = 0.00000$

$\frac{AG^{\frac{0}{5}}}{AB} = 1.00000$

$\frac{AG^{\frac{1}{5}}}{AB} = 1.41421$

$\frac{AG^{\frac{2}{5}}}{AB} = 2.00000$

$\frac{AG^{\frac{3}{5}}}{AB} = 2.82843$

$\frac{AG^{\frac{4}{5}}}{AB} = 4.00000$

$\frac{AG^{\frac{5}{5}}}{AB} = 5.65685$

$\frac{AG^{\frac{1\,2}{5}}}{AB} = 2.00000$

$\frac{AG^{\frac{1\,3}{5}}}{AB} = 2.82843$

$\frac{AG^{\frac{1\,4}{5}}}{AB} = 4.00000$

$\frac{AG^{\frac{1\,5}{5}}}{AB} = 5.65685$

$\frac{AG}{AB} = 5.65685$

$\frac{AF}{AB} = 4.00000$

$\frac{AE}{AB} = 2.82843$

$\frac{AD}{AB} = 2.00000$

$\frac{AC}{AB} = 1.41421$

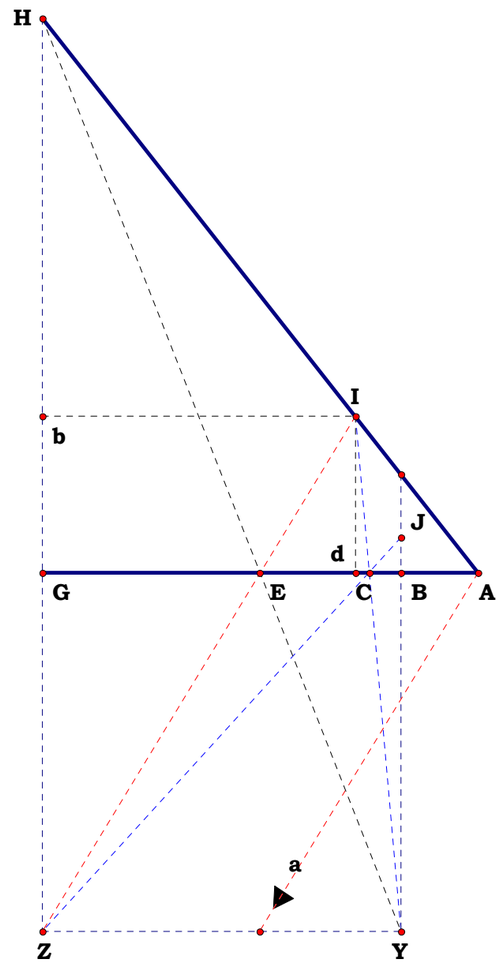
$$Ga := \frac{GZ \cdot AG}{EG} \qquad Hb := \frac{GH \cdot (GH + GZ)}{GH + Ga}$$

$$Gb := GH - Hb \qquad Ib := \frac{AG \cdot (GH + GZ)}{GH + Ga}$$

$$Bd := BG - Ib \qquad BC := \frac{Bd \cdot BY}{BY + Gb}$$

$$AC := AB + BC$$

$$CG := BG - BC \qquad BJ := \frac{GZ \cdot BC}{CG}$$



$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$

$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$$

$$\frac{AG}{AB}^{\frac{1}{5}^{2}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}^{3}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{1}{5}^{4}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}^{5}} = 5.65685$$

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$$(AB^{4} \cdot AG^{1})^{\frac{1}{5}} - AC = 0.00000$$

$$(AB^{3} \cdot AG^{2})^{\frac{1}{5}} - AD = 0.00000$$

$$(AB^{2} \cdot AG^{3})^{\frac{1}{5}} - AE = 0.00000$$

$$(AB^{1} \cdot AG^{4})^{\frac{1}{5}} - AF = 0.00000$$

$$\frac{AG}{AB} = 5.65685$$

$$\frac{AF}{AB} = 4.00000$$

$$\frac{AE}{AB} = 2.82843$$

$$\frac{AD}{AB} = 2.00000$$

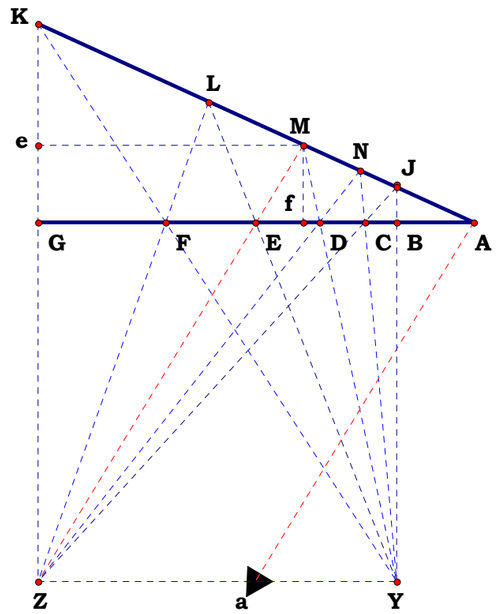
$$\frac{AC}{AB} = 1.41421$$

$$GK := \frac{BJ \cdot AG}{AB} \qquad KZ := GZ + GK$$

$$FG := \frac{YZ \cdot GK}{KZ} \qquad AF := AG - FG$$

$$Ke := \frac{GK \cdot KZ}{GK + Ga} \qquad Me := \frac{AG \cdot KZ}{GK + Ga}$$

$$BD := \frac{(BG - Me) \cdot BY}{KZ - Ke} \qquad AD := AB + BD$$



$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$

$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$

$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$

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$$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$$

$$\frac{AG}{AB}^{\frac{1\,2}{5}} = 2.00000$$

$$\frac{AG}{AB}^{\frac{1\,3}{5}} = 2.82843$$

$$\frac{AG}{AB}^{\frac{1\,4}{5}} = 4.00000$$

$$\frac{AG}{AB}^{\frac{1\,5}{5}} = 5.65685$$

$$\frac{AG}{AB} = 5.65685$$

$$\frac{AF}{AB} = 4.00000$$

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$$\frac{AD}{AB} = 2.00000$$

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$$(AB^3 \cdot AG^2)^{\frac{1}{5}} - AD = 0.00000$$

$$(AB^2 \cdot AG^3)^{\frac{1}{5}} - AE = 0.00000$$

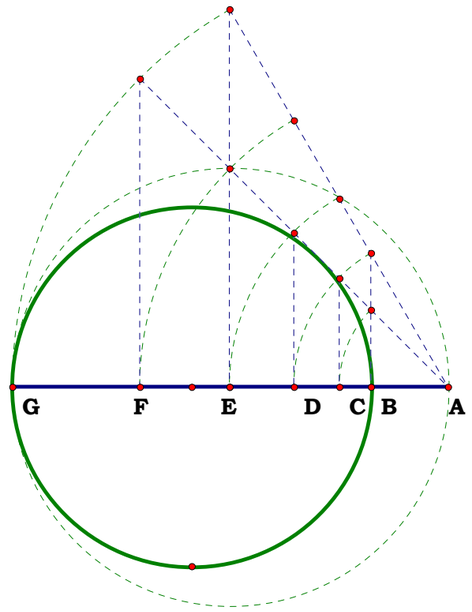
$$(AB^1 \cdot AG^4)^{\frac{1}{5}} - AF = 0.00000$$

If any of a prime root series can be given exactly, every root of the series can be determined exactly.

$$\frac{\left(AB^5 \cdot AG^0\right)^{\frac{1}{5}}}{AB} = 1 \qquad \frac{\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}}}{AC} = 1$$

$$\frac{\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}}}{AD} = 1 \qquad \frac{\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}}}{AE} = 1$$

$$\frac{\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}}}{AF} = 1 \qquad \frac{\left(AB^0 \cdot AG^5\right)^{\frac{1}{5}}}{AG} = 1$$



$AB = 1.29117$ cm
$AG = 7.30394$ cm
$AC = 1.82599$ cm
$AD = 2.58233$ cm
$AE = 3.65197$ cm
$AF = 5.16467$ cm

$$\left(AB^4 \cdot AG^1\right)^{\frac{1}{5}} - AC = 0.00000$$
$$\left(AB^3 \cdot AG^2\right)^{\frac{1}{5}} - AD = 0.00000$$
$$\left(AB^2 \cdot AG^3\right)^{\frac{1}{5}} - AE = 0.00000$$
$$\left(AB^1 \cdot AG^4\right)^{\frac{1}{5}} - AF = 0.00000$$

$$\frac{AG}{AB}^{\frac{0}{5}} = 1.00000$$
$$\frac{AG}{AB}^{\frac{1}{5}} = 1.41421$$
$$\frac{AG}{AB}^{\frac{2}{5}} = 2.00000$$
$$\frac{AG}{AB}^{\frac{3}{5}} = 2.82843$$
$$\frac{AG}{AB}^{\frac{4}{5}} = 4.00000$$
$$\frac{AG}{AB}^{\frac{5}{5}} = 5.65685$$

$$\frac{AG}{AB}^{\frac{1}{5}^2} = 2.00000$$
$$\frac{AG}{AB}^{\frac{1}{5}^3} = 2.82843$$
$$\frac{AG}{AB}^{\frac{1}{5}^4} = 4.00000$$
$$\frac{AG}{AB}^{\frac{1}{5}^5} = 5.65685$$

$$\frac{AG}{AB} = 5.65685$$
$$\frac{AF}{AB} = 4.00000$$
$$\frac{AE}{AB} = 2.82843$$
$$\frac{AD}{AB} = 2.00000$$
$$\frac{AC}{AB} = 1.41421$$

Unit.

$AB := 1$

Given.

$N := 5$

042996

Descriptions.

$AG := AB \cdot N \qquad BG := AG - AB$

$BF := \frac{BG}{2} \qquad FK := BF \qquad FO := BF \qquad AF := BF + AB$

$DF := \frac{FK \cdot FO}{AF} \qquad AK := \sqrt{AF^2 + FK^2} \qquad KO := BG$

$HO := \frac{AF \cdot KO}{AK} \qquad DO := \frac{AK \cdot FO}{AF} \qquad DH := HO - DO$

$DJ := \sqrt{DH \cdot DO}$

Definitions.

$AG - N = 0 \qquad BG - (N - 1) = 0 \qquad BF := \frac{N - 1}{2} \qquad AF - \left(\frac{1}{2} \cdot N + \frac{1}{2}\right) = 0$

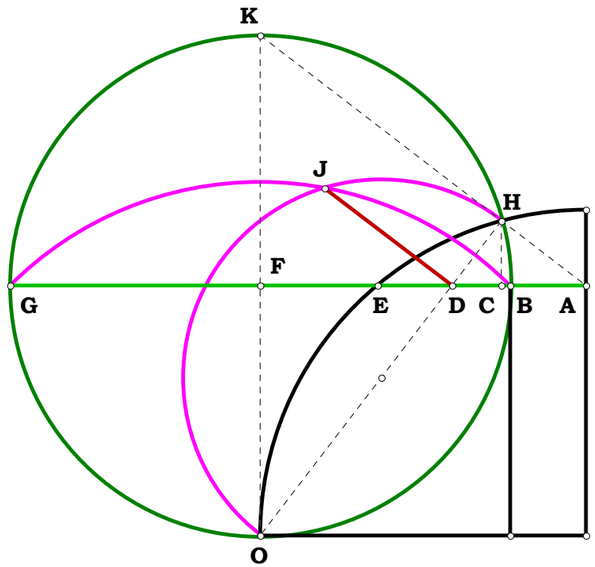
$DF - \frac{(N - 1)^2}{2 \cdot (N + 1)} = 0 \qquad AK - \frac{\sqrt{2 \cdot (N^2 + 1)}}{2} = 0$

$HO - \frac{\sqrt{2} \cdot (N^2 - 1)}{2 \cdot \sqrt{N^2 + 1}} = 0 \qquad DO - \frac{\sqrt{2} \cdot (N - 1) \cdot \sqrt{N^2 + 1}}{2 \cdot (N + 1)} = 0$

$DH - \frac{\sqrt{2} \cdot N \cdot (N - 1)}{(N + 1) \cdot \sqrt{N^2 + 1}} = 0 \qquad DJ - \sqrt{N} \cdot \frac{(N - 1)}{(N + 1)} = 0$

DJ is the Geometric name, what is its Algebraic name?

What is DJ?



043096

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 128 \quad Root := 8$

$\delta := 1 .. Root$

$BG := N_1 \quad AG := AB + BG \quad BO := \frac{BG}{2}$

$AC := \left(AB^{Root-1} \cdot AG\right)^{\frac{1}{Root}} \quad AF := \left(AB \cdot AG^{Root-1}\right)^{\frac{1}{Root}}$

$BC := AC - AB \quad FG := AG - AF \quad FX := \sqrt{AF^2 + AG^2}$

$FY := \frac{AF^2}{FX} \quad BD := \frac{FY \cdot BG}{FX} \quad AD := BD + AB$

$DG := AG - AD \quad DK := \sqrt{BD \cdot DG}$

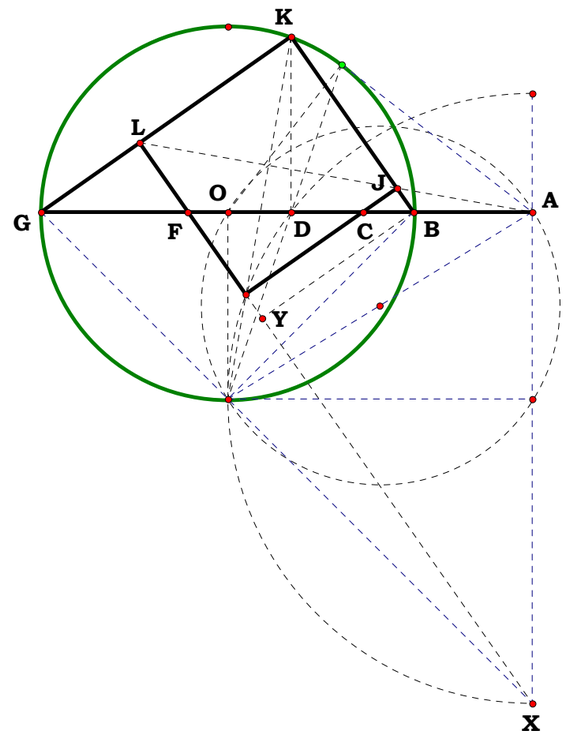
$BK := \sqrt{BD^2 + DK^2} \quad GK := \sqrt{DG^2 + DK^2}$

$BJ := \frac{BK \cdot BC}{BG} \quad GL := \frac{GK \cdot FG}{BG}$

Geometric Exponential Series of the form

$$\frac{\sum_{\delta} N^{\frac{Root-\delta}{Root}}}{N^{\frac{Root-1}{Root}}} \quad , \quad \sum_{\delta} N^{\frac{Root-\delta}{Root}} \quad \text{and} \quad \frac{N^{\frac{\delta+2}{Root}} + N^{\frac{\delta}{Root}}}{N^{\frac{1}{Root}} - N^{\frac{0}{Root}}}$$

Generalize some of the ratios found in 010896 and 011696 for the sides of the right triangle.



Definitions.

$GL = 51.575206$ $BJ = 0.399808$

$\frac{GL}{BJ} = 129$ $\frac{AG}{AB} = 129$

$$\frac{\sum_{\delta}\left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{Root-1}{Root}}} = 2.179442$$

$\frac{GK}{GL} = 2.179442$

$$\sum_{\delta}\left(\frac{AG}{AB}\right)^{\frac{Root-\delta}{Root}} = 153.147965$$

$\frac{BK}{BJ} = 153.147965$

$BM := \frac{BD \cdot BC}{BG}$ $FQ := \frac{BD \cdot FG}{BG}$

$\frac{AG}{FQ} = 9.598866$

$\frac{AG}{BM} = 674.506167$

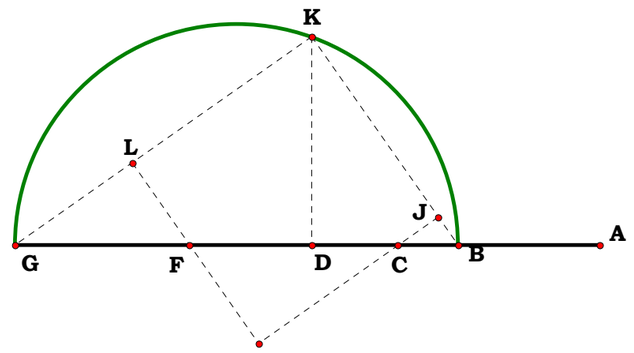
On the left is the first and last of the series, on the right is the entire series.

$\frac{\delta+2}{Root}$	$\frac{\delta}{Root}$	$\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}}$	$\left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}$
0.375	0.125	6.186872	1.835793
0.5	0.25	11.357817	3.370136
0.625	0.375	20.850601	6.186872
0.75	0.5	38.277387	11.357817
0.875	0.625	70.26936	20.850601
1	0.75	129	38.277387
1.125	0.875	236.817299	70.26936
1.25	1	434.747545	129

$\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} = 1.835793$

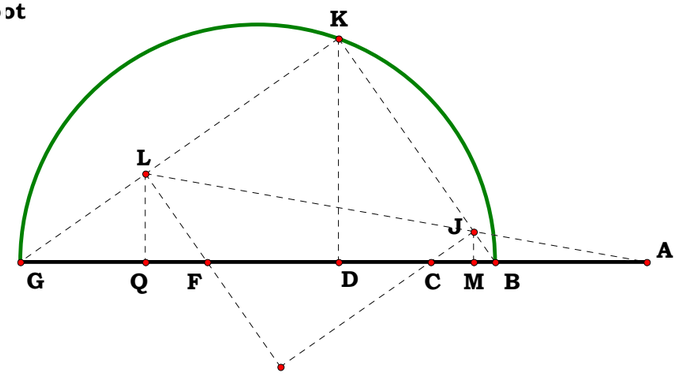
$\frac{1}{Root} = 0.125$

$\frac{0}{Root} = 0$



$$\frac{\left(\frac{AG}{AB}\right)^{\frac{\delta+2}{Root}} + \left(\frac{AG}{AB}\right)^{\frac{\delta}{Root}}}{\left(\frac{AG}{AB}\right)^{\frac{1}{Root}} - \left(\frac{AG}{AB}\right)^{\frac{0}{Root}}} =$$

9.598866
17.621531
32.349484
59.386957
109.02216
200.142121
367.419508
674.506167



122096

Unit.

$AB := 1$

Given.

$N_1 := 2 \quad AL := N_1$

$N_2 := .2$

Descriptions.

$BL := AL - AB \quad BS := BL \quad LT := BL$

$BH := \frac{BL}{2} \quad HL := BH \quad BQ := BS \cdot N_2$

$AF := \sqrt{AB \cdot AL} \quad FL := AL - AF \quad BF := AF - AB$

$FP := \sqrt{BF \cdot FL} \quad FN := \frac{BQ \cdot FP}{BS} \quad EF := \frac{BF \cdot FN}{BQ}$

$EL := EF + FL \quad FG := \frac{EF \cdot FL}{EL} \quad GO := \frac{FN \cdot FG}{EF}$

$GL := FL - FG \quad LR := BQ \quad JL := \frac{GL \cdot LR}{LR + GO}$

$AJ := AL - JL \quad AJ = 1.681793$

Definitions.

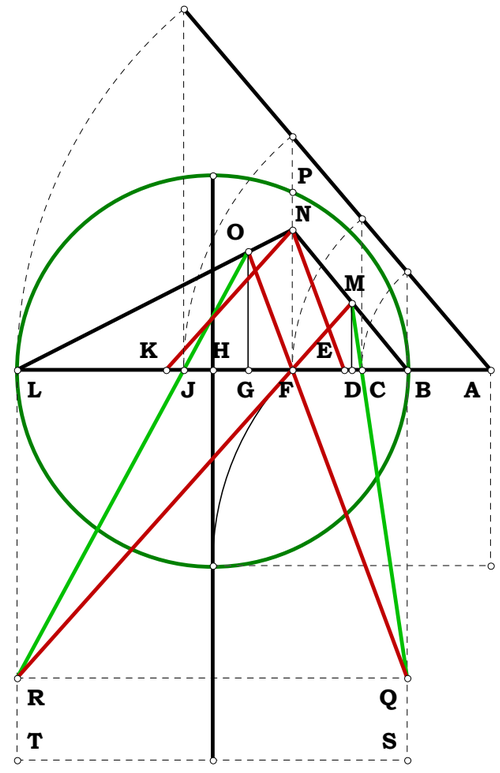
$\left(AL^3\right)^{\frac{1}{4}} - AJ = 0$

$AJ - \left(N_1^{\frac{1}{4}}\right)^3 = 0$

$\left(N_1^3\right)^{\frac{1}{4}} - \left(N_1^{\frac{1}{4}}\right)^3 = 0$

Alternate Method Quad Roots

If FN:FP as BQ:BS then quad roots series can be divided off in the figure.



Unit.

Given.

$n := 1 .. 3$

$$S_1 := \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad S_2 := \begin{pmatrix} b \\ c \\ a \end{pmatrix} \quad S_3 := \begin{pmatrix} c \\ a \\ b \end{pmatrix}$$

040397

Descriptions.

$$Is\_This\_a\_Triangle := \left(S_{1_1} + S_{2_1} > S_{3_1}\right) \cdot \left(S_{1_1} + S_{3_1} > S_{2_1}\right) \cdot \left(S_{2_1} + S_{3_1} > S_{1_1}\right)$$

As was learned in school, the area of a triagle is given by $\frac{1}{2} \cdot B \cdot H$.

From 04_02_97.MCD I show that, for a given side, the height is given by;

$$H_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{2 \cdot S_{1_n}}$$

And since $B := S_1$ Area is defined as

$$A_n := \frac{\sqrt{S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{-S_{1_n} + S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} - S_{2_n} + S_{3_n}} \cdot \sqrt{S_{1_n} + S_{2_n} - S_{3_n}}}{4}$$

$\frac{1 \cdot B_n \cdot H_n}{2} - A_n =$

0
0
0

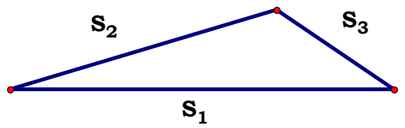
$A_n =$

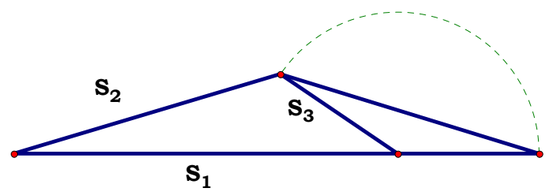
2.904738
2.904738
2.904738

$H_n =$

2.904738
1.936492
1.452369

Not changing the height of a given triangle, or the length of the subtended side, what happens to it's area if we halve the angle of one side?





What is the definition of acute, solely in terms of the sides of a triangle? Basically from this it can be argued that Euclid's definition of acute or obtuse was out of order.

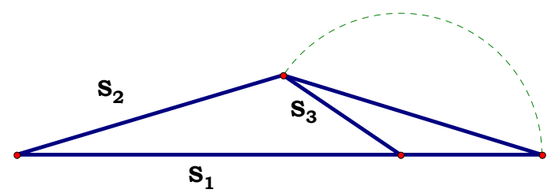
$$\mathbf{Acute}_n := \mathbf{if}\left[\sqrt{\left(\mathbf{S1}_n\right)^2 + \left(\mathbf{S2}_n\right)^2} > \mathbf{S3}_n\ ,\ 1\ ,\ 0\right] \qquad \mathbf{Acute2}_n := \mathbf{if}\left[\sqrt{\left(\mathbf{S1}_n\right)^2 + \left(\mathbf{S3}_n\right)^2} > \mathbf{S2}_n\ ,\ 1\ ,\ 0\right]$$

$\mathbf{Acute}_n =$

0
1
1

$\mathbf{Acute2}_n =$

1
0
1



$\mathbf{H}_n =$

2.904738
1.936492
1.452369

$\mathbf{S1}_n =$

2
3
4

$\mathbf{S2}_n =$

3
4
2

$\mathbf{S3}_n =$

4
2
3

if we halve $\angle \mathbf{S1S2}$ $\dfrac{\left(\mathbf{S1}_n + \mathbf{S2}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n =$

4.357106
3.872983
1.452369

if we halve $\angle \mathbf{S1S3}$ $\dfrac{\left(\mathbf{S1}_n + \mathbf{S3}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n =$

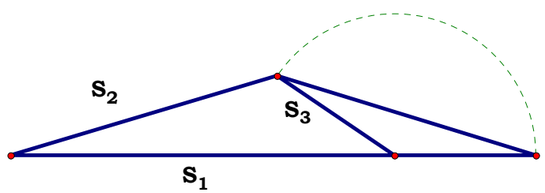
5.809475
1.936492
2.178553

$\mathbf{Is\_This\_a\_Triangle} = 1$

Since the greater angle is subtended by the greater side, halving the lesser angle increases the area of the triangle by the greater amount.

$$\left(\mathbf{S2}_n > \mathbf{S3}_n\right) - \left[\frac{\left(\mathbf{B}_n + \mathbf{S2}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n > \frac{\left(\mathbf{B}_n + \mathbf{S3}_n\right)\cdot \mathbf{H}_n}{2} - \mathbf{A}_n\right] =$$

0
0
0



Given two sides of a triangle, the height and if the angle contained by the two sides is acute or not, find the remaining side. What would happen if you were given just the equation and had no idea what the equation represented? You could not possibly solve it so quickly.

$$H_n = \frac{\sqrt{S1_n + S2_n + S3_n} \cdot \sqrt{-S1_n + S2_n + S3_n} \cdot \sqrt{S1_n - S2_n + S3_n} \cdot \sqrt{S1_n + S2_n - S3_n}}{2 \cdot S1_n}$$

Given S_1, S_2 and $\sqrt{S_1^2 + S_2^2} > S_3$, find S_3.

$Acute_n =$

0
1
1

Is_This_a_Triangle = 1

$$S4_n := \sqrt{(S2_n)^2 - (H_n)^2}$$

$$SX_n := \text{if}(Acute_n, S1_n - S4_n, S1_n + S4_n)$$

$a \equiv 2$ $b \equiv 3$ $c \equiv 4$ ← Plug your values in here.

$$S3_n := \sqrt{(H_n)^2 + (SX_n)^2}$$

$S1_n =$

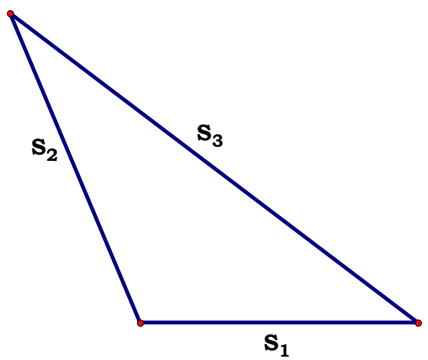
2
3
4

$S2_n =$

3
4
2

$S3_n =$

4
2
3



040497

Unit.

Given.

$N_1 := 5 \quad AB := N_1$

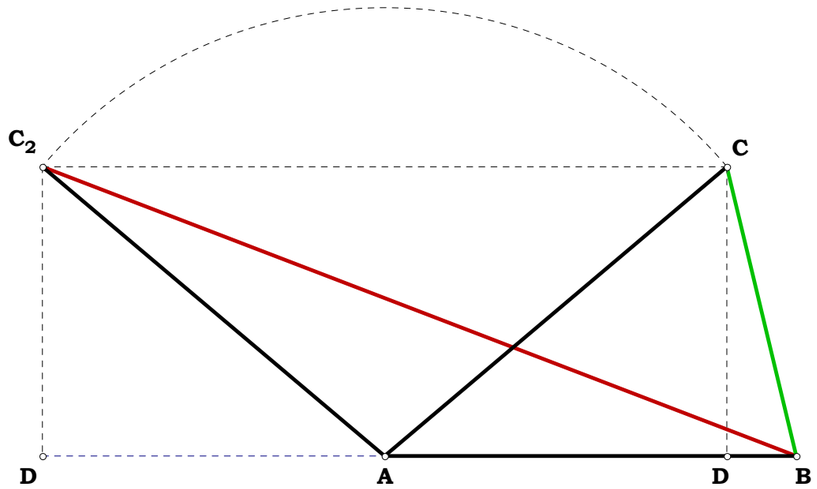
$N_2 := 4 \quad AC := N_2$

$N_3 := 3 \quad CD := N_3$

Descriptions.

Triangles

Given the base, one side and the height of a triangle, find both possible lengths of the remaining side.



$AD := \sqrt{AC^2 - CD^2} \qquad BD_1 := AB + AD \qquad BD_2 := AB - AD$

$BC_1 := \sqrt{CD^2 + BD_1^2} \qquad BC_2 := \sqrt{CD^2 + BD_2^2}$

$BC_1 = 8.213252 \qquad BC_2 = 3.813461$

$BC_1 - \sqrt{N_1^2 + N_2^2 + 2 \cdot N_1 \cdot \sqrt{N_2^2 - N_3^2}} = 0$

$BC_2 - \sqrt{\left(N_1^2 + N_2^2 - 2 \cdot N_1 \cdot \sqrt{N_2^2 - N_3^2}\right)} = 0$

Definitions.

$S_1 := AB \qquad S_2 := AC \qquad S_3 := BC_1$

$$\frac{\sqrt{S_1 + S_2 + S_3} \cdot \sqrt{-S_1 + S_2 + S_3} \cdot \sqrt{S_1 - S_2 + S_3} \cdot \sqrt{S_1 + S_2 - S_3}}{2 \cdot S_1} - CD = 0$$

$S_1 := AB \qquad S_2 := AC \qquad S_3 := BC_2$

$$\frac{\sqrt{S_1 + S_2 + S_3} \cdot \sqrt{-S_1 + S_2 + S_3} \cdot \sqrt{S_1 - S_2 + S_3} \cdot \sqrt{S_1 + S_2 - S_3}}{2 \cdot S_1} - CD = 0$$

$AB = 2.71667$ in. $\quad N_1 = 2.71667$ in.

$AC = 2.95665$ in. $\quad N_2 = 2.95665$ in.

$CD = 1.90833$ in. $\quad N_3 = 1.90833$ in.

$BC = 1.96260$ in. $\quad N_1^2+N_2^2 = 16.12208$ in^2

$BC_2 = 5.32845$ in. $\quad 2 \cdot N_1 \cdot \sqrt{N_2^2-N_3^2} = 12.27028$ in^2

$BC - \sqrt{(N_1^2+N_2^2) - (2 \cdot N_1 \cdot \sqrt{N_2^2-N_3^2})} = 0.00000$ in.

$BC_2 - \sqrt{(N_1^2+N_2^2) + (2 \cdot N_1 \cdot \sqrt{N_2^2-N_3^2})} = 0.00000$ in.

Unit.

Given.

$N_1 := 3.14854$ $AD := N_1$

$N_2 := 6.50875$ $AC := N_2$

042897

Descriptions.

$CD := \sqrt{AD^2 + AC^2}$ $DH := CD$

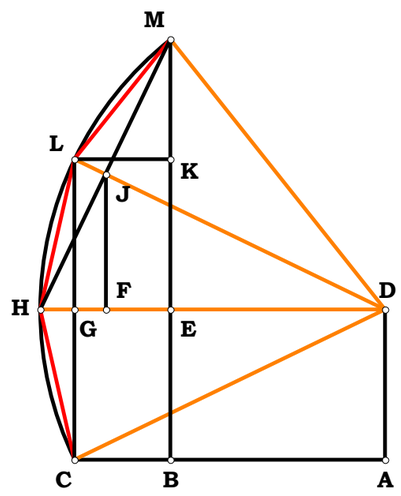
$CG := AD$ $DG := AC$ $GH := DH - DG$ $CH := \sqrt{GH^2 + CG^2}$

$HJ := CG$ $DJ := DG$

$FH := \frac{\left(HJ^2 + DH^2\right) - DJ^2}{2 \cdot DH}$ $EF := FH$ $DE := DH - (EF + FH)$

$AB := DE$ $EG := DG - DE$ $LM := CH$ $LK := EG$

$KM := \sqrt{LM^2 - LK^2}$ $BE := AD$ $BK := 2 \cdot BE$ $BM := BK + KM$



$AD = 2.51883$ cm $N_1 = 2.51883$ cm

$AC = 5.20700$ cm $N_2 = 5.20700$ cm

$CH = 2.58413$ cm $\sqrt{(2 \cdot N_1^2 + 2 \cdot N_2^2) - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} = 2.58413$ cm

$CH - \sqrt{(2 \cdot N_1^2 + 2 \cdot N_2^2) - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} = 0.00000$ cm

$m\angle DCA = 25.81490°$

$m\angle CDM = 77.44471°$

$\frac{m\angle CDM}{m\angle DCA} = 3.00000$

Some definitions:

$\sqrt{N_1^2 + N_2^2} - CD = 0$ $\sqrt{N_1^2 + N_2^2} - N_2 - GH = 0$ $\sqrt{2 \cdot N_1^2 + 2 \cdot N_2^2 - 2 \cdot N_2 \cdot \sqrt{N_1^2 + N_2^2}} - CH = 0$

$\frac{N_1^2}{\sqrt{N_1^2 + N_2^2}} - FH = 0$ $\frac{\left(N_2^2 - N_1^2\right)}{\sqrt{N_1^2 + N_2^2}} - DE = 0$

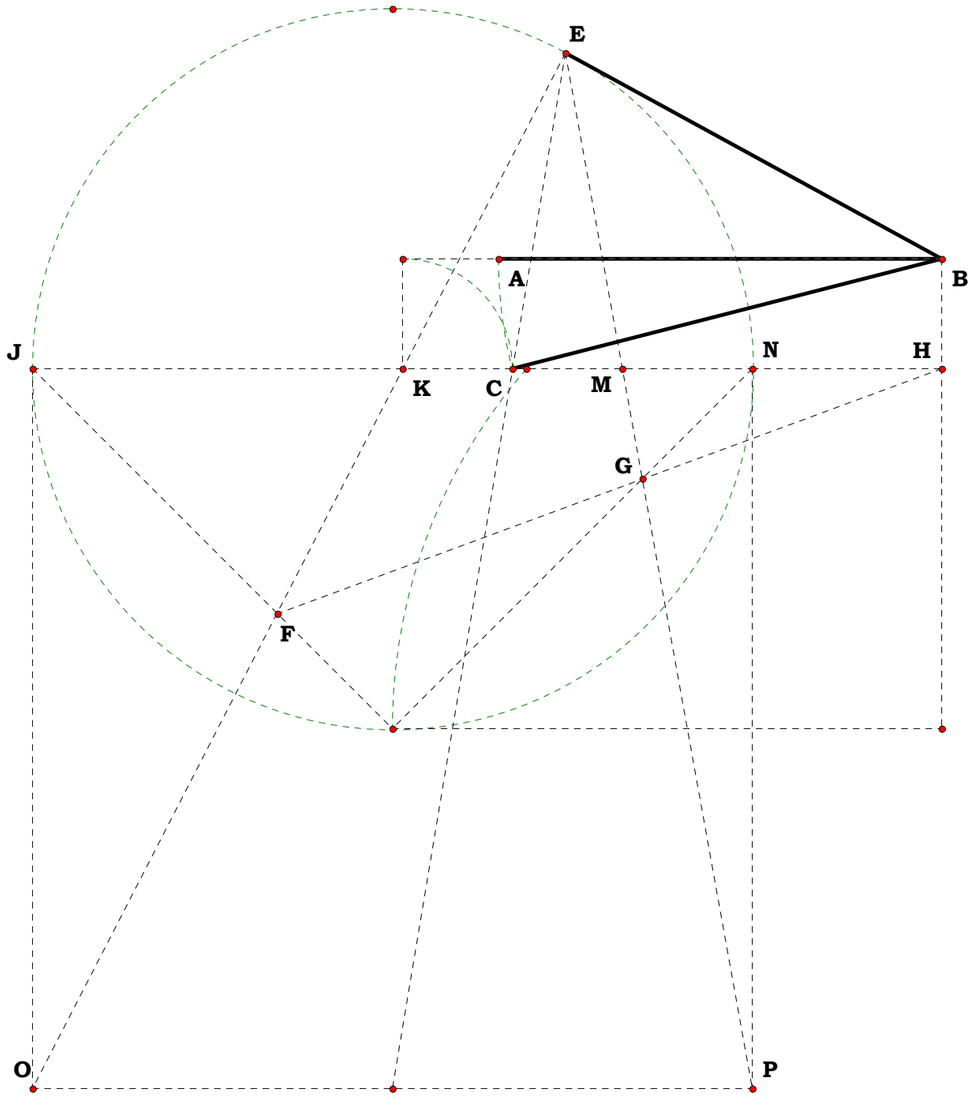
$2 \cdot N_1 + \frac{\sqrt{\left(2 \cdot N_2^3 - 2 \cdot N_1^2 \cdot N_2\right) \cdot \sqrt{N_1^2 + N_2^2} + N_1^4 + 5 \cdot N_1^2 \cdot N_2^2 - 2 \cdot N_2 \cdot \left(\sqrt{N_1^2 + N_2^2}\right)^3}}{\sqrt{N_1^2 + N_2^2}} - BM = 0$

042997

Trisection and the Cube Roots

Exploring trisection produced in the cube root figure. If one places E where they will, they would find the origin of the root series H by projecting through FG. Taking half of KM for BH we find that angle ABC is 1/3 of angle EBC. So, we can not only produce a cube root series from OP but also a trisection as well from the orign which is the same origin for the square root of the figure.

I should write up some plates concerning the point G or F and the different relationships they form with the finished plate.



$m\angle ABC = 14.33586°$

$m\angle EBA = 28.67173°$

$\frac{m\angle EBA}{m\angle ABC} = 2.00000$

$m\angle EBC = 43.00759°$

$\frac{m\angle EBC}{m\angle ABC} = 3.00000$

Unit.

$AB := 1$

Given.

$N := 5$

Descriptions.

$AH := N \qquad BH := AH - AB \qquad BJ := BH$

$AC := \left(AB^2 \cdot AH\right)^{\frac{1}{3}} \qquad AF := \left(AB \cdot AH^2\right)^{\frac{1}{3}}$

$CF := AF - AC \qquad CE := \frac{CF}{2} \qquad AE := AC + CE$

$AU := CE \qquad NV := AU \qquad MW := AU$

(For the next two equations see 042897.)

$$AM := \frac{\left(\frac{AE}{AU} - 1\right) \cdot \left(\frac{AE}{AU} + 1\right) \cdot AU}{\sqrt{\left(\frac{AE}{AU}\right)^2 + 1}}$$

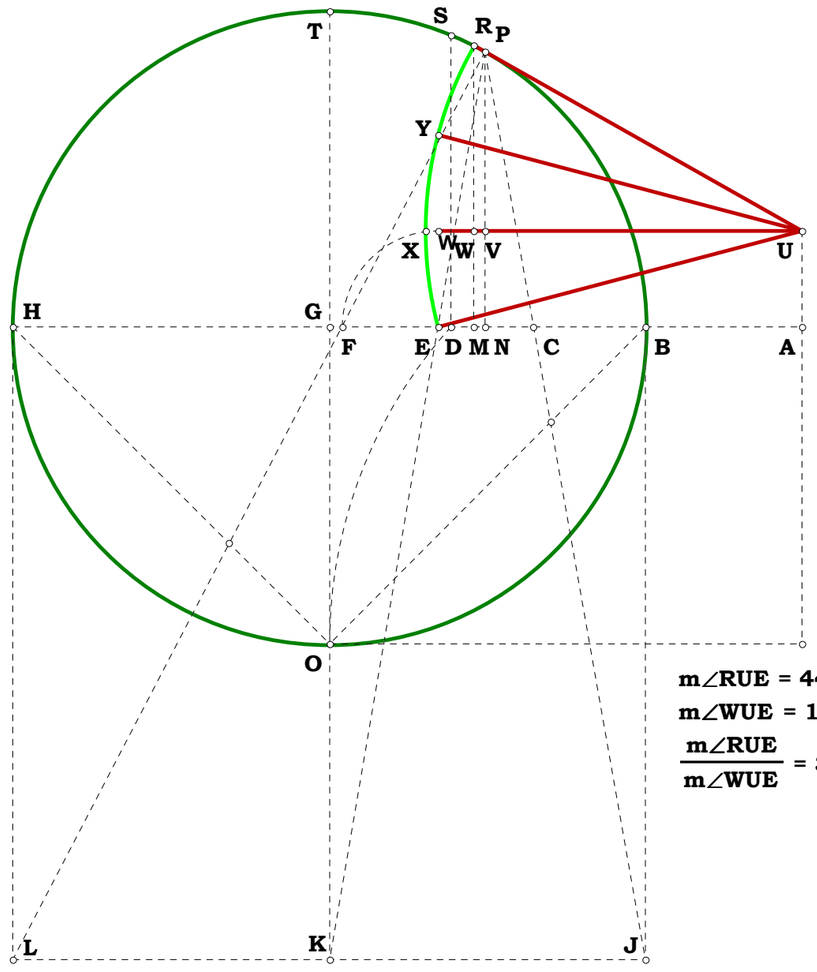
$$MR := 2 \cdot AU + AU \cdot \sqrt{\frac{\left(AU^2\right)^{\frac{3}{2}} + 5 \cdot AE^2 \cdot \sqrt{AU^2} - 4 \cdot AE \cdot AU \cdot \sqrt{AE^2 + AU^2}}{\left(AE^2 + AU^2\right) \cdot \sqrt{AU^2}}}$$

$RW := MR - MW \qquad BC := AC - AB \qquad FH := AH - AF \qquad CN := \frac{BC \cdot CF}{BC + FH}$

$NP := \frac{BJ \cdot CN}{BC} \qquad PV := NP - NV \qquad AN := AC + CN \qquad UV := AN$

$UW := AM \qquad \frac{RW \cdot UV}{UW} - PV = 0$

If trisection can be placed at RUE, then PV is proportional to RW.



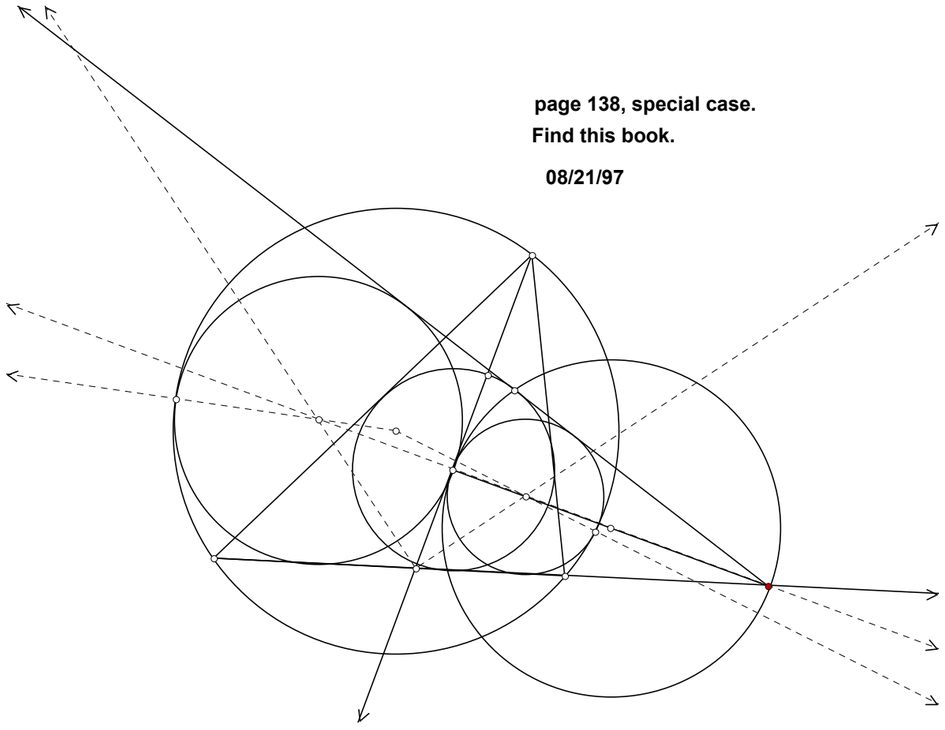
$m\angle RUE = 44.289°$

$m\angle WUE = 14.763°$

$\frac{m\angle RUE}{m\angle WUE} = 3.000$

082197

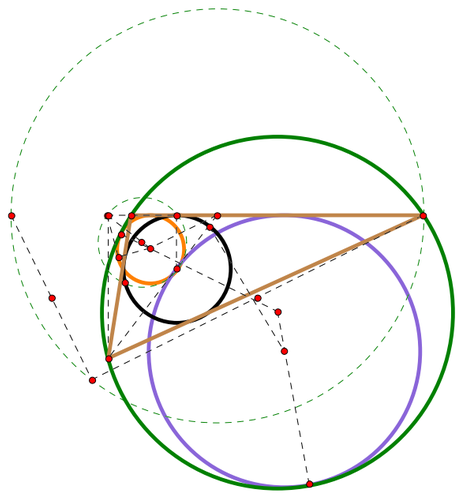
In my files I have the following plate, from what book I do not recall, however, the figure as it was written up there was listed as a "special case," of what I do not recall either. But I want to write it up because it is not a special case of anything, it is actually a plate showing that one can treat every triangle as an eight circle problem, simply add the remaining two sides by recursion of the first. I suspect now that if someone thought this was a special case of something, then they did not comprehend the actual relationships, they are easily found by compass.
The project would start with the equations from 062793 and 040694. So, this is a project I am interested in doing and have been for a long time. I might even find the book it came from. Might be interesting to find the equations for all eight circles. Maybe some day.

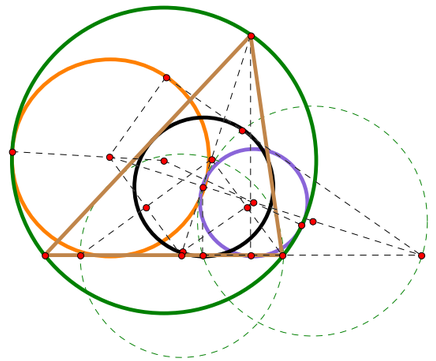


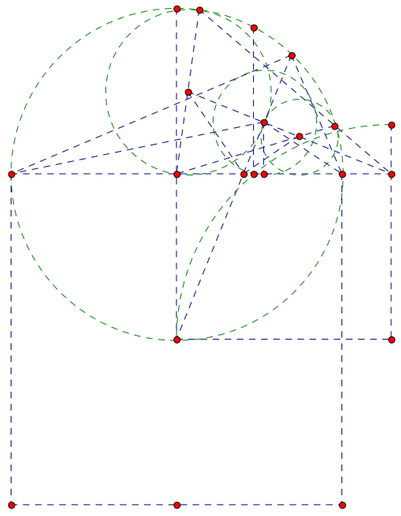
page 138, special case.
Find this book.

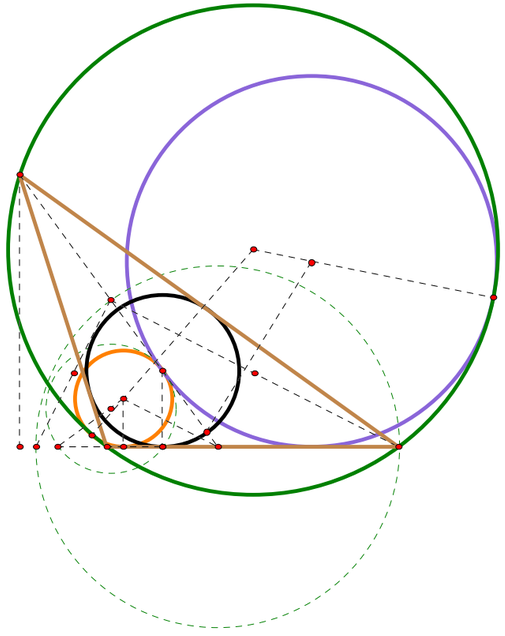
08/21/97

Eight Circles and a Forgotten Book.







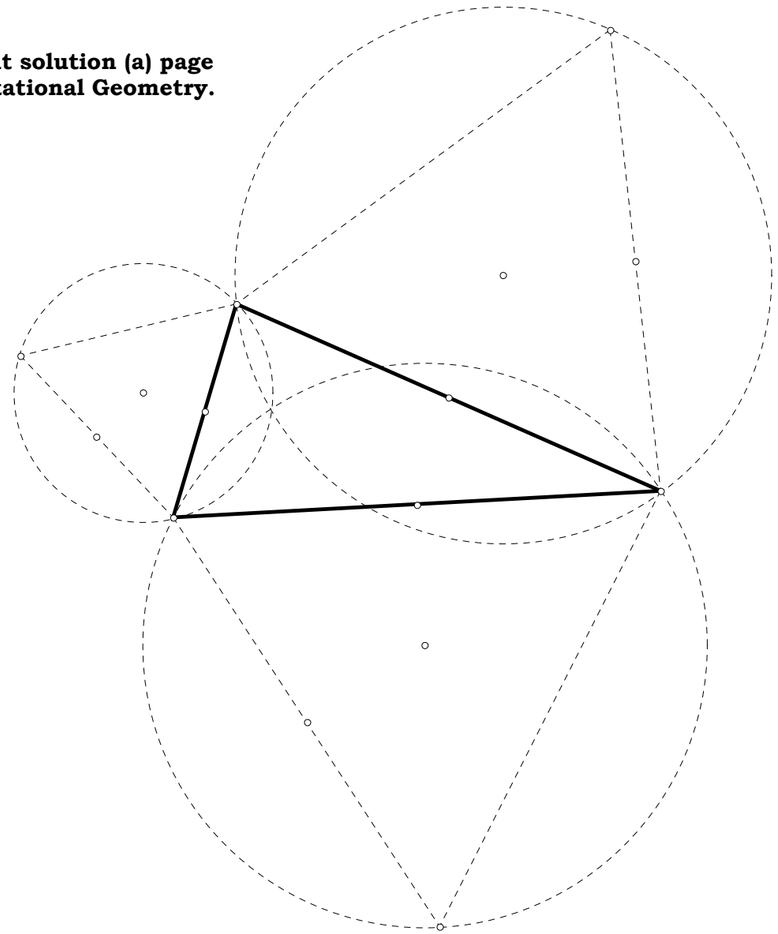


Unit.
Given.

082297
Descriptions.
Definitions.

Steiner Point

Steiner point solution (a) page 361 Computational Geometry.



Needs a proper writeup

Unit.

Given.

$N_1 := 2.66666$ $AB := N_1$

$N_2 := 1.31473$ $EF := N_2$

$N_3 := 1.26711$

091197A

Descriptions.

$$AD := \frac{N_1}{N_3} \quad BD := AB - AD \quad DJ := \sqrt{AD \cdot BD} \quad AC := \frac{AB}{2}$$

$$DC := \frac{(AC - AD)^2}{\sqrt{(AC - AD)^2}} \quad CH := \frac{EF}{2} \quad CJ := AC$$

$$DG := \frac{DJ \cdot CH}{CJ} \quad CG := \sqrt{DG^2 + DC^2} \quad MN := 2 \cdot \sqrt{\left(\frac{AB}{2}\right)^2 - CH^2}$$

Definitions.

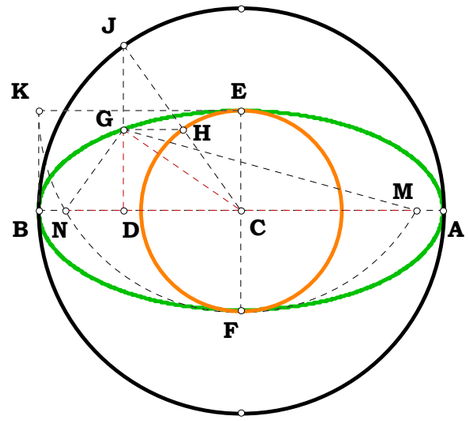
$$BD - \frac{N_1 \cdot (N_3 - 1)}{N_3} = 0 \quad DJ - \frac{N_1 \cdot \sqrt{(N_3 - 1)}}{N_3} = 0 \quad AC - \frac{N_1}{2} = 0$$

$$DC - \frac{N_1 \cdot \sqrt{(N_3 - 2)^2}}{2 \cdot N_3} = 0 \quad CH - \frac{N_2}{2} = 0 \quad CJ - \frac{N_1}{2} = 0 \quad DG - \frac{N_2 \cdot \sqrt{N_3 - 1}}{N_3} = 0$$

$$CG - \frac{\sqrt{(N_3^2 - 4 \cdot N_3 + 4) \cdot N_1^2 + 4 \cdot N_2^2 \cdot (N_3 - 1)}}{2 \cdot N_3} = 0 \quad MN - \sqrt{(N_1 - N_2) \cdot (N_1 + N_2)} = 0$$

The Ellipse

Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.



AB = 2.66667 in.

EF = 1.31473 in.

AD = 2.10453 in.

$\frac{AB}{AD}$ = 1.26711

CH = 0.65736 in.

DG = 0.53625 in.

MN = 2.32004 in.

$MN - 2 \cdot \sqrt{\frac{AB^2}{2} - CH^2}$ = 0.00000 in.

$MN - \sqrt{(AB - EF) \cdot (AB + EF)}$ = 0.00000 in.

091197B

Unit.

Given.

$S_1 := 8.14917 \quad AB := S_1$

$S_2 := 7.23745 \quad AC := S_2$

$S_3 := 2.58277 \quad BC := S_3$

Descriptions.

$DE := AC + BC \quad AH := \frac{DE}{2} \quad AG := \frac{AB}{2} \quad FG := \sqrt{AH^2 - AG^2}$

Definitions.

$$FG - \frac{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}}{2} = 0$$

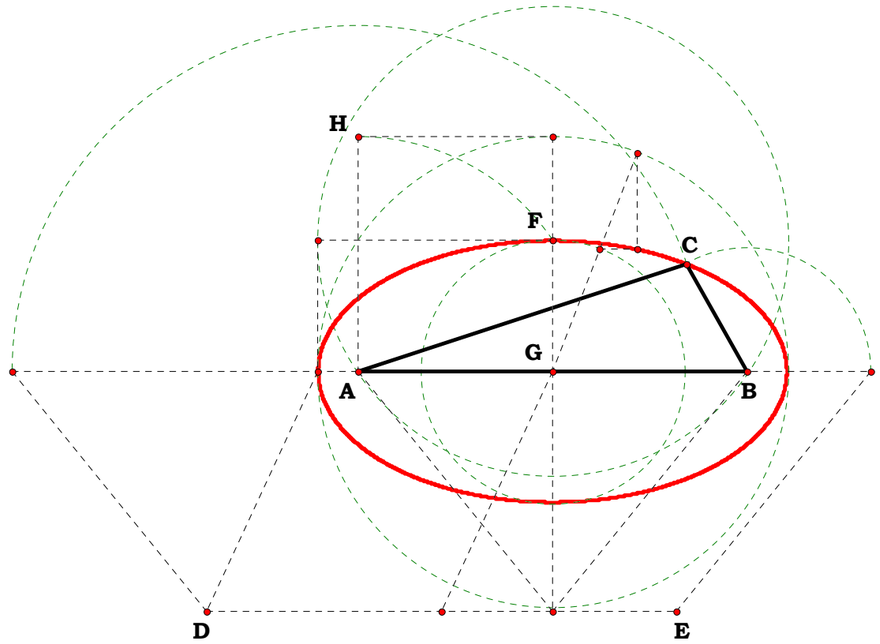
The ratio of the ellipse is thus;

$$\frac{AH}{FG} - \frac{(S_2 + S_3)}{\sqrt{(S_2 - S_1 + S_3)\cdot(S_1 + S_2 + S_3)}} = 0$$

From any point on DE, one can find everything and not once think about x and y.

The Ellipse

Given triangle ABC, and AB as base, describe the Ellipse



Unit.
Given.

$N := .656$

$\Delta := 4$

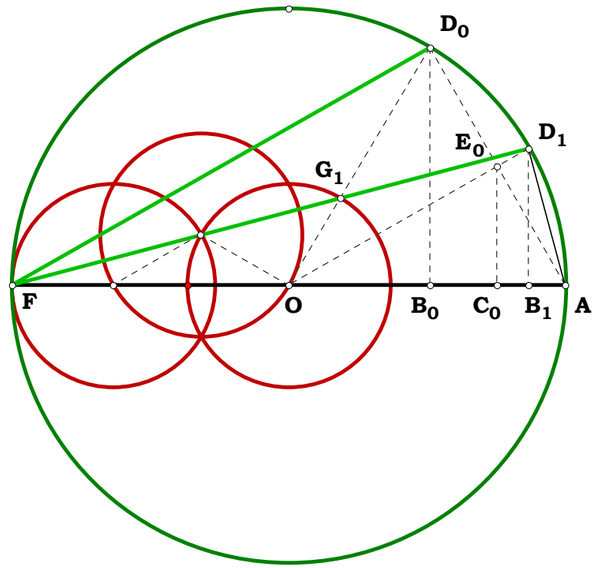
$\delta := 0 .. \Delta - 1$

020298
Descriptions.

$AF := 2.3754 \quad AO := \frac{AF}{2} \quad AB_0 := N \quad BD_0 := \sqrt{AB_0 \cdot (AF - AB_0)}$

$CE_0 := \frac{BD_0}{2} \quad AC_0 := \frac{AB_0}{2} \quad OC_0 := AO - AC_0 \quad OE_0 := \sqrt{(CE_0)^2 + (OC_0)^2}$

$$\begin{pmatrix} OB_{\delta+1} \\ AB_{\delta+1} \\ BD_{\delta+1} \\ CE_{\delta+1} \\ AC_{\delta+1} \\ OC_{\delta+1} \\ OE_{\delta+1} \end{pmatrix} := \begin{bmatrix} OC_\delta \cdot \frac{AO}{OE_\delta} \\ AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{OE_\delta} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{(2 \cdot OE_\delta)} \cdot \sqrt{-AO \cdot (OE_\delta - OC_\delta) \cdot (-AF \cdot OE_\delta + AO \cdot OE_\delta - OC_\delta \cdot AO)} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta - OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot AO \cdot \frac{(OE_\delta + OC_\delta)}{OE_\delta} \\ \frac{1}{2} \cdot \sqrt{AO \cdot \frac{(AF \cdot OE_\delta + 4 \cdot OC_\delta \cdot AO - OC_\delta \cdot AF)}{OE_\delta}} \end{bmatrix}$$



$AD_\delta := \sqrt{(AB_\delta)^2 + (BD_\delta)^2}$

Definitions.

$$AD = \begin{pmatrix} 1.248304 \\ 0.648825 \\ 0.327541 \\ 0.164163 \end{pmatrix}$$

Length of cord by progressive bisections.

I have no idea why I did this figure, it was so long ago. I don't know what I would do with a cord series unless one wanted to see how close one could get to PI starting from some particular angle or the length of some cord of a circle of some lenght at some commensurable angle. I wonder if I will ever get that bored?

Unit.

Given.

$N_1 := 2.98958$ $AE := N_1$

$N_2 := 1.86690$ $EG := N_2$

021098

Descriptions.

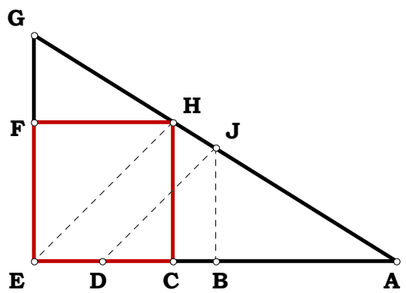
$AB := \frac{AE}{2}$ $BJ := \frac{EG}{2}$ $BD := BJ$

$AD := AB + BD$ $CE := BD \cdot \frac{AE}{AD}$

$AC := AE - CE$ $FG := EG - CE$

A Square In A Triangle

What is the Algebraic Name for the square as given in a right triangle? What is the Algebraic name for the ratio AE/AC?



AE = 2.39167 in. N_1 = 2.39167 in.

EG = 1.49352 in. N_2 = 1.49352 in.

CE = 0.91939 in.

AC = 1.47228 in.

$CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0.00000$ in. $\frac{AE}{AC} - \frac{N_1 + N_2}{N_1} = 0.00000$

Definitions.

$AB - \frac{N_1}{2} = 0$ $BJ - \frac{N_2}{2} = 0$ $AD - \left(\frac{N_1}{2} + \frac{N_2}{2}\right) = 0$

$CE - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$ $AC - \frac{{N_1}^2}{N_1 + N_2} = 0$

$FG - \frac{{N_2}^2}{N_1 + N_2} = 0$ $\frac{AE}{AC} - \left(\frac{N_1 + N_2}{N_1}\right) = 0$ $\frac{EG}{FG} - \frac{N_1 + N_2}{N_2} = 0$

Unit.

Given.

$N_1 := 1.67500$ $AH := N_1$

$N_2 := 1.55441$ $HN := N_1 \cdot N_2$

022598A

Descriptions. Given for the third power.

$HJ := HN - AH$ $FH := \frac{AH \cdot HJ}{AH + HJ}$ $FG := FH$

$AF := AH - FH$ $DF := \frac{AF \cdot FG}{AF + FG}$ $AD := AF - DF$

$DE := DF$ $BD := \frac{AD \cdot DE}{AD + DE}$ $AB := AD - BD$

Definitions.

$\frac{AH}{AF} - N_2^{\,1} = 0$ $\frac{AH}{AD} - N_2^{\,2} = 0$ $\frac{AH}{AB} - N_2^{\,3} = 0$

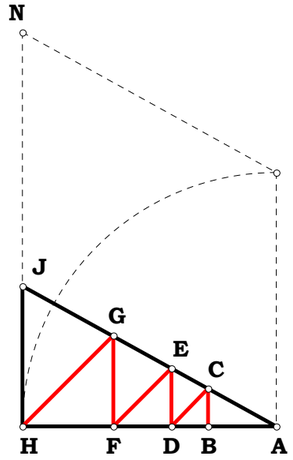
$HN - N_1 \cdot N_2 = 0$ $HJ - (N_1 \cdot N_2 - N_1) = 0$ $FH - \frac{N_1 \cdot (N_2 - 1)}{N_2} = 0$

$FG - \frac{N_1 \cdot (N_2 - 1)}{N_2} = 0$ $AF - \frac{N_1}{N_2} = 0$ $DF - \frac{N_1 \cdot (N_2 - 1)}{N_2^{\,2}} = 0$

$AD - \frac{N_1}{N_2^{\,2}} = 0$ $DE - \frac{N_1 \cdot (N_2 - 1)}{N_2^{\,2}} = 0$ $BD - \frac{N_1 \cdot (N_2 - 1)}{N_2^{\,3}} = 0$ $AB - \frac{N_1}{N_2^{\,3}} = 0$

Alternate Method Root Series

Given a length and a unit, raise that length to any whole power.



AH = 1.67500 in. N_1 = 1.67500 in.

HN = 2.60364 in.

$\frac{HN}{AH}$ = 1.55441 N_2 = 1.55441

HJ = 0.92864 in.

AB = 0.44598 in.

AF = 1.07758 in.

AD = 0.69324 in.

$\frac{AH}{AF}$-N_2 = 0.00000 $\frac{AH}{AD}$-N_2^2 = 0.00000 $\frac{AH}{AB}$-N_2^3 = 0.00000

Unit.

$AH := 1$

Given.

$N_1 := 5$ $\quad HM := N_1$

$N_2 := 6$ $\quad AN := N_2$

022598B

Descriptions.

$$HO := \frac{AH \cdot HM}{AN} \qquad AO := AH + HO \qquad AF := \frac{AH^2}{AO}$$

$$FH := AH - AF \qquad FG := FH \qquad DF := \frac{AF \cdot FG}{AF + FG}$$

$$AD := AF - DF \qquad DE := DF \qquad BD := \frac{AD \cdot DE}{AD + DE}$$

$$AB := AD - BD$$

Definitions.

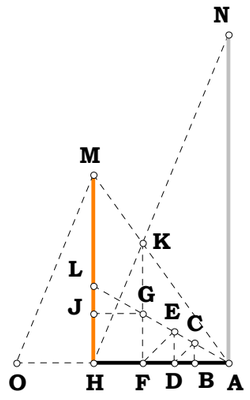
$$\frac{AH}{AB} - \left(\frac{N_1 + N_2}{N_2}\right)^3 = 0 \qquad HO - \frac{N_1}{N_2} = 0 \qquad AO - \frac{N_1 + N_2}{N_2} = 0$$

$$AF - \frac{N_2}{N_1 + N_2} = 0 \qquad FH - \frac{N_1}{N_1 + N_2} = 0 \qquad FG - \frac{N_1}{N_1 + N_2} = 0$$

$$DF - \frac{N_1 \cdot N_2}{(N_1 + N_2)^2} = 0 \qquad AD - \frac{N_2^2}{(N_1 + N_2)^2} = 0 \qquad DE - \frac{N_1 \cdot N_2}{(N_1 + N_2)^2} = 0$$

$$BD - \frac{N_1 \cdot N_2^2}{(N_1 + N_2)^3} = 0 \qquad AB - \frac{N_2^3}{(N_1 + N_2)^3} = 0 \qquad AB^{\frac{1}{3}} - \frac{N_2}{(N_1 + N_2)} = 0$$

Sum Divided by One Powered



$AH = 0.89167$ in.

$HM = 1.24469$ in.

$AN = 2.16687$ in.

$AB = 0.22848$ in.

$\frac{HM}{AH} = 1.39591$ $\quad N_1 = 1.39591$

$\frac{AN}{AH} = 2.43013$ $\quad N_2 = 2.43013$

$\frac{AB}{AH} = 0.25624$ $\quad AB = 0.25624$

$$AB - \frac{N_2^3}{(N_1+N_2)^3} = 0.00000$$

Doing the Math

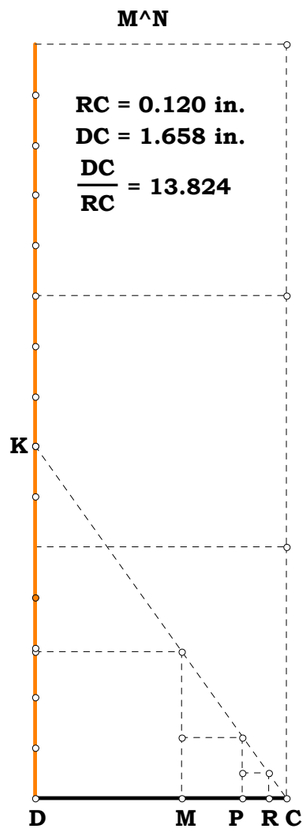
022598C

Descriptions.

These plates sat, I never actually wrote them up, in their directory, but included in the Delian Quest. Because they are so elementary, I assumed that doing math with a geometric figure was known. I was a bit conflicted about this, however, working 12 hours a day for years on end tends to dull the senses. Then, I got to thinking about them again in 2007. I even did a couple of searches on the internet to see if anyone had actually developed doing the math with a simple geometric figure and could not find anything. Then I found scraps in old books found on the Internet Archive where certain operations were fragmented and really undeveloped. Then I started to realize and understand that BAM was not developed as a grammar If it had been, there would be no talk of non-Euclidean Geometry, there would only be embarrassment of its memory.

One can see that it is directly derived from plate A on this date. BAM (Basic Analog Mathematics) has its roots in exponential series.

Definitions.



$4^3 = 64.000$

$\left(3+\frac{4}{5}\right)^3 = 54.872$

$\left(3+\frac{3}{5}\right)^3 = 46.656$

$\left(3+\frac{2}{5}\right)^3 = 39.304$

$\left(3+\frac{1}{5}\right)^3 = 32.768$

$3^3 = 27.000$

$\left(2+\frac{4}{5}\right)^3 = 21.952$

$\left(2+\frac{3}{5}\right)^3 = 17.576$

$\left(2+\frac{2}{5}\right)^3 = 13.824$

$\left(2+\frac{1}{5}\right)^3 = 10.648$

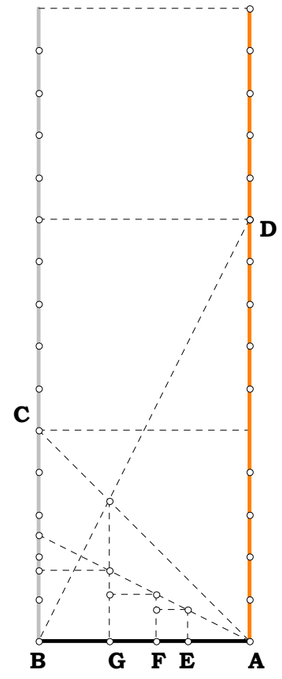
$\left(1+\frac{4}{5}\right)^3 = 5.832$

$\left(1+\frac{3}{5}\right)^3 = 4.096$

$\left(1+\frac{2}{5}\right)^3 = 2.744$

$\left(1+\frac{1}{5}\right)^3 = 1.728$

Given line AB, divide it by the equation $\frac{BC+AD^{W}}{AD} = 5.06250$ **where W is a whole number.**



$\frac{AB}{AG} = 1.50000$

$\frac{AB}{AF} = 2.25000$

$\frac{AB}{AE} = 3.37500$

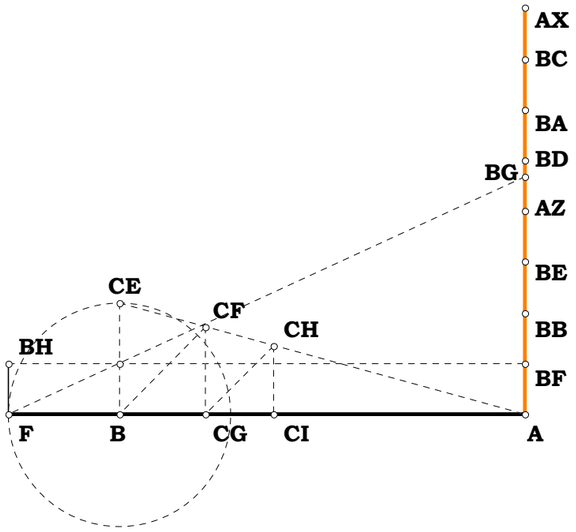
$\frac{BC+AD^{1}}{AD} = 1.50000$

$\frac{BC+AD^{2}}{AD} = 2.25000$

$\frac{BC+AD^{3}}{AD} = 3.37500$

022698

Now ain't this just typically human. Some of the more defined plates that led to Basic Analog Mathematics promptly get wrote up in 0816 2015. I am on the ball! It appears I never even bothered to do a pdf file of these. They are, however, not fundamentally distinct from some previous write ups except, these are series format. So, I think I will forgo the write ups again!



$\frac{AF}{FB} = 4.672$

$AB = 2.679$ in.

$FB = 0.730$ in.

$AF = 3.408$ in.

$\frac{AF}{AB} = 1.272$

$ACG = 2.105$ in.

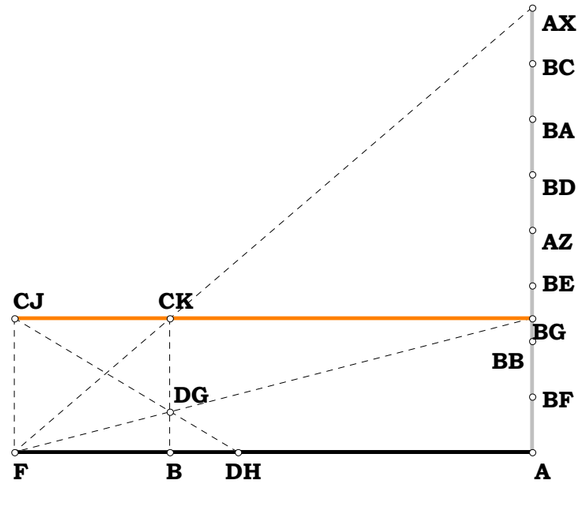
$ACI = 1.655$ in.

$\frac{AF}{ACI} = 2.060$

$\frac{AF}{ACG} = 1.619$

$\frac{AF^3}{AB} = 2.060$

$\frac{AF^2}{AB} = 1.619$



$AF = 3.417$ in.

$AB = 2.388$ in.

$\frac{AF}{AB} = 1.431$

$\frac{AF}{DHF} = 2.321$

$\frac{DHF}{AF} = 0.431$

$DHF = 1.472$ in.

$BF = 1.029$ in.

$\frac{AAX}{BGA} = 3.321$

$\frac{AAX}{BGA} - 1 = 2.321$

$BGA = 0.884$ in.

$AAX = 2.936$ in.

$\left(\frac{1}{\frac{AAX}{BGA} - 1}\right) \cdot BF = 0.443$ in.

Unit.

$AB := 1$

Given.

$N_1 := 4.27 \quad AF := N_1$

042398

Descriptions.

$BF := AF - AB \qquad AD := (AB \cdot AF)^{\frac{1}{2}} \qquad BE := \frac{BF}{2}$

$BD := AD - AB \qquad DE := BE - BD \qquad EQ := BE$

$DQ := \left(DE^2 + EQ^2\right)^{\frac{1}{2}} \qquad PQ := BF \qquad QM := \frac{EQ \cdot PQ}{DQ}$

$DM := QM - DQ \qquad AE := AB + BE \qquad AC := \frac{AE}{2}$

$Db := \frac{DM}{2} \qquad CM := AC \qquad ab := \frac{CM \cdot Db}{DM}$

$CD := AD - AC \qquad Ca := \frac{CD}{2} \qquad Aa := AC + Ca$

$CH := \frac{ab \cdot AC}{Aa} \qquad AM := AD \qquad Ac := \frac{AM \cdot CH}{CM}$

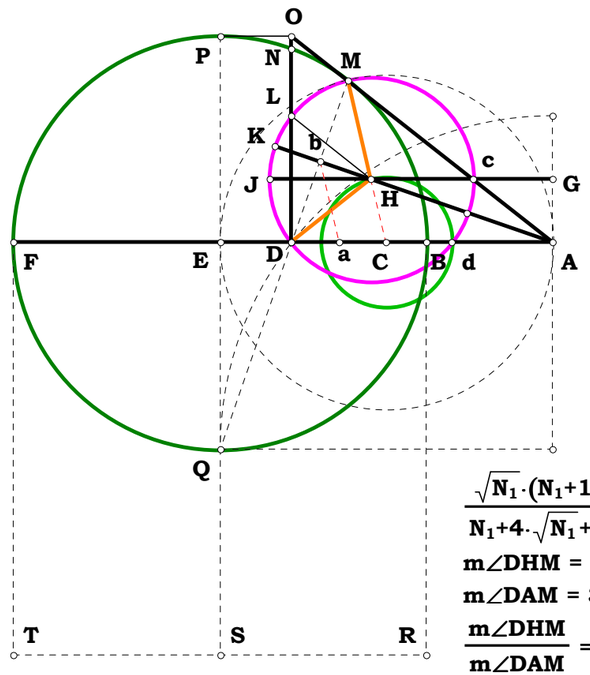
$HM := CM - CH \qquad HM - Ac = 0$

Definitions.

$Ac - \frac{\sqrt{N_1} \cdot \left(N_1 + 1\right)}{N_1 + 4 \cdot \sqrt{N_1} + 1} = 0 \qquad HM - \frac{\sqrt{N_1} \cdot \left(N_1 + 1\right)}{N_1 + 4 \cdot \sqrt{N_1} + 1} = 0$

A Square Root Figure And Triseciton

In this square root figure, what is the radius of the circle with the trisected angle? What is the radius CH?



AB = 2.11667 cm

AF = 9.03817 cm

Ac = 1.70294 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AF}{AB} = 4.27000$ $\qquad N_1 = 4.27000$

$\frac{Ac}{AB} = 0.80454$ $\qquad Ac = 0.80454$

$\frac{\sqrt{N_1} \cdot (N_1+1)}{N_1+4 \cdot \sqrt{N_1}+1} - Ac = 0.00000$

$m\angle DHM = 115.05651°$

$m\angle DAM = 38.35217°$

$\frac{m\angle DHM}{m\angle DAM} = 3.00000$

The traditional paper trisector fits right into this figure.
This figure seems to be just full of surprises.

$$AF - N_1 = 0 \qquad BF - (N_1 - 1) = 0 \qquad AD - \sqrt{N_1} = 0$$

$$BE - \frac{N_1 - 1}{2} = 0 \qquad BD - \left(\sqrt{N_1} - 1\right) = 0 \qquad DE - \frac{\left(\sqrt{N_1} - 1\right)^2}{2} = 0$$

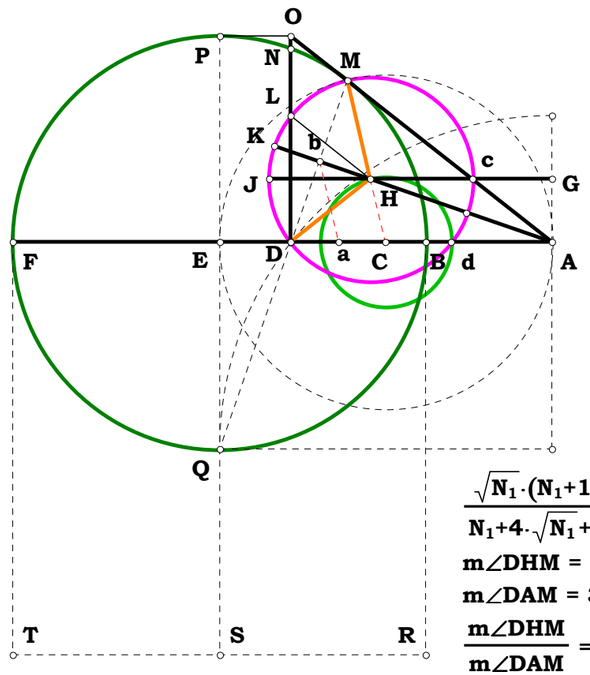
$$DQ - \frac{\sqrt{(N_1 + 1)\cdot\left(\sqrt{N_1} - 1\right)^2}}{\sqrt{2}} = 0 \qquad QM - \frac{\sqrt{2}\cdot\left(\sqrt{N_1} - 1\right)^2\cdot\left(\sqrt{N_1} + 1\right)^2}{2\cdot\sqrt{(N_1 + 1)\cdot\left(\sqrt{N_1} - 1\right)^2}} = 0$$

$$DM - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)^2}{\sqrt{(N_1 + 1)\cdot\left(\sqrt{N_1} - 1\right)^2}} = 0 \qquad AE - \frac{1 + N_1}{2} = 0 \qquad AC - \frac{1 + N_1}{4} = 0$$

$$Db - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1} - 1\right)^2}{2\cdot\sqrt{(N_1 + 1)\cdot\left(N_1 - 2\cdot\sqrt{N_1} + 1\right)}} = 0 \qquad CD - \frac{4\cdot\sqrt{N_1} - N_1 - 1}{4} = 0$$

$$Ca - \frac{4\cdot\sqrt{N_1} - N_1 - 1}{8} = 0 \qquad Aa - \frac{N_1 + 4\cdot\sqrt{N_1} + 1}{8} = 0$$

$$CH - \frac{\left(1 + N_1\right)^2}{4\cdot\left(N_1 + 4\cdot\sqrt{N_1} + 1\right)} = 0$$



AB = 2.11667 cm
AF = 9.03817 cm
Ac = 1.70294 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AF}{AB} = 4.27000$ $\qquad N_1 = 4.27000$

$\frac{Ac}{AB} = 0.80454$ $\qquad Ac = 0.80454$

$\frac{\sqrt{N_1}\cdot(N_1+1)}{N_1+4\cdot\sqrt{N_1}+1}$-Ac = 0.00000

m∠DHM = 115.05651°

m∠DAM = 38.35217°

$\frac{m\angle DHM}{m\angle DAM} = 3.00000$

072499

Unit.

$AB := 1$

Given.

$N_1 := 6.429$ $AD := N_1$

$N_2 := 2$

On Gemini Roots

CE is to EF as CY is to CW

Descriptions.

$BD := AD - AB$ $BC := \frac{BD}{2}$ $CW := BC$ $CT := BC$

$FV := BC$ $AI := \sqrt{AB \cdot AD}$ $BI := AI - AB$ $DI := BD - BI$

$IR := \sqrt{BI \cdot DI}$ $AE := AI$ $AC := AB + BC$ $ED := AD + AE$

$CE := AC + AE$ $EF := \frac{CE}{N_2}$ $BE := AE + AB$ $EI := AE + AI$

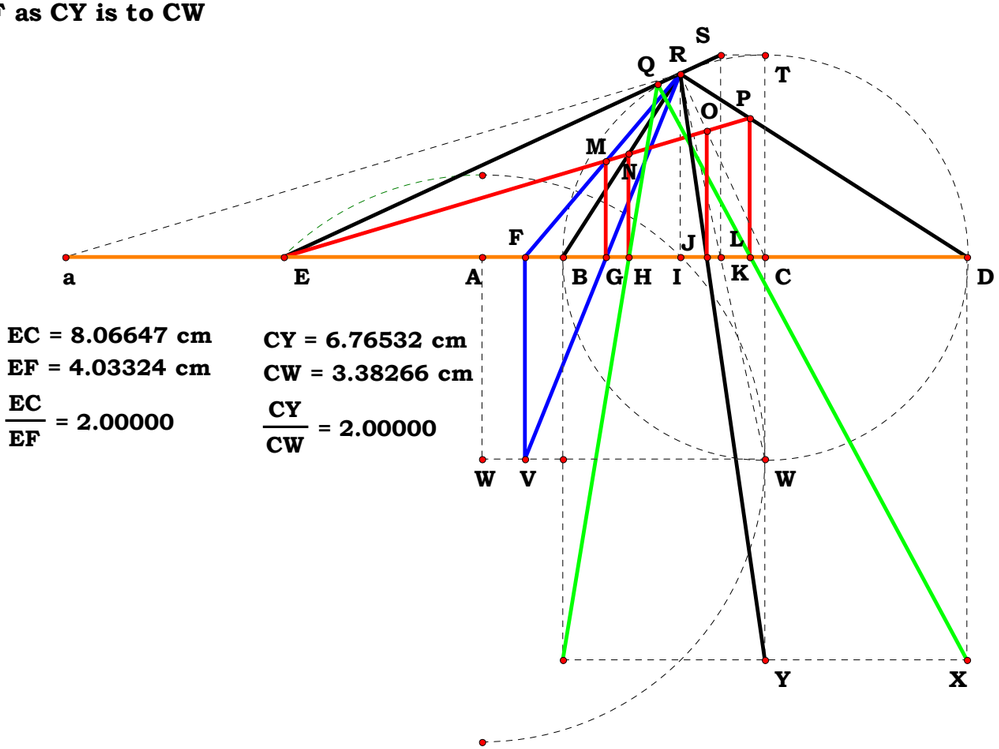
$FI := EI - EF$ $FG := \frac{FI \cdot FV}{FV + IR}$ $EG := EF + FG$ $GI := FI - FG$

$GM := \frac{FV \cdot GI}{FI}$ $Ia := \frac{EG \cdot IR}{GM}$ $EL := \frac{Ia \cdot ED}{Ia + DI}$ $BR := \sqrt{BI^2 + IR^2}$

$Ba := Ia - BI$ $BH := \frac{BI \cdot BE}{Ba}$ $EH := BE + BH$ $CI := AC - AI$

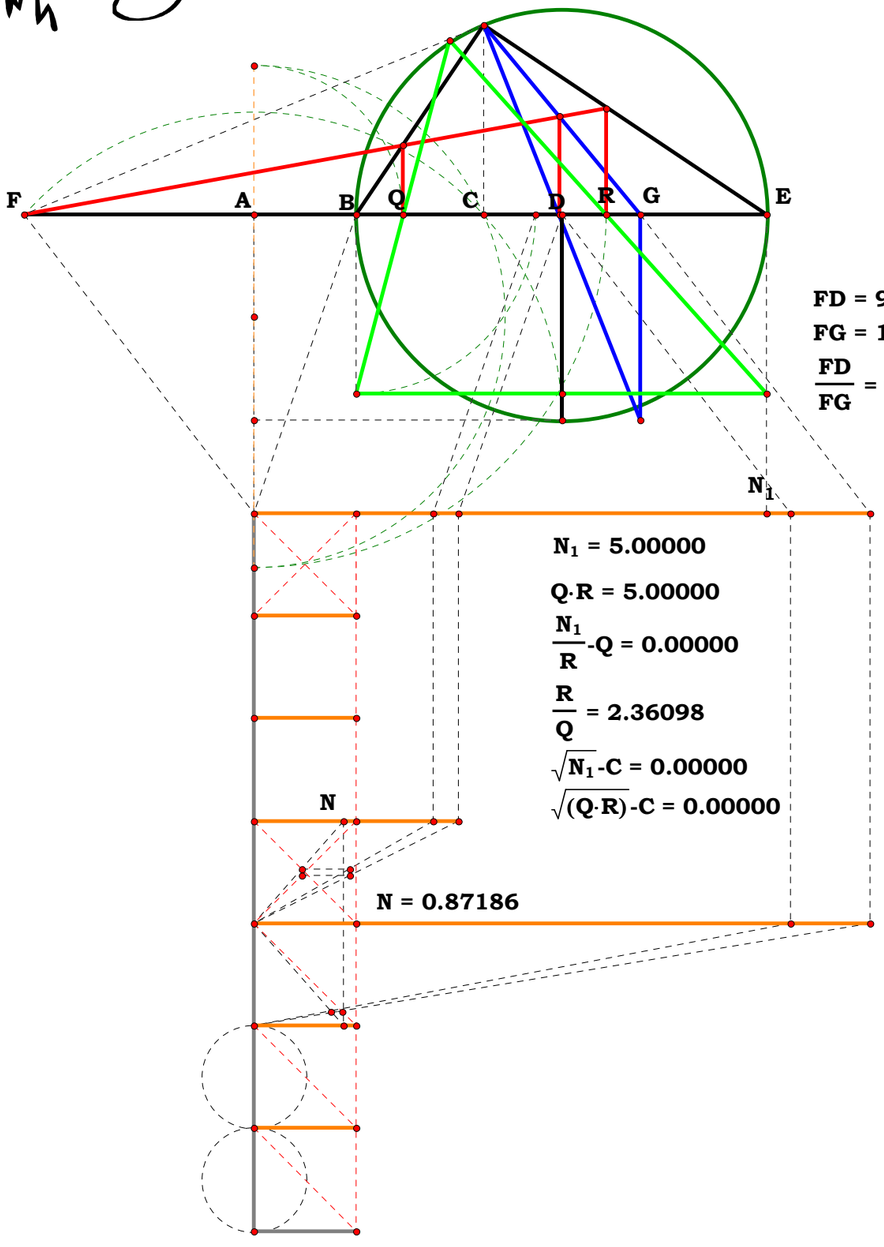
$JO := \frac{IR \cdot CE}{CI + Ia}$ $CJ := \frac{CI \cdot JO}{IR}$ $JI := CI - CJ$

$CY := \frac{IR \cdot CJ}{JI}$ $\frac{CY}{CW} - \frac{CE}{EF} = 0$



EC = 8.06647 cm
EF = 4.03324 cm
$\frac{EC}{EF} = 2.00000$

CY = 6.76532 cm
CW = 3.38266 cm
$\frac{CY}{CW} = 2.00000$



FD = 9.00429 cm

FG = 10.32767 cm

$\frac{FD}{FG}$ = 0.87186

DK = 2.99862 cm

DJ = 3.43933 cm

$\frac{DK}{DJ}$ = 0.87186

$\frac{FD}{FG}-\frac{DK}{DJ}$ = 0.00000

N_1 = 5.00000

Q·R = 5.00000

$\frac{N_1}{R}-Q$ = 0.00000

$\frac{R}{Q}$ = 2.36098

$\sqrt{N_1}-C$ = 0.00000

$\sqrt{(Q\cdot R)}-C$ = 0.00000

N = 0.87186

A = 0.00000

B = 1.00000

C = 2.23607

Q = 1.45525

R = 3.43583

E = 5.00000

$\frac{E}{R}$ = 1.45525

$\frac{E}{R}-Q$ = 0.00000

20XX Edit

The figures on this edit will be produced with a template which allows me to explore the advantage of using a BAM figure to place UZ on the figure exactly and these types of templates allow for a more ordered examination of the figure, which I would do if I actually had the time.

As one can see, it also allows for decimal roots as well to be explored.

A Delian Solution

Show Lines
Points
Show Line

$N_1 = 2.00000$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

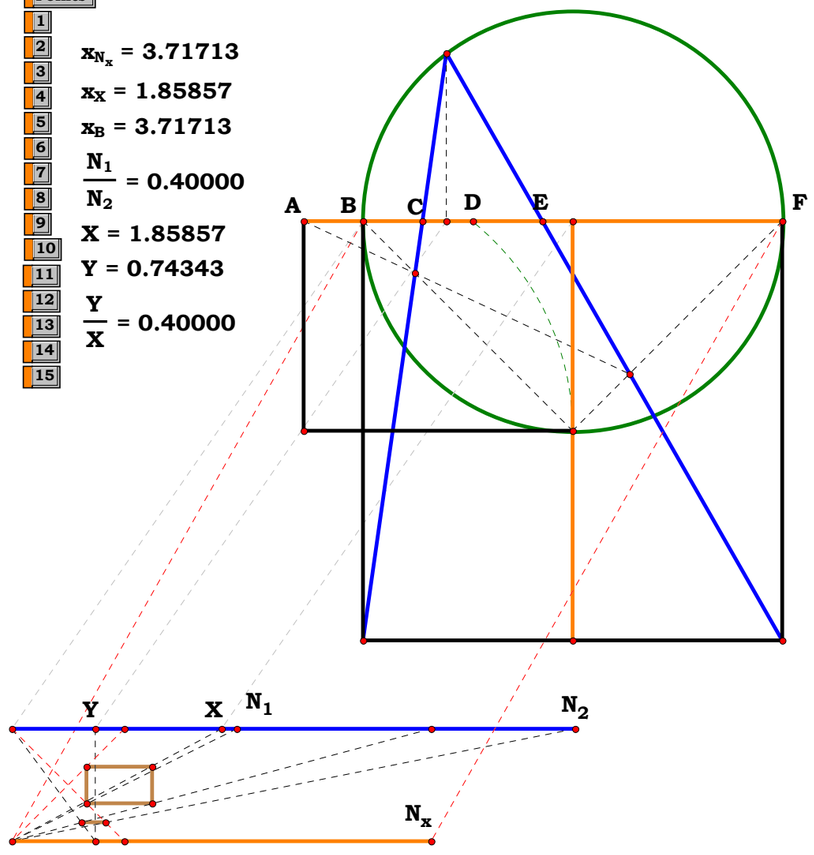
$N_2 = 5.00000$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Circle
Show Lines
Points

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

$x_{N_x} = 3.71713$
$x_X = 1.85857$
$x_B = 3.71713$
$\frac{N_1}{N_2} = 0.40000$
$X = 1.85857$
$Y = 0.74343$
$\frac{Y}{X} = 0.40000$



$AB = 1.00310$ cm
$AC = 2.00619$ cm
$AD = 2.83718$ cm
$AE = 4.01238$ cm
$AF = 8.02476$ cm

$\frac{AB}{AB} = 1.00000$	$AB = 1.00000$	
$\frac{AC}{AB} = 2.00000$	$AC = 2.00000$	$AF^{\frac{1}{3}} - AC = 0.00000$
$\frac{AD}{AB} = 2.82843$	$AD = 2.82843$	$AF^{\frac{1}{2}} - AD = 0.00000$
$\frac{AE}{AB} = 4.00000$	$AE = 4.00000$	$AF^{\frac{2}{3}} - AE = 0.00000$
$\frac{AF}{AB} = 8.00000$	$AF = 8.00000$	
$\frac{AB}{AC} = 0.50000$	$ac = 0.50000$	$af^{\frac{1}{3}} - ac = 0.00000$
$\frac{AB}{AD} = 0.35355$	$ad = 0.35355$	$af^{\frac{1}{2}} - ad = 0.00000$
$\frac{AB}{AE} = 0.25000$	$ae = 0.25000$	$af^{\frac{2}{3}} - ae = 0.00000$
$\frac{AB}{AF} = 0.12500$	$af = 0.12500$	

Unit.

AB := 2.59047

Given.

AG := 11.81347

081199

A Delian Solution

Definitions.

What are the minor and major axis for the ellipse that will give point Z for the cube root?

Descriptions.

$BG := AG - AB$ $BF := \frac{BG}{2}$ $FG := BF$ $AF := AB + BF$

$FX := BF$ $Mf := \frac{\sqrt{AF^2 + FX^2}}{2}$ $Lf := \frac{FX}{2}$ $ML := Mf - Lf$

$FL := \frac{AF}{2}$ $Xd := FL$ $df := Lf$ $IX := FX$ $Md := Mf + df$

Definitions.

BG = 9.22300 BF = 4.61150 FG = 4.61150 AF = 7.20197

FX = 4.61150 Mf = 4.27593 Lf = 2.30575 ML = 1.97018

FL = 3.60099 Xd = 3.60099 df = 2.30575 IX = 4.61150

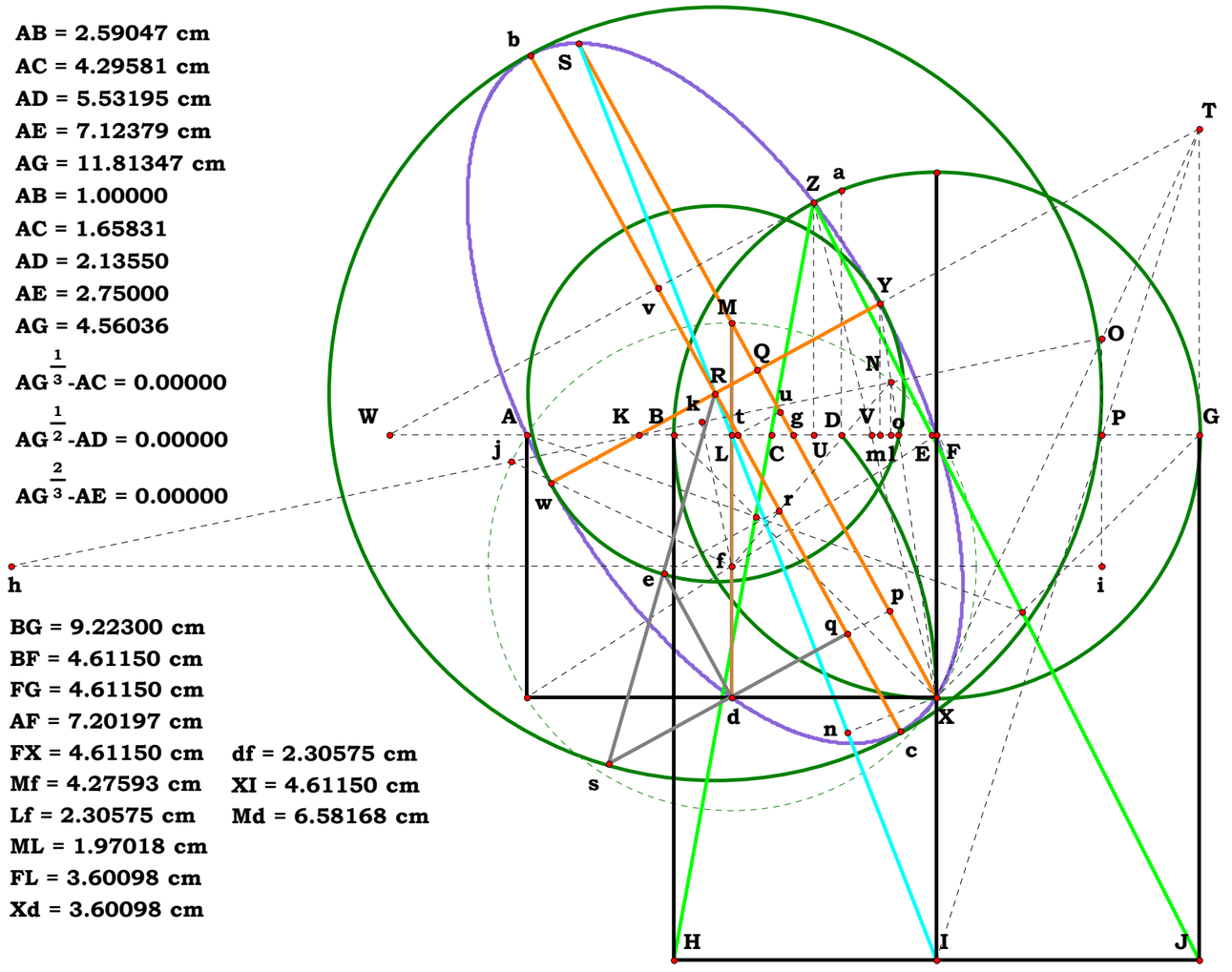
Md = 6.58168

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036

$AG^{\frac{1}{3}}$-AC = 0.00000

$AG^{\frac{1}{2}}$-AD = 0.00000

$AG^{\frac{2}{3}}$-AE = 0.00000



BG = 9.22300 cm
BF = 4.61150 cm
FG = 4.61150 cm
AF = 7.20197 cm
FX = 4.61150 cm
Mf = 4.27593 cm
Lf = 2.30575 cm
ML = 1.97018 cm
FL = 3.60098 cm
Xd = 3.60098 cm

df = 2.30575 cm
XI = 4.61150 cm
Md = 6.58168 cm

Descriptions.

$MX := \sqrt{Xd^2 + Md^2}$ $SX := \frac{MX \cdot IX}{IX - ML}$ $Lg := \frac{FL \cdot ML}{ML + FX}$

$QX := \frac{SX}{2}$ $Fg := FL - Lg$ $Xg := \frac{MX \cdot Fg}{Xd}$ $Qg := QX - Xg$

$Kg := \frac{Xg \cdot Qg}{Fg}$ $GK := FG + Fg + Kg$ $GJ := BG$ $GT := \frac{Fg \cdot GK}{FX}$

$JT := GJ + GT$ $IJ := BF$ $FP := \frac{IJ \cdot GJ}{JT}$ $OP := \frac{IX \cdot GT}{JT}$

Definitions.

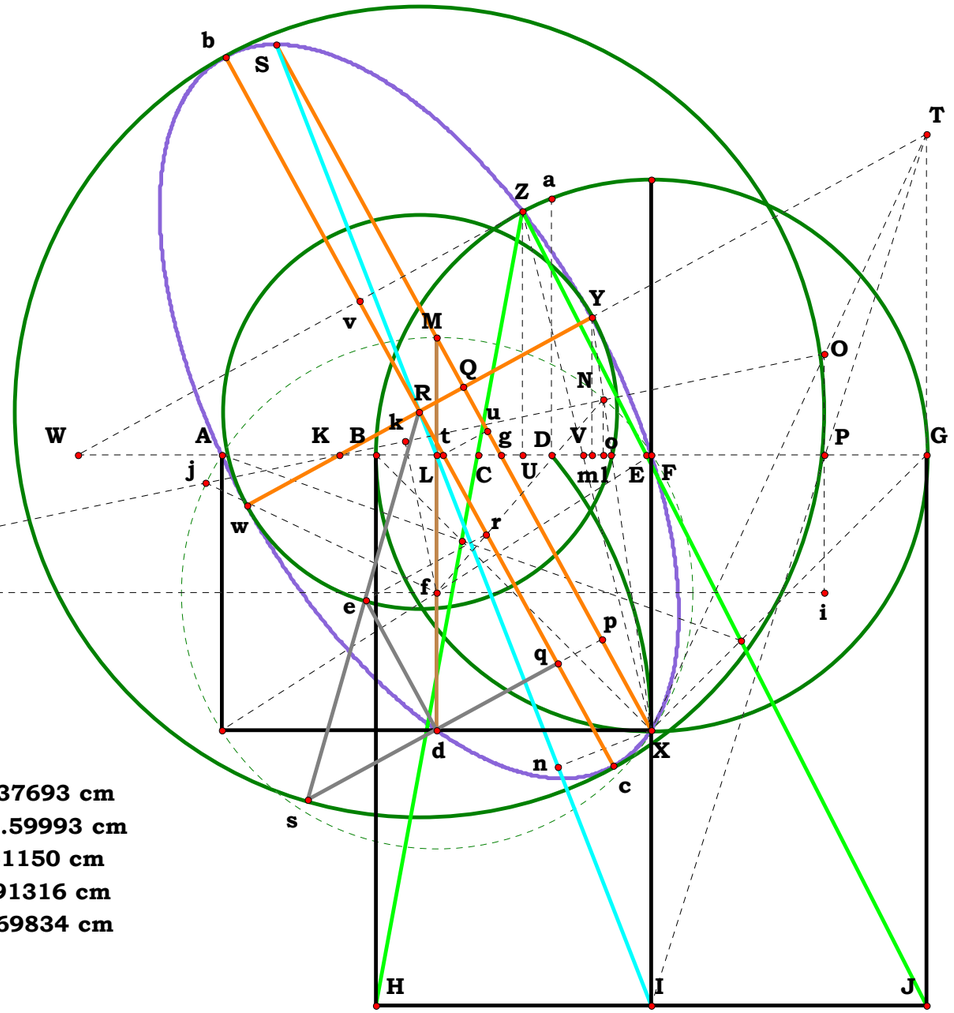
MX = 7.50237 SX = 13.09845 Lg = 1.07793 QX = 6.54922

Fg = 2.52306 Xg = 5.25659 Qg = 1.29263

Kg = 2.69310 GK = 9.82766 GJ = 9.22300 GT = 5.37693

JT = 14.59993 IJ = 4.61150 FP = 2.91315 OP = 1.69835

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036
$AG^{\frac{1}{3}}$-AC = 0.00000
$AG^{\frac{1}{2}}$-AD = 0.00000
$AG^{\frac{2}{3}}$-AE = 0.00000



MX = 7.50237 cm
SX = 13.09844 cm
Lg = 1.07793 cm
QX = 6.54922 cm
Fg = 2.52306 cm
Xg = 5.25659 cm
Qg = 1.29263 cm
Kg = 2.69310 cm
GK = 9.82765 cm
GJ = 9.22300 cm
GT = 5.37693 cm
JT = 14.59993 cm
IJ = 4.61150 cm
FP = 2.91316 cm
OP = 1.69834 cm

Descriptions.

$KP := Fg + Kg + FP$ $\quad Pi := Lf$ $\quad Oi := OP + Pi$ $\quad hi := \frac{KP \cdot Oi}{OP}$

$fi := FP + FL$ $\quad fh := hi - fi$ $\quad KO := \sqrt{KP^2 + OP^2}$ $\quad hk := \frac{KP \cdot fh}{KO}$

$Nf := Mf$ $\quad fk := \frac{OP \cdot fh}{KO}$ $\quad Nk := \sqrt{Nf^2 - fk^2}$ $\quad Nh := hk + Nk$

$Oh := \frac{KO \cdot Oi}{OP}$ $\quad NO := Oh - Nh$ $\quad KN := KO - NO$

Definitions.

$KP = 8.12931$ $\quad Pi = 2.30575$ $\quad Oi = 4.00410$ $\quad hi = 19.16603$

$fi = 6.51414$ $\quad fh = 12.65189$ $\quad KO = 8.30482$ $\quad hk = 12.38451$

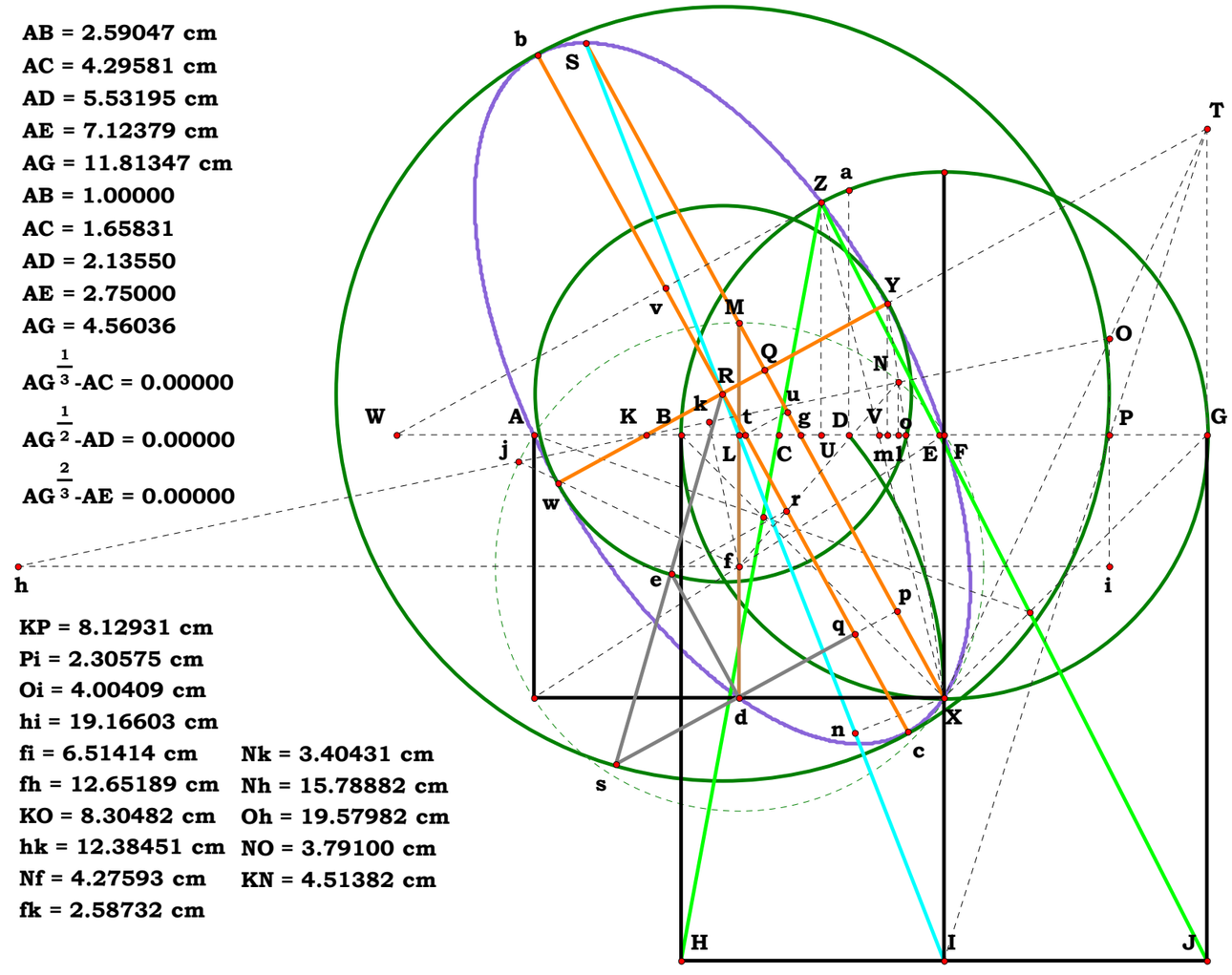
$Nf = 4.27593$ $\quad fk = 2.58733$ $\quad Nk = 3.40431$ $\quad Nh = 15.78882$

$Oh = 19.57983$ $\quad NO = 3.79100$ $\quad KN = 4.51382$

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036
$AG^{\frac{1}{3}}$-AC = 0.00000
$AG^{\frac{1}{2}}$-AD = 0.00000
$AG^{\frac{2}{3}}$-AE = 0.00000

KP = 8.12931 cm
Pi = 2.30575 cm
Oi = 4.00409 cm
hi = 19.16603 cm
fi = 6.51414 cm
fh = 12.65189 cm
KO = 8.30482 cm
hk = 12.38451 cm
Nf = 4.27593 cm
fk = 2.58732 cm

Nk = 3.40431 cm
Nh = 15.78882 cm
Oh = 19.57982 cm
NO = 3.79100 cm
KN = 4.51382 cm



Descriptions.

$$Nl := \frac{OP \cdot KN}{KO} \qquad Kl := \frac{KP \cdot KN}{KO} \qquad FK := KP - FP \qquad Fl := FK - Kl$$

$$NX := \sqrt{(FX + Nl)^2 + Fl^2} \qquad XY := \frac{NX \cdot IX}{IX - Nl} \qquad Fm := \frac{Fl \cdot XY}{NX} \qquad Km := FK - Fm$$

$$Fo := \frac{Fl \cdot FX}{FX + Nl} \qquad Xo := \frac{NX \cdot Fo}{Fl} \qquad mo := Fm - Fo \qquad Ym := \frac{FX \cdot mo}{Fo}$$

$$FI := 2 \cdot BF \qquad IL := \sqrt{FL^2 + FI^2} \qquad IS := \frac{IL \cdot IX}{IX - ML} \qquad In := \frac{IS^2 - SX^2 + IX^2}{2 \cdot IS}$$

Definitions.

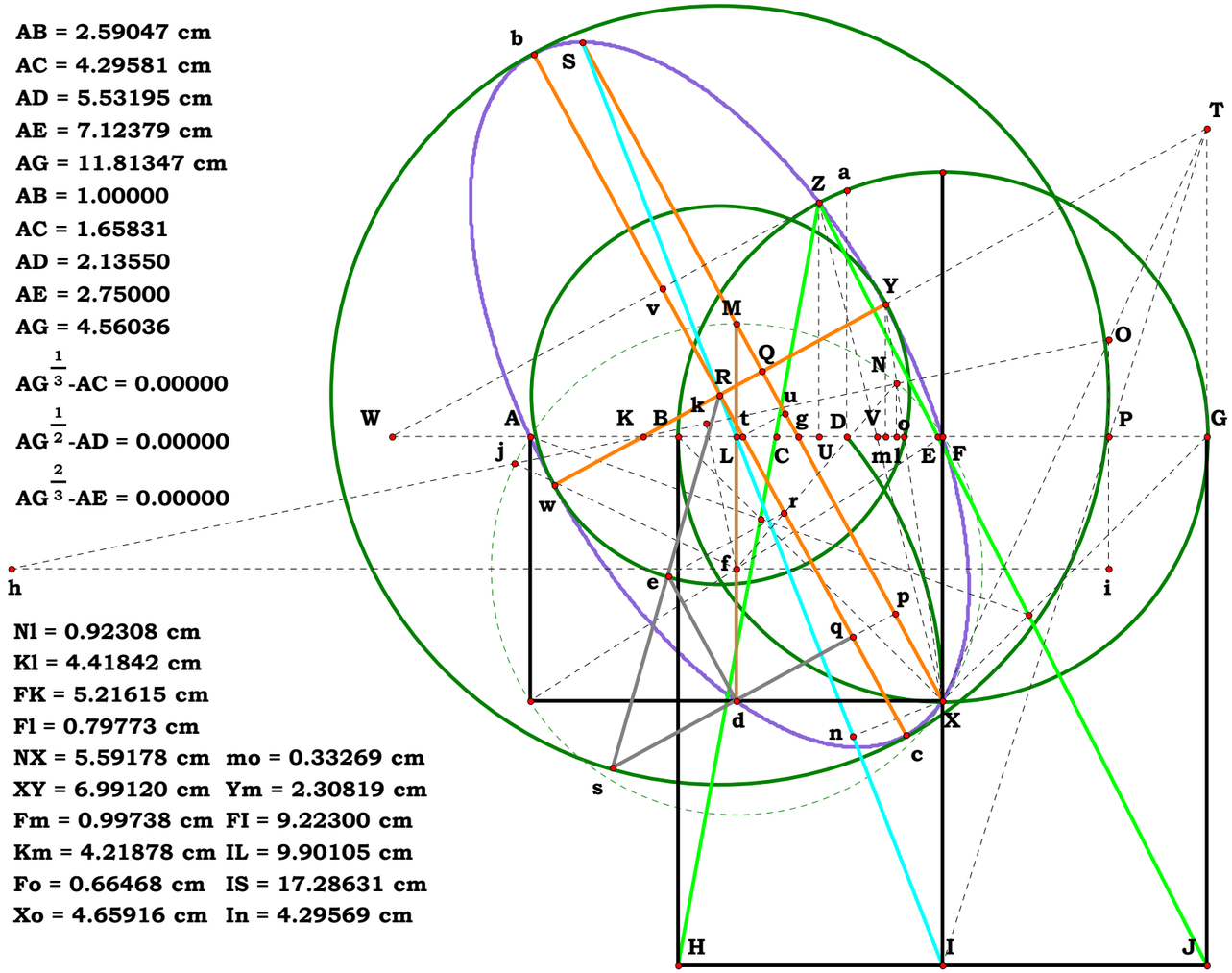
Nl = 0.92308 Kl = 4.41843 FK = 5.21616 Fl = 0.79773

NX = 5.59178 XY = 6.99120 Fm = 0.99738 Km = 4.21878

Fo = 0.66468 Xo = 4.65916 mo = 0.33269 Ym = 2.30819

FI = 9.22300 IL = 9.90105 IS = 17.28632 In = 4.29569

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036
$AG^{\frac{1}{3}}$-AC = 0.00000
$AG^{\frac{1}{2}}$-AD = 0.00000
$AG^{\frac{2}{3}}$-AE = 0.00000



Nl = 0.92308 cm
Kl = 4.41842 cm
FK = 5.21615 cm
Fl = 0.79773 cm
NX = 5.59178 cm mo = 0.33269 cm
XY = 6.99120 cm Ym = 2.30819 cm
Fm = 0.99738 cm FI = 9.22300 cm
Km = 4.21878 cm IL = 9.90105 cm
Fo = 0.66468 cm IS = 17.28631 cm
Xo = 4.65916 cm In = 4.29569 cm

Descriptions.

$Xn := \sqrt{IX^2 - In^2}$ $QR := \frac{Xn \cdot QX}{IS - In}$ $KT := \sqrt{GK^2 + GT^2}$ $KY := \frac{KT \cdot Ym}{GT}$

$KQ := \frac{GK \cdot Qg}{GT}$ $RY := KY - KQ + QR$ $Xd := FL$ $dp := \frac{GK \cdot Xd}{KT}$

$pq := QR$ $dq := dp - pq$ $Xp := \frac{GT \cdot Xd}{KT}$ $Qp := QX - Xp$ $Rq := Qp$

$er := dq$ $Re := RY$ $Rr := \sqrt{Re^2 - er^2}$ $Rs := \frac{Re \cdot Rq}{Rr}$

Definitions.

Xn = 1.67719 QR = 0.84556 KT = 11.20242 KY = 4.80894

KQ = 2.36260 RY = 3.29189 Xd = 3.60099 dp = 3.15907

pq = 0.84556 dq = 2.31352 Xp = 1.72840 Qp = 4.82082
Rq = 4.82082 er = 2.31352 Re = 3.29189 Rr = 2.34183

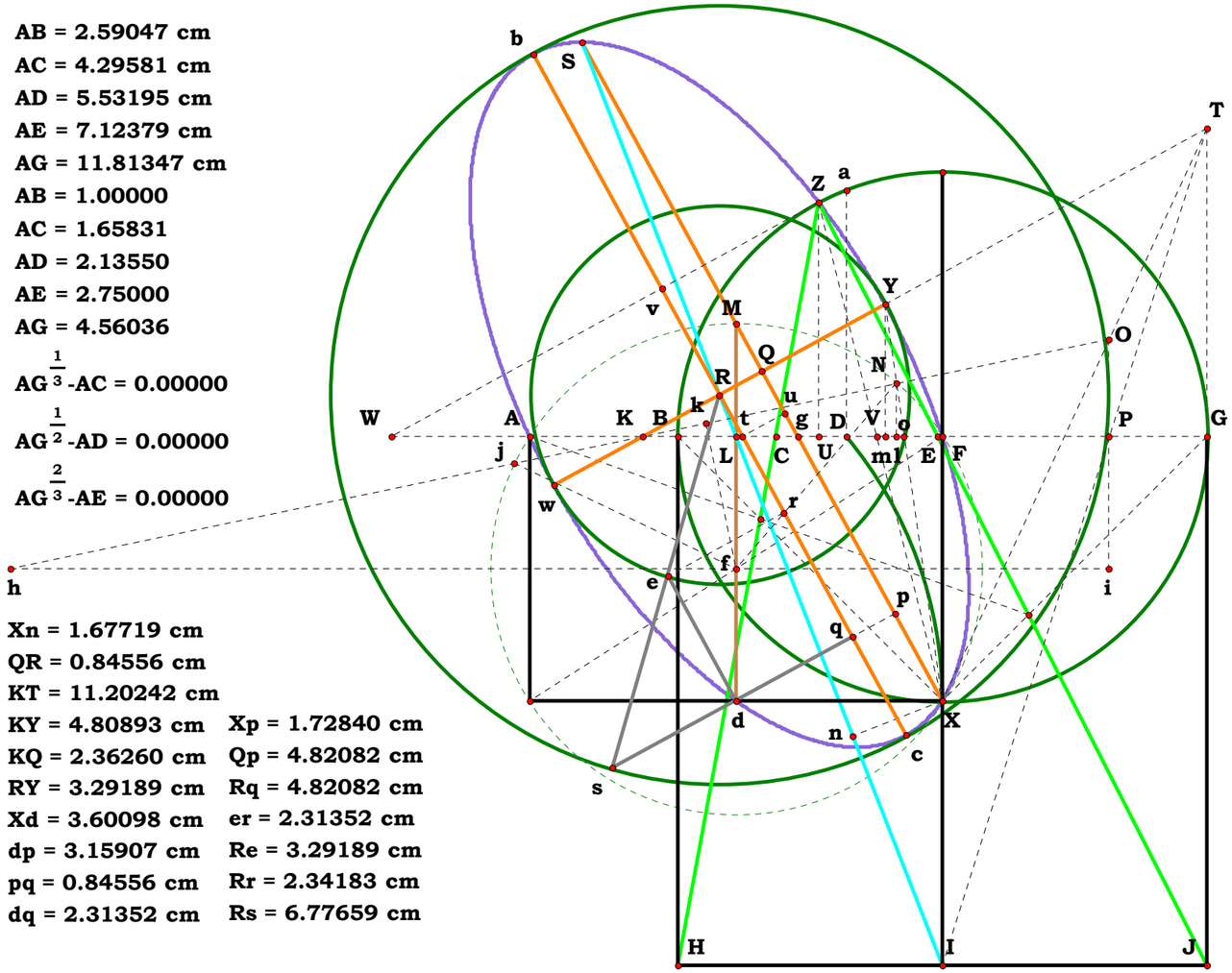
Rs = 6.77659

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036

$AG^{\frac{1}{3}}$-AC = 0.00000

$AG^{\frac{1}{2}}$-AD = 0.00000

$AG^{\frac{2}{3}}$-AE = 0.00000



Xn = 1.67719 cm
QR = 0.84556 cm
KT = 11.20242 cm
KY = 4.80893 cm Xp = 1.72840 cm
KQ = 2.36260 cm Qp = 4.82082 cm
RY = 3.29189 cm Rq = 4.82082 cm
Xd = 3.60098 cm er = 2.31352 cm
dp = 3.15907 cm Re = 3.29189 cm
pq = 0.84556 cm Rr = 2.34183 cm
dq = 2.31352 cm Rs = 6.77659 cm

Descriptions.

$AC := \left(AB^2 \cdot AG\right)^{\frac{1}{3}}$ $AE := \left(AB \cdot AG^2\right)^{\frac{1}{3}}$ $BC := AC - AB$ $BE := AE - AB$

$CE := BE - BC$ $EG := BG - BE$ $CU := \frac{BC \cdot CE}{BC + EG}$ $BU := BC + CU$

$GU := BG - BU$ $UZ := \sqrt{BU \cdot GU}$ $UW := \frac{GK \cdot UZ}{GT}$ $Gg := GK - Kg$

$tu := QR$ $gt := \frac{KT \cdot tu}{GK}$ $Gt := Gg + gt$ $GW := GU + UW$

Is the segment Zv equal to the perpendicular for the ellipse?

Definitions.

AC = 4.29581 AE = 7.12379 BC = 1.70534 BE = 4.53332

CE = 2.82798 EG = 4.68968 CU = 0.75413 BU = 2.45947

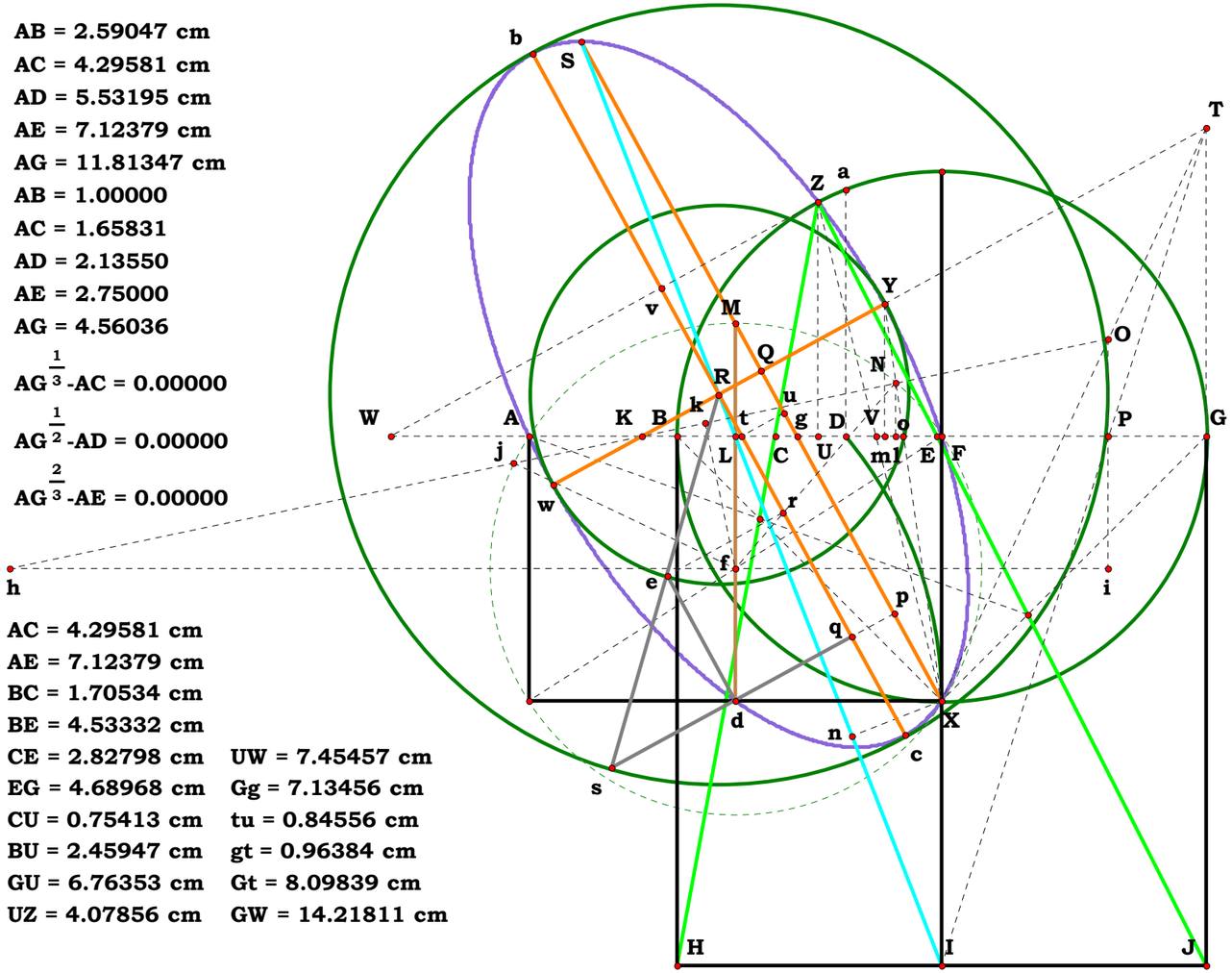
GU = 6.76353 UZ = 4.07856 UW = 7.45457 Gg = 7.13456

tu = 0.84556 gt = 0.96384 Gt = 8.09839 GW = 14.21811

AB = 2.59047 cm
AC = 4.29581 cm
AD = 5.53195 cm
AE = 7.12379 cm
AG = 11.81347 cm
AB = 1.00000
AC = 1.65831
AD = 2.13550
AE = 2.75000
AG = 4.56036
$AG^{\frac{1}{3}}$-AC = 0.00000
$AG^{\frac{1}{2}}$-AD = 0.00000
$AG^{\frac{2}{3}}$-AE = 0.00000

AC = 4.29581 cm
AE = 7.12379 cm
BC = 1.70534 cm
BE = 4.53332 cm
CE = 2.82798 cm
EG = 4.68968 cm
CU = 0.75413 cm
BU = 2.45947 cm
GU = 6.76353 cm
UZ = 4.07856 cm

UW = 7.45457 cm
Gg = 7.13456 cm
tu = 0.84556 cm
gt = 0.96384 cm
Gt = 8.09839 cm
GW = 14.21811 cm



Descriptions.

$Wt := GW - Gt$ $\quad tv := \frac{GT \cdot Wt}{KT}$ $\quad Kt := GK - Gt$ $\quad Rt := \frac{GT \cdot Kt}{KT}$

$Rv := tv - Rt$ $\quad bc := 2 \cdot Rs$ $\quad Rc := Rs$ $\quad cv := Rc + Rv$

$Yw := 2 \cdot RY$ $\quad WZ := \frac{KT \cdot UZ}{GT}$ $\quad Wv := \frac{GK \cdot tv}{GT}$ $\quad Zv := |WZ - Wv|$

Definitions.

$Wt = 6.11971$ $\quad tv = 2.93734$ $\quad Kt = 1.72926$ $\quad Rt = 0.83001$

$Rv = 2.10733$ $\quad bc = 13.55318$ $\quad Rc = 6.77659$ $\quad cv = 8.88391$

$Yw = 6.58377$ $\quad WZ = 8.49737$ $\quad Wv = 5.36870$ $\quad Zv = 3.12867$

Given.

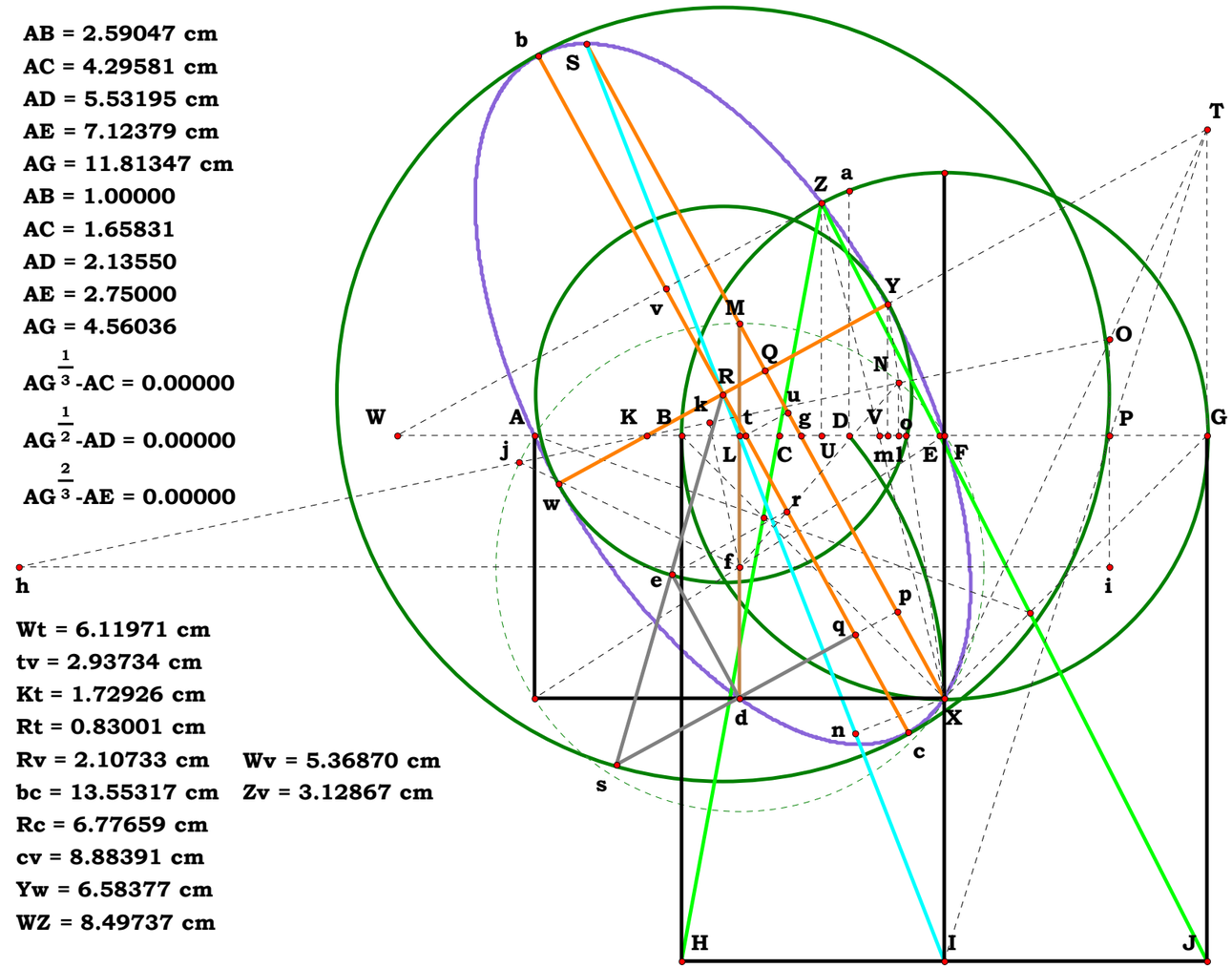
$N_1 := Yw$ $\quad N_2 := bc$ $\quad N_3 := cv$ $\quad N_4 := bc$

Definitions.

$\sqrt{N_3 \cdot (N_4 - N_3)} \cdot \frac{N_1}{N_2} - Zv = 0.00000$

$\sqrt{N_3 \cdot (N_4 - N_3)} \cdot \frac{N_1}{N_2}$ **is from 09/11/97 The Ellipse for the segment Zv (BG), units divided out.**

$AB = 2.59047$ cm
$AC = 4.29581$ cm
$AD = 5.53195$ cm
$AE = 7.12379$ cm
$AG = 11.81347$ cm
$AB = 1.00000$
$AC = 1.65831$
$AD = 2.13550$
$AE = 2.75000$
$AG = 4.56036$
$AG^{\frac{1}{3}}\text{-}AC = 0.00000$
$AG^{\frac{1}{2}}\text{-}AD = 0.00000$
$AG^{\frac{2}{3}}\text{-}AE = 0.00000$



$Wt = 6.11971$ cm
$tv = 2.93734$ cm
$Kt = 1.72926$ cm
$Rt = 0.83001$ cm
$Rv = 2.10733$ cm $\quad Wv = 5.36870$ cm
$bc = 13.55317$ cm $\quad Zv = 3.12867$ cm
$Rc = 6.77659$ cm
$cv = 8.88391$ cm
$Yw = 6.58377$ cm
$WZ = 8.49737$ cm

081899

Unit.

Given.

$N_1 := 5.05354$ $AC := N_1$

$N_2 := 14.49917$ $AF := N_2$

$N_3 := 5.71500$ $FN := N_3$

Descriptions.

$CF := AF - AC$ $CG := \frac{FN \cdot AC}{AF}$ $EF := \frac{CF}{2}$ $EH := \frac{FN \cdot (AC + EF)}{AF}$

$KN := \frac{EF \cdot FN}{EH}$ $JK := CF - KN$ $BC := \frac{JK \cdot CG}{FN - CG}$ $BF := BC + CF$

$AD := AC + JK$ $BD := BC + JK$

Definitions.

$CF - (N_2 - N_1) = 0$ $\left(\frac{AF}{AC}\right)^2 - \frac{BF}{BC} = 0$ $\sqrt{\frac{BF}{BC}} - \frac{AF}{AC} = 0$

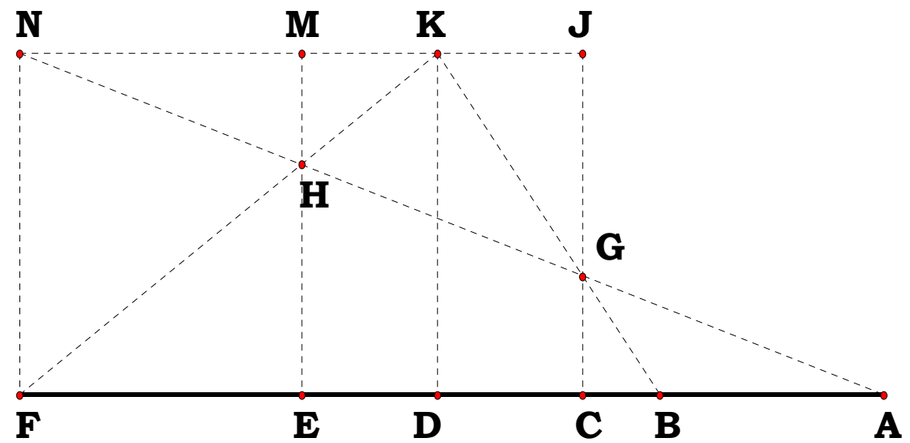
$CG - \frac{N_3 \cdot N_1}{N_2} = 0$ $EF - \frac{N_2 - N_1}{2} = 0$ $EH - \frac{N_3 \cdot (N_1 + N_2)}{2 \cdot N_2} = 0$

$KN - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$ $JK - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$ $BC - \frac{N_1^2}{N_1 + N_2} = 0$

$BF := \frac{N_2^2}{N_1 + N_2}$ $AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0$ $BD - \frac{N_1 \cdot N_2}{N_1 + N_2} = 0$

Promptly writing this up in 0816 2015

Exponential series by changing the unit, in other words, the same way as done inside a circle only this circle is getting smaller.



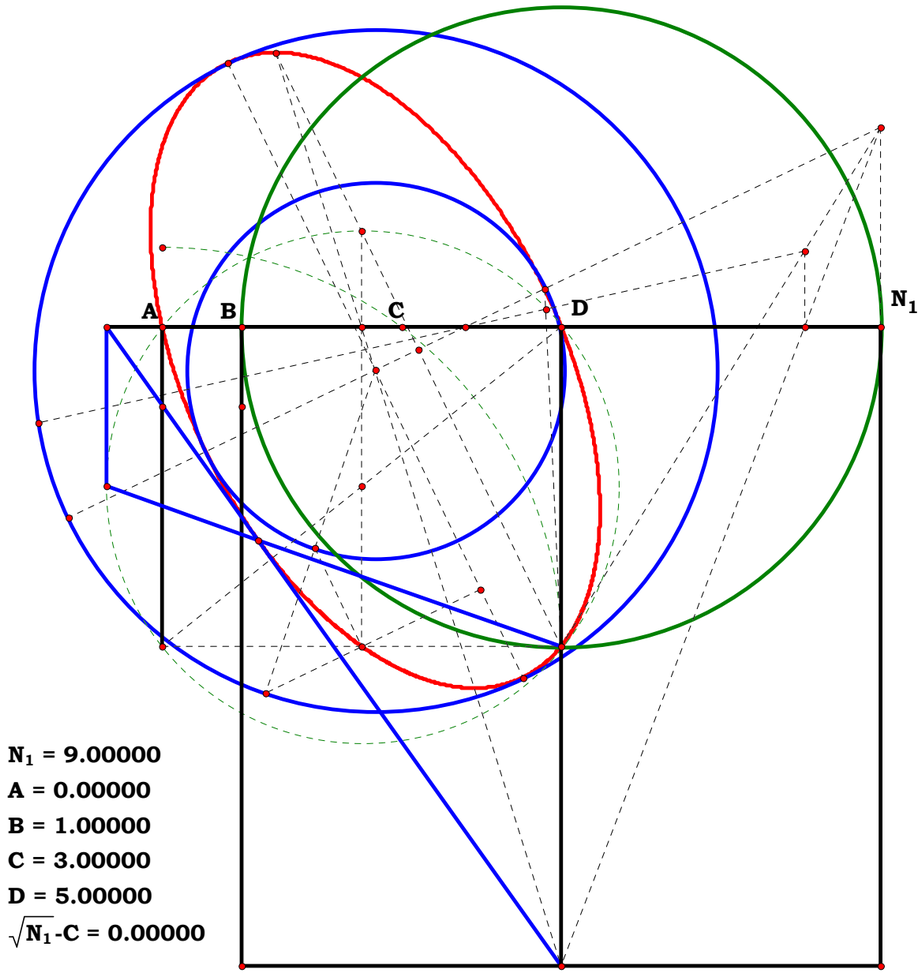
081999

What about the Names?

Geometry, Algebra and Arithmetic are each and all binary grammars. All we do when writing up a figure is pair the name for any particular thing or the parts of a thing in terms of each of the convention of names provided by these three systems of grammar to each other. Each of these names are a binary expression, but they each use the convention of names given by a particular grammar. Traditionally, people have approached the topic in terms of precision, which is not right. The distinction is between the perceptible and the intelligible provided by each system of grammar which is expressible by our ability in that grammar system. Grammar cannot, in any wise, change the facts, nor can our ability with a grammar. Stupid people speak of proof as if proof determines the reality, the facts, which is wholly bizzar. All we are doing when pairing names is exercising our ability to do so. Proofing is only our ability to follow the intelligible using perceptible systems of grammar. So, by writing up complex figures one not only pushes their limits, but also learn how to fall back and go on naming in all three conventions. We go from universal expressions to expressions particular to where we are naming.

It is quite natural to become wholly frustrated when on reaches their own particular limits and the limits of even the computer that they may be using.

What is the name of the Ellipse which gives us the cube roots of any number?



N_1 = 9.00000
A = 0.00000
B = 1.00000
C = 3.00000
D = 5.00000
$\sqrt{N_1}$-C = 0.00000

Unit.

$AB := 1$

Given.

$N_1 := 9$

Descriptions.

$BN_1 := N_1 - AB \qquad BD := \frac{BN_1}{2}$

$DO := BD \qquad AD := BD + AB$

$AC := \sqrt{AD^2 - DO^2}$

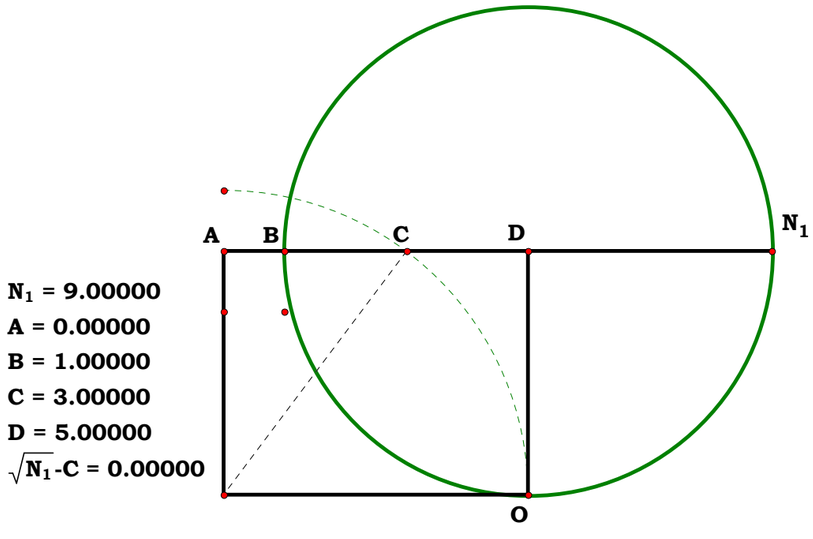
Definitions.

$BN_1 - (N_1 - 1) = 0 \qquad BD - \frac{N_1 - 1}{2} = 0$

$DO - \frac{N_1 - 1}{2} = 0 \qquad AD - \frac{N_1 + 1}{2} = 0$

$AC - \sqrt{\left(\frac{N_1 + 1}{2}\right)^2 - \left(\frac{N_1 - 1}{2}\right)^2} = 0$

$AC - \sqrt{N_1} = 0$



N_1 = 9.00000
A = 0.00000
B = 1.00000
C = 3.00000
D = 5.00000
$\sqrt{N_1}$-C = 0.00000

AC is the square root of N_1

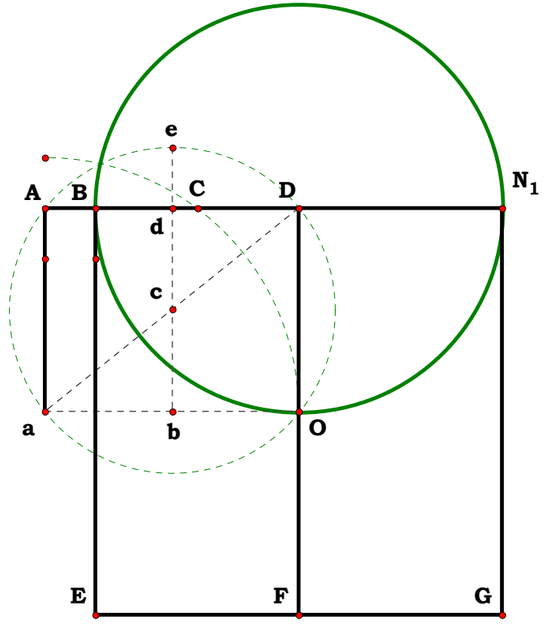
Descriptions.

$Dd := \frac{AD}{2} \qquad Da := \sqrt{AD^2 + DO^2} \qquad ce := \frac{Da}{2} \qquad de := ce - \frac{BD}{2}$

Definitions.

$Dd - \frac{N_1 + 1}{4} = 0 \qquad Da - \frac{\sqrt{N_1^2 + 1}}{\sqrt{2}} = 0 \qquad ce - \frac{\sqrt{2 \cdot (N_1^2 + 1)}}{4} = 0$

$de - \frac{\sqrt{2 \cdot (N_1^2 + 1)} - N_1 + 1}{4} = 0$



N_1 = 9.00000

Dd = 2.50000 $\frac{N_1+1}{4}$-Dd = 0.00000

Da = 6.40312 $\frac{\sqrt{N_1^2+1}}{\sqrt{2}}$-Da = 0.00000

ce = 3.20156 $\frac{\sqrt{2\cdot(N_1^2+1)}}{4}$-ce = 0.00000

de = 1.20156 $\frac{(\sqrt{2\cdot(N_1^2+1)}-N_1)+1}{4}$-de = 0.00000

Descriptions.

$DF := BN_1$ $\quad Fd := \sqrt{Dd^2 + DF^2}$

$Df := DO + de$ $\quad df := \sqrt{Dd^2 + Df^2}$ $\quad Ff := DF - Df$

$Og := \dfrac{df \cdot DO}{Ff}$

Definitions.

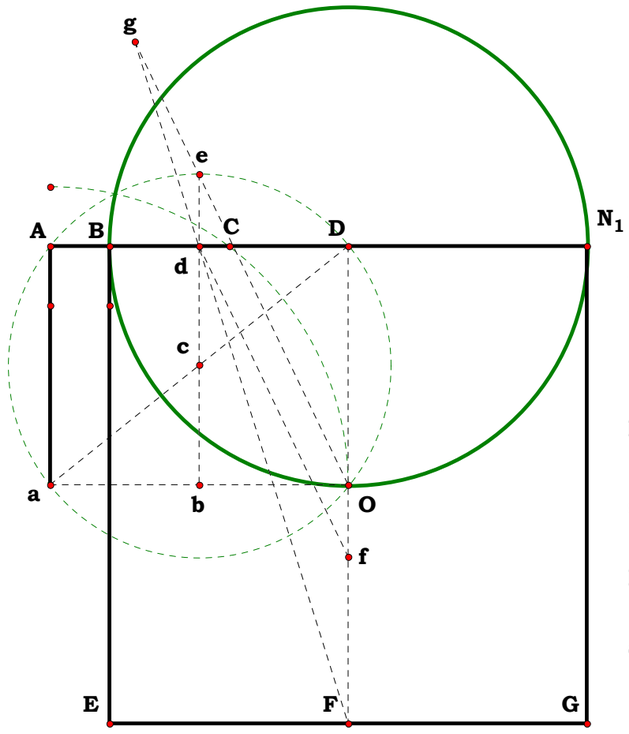
$$DF - (N_1 - 1) = 0 \qquad Fd - \frac{\sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}}{4} = 0$$

$$Df - \frac{N_1 + \sqrt{2 \cdot (N_1^2 + 1)} - 1}{4} = 0$$

$$df - \frac{\sqrt{2 \cdot N_1^2 + \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1) + 2}}{\sqrt{8}} = 0$$

$$Ff - \frac{3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3}{4} = 0$$

$$Og - \frac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)} + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{2 \cdot \left[3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3\right]} = 0$$



$N_1 = 9.00000$

$DF = 8.00000$ $\quad N_1 - 1 = 8.00000$

$Fd = 8.38153$ $\quad \dfrac{\sqrt{(17 \cdot N_1^2 - 30 \cdot N_1) + 17}}{4} - Fd = 0.00000$

$Df = 5.20156$ $\quad \dfrac{\sqrt{2 \cdot N_1^2 + \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1) + 2}}{\sqrt{8}} - df = 0.00000$

$df = 5.77116$ $\quad \dfrac{(N_1 + \sqrt{2 \cdot (N_1^2 + 1)}) - 1}{4} - Df = 0.00000$

$Ff = 2.79844$ $\quad \dfrac{3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3}{4} - Ff = 0.00000$

$Og = 8.24911$ $\quad \dfrac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{(2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)}) + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{2 \cdot (3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3)} - Og = 0.00000$

Descriptions.

$$OI := \frac{Og}{2} \qquad FO := DO \qquad Fh := \frac{DF \cdot FO}{Fd}$$

$$Oh := \sqrt{FO^2 - Fh^2} \qquad gh := \sqrt{Og^2 - Oh^2}$$

$$IJ := \frac{Oh \cdot OI}{gh}$$

Definitions.

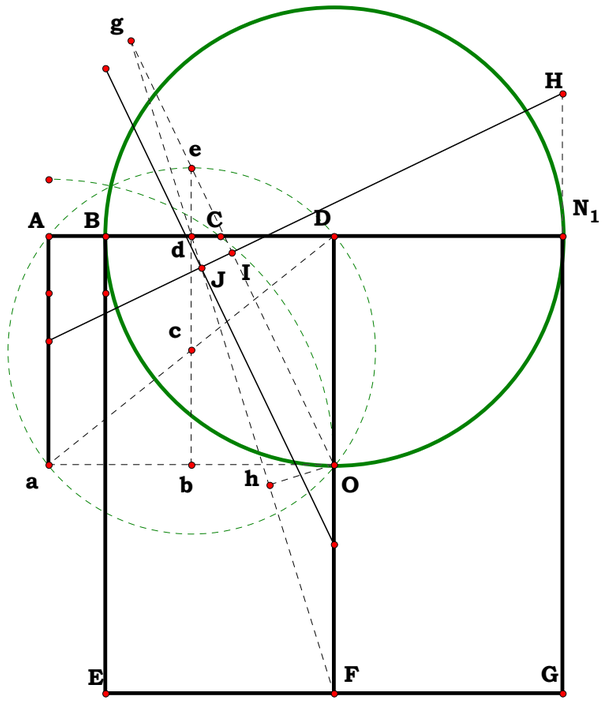
$$FO - \frac{N_1 - 1}{2} = 0 \qquad Fh - \frac{2 \cdot (N_1 - 1)^2}{\sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$

$$Oh - \frac{\sqrt{(N_1^2 - 1)^2}}{2 \cdot \sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}} = 0$$

$$OI - \frac{\sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2 \cdot (N_1^2 + 1)} + N_1 \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 2}}{4 \cdot \left[3 \cdot N_1 - \sqrt{2 \cdot (N_1^2 + 1)} - 3\right]} = 0$$

$$gh - \frac{\sqrt{(N_1 - 1)^2 \cdot \left[\left[8 \cdot (N_1 - 1) \cdot (5 \cdot N_1^2 - 6 \cdot N_1 + 5)\right] \cdot \sqrt{2 \cdot (N_1^2 + 1)} + 57 \cdot (N_1^4 + 1) + 150 \cdot N_1^2 - 124 \cdot N_1^3 - 124 \cdot N_1\right]}}{\sqrt{\left[4 \cdot (17 \cdot N_1^2 - 30 \cdot N_1 + 17) \cdot \left[\left[11 \cdot (N_1^2 + 1) - 18 \cdot N_1\right] - 6 \cdot \sqrt{2 \cdot (N_1^2 + 1)} \cdot (N_1 - 1)\right]\right]}} = 0$$

$$IJ - \frac{\sqrt{2} \cdot \sqrt{-(68 \cdot N_1^2 - 120 \cdot N_1 + 68)} \cdot \left[18 \cdot N_1 - 11 \cdot N_1^2 + 6 \cdot \sqrt{2} \cdot (N_1 - 1) \cdot \sqrt{N_1^2 + 1} - 11\right] \cdot \sqrt{(N_1^2 - 1)^2} \cdot (N_1 - 1) \cdot \sqrt{2 \cdot N_1^2 - \sqrt{2} \cdot \sqrt{N_1^2 + 1} + \sqrt{2} \cdot N_1 \cdot \sqrt{N_1^2 + 1} + 2}}{8 \cdot \sqrt{(N_1 - 1)^2 \cdot \left[150 \cdot N_1^2 - 124 \cdot N_1 - 124 \cdot N_1^3 + 57 \cdot N_1^4 + \sqrt{2} \cdot (8 \cdot N_1 - 8) \cdot \sqrt{N_1^2 + 1} \cdot (5 \cdot N_1^2 - 6 \cdot N_1 + 5) + 57\right] \cdot \left(3 \cdot N_1 - \sqrt{2} \cdot \sqrt{N_1^2 + 1} - 3\right) \cdot \sqrt{17 \cdot N_1^2 - 30 \cdot N_1 + 17}}} = 0$$



$N_1 = 9.00000$

$\frac{OI}{AB} = 4.12456$

$\frac{Fh}{AB} = 3.81792$

$\frac{gh}{AB} = 8.16237$

$\frac{FO}{AB} = 4.00000$

$\frac{Oh}{AB} = 1.19310$

$\frac{IJ}{AB} = 0.60289$

Definitions (Arithmetic).

$OI = 4.124556$	$FO = 4$
$Fh = 3.81792$	$Oh = 1.1931$
$gh = 8.162374$	$IJ = 0.602889$

At this point of complexity, I have to set the definitions aside and just compare the arithmetic. I really do not have all the time there is. This is the major reason I set this write-up aside for a later date, one which is after mine. As we find in the name of Shadows (Babylon 5,) a definition could become over 10,000 symbols long.

$Oj := \frac{Df \cdot OI}{df}$ $Ij := \frac{Dd \cdot Oj}{Df}$ $Ik := DO + Ij$

$Hk := \frac{Dd \cdot Ik}{Df}$ $Dj := DO - Oj$ $HN_1 := Hk - Dj$

$HI := \frac{df \cdot Ik}{Df}$ $HJ := HI + IJ$ $lm := \frac{DO \cdot HN_1}{DF + HN_1}$

$nN_1 := \frac{Df \cdot HN_1}{Dd}$

Definitions (Arithmetic).

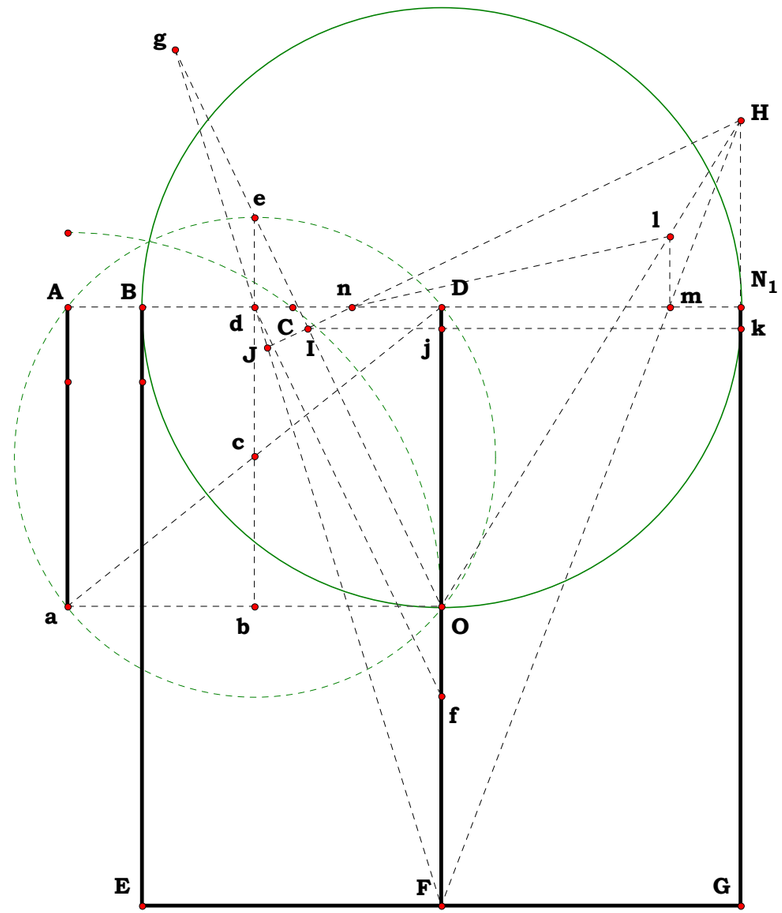
$Oj = 3.717475$ $Ij = 1.786711$

$Ik = 5.786711$ $Hk = 2.781237$

$Dj = 0.282525$ $HN_1 = 2.498713$

$HI = 6.420382$ $HJ = 7.023271$

$lm = 0.952007$ $nN_1 = 5.198884$



$N_1 = 9.00000$

$AB = 1.25400$ cm

$\frac{Oj}{AB} = 3.71748$ $\frac{Ij}{AB} = 1.78671$

$\frac{Ik}{AB} = 5.78671$ $\frac{Hk}{AB} = 2.78124$

$\frac{Dj}{AB} = 0.28252$ $\frac{HN_1}{AB} = 2.49871$

$\frac{HI}{AB} = 6.42038$ $\frac{HJ}{AB} = 7.02327$

$\frac{ml}{AB} = 0.95201$ $\frac{nN_1}{AB} = 5.19888$

082999A

The first section of this is simply demonstrating that cube roots are producable as a ratio between BE and BO. In fact, using whole numbers, and a lot of time, one can actually construct an Arithmetic dictionary of such ratios.

Then sometime I am want to I show that AW passes through R, Y, S, V, S and T.

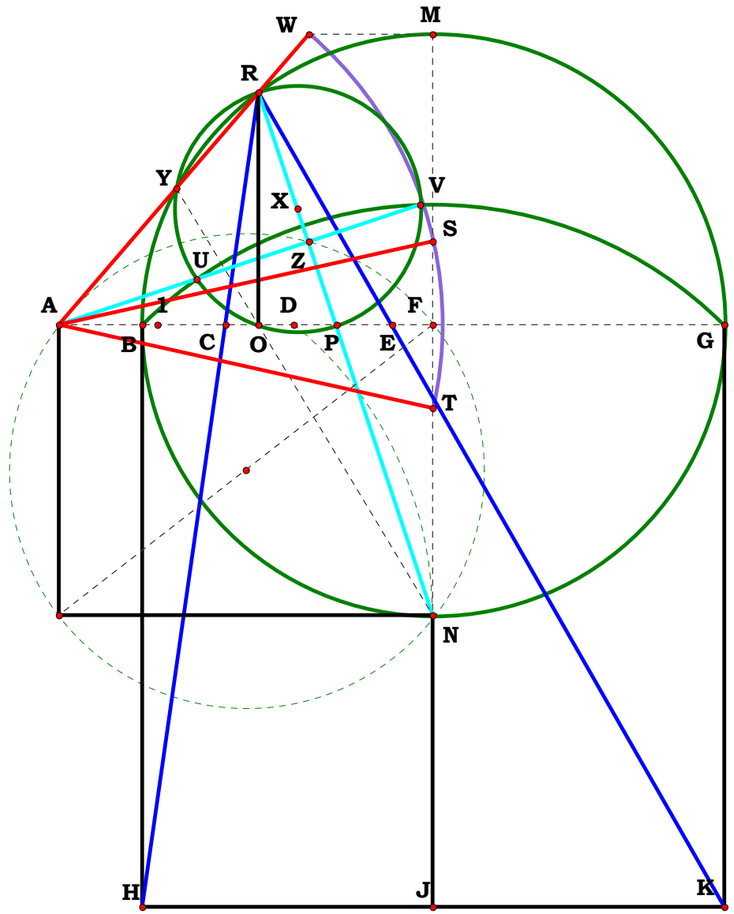
UV originates at A.

and;

The cord ST is equal to CE.

The circle RX pass through R, Y, O, P and V.

See E Go!



Unit.

$BG := 1$

Given.

$N_1 := 2$

$N_2 := 5$

Descriptions.

$BF := \frac{BG}{2}$ $BO := \frac{N_1 \cdot BF}{N_2}$ $GO := BG - BO$ $OR := \sqrt{BO \cdot GO}$

$FN := BF$ $FO := BF - BO$ $FP := \frac{FO \cdot FN}{FN + OR}$ $GK := BG$ $BH := BG$

$BC := \frac{BO \cdot BH}{BH + OR}$ $GE := \frac{GO \cdot GK}{GK + OR}$ $CE := BG - (BC + GE)$

$BE := BC + CE$ $AB := \frac{BC^2}{CE - BC}$ $AG := AB + BG$ $\frac{AG}{AB} = 8$

Definitions.

$BF - \frac{1}{2} = 0$ $BO - \frac{N_1}{2 \cdot N_2} = 0$ $GO - \frac{2 \cdot N_2 - N_1}{2 \cdot N_2} = 0$

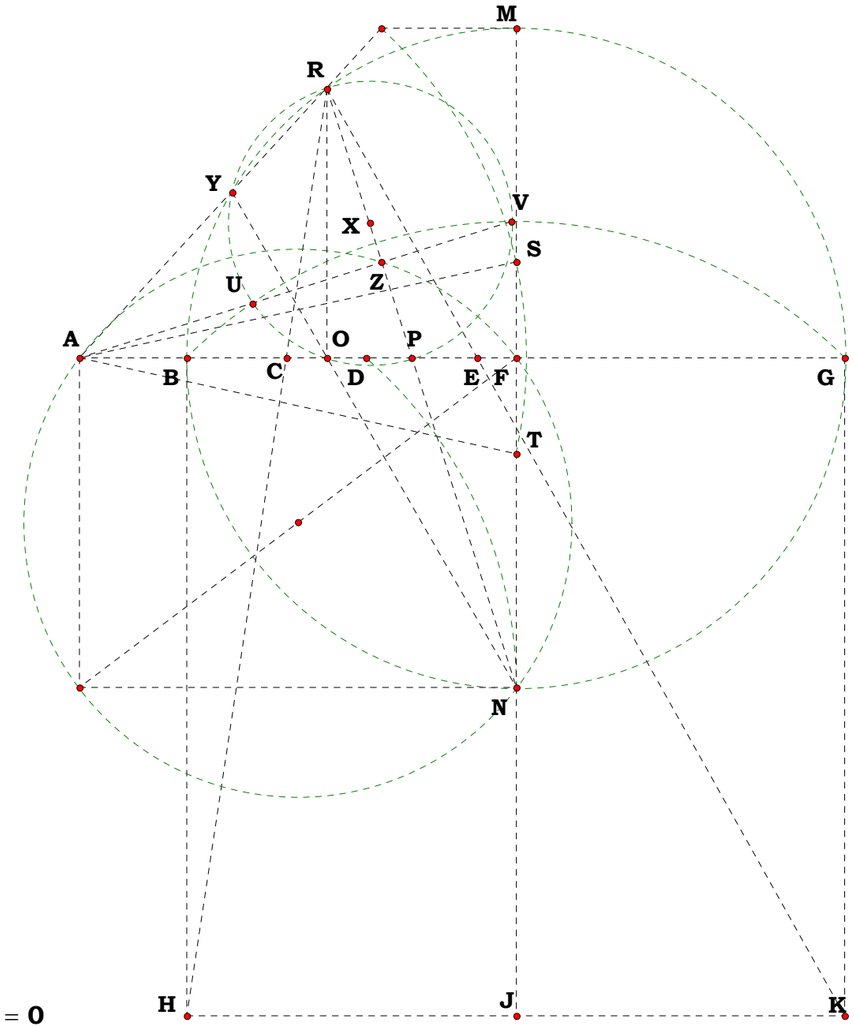
$OR - \frac{\sqrt{N_1 \cdot (2 \cdot N_2 - N_1)}}{2 \cdot N_2} = 0$ $FN - \frac{1}{2} = 0$ $FO - \frac{N_2 - N_1}{2 \cdot N_2} = 0$

$FP - \frac{N_2 - N_1}{2 \cdot \left(N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}\right)} = 0$ $GK - 1 = 0$ $BH - 1 = 0$

$BC - \frac{N_1}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $GE - \frac{(N_1 - 2 \cdot N_2) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - 2 \cdot N_2\right)}{N_1^2 - 2 \cdot N_1 \cdot N_2 + 4 \cdot N_2^2} = 0$

$CE - \frac{\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $BE - \frac{N_1 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}}{2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$ $AB - \frac{N_1^2}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}\right) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - N_1\right)} = 0$

$AG - \frac{\sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} \cdot (N_1 - 2 \cdot N_2)}{N_1^2 + N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2} - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^2}} = 0$

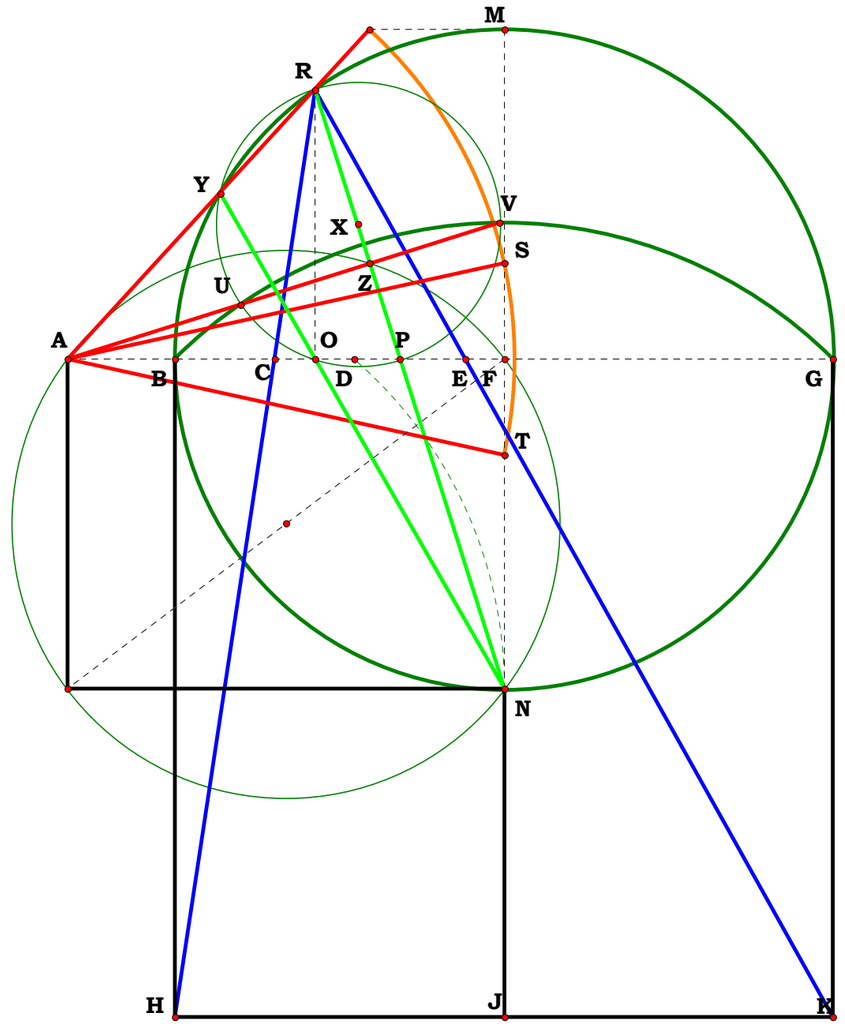


$$AC := AB + BC$$
$$AE := AB + BE$$

$$\left(\frac{AG}{AB}\right)^{\left(\frac{1}{3}\right)} - \frac{AC}{AB} = 0 \qquad \left(\frac{AG}{AB}\right)^{\left(\frac{2}{3}\right)} - \frac{AE}{AB} = 0$$

I find it very strange that so called mathematicians claim that one cannot abstract cube roots in geomety when every grammar is a binary expression and, since cube roots is simply a two dimensional ratio. I grant that the process to most is complicated, however, complicated and impossible are not the same concept.

I have, by no means, finished the write-up of this figure as I want to find the equations to the remaining structures pointed out in the opening graphic, however, I am still in the early stages of this revision and may come back to it at some later date.



$$N_1 := 2 \qquad N_2 := 5$$

$$AB - \frac{N_1^{\,2}}{\left(2 \cdot N_2 + \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}}\right) \cdot \left(\sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - N_1\right)} = 0$$

$$AC - \frac{N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}}}{\left(2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - N_1^{\,2}\right)} = 0$$

$$AE - \frac{N_1 \cdot \left(N_1 - 2 \cdot N_2\right)}{N_1^{\,2} + N_1 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}} - 2 \cdot N_2 \cdot \sqrt{2 \cdot N_1 \cdot N_2 - N_1^{\,2}}} = 0$$

Unit.

$A := 1$

Given.

$N := 8 \qquad E := N$

083099

Descriptions.

$AE := E - A \qquad B := E^{\left(\frac{1}{3}\right)} \qquad B = 2 \qquad D := E^{\left(\frac{2}{3}\right)}$

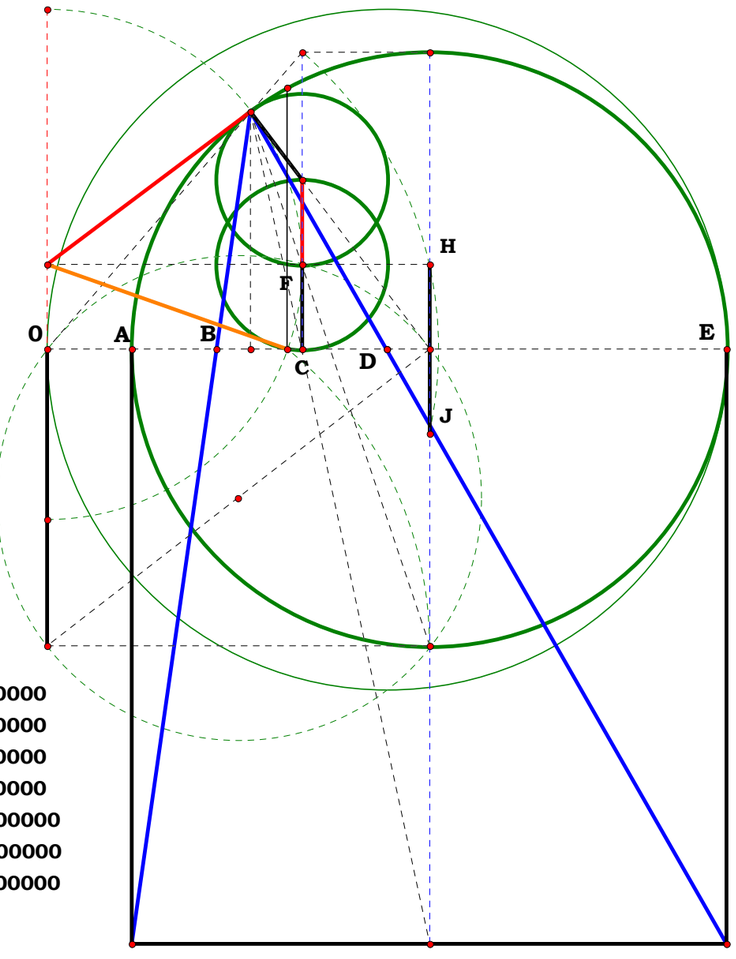
$D = 4 \qquad BD := D - B \qquad CF := \frac{BD}{2} \qquad HJ := 2 \cdot CF$

$CF = 1$

$BD = 2$

$HJ = 2$

To Be or Not to Be Project:
It took me a couple of seconds to remember what I wanted to demonstrate here. It is the answer is now in the graphic.



$A = 1.00000$

$B = 2.00000$

$C = 4.00000$

$D = 8.00000$

$CF = 1.00000$

$BD = 2.00000$

$HJ = 2.00000$

100299

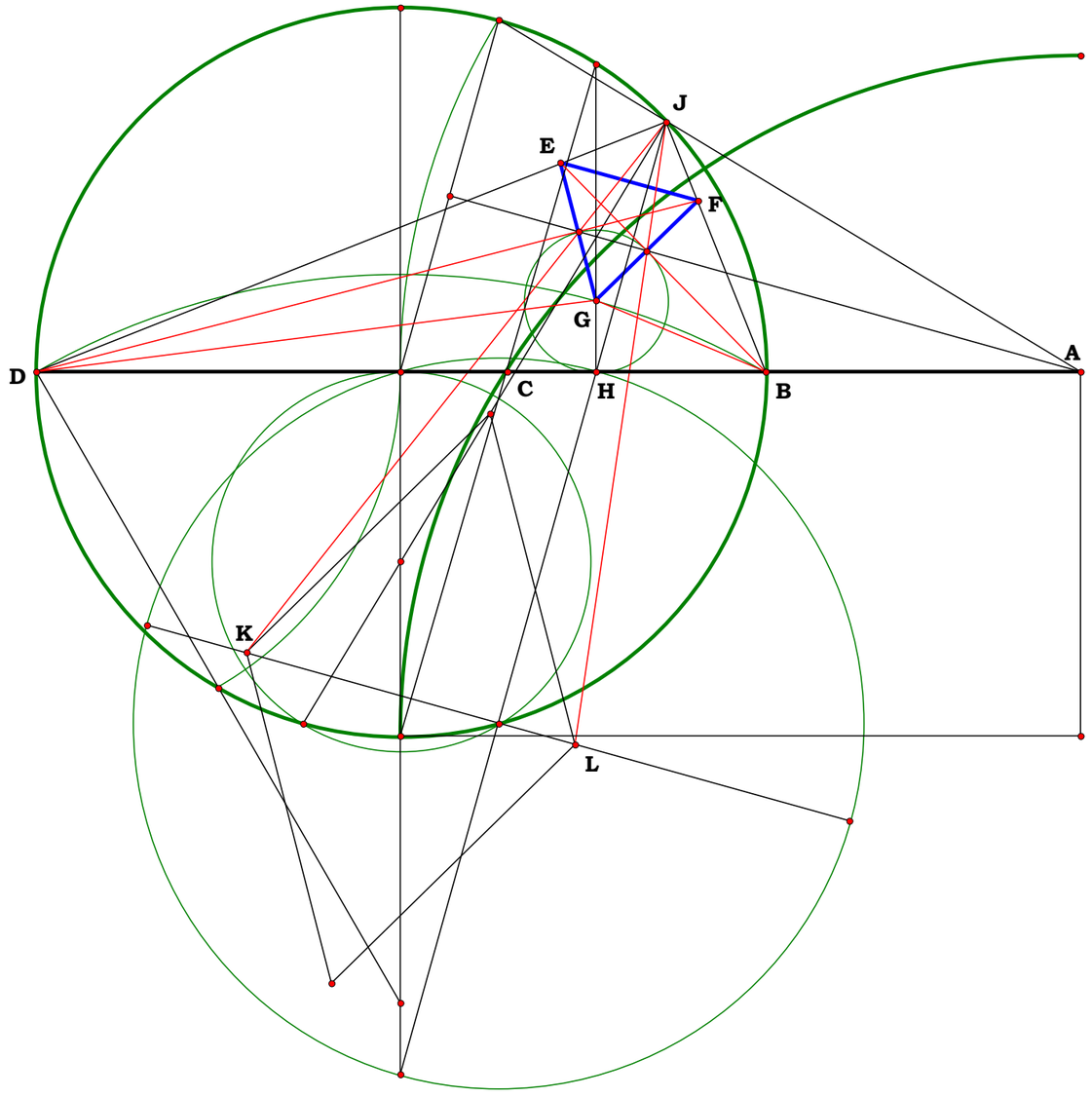
Enough here to keep one busy for a while.

The Circle and Segment, which if one can see it is fundamental to Jacob's Ladder which I use for BAM and BAG. They represent induction and deduction, arithmetic and geometric processing. This is why so many of my plates use it.

Again we see that trisection is directly related to square roots. However, there is just a lot of beauty seeing all of the interactions in the figure.

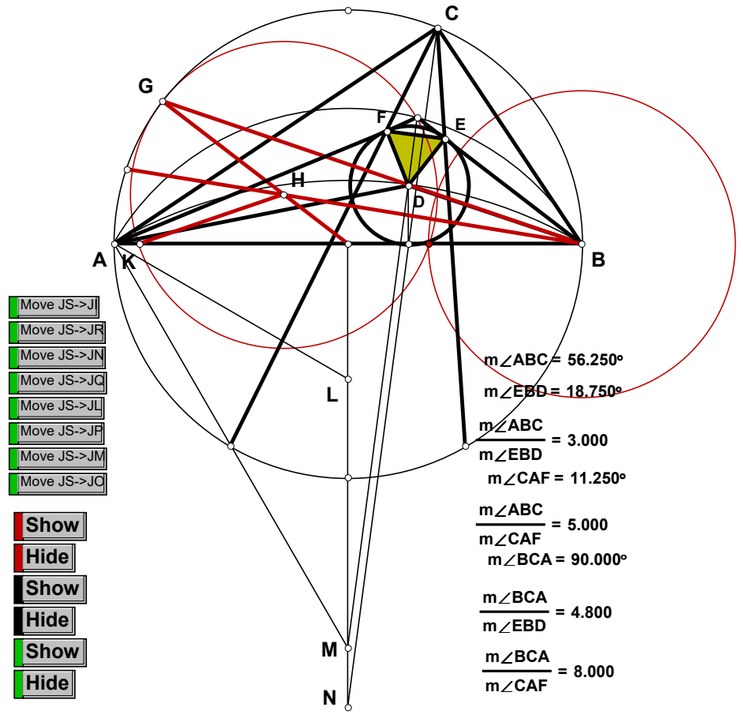
AB = 5.42961 cm
AC = 9.87931 cm
AD = 17.97565 cm
$\sqrt{AB \cdot AD}$-AC = 0.00000 cm
m∠DJB = 90.00000°
m∠KJL = 30.00000°
$\frac{m\angle DJB}{m\angle KJL}$ = 3.00000
m∠JBH = 68.31951°
m∠JBE = 22.77317°
m∠EBG = 22.77317°
m∠GBH = 22.77317°
$\frac{m\angle JBH}{m\angle JBE}$ = 3.00000
m∠JDH = 21.68049°
m∠JDF = 7.22683°
m∠FDG = 7.22683°
m∠GDH = 7.22683°
$\frac{m\angle JDH}{m\angle JDF}$ = 3.00000

Animate Point

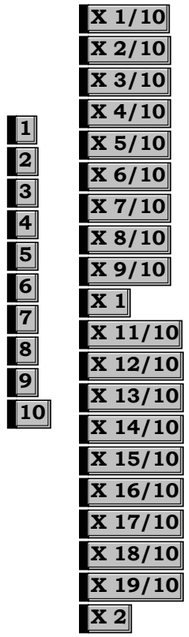


100499

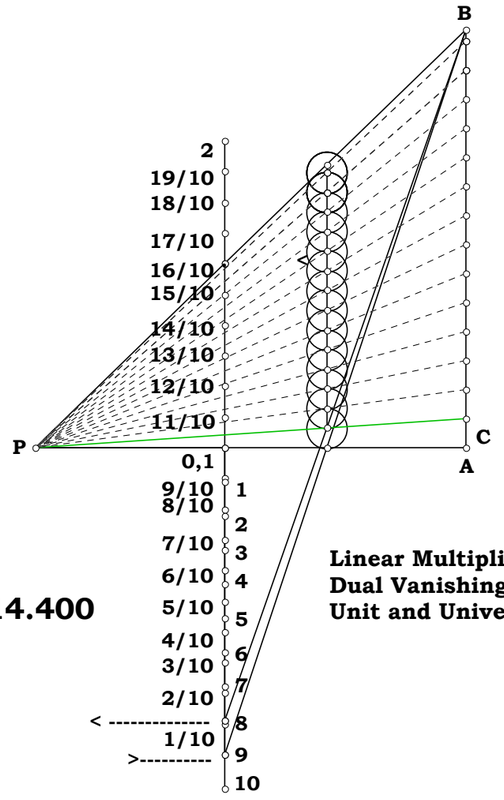
Parcing project



100799

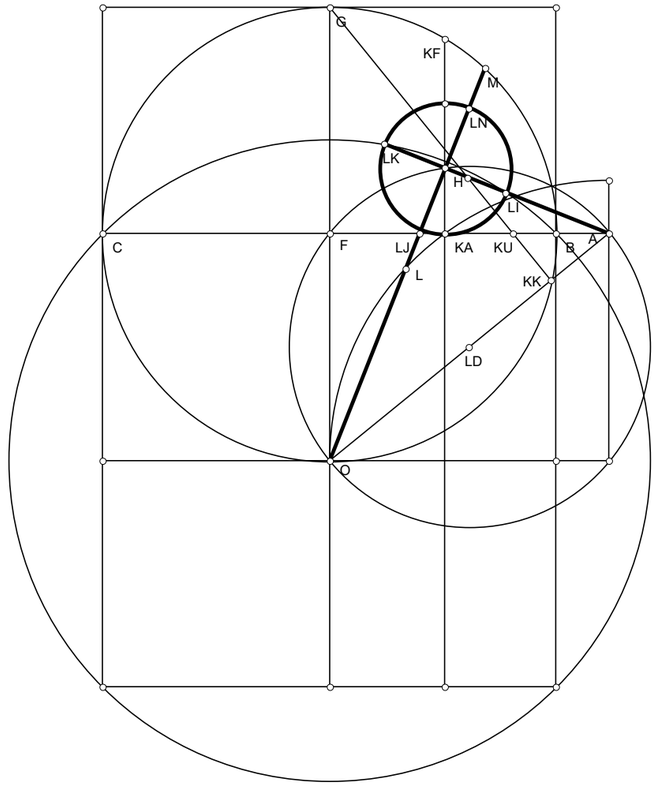


$$\frac{AB}{AC} = 14.400$$



Linear Multiplication and Division,
Dual Vanishing Point Method
Unit and Universe of Discourse.

101799



070200

Unit.

$AC := 1$

Given.

$N_1 := 4$

$N_2 := 5$ $\delta := 1 .. N_2$

Descriptions.

$AB := \frac{AC}{N_1}$ $BC := AC - AB$ $BD := \sqrt{AB \cdot BC}$ $BE_\delta := \frac{BD \cdot \delta}{N_2}$ $CE_\delta := \sqrt{(BE_\delta)^2 + BC^2}$

$CG_\delta := \frac{BC \cdot AC}{CE_\delta}$ $AD := \sqrt{AB^2 + BD^2}$ $AG_\delta := \frac{BE_\delta \cdot AC}{CE_\delta}$ $EG_\delta := CG_\delta - CE_\delta$ $EH_\delta := \frac{BE_\delta \cdot EG_\delta}{CE_\delta}$

$GH_\delta := \frac{BC \cdot EH_\delta}{BE_\delta}$ $BH_\delta := BE_\delta + EH_\delta$ $DH_\delta := BD - BH_\delta$ $DG_\delta := \sqrt{(GH_\delta)^2 + (DH_\delta)^2}$

$BK_\delta := \frac{AB \cdot BE_\delta}{BD}$ $CK_\delta := BC + BK_\delta$ $CJ_\delta := \frac{CE_\delta \cdot AC}{CK_\delta}$ $EJ_\delta := CJ_\delta - CE_\delta$

Definitions.

$AG_\delta =$

0.114708
0.225018
0.327327
0.419314
0.5

$\frac{\delta}{\sqrt{\delta^2 + N_1 \cdot N_2^2 - N_2^2}} =$

0.114708
0.225018
0.327327
0.419314
0.5

$DG_\delta =$

0.39736
0.292306
0.188982
0.090784
0

$(N_2 - \delta) \cdot \sqrt{\frac{(N_1 - 1)}{\left[N_1 \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)\right]}} =$

0.39736
0.292306
0.188982
0.090784
0

$EG_\delta =$

0.238416
0.204614
0.151186
0.081706
0

$\frac{(N_2^2 - \delta^2) \cdot \sqrt{N_1 - 1}}{N_2 \cdot N_1 \cdot \sqrt{\delta^2 + N_2^2 \cdot N_1 - N_2^2}} =$

0.238416
0.204614
0.151186
0.081706
0

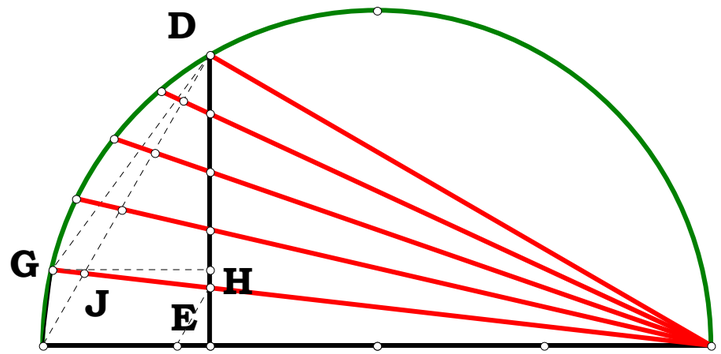
$CG_\delta =$

0.993399
0.974355
0.944911
0.907841
0.866025

$N_2 \cdot \frac{(N_1 - 1)}{\sqrt{(N_1 - 1) \cdot \left(\delta^2 + N_2^2 \cdot N_1 - N_2^2\right)}} =$

0.993399
0.974355
0.944911
0.907841
0.866025

In process. POR something or other.



$BE_\delta =$

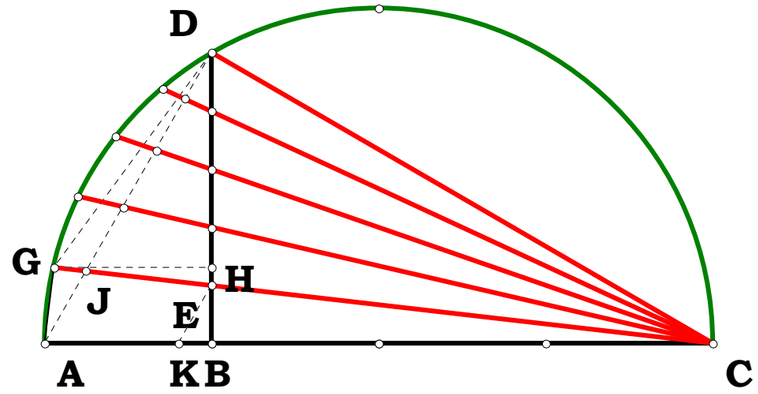
0.086603
0.173205
0.259808
0.34641
0.433013

$\sqrt{\frac{(N_1 - 1)}{(N_1 \cdot N_1)}} \cdot \frac{\delta}{N_2} = EJ_\delta =$

0.086603	0.188746
0.173205	0.135837
0.259808	0.088192
0.34641	0.043481
0.433013	0

$\frac{(N_2 - \delta) \cdot \sqrt{(N_1 - 1) \cdot (\delta^2 + N_2^2 \cdot N_1 - N_2^2)}}{N_2 \cdot N_1 \cdot (N_2 \cdot N_1 - N_2 + \delta)} =$

0.188746
0.135837
0.088192
0.043481
0



$AD = 0.5 \quad \sqrt{\frac{1}{N_1}} = 0.5$

$CJ_\delta =$

0.943729
0.905577
0.881917
0.869616
0.866025

$\frac{\sqrt{(N_1 - 1) \cdot (\delta^2 + N_2^2 \cdot N_1 - N_2^2)}}{N_2 \cdot N_1 - N_2 + \delta}$

0.943729
0.905577
0.881917
0.869616
0.866025

$\frac{AD}{AG_\delta} =$

4.358899
2.222049
1.527525
1.192424
1

$\frac{\sqrt{\frac{(\delta^2 + N_2^2 \cdot N_1 - N_2^2)}{N_1}}}{\delta} =$

4.358899
2.222049
1.527525
1.192424
1

$\frac{CG_\delta}{EG_\delta} =$

4.166667
4.761905
6.25
11.111111
$7.800463 \cdot 10^{15}$

$\text{if}\left(N_2^2 - \delta^2, \frac{N_2^2 \cdot N_1}{N_2^2 - \delta^2}, 0\right) =$

4.166667
4.761905
6.25
11.111111
0

$\text{if}\left(EJ_\delta, \frac{CJ_\delta}{EJ_\delta}, 0\right) =$

5
6.666667
10
20
0

$\text{if}\left(EJ_\delta, \frac{N_2 \cdot N_1}{N_2 - \delta}, 0\right) =$

5
6.666667
10
20
0

Unit.

Given.

$N_1 := 1.79201 \quad AB := N_1$

$N_2 := 10.41743 \quad AG := N_2$

Alternate Method Quad Roots

070900

Descriptions.

$AD := \sqrt{AB \cdot AG}$

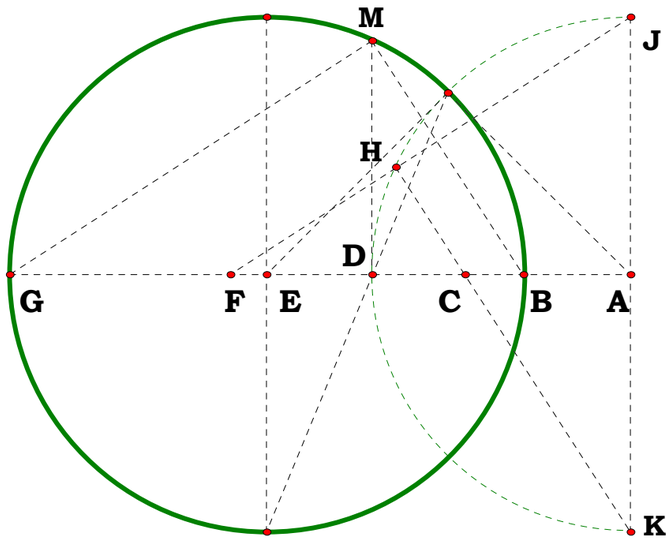
$BD := AD - AB \qquad BG := AG - AB$

$DG := BG - BD \qquad DM := \sqrt{BD \cdot DG}$

$AJ := AD \qquad AK := AD \qquad BM := \sqrt{BD^2 + DM^2}$

$GM := \sqrt{DG^2 + DM^2} \qquad AF := \frac{GM \cdot AJ}{BM}$

$AC := \frac{BM \cdot AK}{GM}$



Definitions.

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0 \qquad \left(AB^3 \cdot AG\right)^{\frac{1}{4}} - AC = 0$

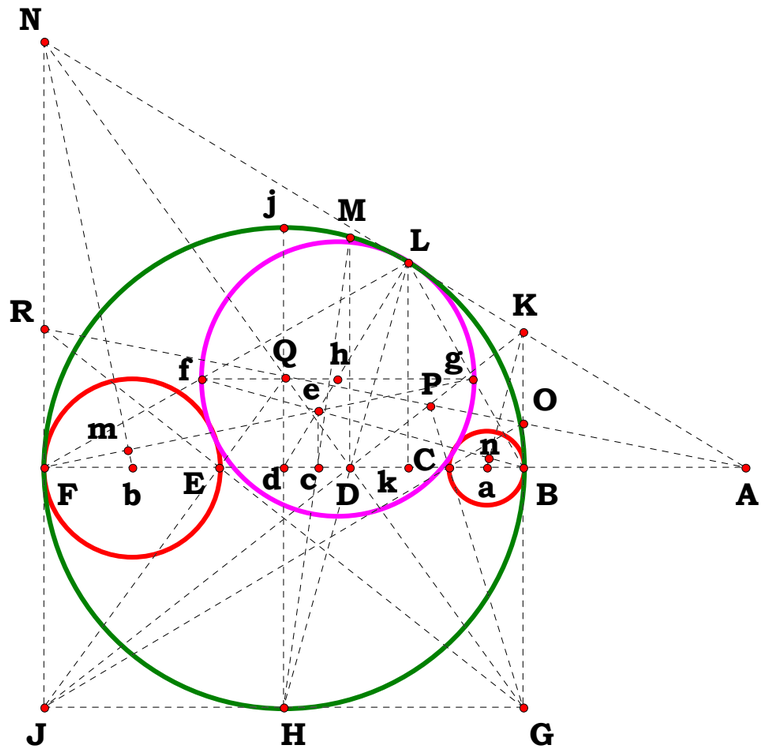
Unit.

Given.

$N_1 := 3.73926$ $AB := N_1$

$N_2 := 11.78259$ $AF := N_2$

Quad Roots via Tangent Circles.



i.e., A, K, L and N are colinear.

Descriptions.

000720a

$AD := \sqrt{AB \cdot AF}$ $BF := AF - AB$ $Bd := \frac{BF}{2}$ $BD := AD - AB$

$Dd := Bd - BD$ $DH := \sqrt{Bd^2 + Dd^2}$ $HL := \frac{Bd \cdot BF}{DH}$

$DL := HL - DH$ $Dk := \frac{Dd \cdot DL}{DH}$ $Bk := Bd - (Dd + Dk)$ $Lk := \frac{Bd \cdot Dk}{Dd}$

$JN := \frac{Bd \cdot BF}{BD}$ $GK := \frac{Bd \cdot BF}{(Bd + Dd)}$ $FN := JN - Bd$ $BK := GK - Bd$

$Ak := Bk + AB$ $AF - \frac{BF \cdot FN}{FN - BK} = 0$ $\frac{AF}{FN} - \frac{Ak}{Lk} = 0$

$DF := Bd + Dd$ $DM := \sqrt{BD \cdot DF}$ $cd := \frac{Dd \cdot Bd}{Bd + DM}$ $dk := Bd - Bk$

$ce := \frac{Lk \cdot cd}{dk}$ $Fb := \frac{ce \cdot FN}{Bd + cd}$ $Ba := \frac{ce \cdot BK}{Bd - cd}$ $AE := AF - 2 \cdot Fb$

$AC := AB + 2 \cdot Ba$

Definitions.

$\left(AB^3 \cdot AF\right)^{\frac{1}{4}} - AC = 0$ $\left(AB \cdot AF^3\right)^{\frac{1}{4}} - AE = 0$ **etc., etc.**

Unit.

Given.

$N_1 := 2.07320$ $AB := N_1$

$N_2 := 10.53987$ $AF := N_2$

Quad Roots by equal angles.

OOO720b

Descriptions.

$AD := \sqrt{AB \cdot AF}$ $BF := AF - AB$

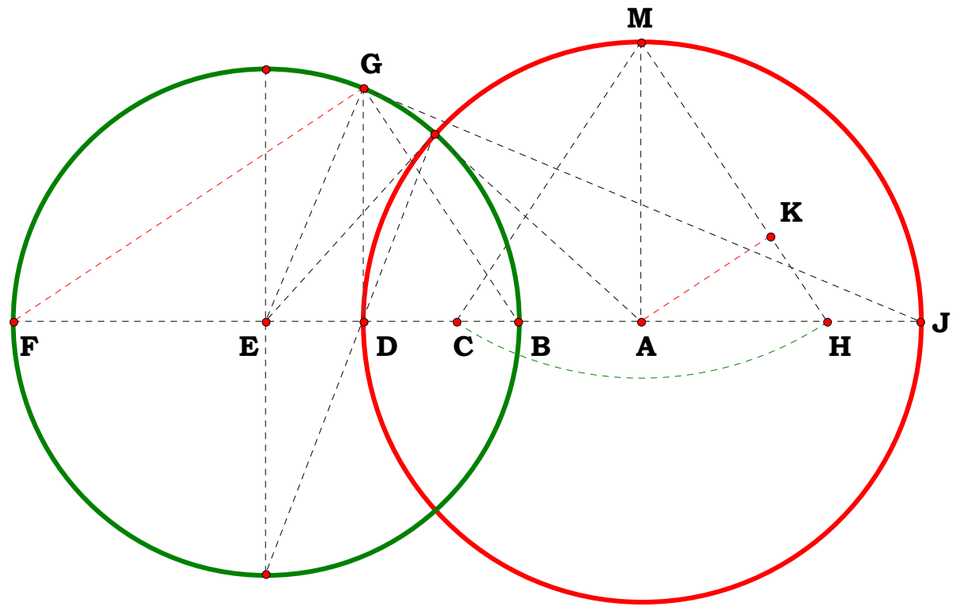
$AM := AD$ $DF := AF - AD$

$BD := AD - AB$ $DG := \sqrt{DF \cdot BD}$

$BG := \sqrt{DG^2 + BD^2}$ $AC := \frac{BD \cdot AD}{DG}$

Definitions.

$$AC - \left(AB^3 \cdot AF\right)^{\frac{1}{4}} = 0$$



Unit.

$AB := 1$

Given.

$N := 5 \qquad AG := N$

000801a

Descriptions.

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$AE := AB + BE \qquad DE := AE - AD \qquad NY := DE$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad NQ := \sqrt{DE^2 + (DN + EQ)^2}$

$QR := AE \qquad OQ := BG \qquad NO := \sqrt{OQ^2 - NQ^2}$

$PQ := \frac{NO \cdot 2 \cdot QR}{OQ} \qquad NP := NQ - PQ \qquad MN := \sqrt{\frac{NP^2}{2}}$

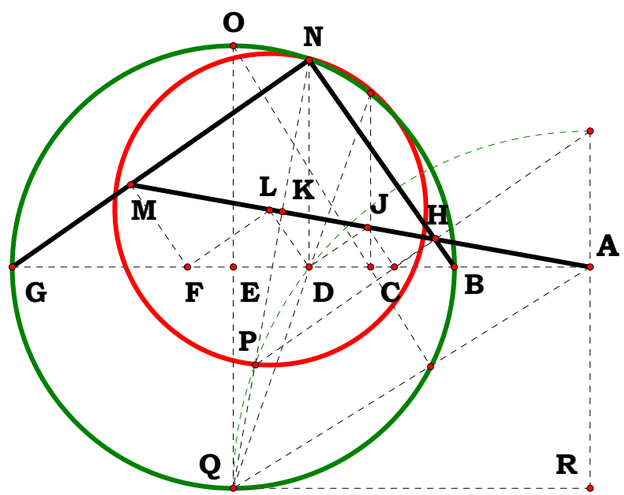
$BN := \sqrt{BD^2 + DN^2} \qquad GN := \sqrt{DG^2 + DN^2}$

$GM := GN - MN \qquad FG := \frac{BG \cdot GM}{GN} \qquad AF := AG - FG$

Definitions.

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

Alternate Method Quad Roots



Unit.

$AB := 1$

Given.

$N := 5 \qquad AG := N$

080100B

Descriptions.

$BG := AG - AB \qquad BE := \frac{BG}{2} \qquad AD := \sqrt{AB \cdot AG}$

$BD := AD - AB \qquad DG := BG - BD \qquad EQ := BE$

$DN := \sqrt{BD \cdot DG} \qquad BN := \sqrt{BD^2 + DN^2}$

$GN := \sqrt{DG^2 + DN^2} \qquad DE := BE - BD$

$NQ := \sqrt{(DN + EQ)^2 + DE^2} \qquad AN := \sqrt{AD^2 + DN^2}$

$AE := AB + BE \qquad AQ := \sqrt{AE^2 + EQ^2}$

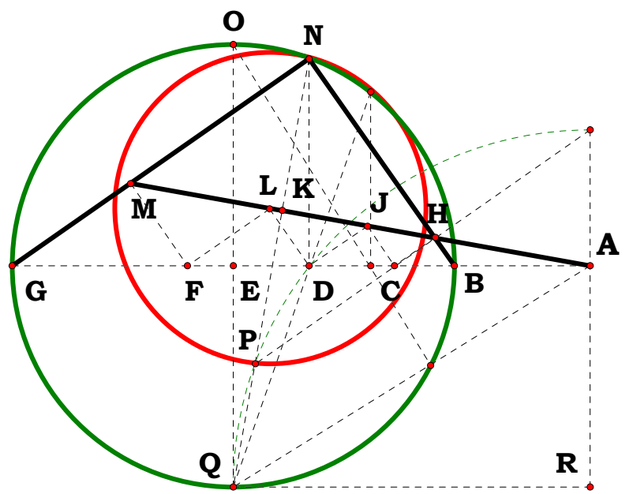
$KN := \frac{NQ^2 + AN^2 - AQ^2}{2 \cdot NQ} \qquad KM := KN \qquad MN := \sqrt{KN^2 + KM^2}$

$GM := GN - MN \qquad GF := \frac{BG \cdot GM}{GN} \qquad AF := AG - GF$

Definitions.

$\left(AB \cdot AG^3\right)^{\frac{1}{4}} - AF = 0$

Alternate Method Quad Roots



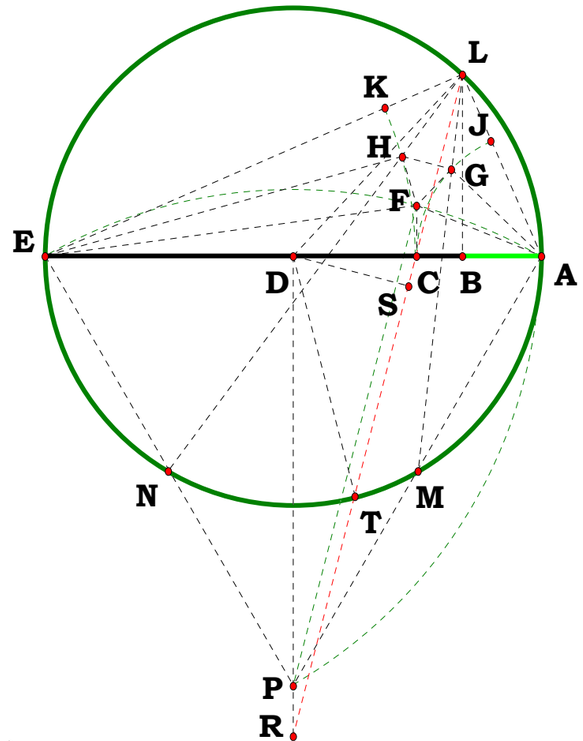
In Trisection What Is AB?

Given.

$AC := .884$

$AE := 3.521$

In the trisection figure given and given AC as the Unit what is AB?



080200

Descriptions.

$AD := \frac{AE}{2}$ $EP := AE$ $DE := AD$ $DP := \sqrt{EP^2 - DE^2}$

$FP := EP$ $CE := AE - AC$ $CD := CE - DE$ $CF := \sqrt{FP^2 - CD^2} - DP$

$PR := CF$ $DR := DP + PR$ $CR := \sqrt{CD^2 + DR^2}$ $CS := \frac{CD^2}{CR}$

$DS := \sqrt{CD^2 - CS^2}$ $DL := AD$ $LS := \sqrt{DL^2 - DS^2}$ $RS := CR - CS$

$LR := RS + LS$ $BD := \frac{CD \cdot LR}{CR}$ $AB := AD - BD$ $ST := LS$ $RT := RS - ST$

In trisection the length RT to the similarity point is equal to the radius of the circle.

$$RT - \left(\frac{1}{2}\right) \cdot AE = 0$$

Definitions.

$$AD - \frac{AE}{2} = 0 \qquad DP - \frac{AE}{2} \cdot \sqrt{3} = 0 \qquad CF - \left(\frac{\sqrt{4 \cdot AC \cdot AE - 4 \cdot AC^2 + 3 \cdot AE^2}}{2} - \frac{\sqrt{3} \cdot AE}{2}\right) = 0 \qquad CE - (AE - AC) = 0 \qquad CD - \left(\frac{1}{2} \cdot AE - AC\right) = 0$$

$$DR - \frac{1}{2} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad CR - AE = 0 \qquad CS - \left(\frac{1}{4}\right) \cdot \frac{(-AE + 2 \cdot AC)^2}{AE} = 0 \qquad LS - \frac{1}{4} \cdot \frac{\left(-4 \cdot AC^2 + 4 \cdot AE \cdot AC + AE^2\right)}{AE} = 0$$

$$DS - \frac{1}{4} \cdot \frac{(AE - 2 \cdot AC)}{AE} \cdot \sqrt{(AE + 2 \cdot AC) \cdot (3 \cdot AE - 2 \cdot AC)} = 0 \qquad LR - \frac{\left(AE^2 - 2 \cdot AC^2 + 2 \cdot AE \cdot AC\right)}{AE} = 0 \qquad RS - \frac{1}{4} \cdot (AE + 2 \cdot AC) \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE} = 0$$

$$BD - \left(\frac{1}{2} \cdot AE - \frac{3}{AE} \cdot AC^2 + \frac{2}{AE^2} \cdot AC^3\right) = 0 \qquad AB - AC^2 \cdot \frac{(3 \cdot AE - 2 \cdot AC)}{AE^2} = 0 \qquad AB \cdot AE^2 - AC^2(3 \cdot AE - 2 \cdot AC) = 0$$

Unit.

$AB := 1$

Given.

$N_1 := 3$

$N_2 := 2$

080300A

Descriptions.

$AD := AB$ $\quad AP := \frac{AD}{2}$ $\quad BP := AB + AP$ $\quad BO := \frac{BP}{N_1}$ $\quad AE := AB$

$DO := N_2 - BO$ $\quad GO := \sqrt{BO \cdot DO}$ $\quad BG := \sqrt{GO^2 + BO^2}$ $\quad BS := \frac{BG}{2}$ $\quad ER := BS$ $\quad TO := ER$

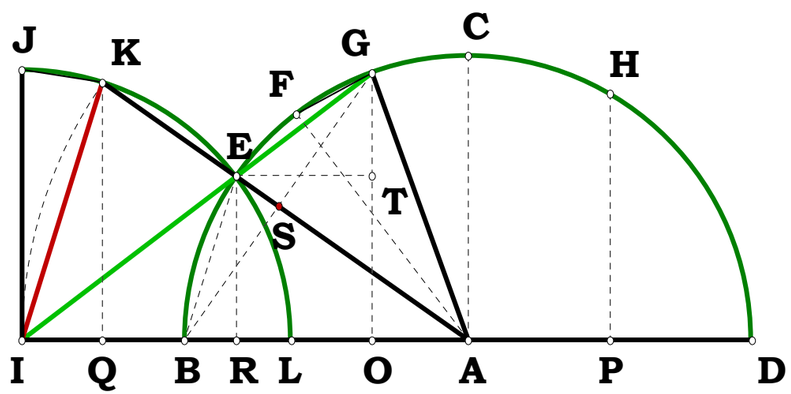
$GT := GO - TO$ $\quad AS := \sqrt{AB^2 - BS^2}$ $\quad ES := AE - AS$ $\quad BR := ES$ $\quad OR := BO - BR$

$ET := OR$ $\quad IO := \frac{ET \cdot GO}{GT}$ $\quad BI := IO - BO$ $\quad AI := BI + AB$ $\quad BE := \sqrt{ER^2 + BR^2}$

$GE := BE$ $\quad GI := \frac{GE \cdot GO}{GT}$ $\quad EI := GI - GE$ $\quad AK := AI$ $\quad IK := EI$ $\quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

$EK := AK - AE$ $\quad \frac{EK}{IQ} = 2$

On Trisection



If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.

Definitions.

$\frac{3 \cdot N_2}{4 \cdot N_1} - BO = 0$ $\quad \frac{N_2}{4} \cdot \frac{(4 \cdot N_1 - 3)}{N_1} - DO = 0$ $\quad \frac{N_2}{(4 \cdot N_1)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N_1 - 3} - GO = 0$ $\quad \frac{N_2}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BG = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BS = 0$

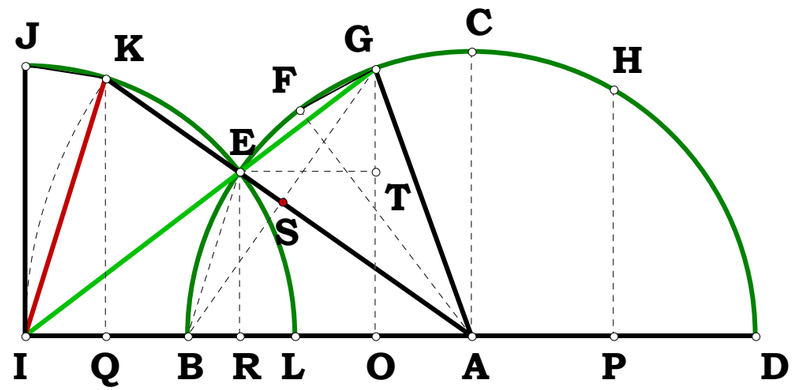
$\frac{\sqrt{3} \cdot N_2 \cdot \left(\sqrt{4 \cdot N_1 - 3} - \sqrt{N_1}\right)}{4 \cdot N_1} - GT = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} - AS = 0$ $\quad \frac{N_2}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}}\right] - ES = 0$ $\quad \frac{N_2}{4} \cdot \frac{\left[3 - 2 \cdot N_1 + \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1} \cdot N_1}\right]}{N_1} - OR = 0$

$$\frac{-N_2}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N_1-3}\cdot\sqrt{N_1}-2\cdot\sqrt{4\cdot N_1-3}\cdot N_1^{\left(\frac{3}{2}\right)}+4\cdot {N_1}^2-3\cdot N_1\right]}{\left[N_1^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)\right]}-IO=0$$

$$\frac{-N_2}{2}\cdot\frac{\left(\sqrt{4\cdot N_1-3}-2\cdot\sqrt{N_1}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-BI=0$$

$$\frac{-N_2}{\left(-2\cdot\sqrt{4\cdot N_1-3}+2\cdot\sqrt{N_1}\right)}\cdot\sqrt{N_1}-AI=0$$

$$\frac{N_2}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N_1}}\cdot\sqrt{4\cdot N_1-3}}-BE=0$$



$$\frac{N_2}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N_1}+\sqrt{4\cdot N_1-3}\right)}{\sqrt{N_1}}}\cdot\frac{\sqrt{4\cdot N_1-3}}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-GI=0$$

$$\frac{-N_2}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N_1-3}}{\sqrt{N_1}}}\cdot\frac{\sqrt{N_1}}{\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)}-EI=0$$

$$\frac{N_2}{4}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-IQ=0$$

$$\frac{N_2}{2}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-EK=0$$

Unit.

$AB := 1$

Given.

$N_1 := 3$

$N_2 := 2$

080300B

Descriptions.

$AD := AB$ $\quad AP := \frac{AD}{2}$ $\quad BP := AB + AP$ $\quad BO := \frac{BP}{N_1}$ $\quad AE := AB$

$DO := N_2 - BO$ $\quad GO := \sqrt{BO \cdot DO}$ $\quad BG := \sqrt{GO^2 + BO^2}$ $\quad BS := \frac{BG}{2}$ $\quad ER := BS$ $\quad TO := ER$

$GT := GO - TO$ $\quad AS := \sqrt{AB^2 - BS^2}$ $\quad ES := AE - AS$ $\quad BR := ES$ $\quad OR := BO - BR$

$ET := OR$ $\quad IO := \frac{ET \cdot GO}{GT}$ $\quad BI := IO - BO$ $\quad AI := BI + AB$ $\quad BE := \sqrt{ER^2 + BR^2}$

$GE := BE$ $\quad GI := \frac{GE \cdot GO}{GT}$ $\quad EI := GI - GE$ $\quad AK := AI$ $\quad IK := EI$ $\quad IQ := \frac{IK^2 + AI^2 - AK^2}{2 \cdot AI}$

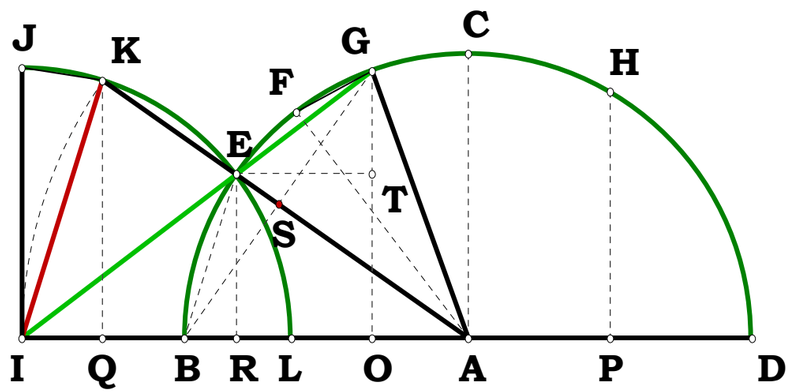
$EK := AK - AE$ $\quad \frac{EK}{IQ} = 2$

Definitions.

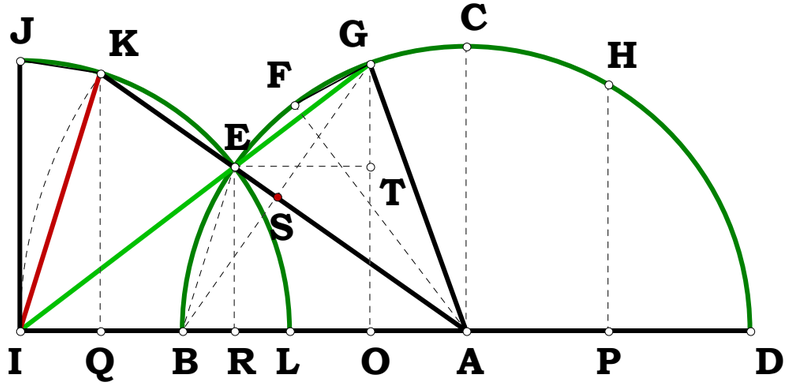
$\frac{3 \cdot N_2}{4 \cdot N_1} - BO = 0$ $\quad \frac{N_2}{4} \cdot \frac{(4 \cdot N_1 - 3)}{N_1} - DO = 0$ $\quad \frac{N_2}{(4 \cdot N_1)} \cdot \sqrt{3} \cdot \sqrt{4 \cdot N_1 - 3} - GO = 0$ $\quad \frac{N_2}{2} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BG = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{3} \cdot \sqrt{\frac{1}{N_1}} - BS = 0$

$\frac{\sqrt{3} \cdot N_2 \cdot \left(\sqrt{4 \cdot N_1 - 3} - \sqrt{N_1}\right)}{4 \cdot N_1} - GT = 0$ $\quad \frac{N_2}{4} \cdot \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}} - AS = 0$ $\quad \frac{N_2}{4} \cdot \left[2 - \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1}}\right] - ES = 0$ $\quad \frac{N_2}{4} \cdot \frac{\left[3 - 2 \cdot N_1 + \sqrt{\frac{(4 \cdot N_1 - 3)}{N_1} \cdot N_1}\right]}{N_1} - OR = 0$

On Trisection



If 2 IQ = EK then 2 JK = EK and the figure projected from BCD will yield a trisected figure JKL.



$$\frac{-N_2}{4}\cdot\frac{\left[3\cdot\sqrt{4\cdot N_1-3}\cdot\sqrt{N_1}-2\cdot\sqrt{4\cdot N_1-3}\cdot N_1^{\left(\frac{3}{2}\right)}+4\cdot {N_1}^2-3\cdot N_1\right]}{\left[N_1^{\left(\frac{3}{2}\right)}\cdot\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)\right]}-IO=0$$

$$\frac{-N_2}{2}\cdot\frac{\left(\sqrt{4\cdot N_1-3}-2\cdot\sqrt{N_1}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-BI=0$$

$$\frac{-N_2}{\left(-2\cdot\sqrt{4\cdot N_1-3}+2\cdot\sqrt{N_1}\right)}\cdot\sqrt{N_1}-AI=0$$

$$\frac{N_2}{2}\cdot\sqrt{2-\frac{1}{\sqrt{N_1}}\cdot\sqrt{4\cdot N_1-3}}-BE=0$$

$$\frac{N_2}{2}\cdot\sqrt{\frac{-\left(-2\cdot\sqrt{N_1}+\sqrt{4\cdot N_1-3}\right)}{\sqrt{N_1}}}\cdot\frac{\sqrt{4\cdot N_1-3}}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-GI=0$$

$$\frac{-N_2}{2}\cdot\sqrt{2-\frac{\sqrt{4\cdot N_1-3}}{\sqrt{N_1}}}\cdot\frac{\sqrt{N_1}}{\left(-\sqrt{4\cdot N_1-3}+\sqrt{N_1}\right)}-EI=0$$

$$\frac{N_2}{4}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-IQ=0$$

$$\frac{N_2}{2}\cdot\frac{\left(2\cdot\sqrt{N_1}-\sqrt{4\cdot N_1-3}\right)}{\left(\sqrt{4\cdot N_1-3}-\sqrt{N_1}\right)}-EK=0$$

080400A

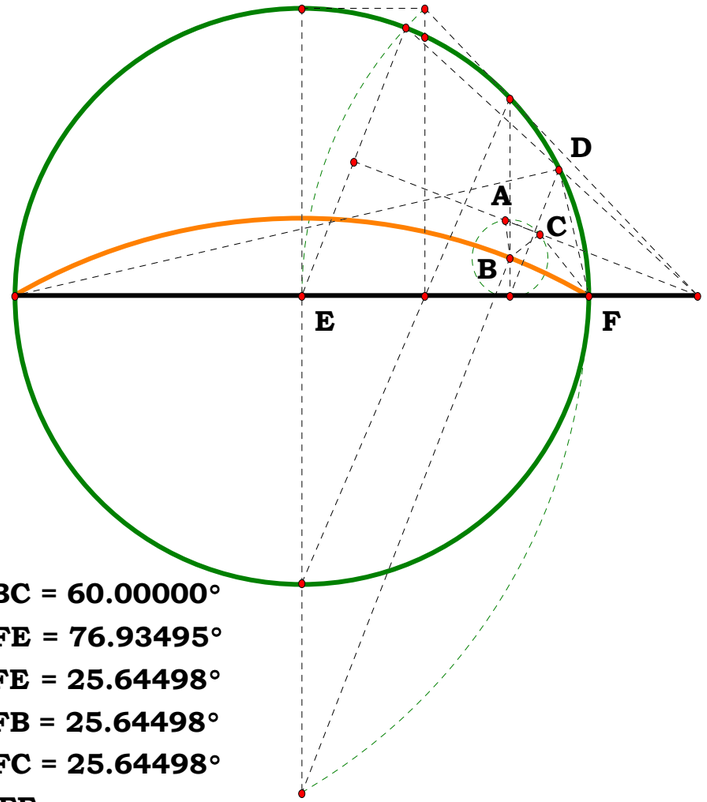
With the following construction, one can see that square roots is directly involved with what is called angle trisection. Is there a reasonable method of projecting to the square? Probably not, but what the heck?

I am going to figure this out and then I am going to order the equations a little different at the start to see what happens to all the definitions.

This is the A plate, or the first of six.

What will be demonstrated are the differences in the choice of what one uses for a unit to write the figure up.

Trisection and Square Roots



$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

Unit.

Given.

$N_1 := 1.90557$ $\quad AB := N_1$

$N_2 := 12.01265$ $\quad AF := N_2$

Descriptions.

$AD := \sqrt{AB \cdot AF}$ $\quad BD := AD - AB$

$BF := AF - AB$ $\quad DF := AF - AD$ $\quad DJ := \sqrt{BD \cdot DF}$

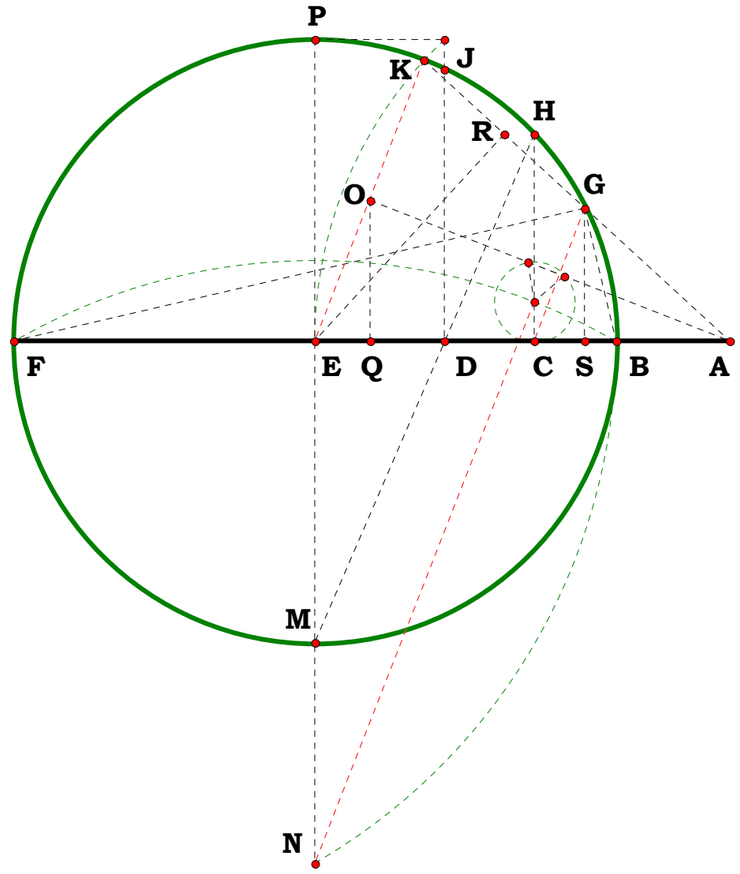
$BE := \frac{BF}{2}$ $\quad EO := \frac{BE}{2}$ $\quad AE := BE + AB$ $\quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD$ $\quad DM := \sqrt{DE^2 + BE^2}$ $\quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM$ $\quad CD := \frac{DE \cdot DH}{DM}$ $\quad CE := DE + CD$ $\quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2}$ $\quad KG := \frac{2 \cdot EQ \cdot BE}{EO}$ $\quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE}$ $\quad AS := AE - (DE + CD + CS)$ $\quad BS := AS - AB$



Definitions:

$AB - N_1 = 0$ $\quad AF - N_2 = 0$ $\quad AD - \sqrt{N_1 \cdot N_2} = 0$ $\quad BD - \left(\sqrt{N_1 \cdot N_2} - N_1\right) = 0$

$BF - \left(N_2 - N_1\right) = 0$ $\quad DF - \left(N_2 - \sqrt{N_1 \cdot N_2}\right) = 0$ $\quad DJ - \sqrt{\sqrt{N_1 \cdot N_2} \cdot \left(N_1 + N_2\right) - 2 \cdot N_1 \cdot N_2} = 0$

$BE - \frac{N_2 - N_1}{2}$ $\quad EO - \frac{N_2 - N_1}{4} = 0$ $\quad AE - \frac{N_1 + N_2}{2} = 0$ $\quad EQ - \frac{\left(N_1 - N_2\right)^2}{8 \cdot \left(N_1 + N_2\right)} = 0$

$$DE - \frac{N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2}}{2} = 0 \qquad DM - \frac{\sqrt{\left[N_1^{\,2} + N_2^{\,2} + 2\cdot N_1\cdot N_2 - 2\cdot\sqrt{N_1\cdot N_2}\cdot(N_1 + N_2)\right]}}{\sqrt{2}} = 0$$

$$HM - \frac{\sqrt{2}\cdot(N_1 - N_2)^2}{2\cdot\sqrt{N_1^{\,2} + N_2^{\,2} - 2\cdot\sqrt{N_1\cdot N_2}\cdot(N_1 + N_2) + 2\cdot N_1\cdot N_2}} = 0$$

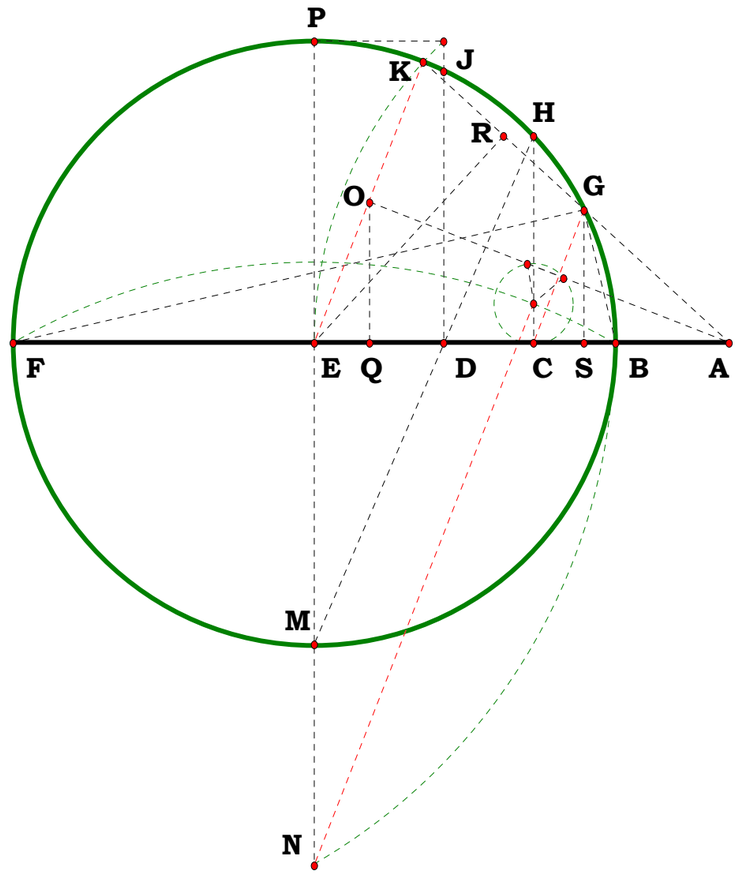
$$DH - \frac{(N_1 + N_2)\cdot\sqrt{2\cdot N_1\cdot N_2} - 2\cdot\sqrt{2}\cdot N_1\cdot N_2}{\sqrt{(N_1 + N_2)\cdot(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2})}} = 0$$

$$CD - \frac{\sqrt{N_1\cdot N_2}\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right) - 4\cdot N_1^{\,2}\cdot N_2 - 4\cdot N_1\cdot N_2^{\,2}}{(N_1 + N_2)\cdot(N_1 + N_2 - 2\cdot\sqrt{N_1\cdot N_2})} = 0$$

$$CE - \frac{(N_1 - N_2)^2}{2\cdot(N_1 + N_2)} = 0 \qquad BC - \frac{N_1\cdot(N_2 - N_1)}{N_1 + N_2} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{(N_1 - N_2)^2}}{2} = 0$$

$$KG - \frac{(N_1 - N_2)^2}{2\cdot(N_1 + N_2)} = 0 \qquad AG - \frac{2\cdot N_1\cdot N_2}{N_1 + N_2} = 0 \qquad CS - \frac{N_1\cdot N_2\cdot(N_1 - N_2)^2}{(N_1 + N_2)^3} = 0$$

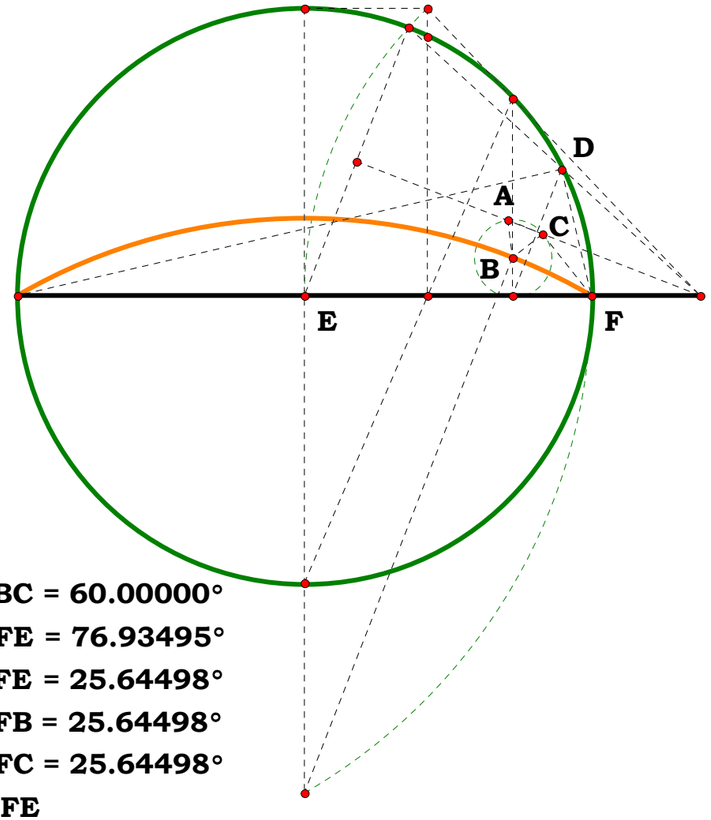
$$AS - \frac{N_1\cdot N_2\cdot\left(N_1^{\,2} + 6\cdot N_1\cdot N_2 + N_2^{\,2}\right)}{(N_1 + N_2)^3} = 0 \qquad BS - \frac{N_1^{\,2}\cdot(N_2 - N_1)\cdot(N_1 + 3\cdot N_2)}{(N_1 + N_2)^3} = 0$$



080400B

This is the B plate, or the second of six.

Trisection and Square Roots



$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

Unit.

Given.

$N_1 := 1.90557$ $AB := N_1$

$N_2 := 10.10708$ $BF := N_2$

Descriptions.

$AF := AB + BF$ $AD := \sqrt{AB \cdot AF}$

$BD := AD - AB$ $DF := AF - AD$ $DJ := \sqrt{BD \cdot DF}$

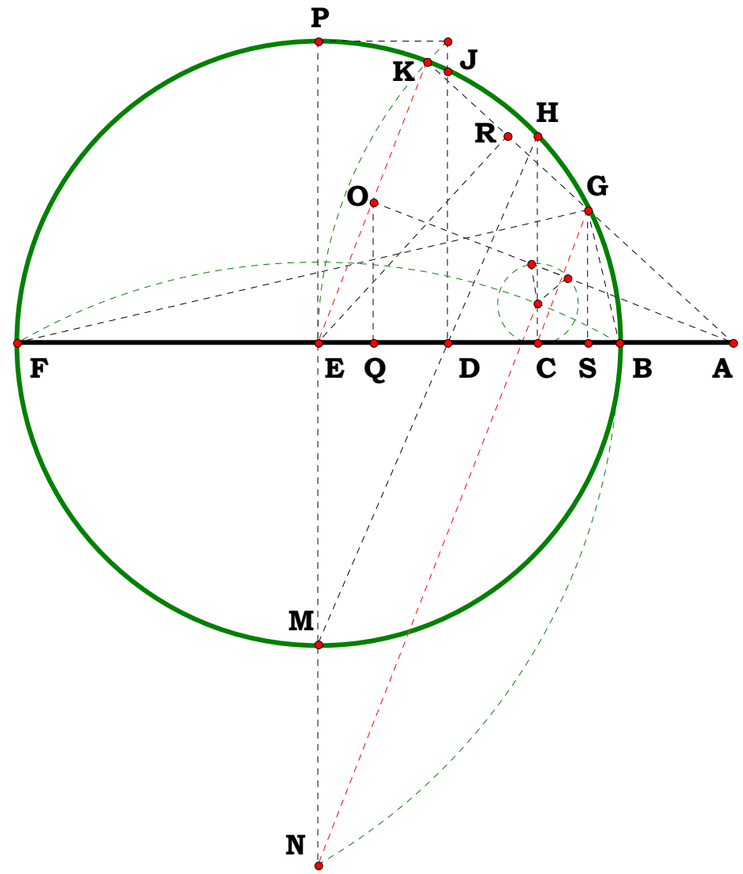
$BE := \frac{BF}{2}$ $EO := \frac{BE}{2}$ $AE := BE + AB$ $EQ := \frac{EO^2}{AE}$

$DE := AE - AD$ $DM := \sqrt{DE^2 + BE^2}$ $HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM$ $CD := \frac{DE \cdot DH}{DM}$ $CE := DE + CD$ $BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2}$ $KG := \frac{2 \cdot EQ \cdot BE}{EO}$ $AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE}$ $AS := AE - (DE + CD + CS)$ $BS := AS - AB$



Definitions:

$AB - N_1 = 0$ $BF - N_2 = 0$ $AF - (N_1 + N_2) = 0$

$AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0$ $BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right]$ $DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot (N_1 + N_2)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0$ $BE - \frac{N_2}{2} = 0$ $EO - \frac{N_2}{4} = 0$

$AE - \frac{2 \cdot N_1 + N_2}{2} = 0$ $EQ - \frac{N_2^2}{8 \cdot (2 \cdot N_1 + N_2)} = 0$ $DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$

$$DM - \frac{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}{2}=0$$

$$HM - \frac{N_2^{\,2}}{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}=0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)-2\cdot N_1^{\,2}-2\cdot N_1\cdot N_2\right]}{\sqrt{\left(2\cdot N_1+N_2\right)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot\left(N_1+N_2\right)}\right]}}=0$$

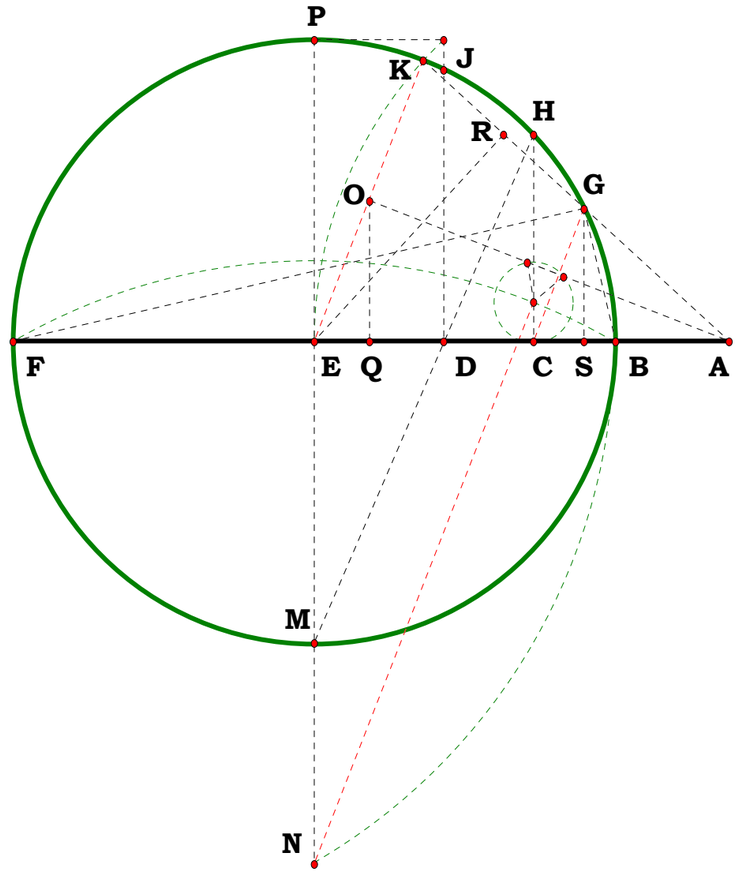
$$CD - \frac{\sqrt{N_1^{\,2}+N_2\cdot N_1}\cdot\left(8\cdot N_1^{\,2}+8\cdot N_1\cdot N_2+N_2^{\,2}\right)-4\cdot N_1\cdot\left(2\cdot N_1+N_2\right)\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{N_1^{\,2}+N_2\cdot N_1}\right)}=0$$

$$CE - \frac{N_2^{\,2}}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{N_2^{\,2}}}{2}=0$$

$$KG - \frac{N_2^{\,2}}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad AG - \frac{2\cdot N_1\cdot\left(N_1+N_2\right)}{2\cdot N_1+N_2}=0 \qquad CS - \frac{N_1\cdot N_2^{\,2}\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$AS - \frac{N_1\cdot\left(N_1+N_2\right)\cdot\left(8\cdot N_1^{\,2}+8\cdot N_1\cdot N_2+N_2^{\,2}\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

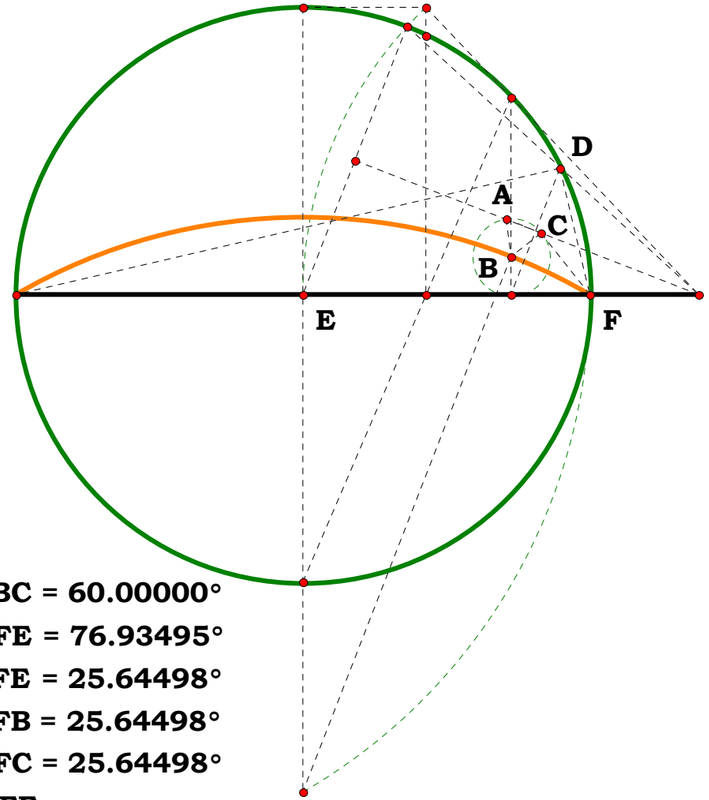
$$BS - \frac{N_1^{\,2}\cdot N_2\cdot\left(4\cdot N_1+3\cdot N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$



080400C

This is the C plate, or the third of six.

Trisection and Square Roots



m∠ABC = 60.00000°
m∠DFE = 76.93495°
m∠BFE = 25.64498°
m∠CFB = 25.64498°
m∠DFC = 25.64498°

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

m∠DFE-(m∠BFE+m∠CFB+m∠DFC) = 0.00000°

Unit.

$AB := 1$

Given.

$N_1 := 12.01265 \qquad AF := N_1$

Descriptions.

$AD := \sqrt{AB \cdot AF} \qquad BD := AD - AB$

$BF := AF - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

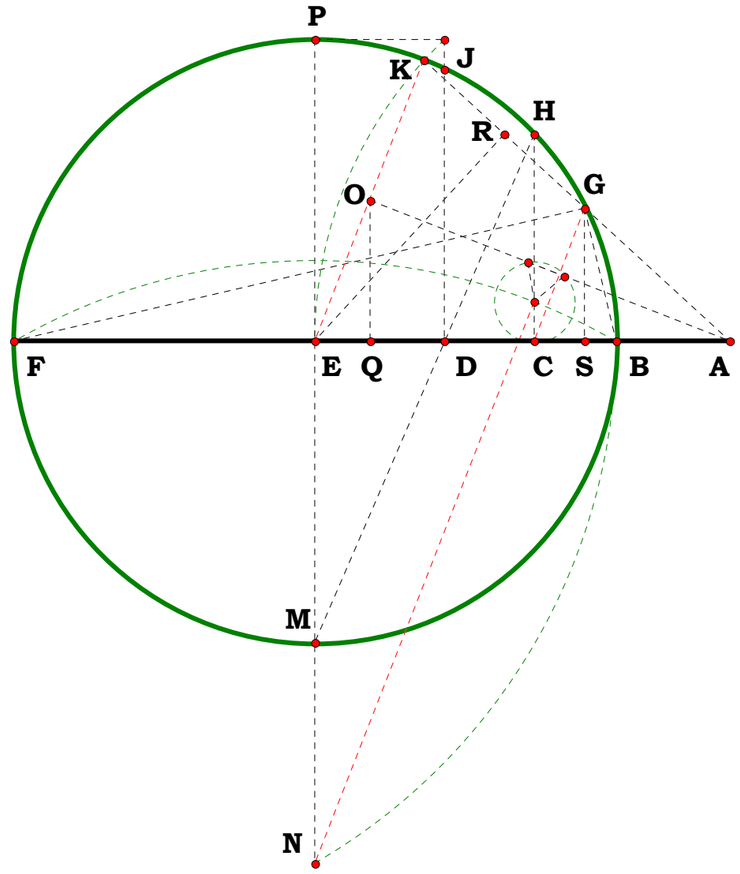
$BE := \frac{BF}{2} \qquad EO := \frac{BE}{2} \qquad AE := BE + AB \qquad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \frac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \frac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$



Definitions:

$1 - 1 = 0 \qquad AF - N_1 = 0 \qquad AD - \sqrt{N_1} = 0 \qquad BD - \left(\sqrt{N_1} - 1\right) = 0$

$BF - \left(N_1 - 1\right) = 0 \qquad DF - \left(N_1 - \sqrt{N_1}\right) = 0 \qquad DJ - \sqrt{\sqrt{N_1} \cdot \left(\sqrt{N_1} - 1\right)^2} = 0$

$BE - \frac{N_1 - 1}{2} \qquad EO - \frac{N_1 - 1}{4} = 0 \qquad AE - \frac{1 + N_1}{2} = 0 \qquad EQ - \frac{\left(1 - N_1\right)^2}{8 \cdot \left(1 + N_1\right)} = 0$

$$DE - \frac{1 + N_1 - 2\cdot\sqrt{N_1}}{2} = 0 \qquad DM - \frac{\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}}{\sqrt{2}} = 0$$

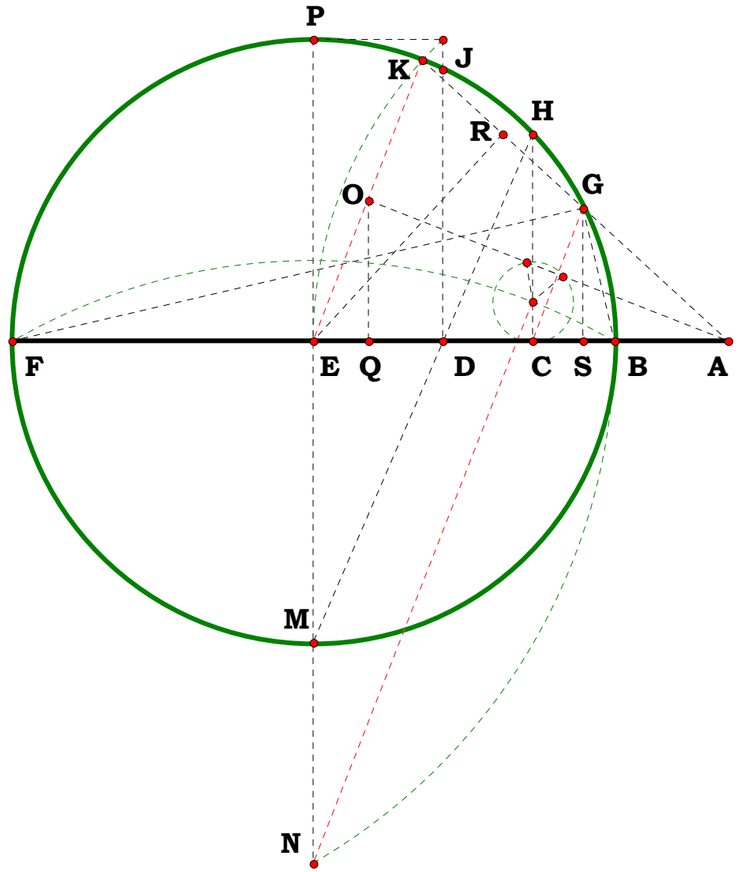
$$HM - \frac{\sqrt{2}\cdot(1-N_1)^2}{2\cdot\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}} = 0 \qquad DH - \frac{\sqrt{2}\cdot\sqrt{N_1}\cdot\left(\sqrt{N_1}-1\right)^2}{\sqrt{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2}} = 0$$

$$CD - \frac{\sqrt{N_1}\cdot\left(\sqrt{N_1}-1\right)^4}{(N_1+1)\cdot\left(\sqrt{N_1}-1\right)^2} = 0 \qquad CE - \frac{(1-N_1)^2}{2\cdot(1+N_1)} = 0$$

$$BC - \frac{1\cdot(N_1-1)}{1+N_1} = 0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{(1-N_1)^2}}{2} = 0 \qquad KG - \frac{(1-N_1)^2}{2\cdot(1+N_1)} = 0$$

$$AG - \frac{2\cdot 1\cdot N_1}{1+N_1} = 0 \qquad CS - \frac{1\cdot N_1\cdot(1-N_1)^2}{(1+N_1)^3} = 0$$

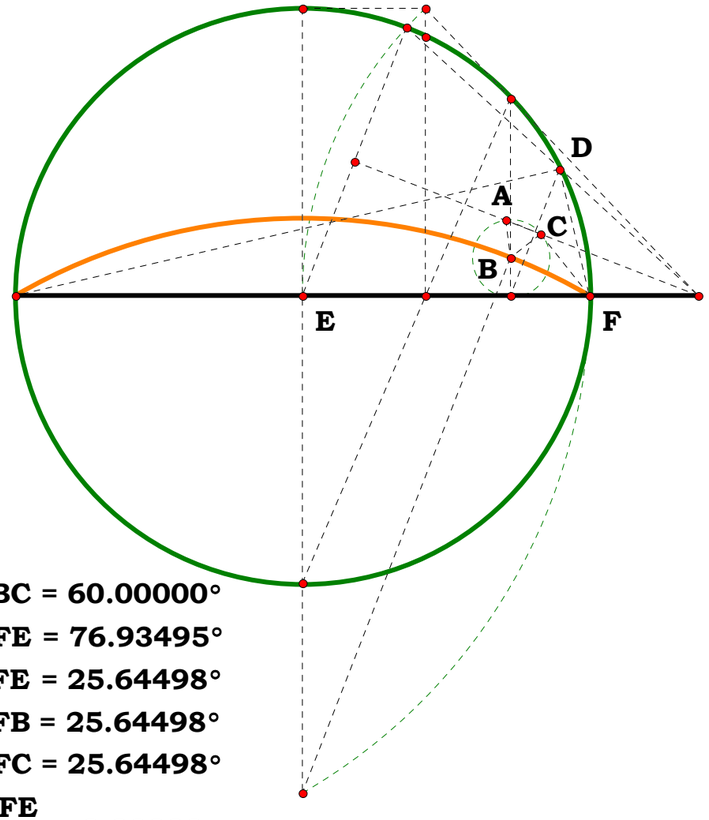
$$AS - \frac{N_1\cdot\left(N_1^2+6\cdot N_1+1\right)}{(1+N_1)^3} = 0 \qquad BS - \frac{(N_1-1)\cdot(3\cdot N_1+1)}{(1+N_1)^3} = 0$$



080400D

This is the D plate, or the forth of six.

Trisection and Square Roots



$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

Unit.

Given.

$N_1 := 1.90557$ $\quad AB := N_1$

$N_2 := 10.10708$ $\quad BF := N_2$

Descriptions.

$AF := AB + BF$ $\quad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB$ $\quad DF := AF - AD$ $\quad DJ := \sqrt{BD \cdot DF}$

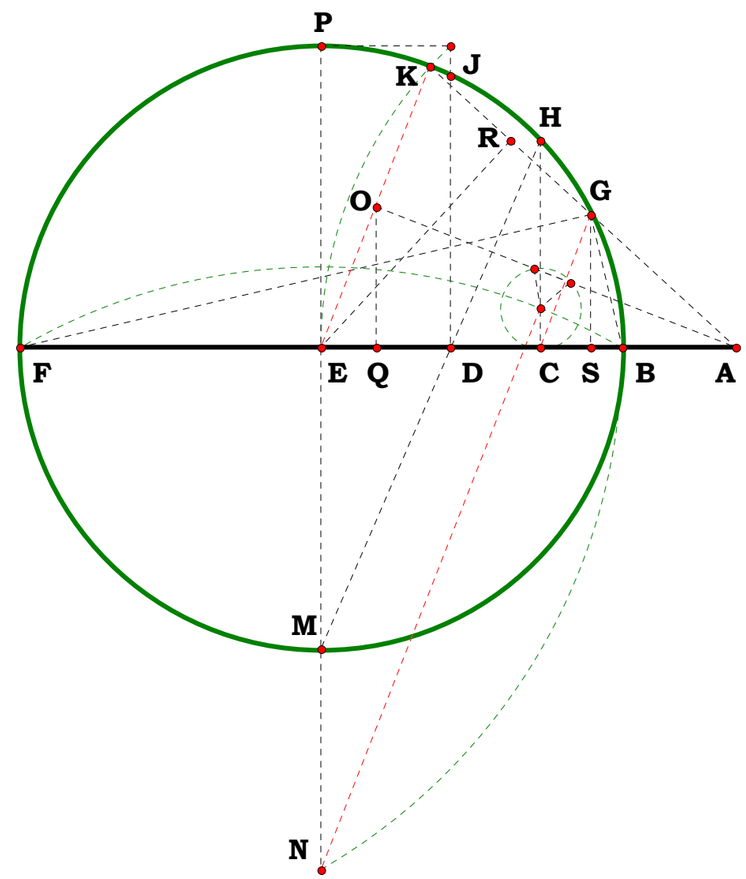
$BE := \frac{BF}{2}$ $\quad EO := \frac{BE}{2}$ $\quad AE := BE + AB$ $\quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD$ $\quad DM := \sqrt{DE^2 + BE^2}$ $\quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM$ $\quad CD := \frac{DE \cdot DH}{DM}$ $\quad CE := DE + CD$ $\quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2}$ $\quad KG := \frac{2 \cdot EQ \cdot BE}{EO}$ $\quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE}$ $\quad AS := AE - (DE + CD + CS)$ $\quad BS := AS - AB$



Definitions:

$AB - N_1 = 0$ $\quad BF - N_2 = 0$ $\quad AF - (N_1 + N_2) = 0$

$AD - \sqrt{N_1 \cdot (N_1 + N_2)} = 0$ $\quad BD - \left[\sqrt{N_1 \cdot (N_1 + N_2)} - N_1\right]$ $\quad DF - \left[N_1 + N_2 - \sqrt{N_1 \cdot (N_1 + N_2)}\right] = 0$

$DJ - \sqrt{\left[\sqrt{N_1^2 + N_2 \cdot N_1} \cdot (2 \cdot N_1 + N_2) - 2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right]} = 0$ $\quad BE - \frac{N_2}{2} = 0$ $\quad EO - \frac{N_2}{4} = 0$

$AE - \frac{2 \cdot N_1 + N_2}{2} = 0$ $\quad EQ - \frac{N_2^2}{8 \cdot (2 \cdot N_1 + N_2)} = 0$ $\quad DE - \frac{2 \cdot N_1 + N_2 - 2 \cdot \sqrt{N_1^2 + N_2 \cdot N_1}}{2} = 0$

$$DM - \frac{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}{2}=0$$

$$HM - \frac{{N_2}^2}{\sqrt{2\cdot\left(2\cdot N_1+N_2\right)^2-4\cdot\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)}}=0$$

$$DH - \frac{\sqrt{2}\cdot\left[\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(2\cdot N_1+N_2\right)-2\cdot{N_1}^2-2\cdot N_1\cdot N_2\right]}{\sqrt{\left(2\cdot N_1+N_2\right)\cdot\left[2\cdot N_1+N_2-2\cdot\sqrt{N_1\cdot\left(N_1+N_2\right)}\right]}}=0$$

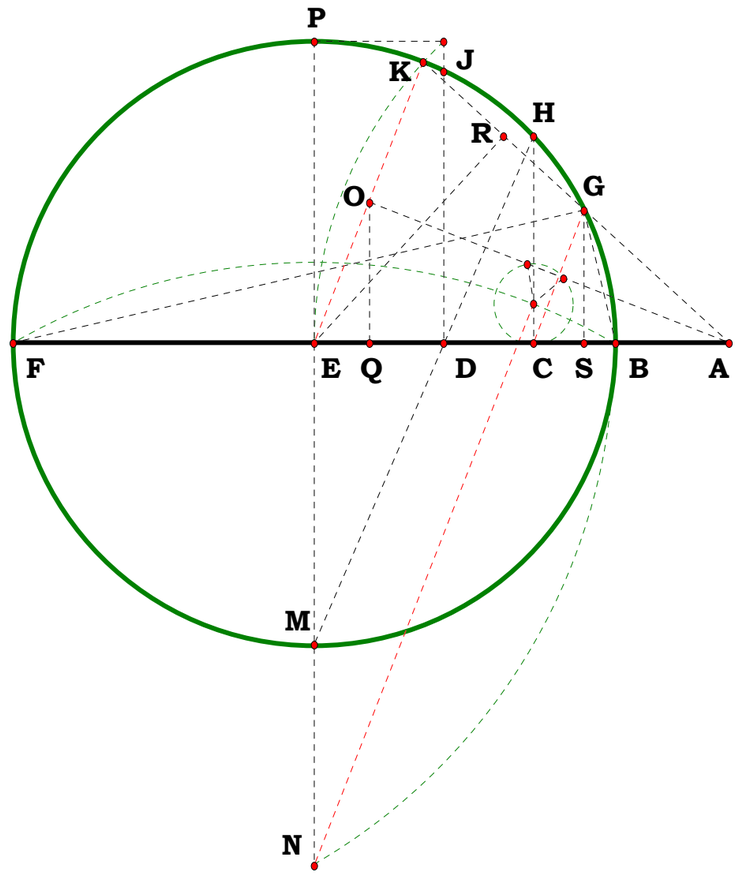
$$CD - \frac{\sqrt{{N_1}^2+N_2\cdot N_1}\cdot\left(8\cdot{N_1}^2+8\cdot N_1\cdot N_2+{N_2}^2\right)-4\cdot N_1\cdot\left(2\cdot N_1+N_2\right)\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)\cdot\left(2\cdot N_1+N_2-2\cdot\sqrt{{N_1}^2+N_2\cdot N_1}\right)}=0$$

$$CE - \frac{{N_2}^2}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad BC - \frac{N_1\cdot N_2}{2\cdot N_1+N_2}=0 \qquad EN - \frac{\sqrt{3}\cdot\sqrt{{N_2}^2}}{2}=0$$

$$KG - \frac{{N_2}^2}{2\cdot\left(2\cdot N_1+N_2\right)}=0 \qquad AG - \frac{2\cdot N_1\cdot\left(N_1+N_2\right)}{2\cdot N_1+N_2}=0 \qquad CS - \frac{N_1\cdot{N_2}^2\cdot\left(N_1+N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

$$AS - \frac{N_1\cdot\left(N_1+N_2\right)\cdot\left(8\cdot{N_1}^2+8\cdot N_1\cdot N_2+{N_2}^2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$

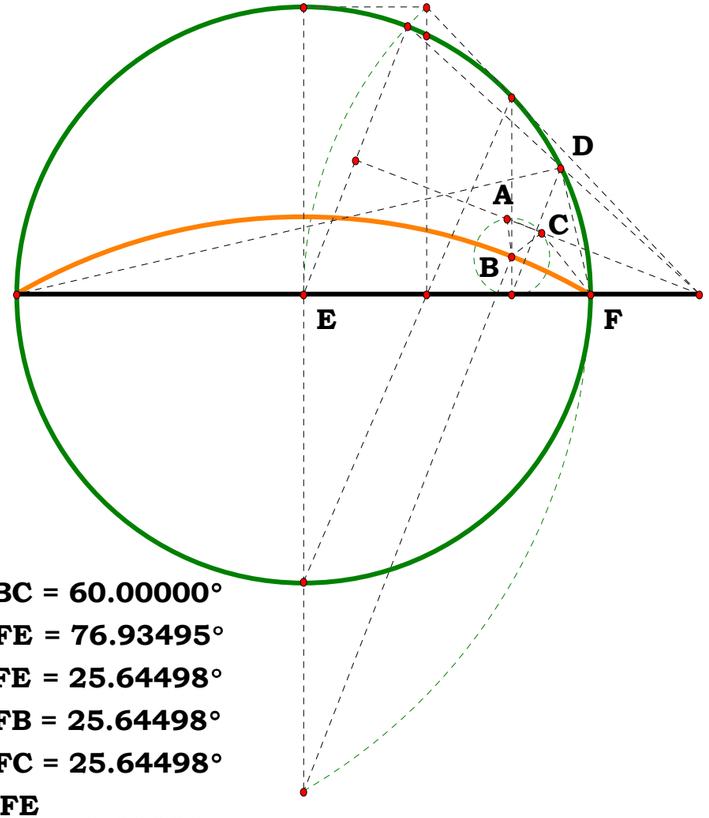
$$BS - \frac{{N_1}^2\cdot N_2\cdot\left(4\cdot N_1+3\cdot N_2\right)}{\left(2\cdot N_1+N_2\right)^3}=0$$



080400E

Trisection and Square Roots

This is the E plate, or the fifth of six.



$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$\frac{m\angle DFE}{m\angle BFE} = 3.00000$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

P

Unit.

$BF := 1$

Given.

$N := 4$

$AF := N + BF \quad AD := \sqrt{N \cdot AF} \quad BD := AD - N$

$BF := AF - N \quad DF := AF - AD \quad DJ := \sqrt{BD \cdot DF}$

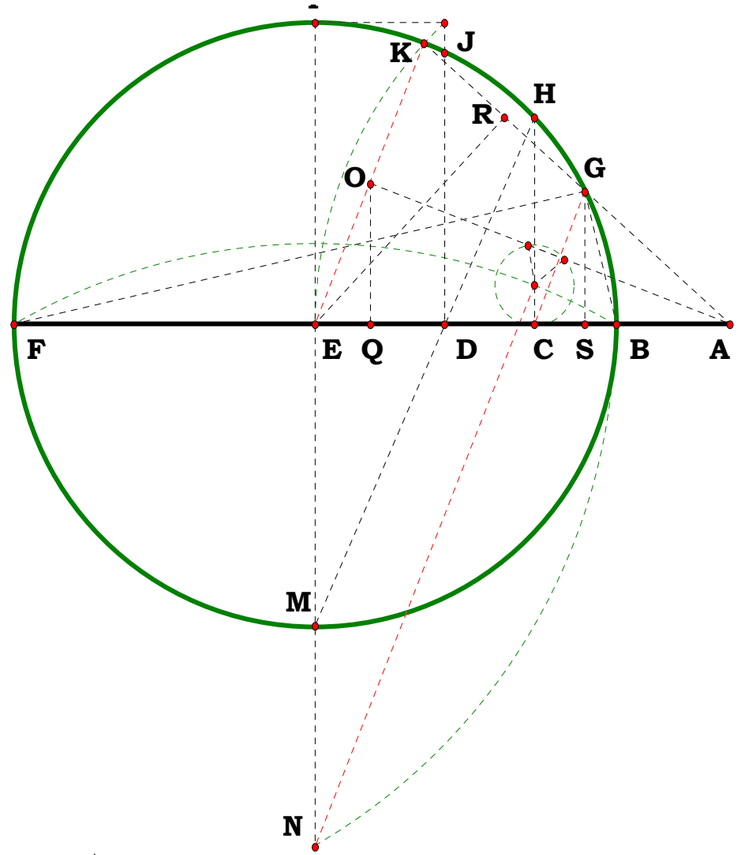
$BE := \frac{BF}{2} \quad EO := \frac{BE}{2} \quad AE := BE + N \quad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \quad DM := \sqrt{DE^2 + BE^2} \quad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \quad CD := \frac{DE \cdot DH}{DM} \quad CE := DE + CD \quad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \quad KG := \frac{2 \cdot EQ \cdot BE}{EO} \quad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \quad AS := AE - (DE + CD + CS) \quad BS := AS - N$



Definitions:

$N - N = 0 \quad AF - (N + 1) = 0 \quad AD - \sqrt{N^2 + N} = 0 \quad BD - \left(\sqrt{N^2 + N} - N\right) = 0$

$BF - (N + 1 - N) = 0 \quad DF - \left[N + 1 - \sqrt{N \cdot (N + 1)}\right] = 0 \quad DJ - \sqrt{(2 \cdot N + 1) \cdot \sqrt{N^2 + N} - \left(2 \cdot N^2 + 2 \cdot N\right)} = 0$

$BE - 2^{-1} \quad EO - 2^{-2} = 0 \quad AE - \frac{2 \cdot N + 1}{2} = 0 \quad EQ - \frac{1}{8 \cdot (2 \cdot N + 1)} = 0$

$$DE - \frac{2\cdot N - 2\cdot\sqrt{N^2+N}+1}{2} = 0 \qquad DM - \frac{\sqrt{2\cdot(2\cdot N+1)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)}+1\right]}}{2} = 0$$

$$HM - \left[(4\cdot N+2)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)}+1\right]\right]^{\frac{-1}{2}} = 0$$

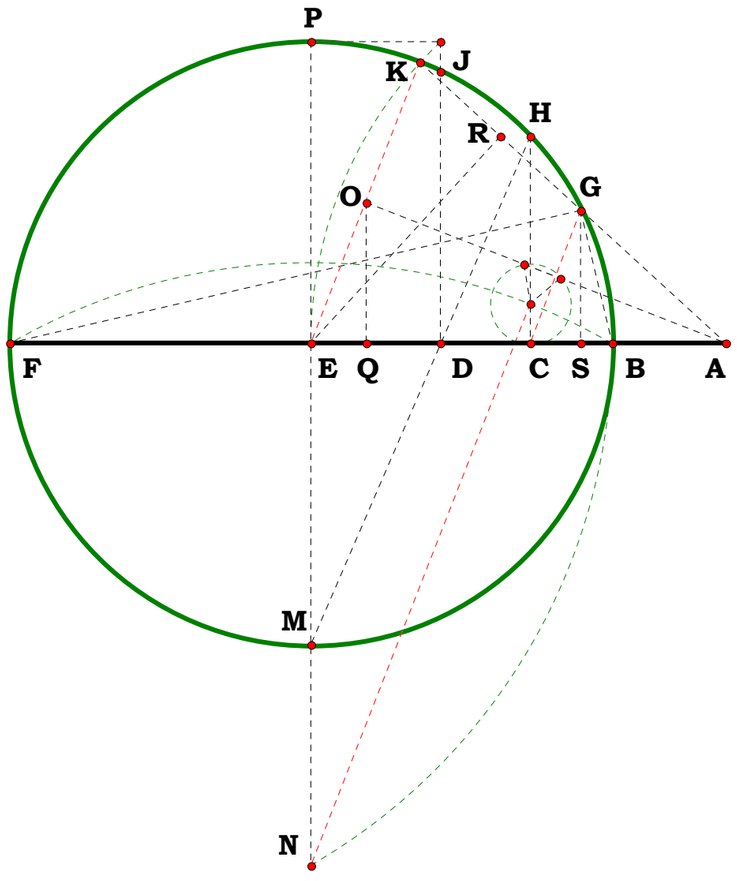
$$DH - \frac{(2\cdot N+1)\cdot\left(\sqrt{2}\cdot\sqrt{N^2+N}\right) - 2\cdot\sqrt{2}\cdot N\cdot(N+1)}{\sqrt{\left[(2\cdot N+1)\cdot\left[2\cdot N - 2\cdot\sqrt{N\cdot(N+1)}+1\right]\right]}} = 0$$

$$CD - \frac{\left(8\cdot N^2+8\cdot N+1\right)\cdot\sqrt{N^2+N} - \left(8\cdot N^3+12\cdot N^2+4\cdot N\right)}{(2\cdot N+1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2+N}+1\right)} = 0$$

$$CE - \frac{1}{2\cdot(2\cdot N+1)} = 0 \qquad BC - \frac{N}{2\cdot N+1} = 0 \qquad EN - \frac{\sqrt{3}}{2} = 0$$

$$KG - \frac{1}{2\cdot(2\cdot N+1)} = 0 \qquad AG - \frac{2\cdot N\cdot(N+1)}{2\cdot N+1} = 0 \qquad CS - \frac{N\cdot(N+1)}{(2\cdot N+1)^3} = 0$$

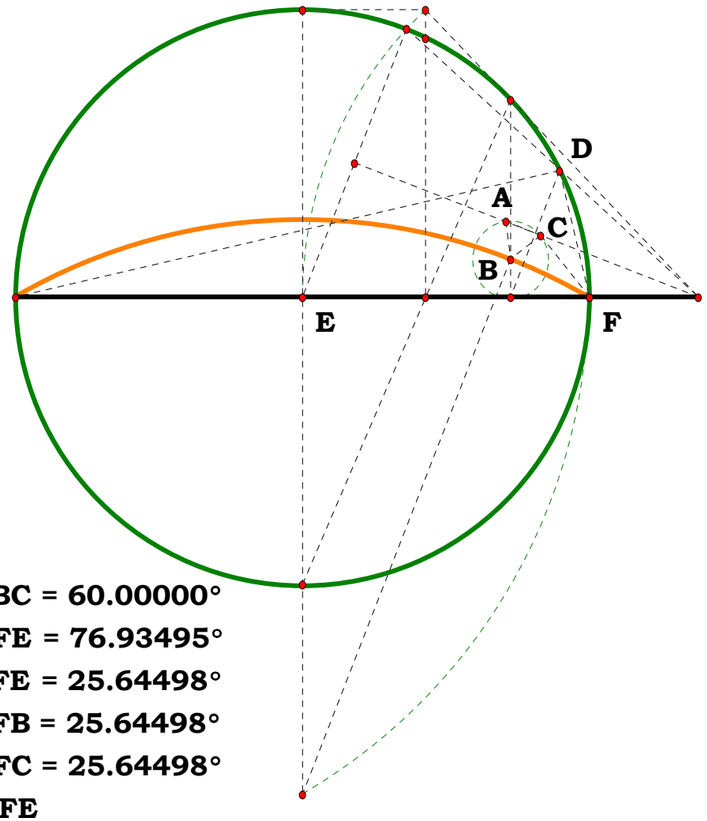
$$AS - \frac{N\cdot(N+1)\cdot\left(8\cdot N^2+8\cdot N+1\right)}{(2\cdot N+1)^3} = 0 \qquad BS - \frac{N^2\cdot(4\cdot N+3)}{(2\cdot N+1)^3} = 0$$



080400F

This is the F plate, or the sixth of six.

Trisection and Square Roots



$m\angle ABC = 60.00000°$

$m\angle DFE = 76.93495°$

$m\angle BFE = 25.64498°$

$m\angle CFB = 25.64498°$

$m\angle DFC = 25.64498°$

$$\frac{m\angle DFE}{m\angle BFE} = 3.00000$$

$m\angle DFE-(m\angle BFE+m\angle CFB+m\angle DFC) = 0.00000°$

Unit

$BF := 1$

Given.

$N := 3 \qquad AB := N$

$AF := AB + BF \qquad AD := \sqrt{AB \cdot AF}$

$BD := AD - AB \qquad DF := AF - AD \qquad DJ := \sqrt{BD \cdot DF}$

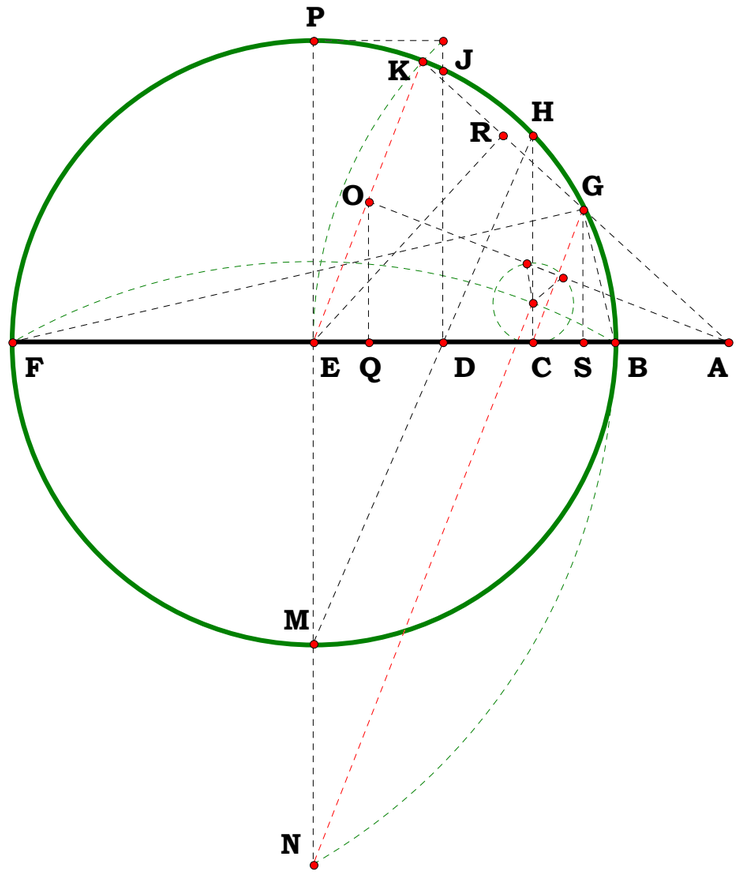
$BE := \frac{BF}{2} \qquad EO := \frac{BE}{2} \qquad AE := BE + AB \qquad EQ := \frac{EO^2}{AE}$

$DE := AE - AD \qquad DM := \sqrt{DE^2 + BE^2} \qquad HM := \frac{BE \cdot BF}{DM}$

$DH := HM - DM \qquad CD := \frac{DE \cdot DH}{DM} \qquad CE := DE + CD \qquad BC := BE - CE$

$EN := \sqrt{BF^2 - BE^2} \qquad KG := \frac{2 \cdot EQ \cdot BE}{EO} \qquad AG := AE - KG$

$CS := \frac{2 \cdot EQ \cdot AG}{AE} \qquad AS := AE - (DE + CD + CS) \qquad BS := AS - AB$



Definitions:

$AB - N = 0 \qquad BF - 1 = 0 \qquad AF - (N + 1) = 0$

$AD - \sqrt{N \cdot (N + 1)} = 0 \qquad BD - \sqrt{N \cdot (N + 1)} - N \qquad DF - \left[N + 1 - \sqrt{N \cdot (N + 1)}\right] = 0$

$DJ - \sqrt{\sqrt{N^2 + 1 \cdot N} \cdot (2 \cdot N + 1) - 2 \cdot N - 2 \cdot N^2} = 0 \qquad BE - \frac{1}{2} = 0 \qquad EO - \frac{1}{4} = 0$

$AE - \frac{2 \cdot N + 1}{2} = 0 \qquad EQ - \frac{1^2}{8 \cdot (2 \cdot N + 1)} = 0 \qquad DE - \frac{2 \cdot N + 1 - 2 \cdot \sqrt{N^2 + 1 \cdot N}}{2} = 0$

$$DM - \frac{\sqrt{(4\cdot N+2)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}}{2} = 0$$

$$HM - \frac{\sqrt{2}}{2\cdot\sqrt{(2\cdot N+1)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}} = 0$$

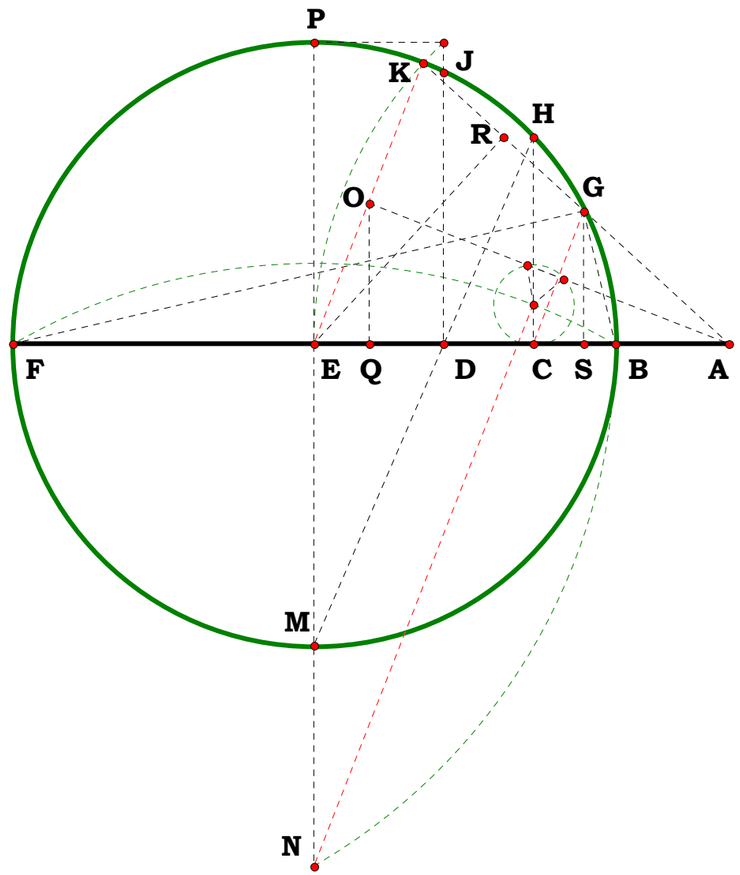
$$DH - \frac{(2\cdot N+1)\cdot\sqrt{2\cdot N^2+2\cdot N}-\left[2\cdot\sqrt{2}\cdot N\cdot(N+1)\right]}{\sqrt{(2\cdot N+1)\cdot\left(2\cdot N-2\cdot\sqrt{N^2+N}+1\right)}} = 0$$

$$CD - \frac{\left(8\cdot N^2+8\cdot N+1\right)\cdot\sqrt{N^2+N}-4\cdot N\cdot(N+1)\cdot(2\cdot N+1)}{(2\cdot N+1)\cdot\left(2\cdot N+1-2\cdot\sqrt{N^2+N}\right)} = 0$$

$$CE - \frac{1}{4\cdot N+2} = 0 \qquad BC - \frac{N}{2\cdot N+1} = 0 \qquad EN - \frac{\sqrt{3}}{2} = 0$$

$$KG - \frac{1}{4\cdot N+2} = 0 \qquad AG - \frac{2\cdot N\cdot(N+1)}{2\cdot N+1} = 0 \qquad CS - \frac{N\cdot(N+1)}{(2\cdot N+1)^3} = 0$$

$$AS - \frac{N\cdot(N+1)\cdot\left(8\cdot N^2+8\cdot N+1\right)}{(2\cdot N+1)^3} = 0 \qquad BS - \frac{N^2\cdot(4\cdot N+3)}{(2\cdot N+1)^3} = 0$$



080700

Unit.

$AB := 1$

Given.

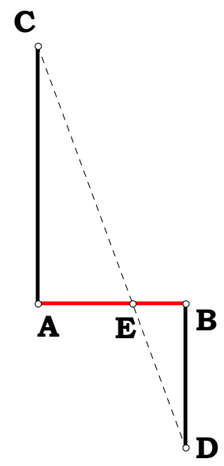
$N_1 := 9 \quad AC := N_1$

$N_2 := 2 \quad BD := N_2$

Descriptions.

Proportion Series II

Divide AB into the same ratio as AB:CD.



$$BE := \frac{AC \cdot AB}{AC + BD} \qquad CE := \frac{BD \cdot AB}{AC + BD}$$

$$BE + CE - AB = 0 \qquad \frac{AC}{BD} - \frac{BE}{CE} = 0$$

Definitions.

$$BE - \frac{N_1}{N_1 + N_2} = 0 \qquad CE - \frac{N_2}{N_1 + N_2} = 0$$

Unit.

$AB := 1$

Given.

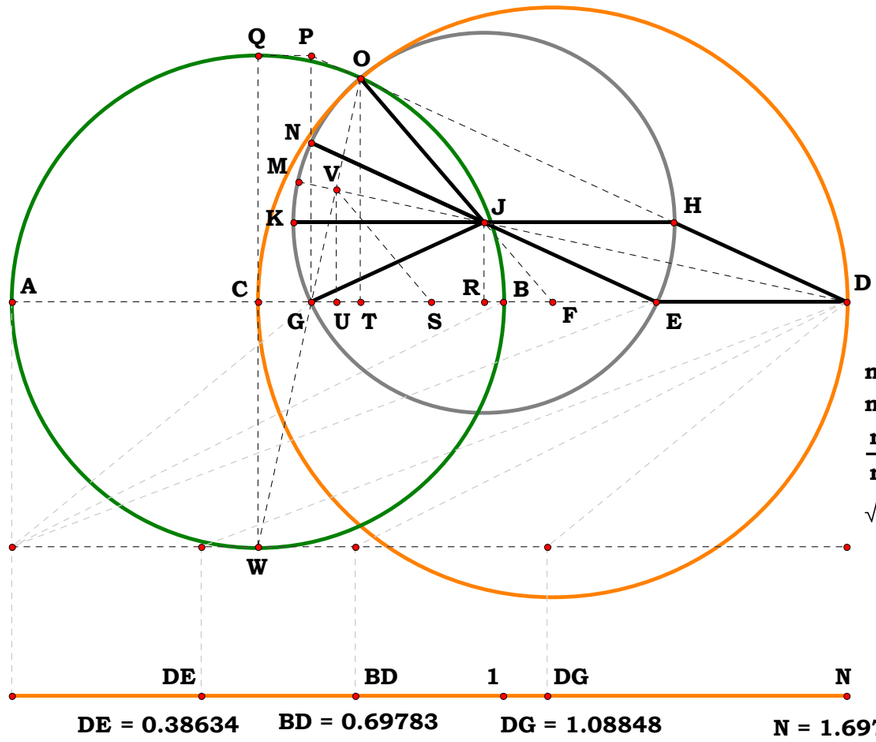
$N := 3 \qquad AD := N$

000822A

Descriptions.

This square root figure affords another approach to proofing the Archimedean Paper Trisecter.

Square Root and the Archimedean Paper Trisecter.



$m\angle OJG = 74.01600°$

$m\angle NJK = 24.67200°$

$\frac{m\angle OJG}{m\angle NJK} = 3.00000$

$\sqrt{N \cdot BD} - DG = 0.00000$

$DE = 0.38634 \qquad BD = 0.69783 \qquad DG = 1.08848 \qquad N = 1.69783$

$BD := AD - AB \qquad DG := \sqrt{BD \cdot AD} \qquad BC := \frac{AB}{2}$

$CD := BD + BC \qquad DF := \frac{CD}{2} \qquad BG := DG - BD$

$CG := BC - BG \qquad GW := \sqrt{CG^2 + BC^2} \qquad OW := \frac{BC \cdot AB}{GW}$

$GO := OW - GW \qquad GU := \frac{CG \cdot GO}{2 \cdot GW} \qquad DU := DG - GU$

$OT := \frac{BC \cdot GO}{GW} \qquad US := \sqrt{\left(\frac{DF}{2}\right)^2 - \left(\frac{OT}{2}\right)^2} \qquad DS := DU - US$

$JR := \frac{OT \cdot DF}{2 \cdot DS} \qquad JF := \frac{DF \cdot JR}{OT} \qquad EJ := DF - JF$

$FR := \frac{2 \cdot US \cdot JR}{OT} \qquad DR := DF + FR \qquad ER := \sqrt{EJ^2 - JR^2}$

$DE := DR - ER \qquad DE - EJ = 0$

$\frac{(2 \cdot N-1) \cdot (((2 \cdot N-1) \cdot \sqrt{N^2-N}+2 \cdot N)-2 \cdot N^2)}{(((4 \cdot N-2) \cdot \sqrt{N^2-N}+4 \cdot N)-4 \cdot N^2)+1} = 0.38634 \qquad \frac{(2 \cdot N-1) \cdot (((2 \cdot N-1) \cdot \sqrt{N^2-N}+2 \cdot N)-2 \cdot N^2)}{(((4 \cdot N-2) \cdot \sqrt{N^2-N}+4 \cdot N)-4 \cdot N^2)+1} - DE = 0.00000$

As one can see, the APT actually multiplies an angle.

Definitions.

$BD - (N - 1) = 0 \qquad DG - \sqrt{N^2 - N} = 0 \qquad BC - \frac{1}{2} = 0 \qquad CD - \frac{2 \cdot N - 1}{2} = 0 \qquad DF - \frac{2 \cdot N - 1}{4} = 0 \qquad BG - \left(\sqrt{N^2 - N} - N + 1\right) = 0 \qquad CG - \frac{2 \cdot N - 2 \cdot \sqrt{N^2 - N} - 1}{2} = 0$

$GW - \frac{\sqrt{(2 \cdot N - 1) \cdot \left(2 \cdot N - 2 \cdot \sqrt{N^2 - N} - 1\right)}}{\sqrt{2}} = 0 \qquad OW - \frac{\sqrt{2}}{2 \cdot \sqrt{(2 \cdot N - 1) \cdot \left(2 \cdot N - 2 \cdot \sqrt{N^2 - N} - 1\right)}} = 0 \qquad GO - \frac{(2 \cdot N - 1) \cdot \left(\sqrt{2} \cdot \sqrt{N^2 - N}\right) - 2 \cdot \sqrt{2} \cdot N \cdot (N - 1)}{\sqrt{(2 - 4 \cdot N) \cdot \sqrt{N^2 - N} + (2 \cdot N - 1)^2}} = 0$

$$GU - \frac{\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot\sqrt{N^2 - N} + \left(12\cdot N^2 - 8\cdot N^3 - 4\cdot N\right)}{2\cdot\left[(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N + 1\right]} = 0 \qquad DU - \frac{\sqrt{N^2 - N}}{2\cdot(2\cdot N - 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0 \qquad OT - \frac{(2\cdot N - 1)\cdot\sqrt{N\cdot(N-1)} + 2\cdot N - 2\cdot N^2}{(2\cdot N - 1)\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0$$

$$US - \frac{4\cdot N^2 - 4\cdot N - 1}{8\cdot(2\cdot N - 1)} = 0 \qquad DS - \frac{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1}{8\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)} = 0 \qquad JR - \frac{(1 - 2\cdot N)\cdot\sqrt{N\cdot(N-1)} + 2\cdot N^2 - 2\cdot N}{(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N - 1} = 0 \qquad JF - \frac{(2\cdot N - 1)^2\cdot\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)}{4\cdot\left[(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1\right]} = 0$$

$$EJ - \frac{(2\cdot N - 1)\cdot\left[(2\cdot N - 1)\cdot\sqrt{N^2 - N} + 2\cdot N - 2\cdot N^2\right]}{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1} = 0 \qquad FR - \frac{\left(2\cdot N - 2\cdot\sqrt{N^2 - N} - 1\right)\cdot\left(4\cdot N^2 - 4\cdot N - 1\right)}{4\cdot\left[(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1\right]} = 0 \qquad DR - \frac{\sqrt{N^2 - N}}{(4\cdot N - 2)\cdot\sqrt{N^2 - N} + 4\cdot N - 4\cdot N^2 + 1} = 0$$

$$ER - \frac{\sqrt{4\cdot N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N^2 - N} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N^2 - N} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}} = 0$$

Here is where Mathcad 15 Uncle call Uncle! It cannot reduce the following equation and I am at a loss as to how to effect reductions for it in the time I am willing to spend on it.

$$DE - \left[\frac{\sqrt{N\cdot(N-1)}}{(4\cdot N - 2)\cdot\sqrt{N\cdot(N-1)} + 4\cdot N - 4\cdot N^2 + 1} - \frac{2\cdot\sqrt{N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N\cdot(N-1)} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N\cdot(N-1)} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}}\right] = 0$$

DE − EJ = 0

$$\left[\frac{\sqrt{N\cdot(N-1)}}{(4\cdot N - 2)\cdot\sqrt{N\cdot(N-1)} + 4\cdot N - 4\cdot N^2 + 1} - \frac{2\cdot\sqrt{N^2\cdot(N-1)^2\cdot\left[(4 - 8\cdot N)\cdot\sqrt{N\cdot(N-1)} + 8\cdot N^2 - 8\cdot N + 1\right]}}{\sqrt{\left(48\cdot N^2 - 32\cdot N^3 - 8\cdot N - 4\right)\cdot\sqrt{N\cdot(N-1)} + 32\cdot N^4 - 64\cdot N^3 + 28\cdot N^2 + 4\cdot N + 1}} - \frac{(2\cdot N - 1)\cdot\left[(1 - 2\cdot N)\cdot\sqrt{N^2 - N} + 2\cdot N^2 - 2\cdot N\right]}{(2 - 4\cdot N)\cdot\sqrt{N^2 - N} + 4\cdot N^2 - 4\cdot N - 1}\right] = 0$$

If you ask Mathcad to reduce the above equation, it will simply spred it out over several pages and quit.

Unit.

$AB := 1$

Given.

$N := 1.43693 \quad AC := N$

000822B

In the 2015 revision of the DQ, I got as fars as a blank Mathcad template like this one. So, yea, I have put doing this off for a special long time.

Descriptions.

$$BO := \frac{AB}{2} \quad BC := AC - AB \quad DO := BO$$

$$CO := BO + BC \quad CD := \sqrt{CO^2 + DO^2}$$

$$DF := \frac{DO \cdot AB}{CD} \quad CF := CD - DF \quad CH := \frac{CD \cdot CF}{CO}$$

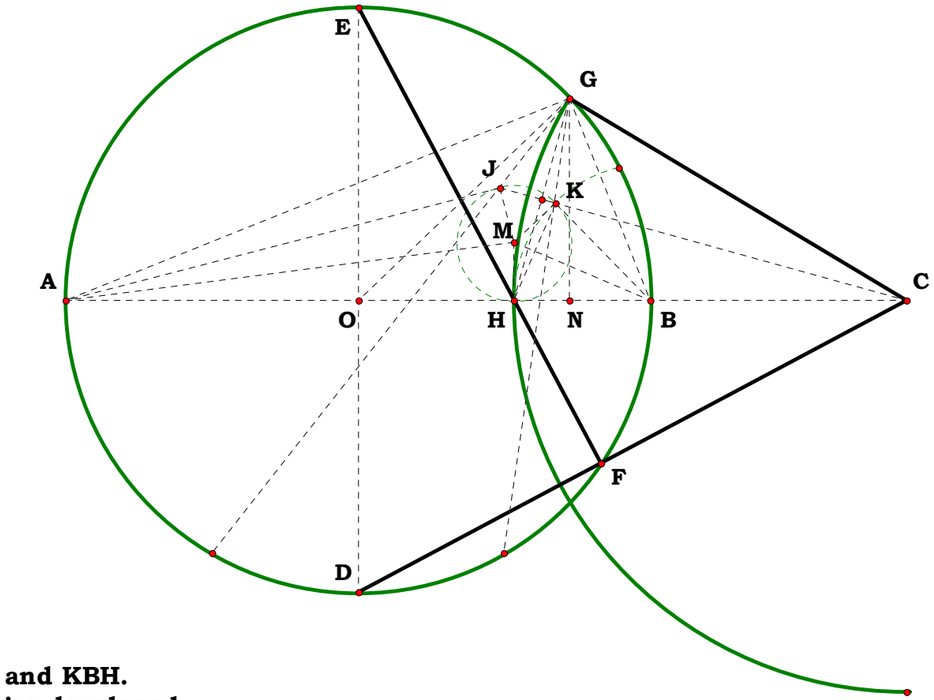
$$BH := CH - BC \quad GO := BO \quad CN := \frac{CO^2 - GO^2 + CH^2}{2 \cdot CO}$$

$$GN := \sqrt{CH^2 - CN^2} \quad BN := CN - BC$$

To be completed by simply dividing the cords, GCH and KBH. This should help show that angle trisection, the entire developed figure is a proportion to the square root figure.

Definitions.

Show the trisection in a circle for any square root that also divides the circle into six equal cords.



$m\angle KCH = 15.47574°$

$m\angle MBH = 22.73787°$

$m\angle GBH = 68.21360°$

$m\angle MBH = 22.73787°$

$\frac{m\angle GBH}{m\angle MBH} = 3.00000$

$m\angle EGF = 30.00000°$

$m\angle KJM = 60.00000°$

$N = 1.43693$

Unit.

$AB := 1$

Given.

$N := .3 \quad BC := N$

082300

Descriptions.

$AC := AB - N \qquad BE := \frac{BC}{2} \qquad AE := AC + BE$

$AN := \sqrt{BE^2 + AE^2} \qquad FN := \frac{BE \cdot BC}{AN} \qquad DE := \frac{BE^2}{AE}$

$BD := BE + DE \quad AD := AB - BD \qquad AH := \frac{AE^2 - BE^2 + AD^2}{2 \cdot AE}$

$BH := AB - AH \qquad CH := BC - BH$

Definitions.

$AC - (1 - N) = 0 \qquad BE - \frac{N}{2} = 0 \qquad AE - \left[\frac{(2 - N)}{2}\right] = 0$

$AN - \frac{\sqrt{N^2 - 2 \cdot N + 2}}{\sqrt{2}} = 0 \qquad FN - \frac{\sqrt{2} \cdot N^2}{2 \cdot \sqrt{N^2 - 2 \cdot N + 2}} = 0$

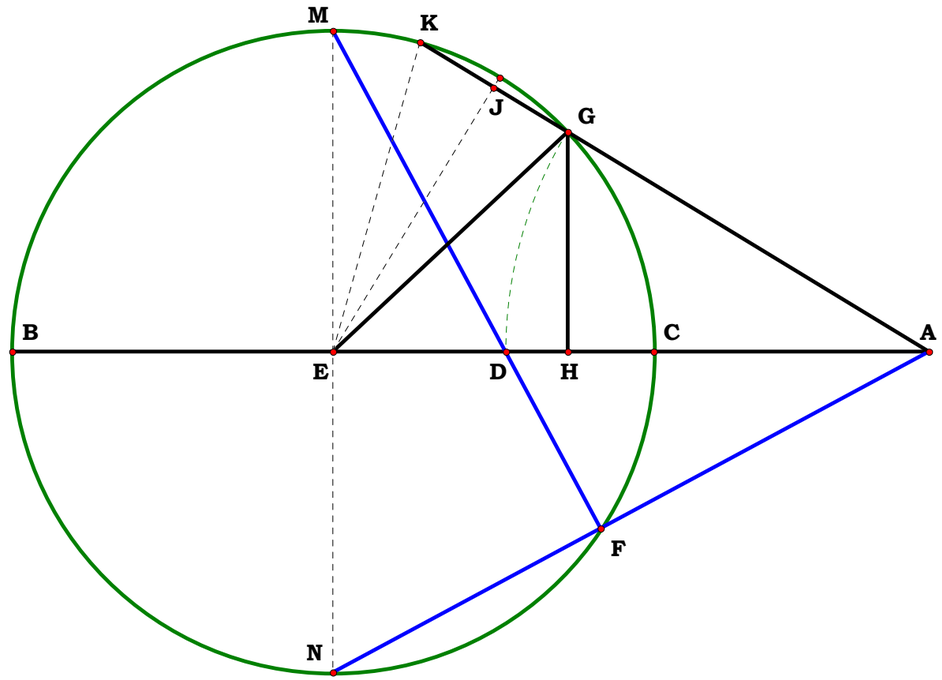
$DE - \frac{N^2}{2 \cdot (2 - N)} = 0$

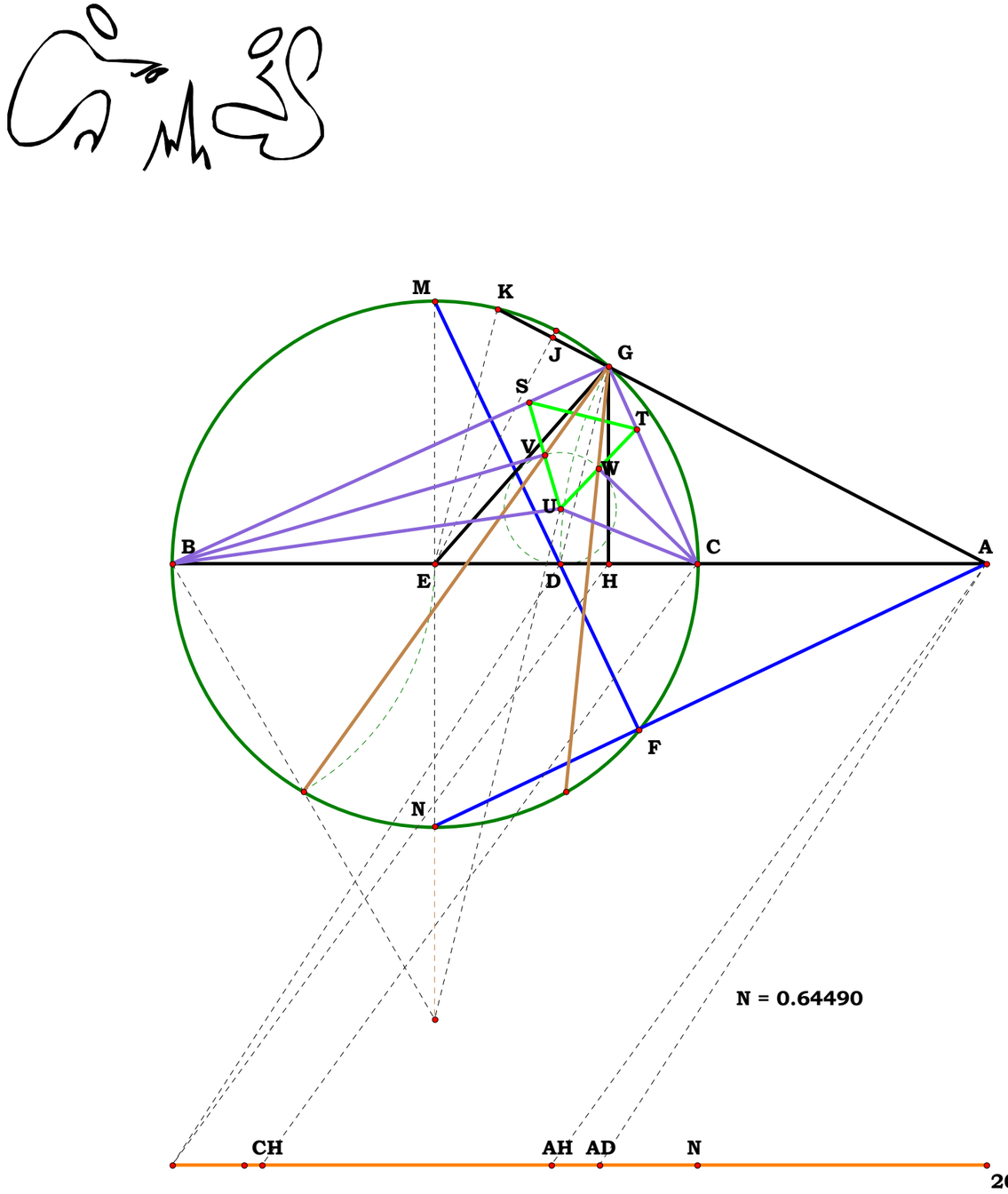
$BD - \left[\frac{N}{(2 - N)}\right] = 0 \qquad AD - \left[\frac{2 \cdot (1 - N)}{(2 - N)}\right] = 0 \qquad AH - \frac{(1 - N) \cdot \left(N^2 - 8 \cdot N + 8\right)}{(2 - N)^3} = 0$

$BH - \left[\frac{N \cdot (4 - 3 \cdot N)}{(2 - N)^3}\right] = 0 \qquad CH - \frac{N \cdot (4 - N) \cdot (1 - N)^2}{(2 - N)^3} = 0$

Trisection In A Square Root Figure

Given the square root figure drawn for trisection, what is AR given AB and AD? A slightly different apprach than the one on 04.





BD = 0.47590

$\frac{N}{2-N}$ = 0.47590

$\frac{N}{2-N}$-BD = 0.00000

AD = 0.52410

$\frac{2\cdot(N-1)}{N-2}$ = 0.52410

$\frac{2\cdot(N-1)}{N-2}$-AD = 0.00000

AH = 0.46475

$\frac{(N-1)\cdot((N^2-8\cdot N)+8)}{(N-2)^3}$ = 0.46475

$\frac{(N-1)\cdot((N^2-8\cdot N)+8)}{(N-2)^3}$-AH = 0.00000

BH = 0.53525

$\frac{N\cdot(3\cdot N-4)}{(N-2)^3}$ = 0.53525

$\frac{N\cdot(3\cdot N-4)}{(N-2)^3}$-BH = 0.00000

CH = 0.10965

$\frac{N\cdot(N-4)\cdot(N-1)^2}{(N-2)^3}$ = 0.10965

$\frac{N\cdot(N-4)\cdot(N-1)^2}{(N-2)^3}$-CH = 0.00000

m∠GEM = 41.29679°

m∠KEM = 13.76560°

$\frac{m\angle GEM}{m\angle KEM}$ = 3.00000

m∠BGC = 90.00000°

m∠VGW = 30.00000°

$\frac{m\angle BGC}{m\angle VGW}$ = 3.00000

m∠GCD = 65.64840°

m∠UCD = 21.88280°

$\frac{m\angle GCD}{m\angle UCD}$ = 3.00000

m∠GBD = 24.35160°

m∠GBV = 8.11720°

$\frac{m\angle GBD}{m\angle GBV}$ = 3.00000

m∠UST = 60.00000°

BC = 0.64490

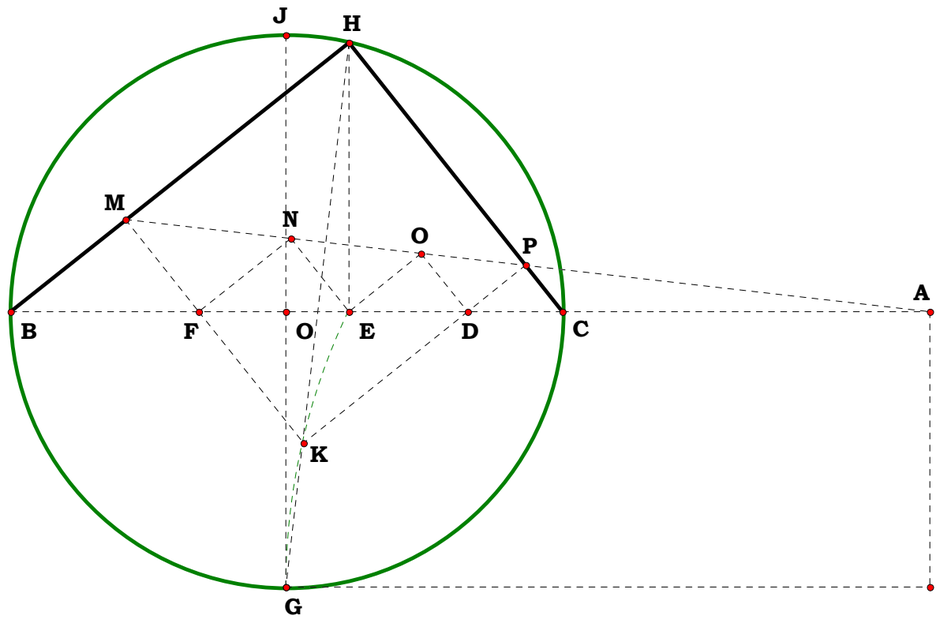
BC-N = 0.00000

Unit.
Given.
Descriptions.
Definitions.

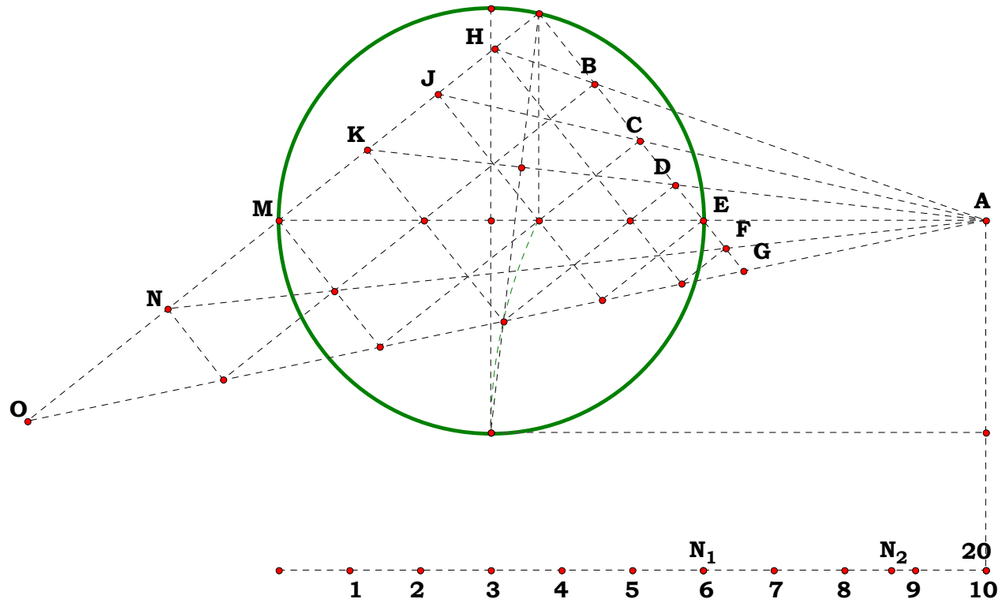
Quad Roots Etc.

090100Z

I have had this plate in and out of revisions since 00 but it is so simple and straight forward, I just like looking at it.



And the figure can be expanded and this I find interesting.



$\frac{AB}{AC}^{\frac{1}{4}} = 1.25743$ $\frac{AB}{AC}^{\frac{1}{4}} - \frac{AD}{AC} = 0.00000$

$\frac{AB}{AC}^{\frac{2}{4}} = 1.58114$ $\frac{AB}{AC}^{\frac{2}{4}} - \frac{AE}{AC} = 0.00000$

$\frac{AB}{AC}^{\frac{3}{4}} = 1.98818$ $\frac{AB}{AC}^{\frac{3}{4}} - \frac{AF}{AC} = 0.00000$

$\frac{AC}{AC} = 1.00000$

$\frac{AD}{AC} = 1.25743$

$\frac{AE}{AC} = 1.58114$

$\frac{AF}{AC} = 1.98818$

$\frac{AB}{AC} = 2.50000$

AC = 6.16580 cm
AD = 7.75308 cm
AE = 9.74899 cm
AO = 10.79015 cm
AF = 12.25870 cm
AB = 15.41450 cm

Unit.

$BG := 1$

Given.

$N_1 := 8$

$N_2 := 10$

090300A

Descriptions.

Ratios In Trisection

How does BF vary with BC? How does DF vary with BC?

$BE := \frac{BG}{2}$ $EM := BE$ $BO := \sqrt{2 \cdot BE^2}$ $EN := BE$ $EK := \frac{BE \cdot BE}{BO}$

$KN := EN - EK$ $BK := \frac{BO}{2}$ $BN := \sqrt{BK^2 + KN^2}$ $BD := \frac{BN^2}{BG}$ $BC := BD - BD \cdot \frac{N_1}{N_2}$

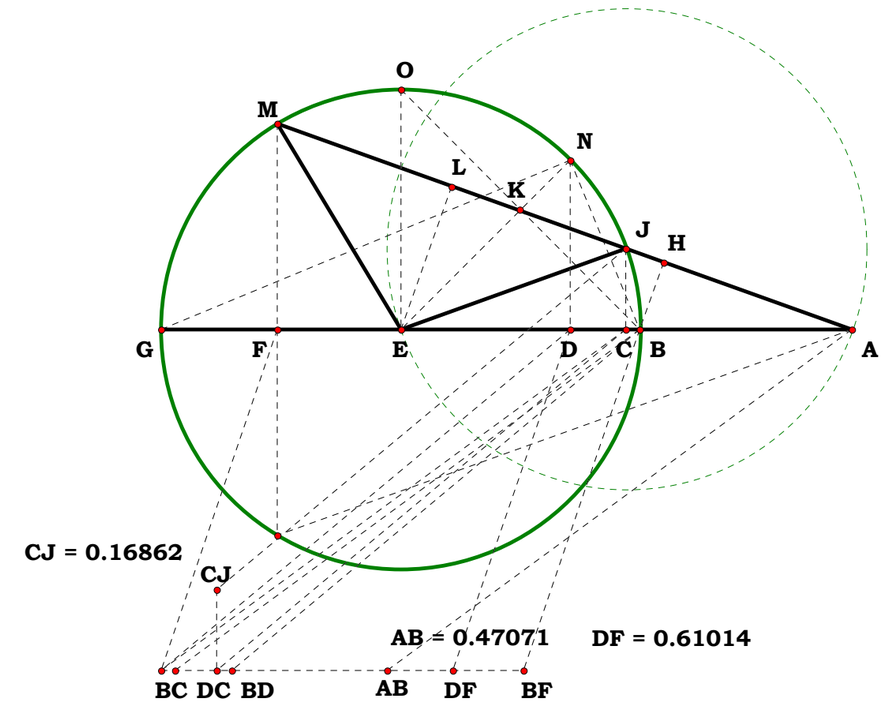
$CG := BG - BC$ $CJ := \sqrt{BC \cdot CG}$ $AJ := BE$ $AC := \sqrt{AJ^2 - CJ^2}$ $AB := AC - BC$

$AE := AB + BE$ $JH := \frac{CJ^2}{AJ}$ $AH := AJ - JH$ $AL := \frac{AH \cdot AE}{AC}$ $JL := AL - AJ$

$LM := JL$ $AM := AL + LM$ $AF := \frac{AH \cdot AM}{AC}$ $BF := AF - AB$ $DF := BF - BD$

$CJ = 0.168616$ $BD = 0.146447$ $DF = 0.610136$ $BF = 0.756583$

$BC = 0.029289$ $CG = 0.970711$ $AB = 0.441421$ $DF = 0.610136$



$BC = 0.02929$ $BD = 0.14645$ $BF = 0.75658$

$CG = 0.97071$ $DC = 0.11716$

Definitions.

$BC - \frac{\left(\sqrt{2} - 2\right) \cdot \left(N_1 - N_2\right)}{4 \cdot N_2} = 0$ $BE - \frac{1}{2} = 0$ $EM - \frac{1}{2} = 0$ $BO - \frac{\sqrt{2}}{2} = 0$ $EN - \frac{1}{2} = 0$

$EK - \frac{\sqrt{2}}{4} = 0$ $KN - \left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) = 0$ $BK - \frac{\sqrt{2}}{4} = 0$ $BN - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0$

$BD - \frac{2 - \sqrt{2}}{4} = 0$ $BC - \frac{\left(\sqrt{2} - 2\right) \cdot \left(N_1 - N_2\right)}{4 \cdot N_2} = 0$ $CG - \frac{\left(N_2 - N_1\right) \cdot \left(\sqrt{2} + 2\right) + 4 \cdot N_1}{4 \cdot N_2} = 0$ $CJ - \frac{\sqrt{\left(N_1 - N_2\right) \cdot \left[N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right) - N_2\right]}}{2\sqrt{2} \cdot N_2} = 0$ $AJ - \frac{1}{2} = 0$ $AC - \frac{N_2 + N_1 \cdot \left(\sqrt{2} - 1\right)}{2\sqrt{2} \cdot N_2} = 0$

$AB - \frac{N_2 \cdot \left(\sqrt{2} - 1\right) - N_1 \cdot \left(\sqrt{2} - 2\right)}{2 \cdot N_2} = 0$ $AE - \frac{\sqrt{2} \cdot N_2 - N_1 \cdot \left(\sqrt{2} - 2\right)}{2 \cdot N_2} = 0$ $JH - \frac{\left(N_1 - N_2\right) \cdot \left[N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right) - N_2\right]}{4 \cdot N_2^2} = 0$ $AH - \frac{\left[N_2 + N_1 \cdot \left(\sqrt{2} - 1\right)\right]^2}{4 \cdot N_2^2} = 0$

$AL - \frac{\left(2 \cdot N_1 \cdot N_2 - 2 \cdot N_1^2\right) \cdot \sqrt{2} + 3 \cdot N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2}{2 \cdot N_2^2} = 0$ $JL - \frac{N_1 \cdot \left[N_2 \cdot \left(2 \cdot \sqrt{2} - 2\right) - N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right)\right]}{2 \cdot N_2^2} = 0$ $LM - \frac{N_1 \cdot \left[N_2 \cdot \left(2 \cdot \sqrt{2} - 2\right) - N_1 \cdot \left(2 \cdot \sqrt{2} - 3\right)\right]}{2 \cdot N_2^2} = 0$

$$BF - \frac{\left(\frac{7\cdot\sqrt{2}}{4}+\frac{5}{2}\right)\cdot\left(3\cdot N_1+N_2-2\cdot\sqrt{2}\cdot N_1\right)\cdot\left(4\cdot N_1-3\cdot N_2-3\cdot\sqrt{2}\cdot N_1+2\cdot\sqrt{2}\cdot N_2\right)^2}{{N_2}^3}=0$$

$$AM - \left[\frac{\left(N_2+\sqrt{2}\cdot N_1\right)\cdot\left[N_2+N_1\cdot\left(3\cdot\sqrt{2}-4\right)\right]}{2\cdot{N_2}^2}\right]=0 \qquad AF - \frac{\sqrt{2}\cdot\left(N_2+\sqrt{2}\cdot N_1\right)\cdot\left(N_2-N_1+\sqrt{2}\cdot N_1\right)\cdot\left(N_2-4\cdot N_1+3\cdot\sqrt{2}\cdot N_1\right)}{4\cdot{N_2}^3}=0$$

$$DF - \left[\frac{3\cdot N_1\cdot\left[\left(\frac{20}{3}-\frac{14\cdot\sqrt{2}}{3}\right)\cdot{N_1}^2+\left(6\cdot\sqrt{2}-8\right)\cdot N_1\cdot N_2+\left(2-\sqrt{2}\right)\cdot{N_2}^2\right]}{4\cdot{N_2}^3}\right]=0$$

$$\frac{BF}{BC}=25.831354 \qquad \frac{DF}{BC}=20.831354$$

$$\frac{BF}{BC} - \frac{4\cdot\left(\frac{7\cdot\sqrt{2}}{4}+\frac{5}{2}\right)\cdot\left(3\cdot N_1+N_2-2\cdot\sqrt{2}\cdot N_1\right)\cdot\left(4\cdot N_1-3\cdot N_2-3\cdot\sqrt{2}\cdot N_1+2\cdot\sqrt{2}\cdot N_2\right)^2}{{N_2}^2\cdot\left(\sqrt{2}-2\right)\cdot\left(N_1-N_2\right)}=0$$

$$\frac{DF}{BC} - \frac{\left(6\cdot{N_1}^2\cdot N_2-3\cdot N_1\cdot{N_2}^2+4\cdot\sqrt{2}\cdot{N_1}^3-6\cdot{N_1}^3-6\cdot\sqrt{2}\cdot{N_1}^2\cdot N_2\right)}{{N_2}^2\cdot\left(N_1-N_2\right)}=0$$

Unit.

$BC := 1$

Given.

$N := 2$

091600

Descriptions.

$BJ := N \quad BE := \sqrt{BC \cdot BJ} \quad CJ := BJ - BC \quad CI := \frac{CJ}{2}$

$IO := CI \quad NO := CJ \quad CR := CJ \quad CE := BE - BC$

$EI := CI - CE \quad EJ := CJ - CE \quad EL := \sqrt{CE \cdot EJ}$

$EG := \frac{EI \cdot EL}{EL + IO} \quad GI := EI - EG \quad GO := \sqrt{GI^2 + IO^2}$

$OP := GO \quad IP := IO + OP \quad EF := \frac{EI \cdot EL}{EL + IP} \quad FI := EI - EF$

$FO := \sqrt{FI^2 + IO^2} \quad OK := \frac{IO \cdot NO}{FO} \quad FK := OK - FO$

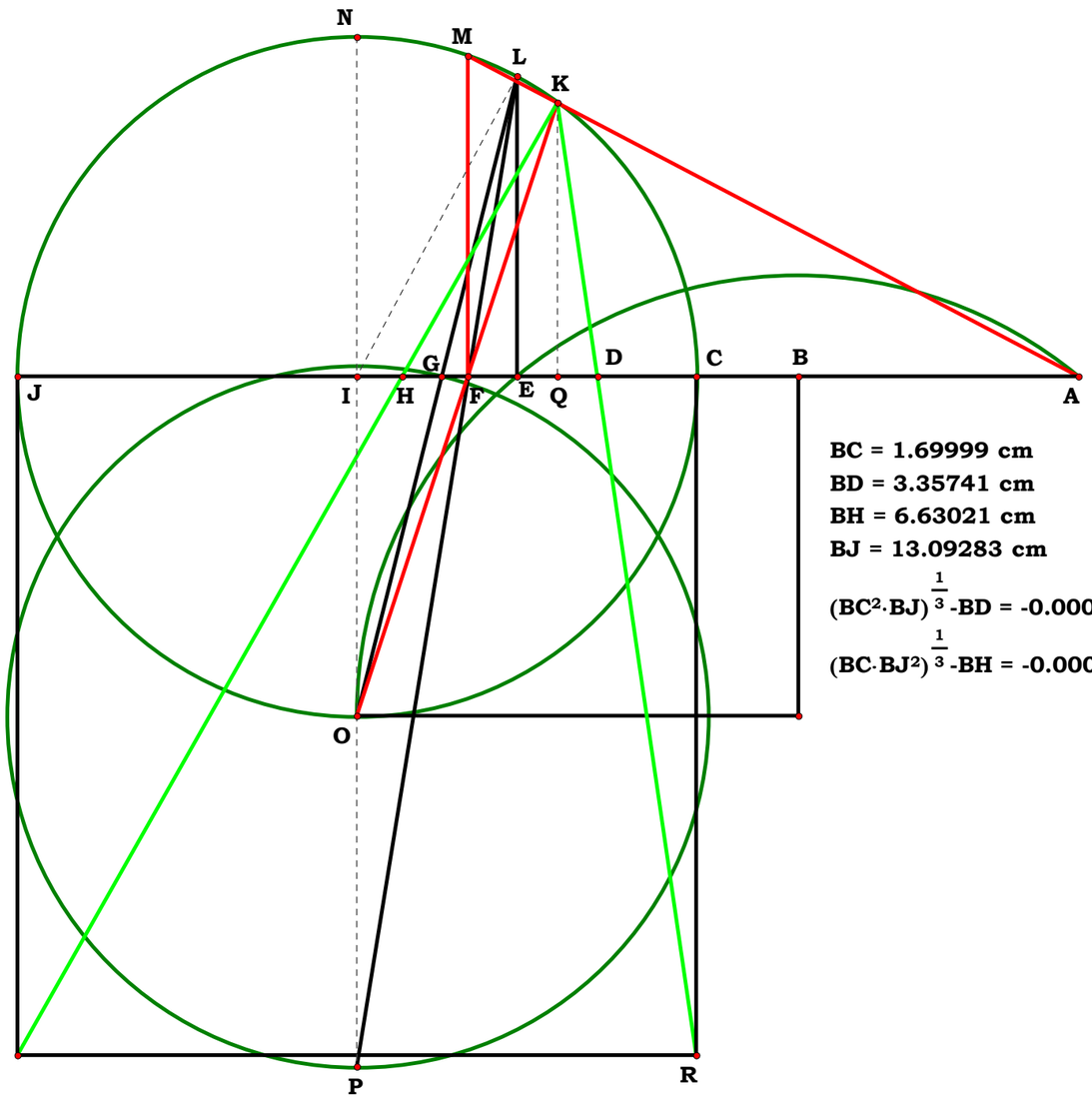
$FQ := \frac{FI \cdot FK}{FO} \quad QI := FQ + FI \quad CQ := CI - QI \quad QJ := CJ - CQ$

$QK := \sqrt{CQ \cdot QJ} \quad CD := \frac{CQ \cdot CR}{CR + QK} \quad BD := CD + BC$

$\left(BC^2 \cdot BJ\right)^{\frac{1}{3}} - BD = 4.486958 \times 10^{-6}$

Definitions.

Goshdarn Good Pencil

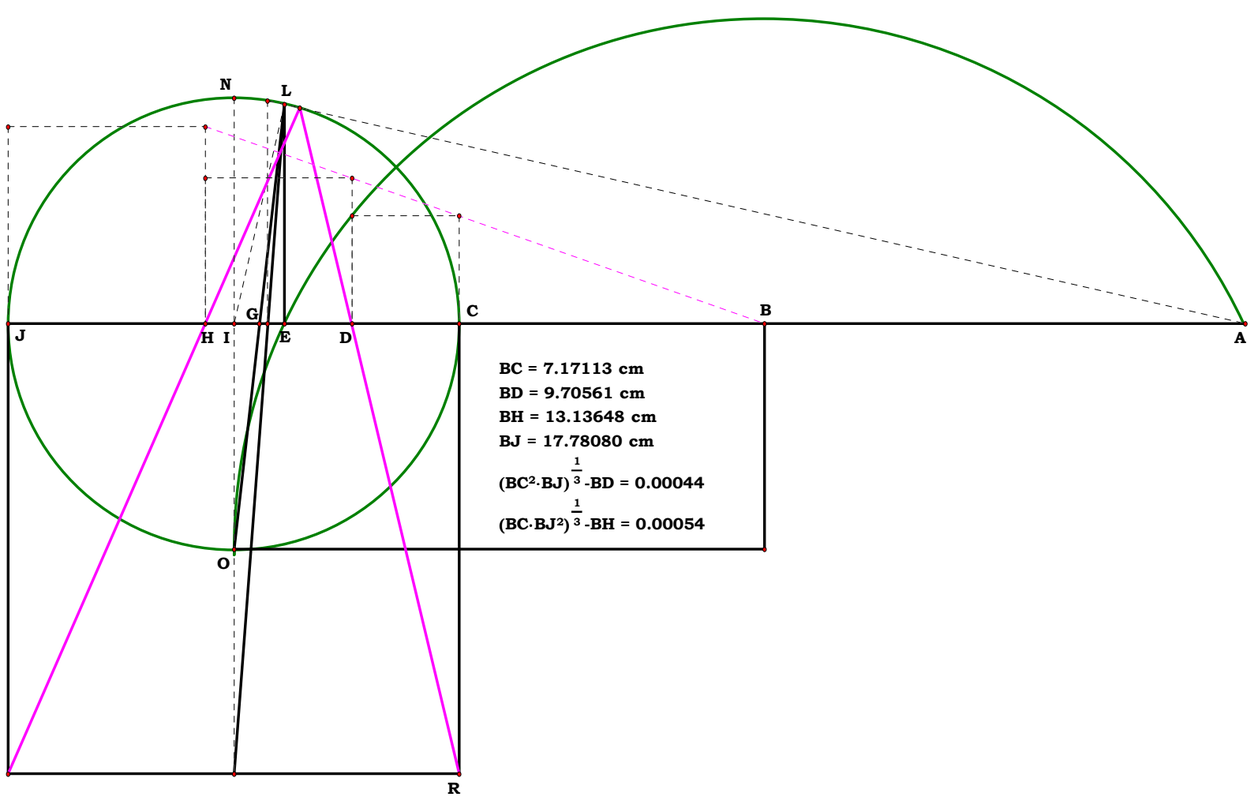


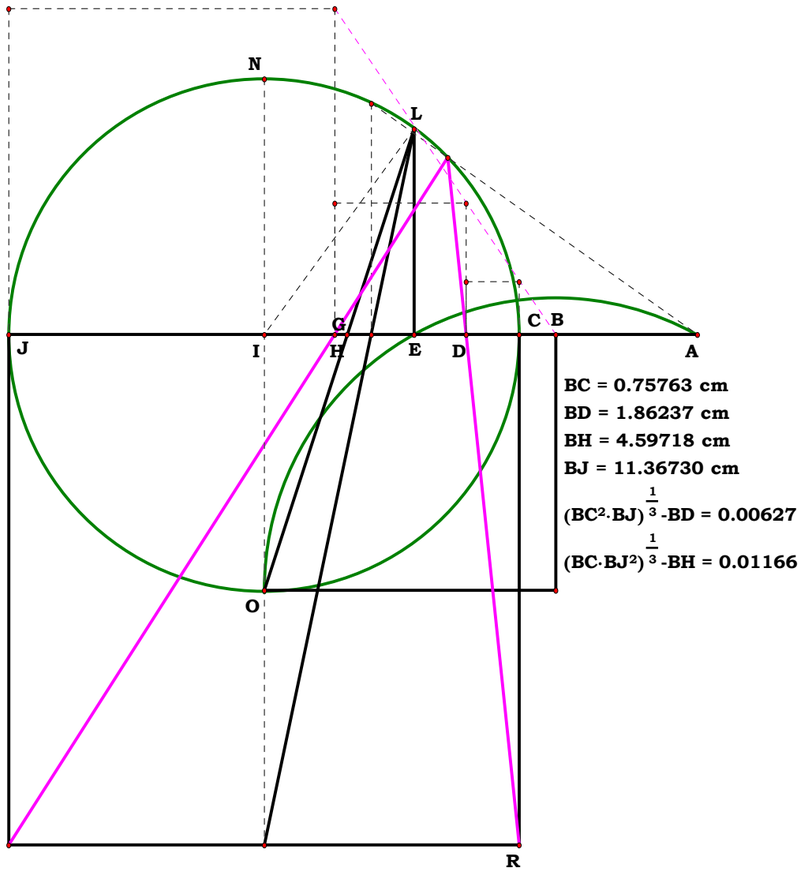
BC = 1.69999 cm
BD = 3.35741 cm
BH = 6.63021 cm
BJ = 13.09283 cm

$(BC^2 \cdot BJ)^{\frac{1}{3}} - BD = -0.00022$

$(BC \cdot BJ^2)^{\frac{1}{3}} - BH = -0.00035$

Compare to the following which is a great deal less accurate. When you work the figure, you cannot tell it is off:





091800A

Unit.

Given.

$N_1 := 2$ $BC := N_1$ $CJ := BC$ $GK := CJ$

$N_2 := 3$ $GH := N_2$ $GM := GH$

$N_3 := 8$ $CG := N_3$ $JK := CG$

Descriptions.

$BH := BC + CG + GH$ $BE := \frac{BH}{2}$ $KM := GM - GK$

$JK := CG$ $AG := \frac{JK \cdot GM}{KM}$ $AH := AG + GH$

$AB := AH - BH$ $AE := AB + BE$

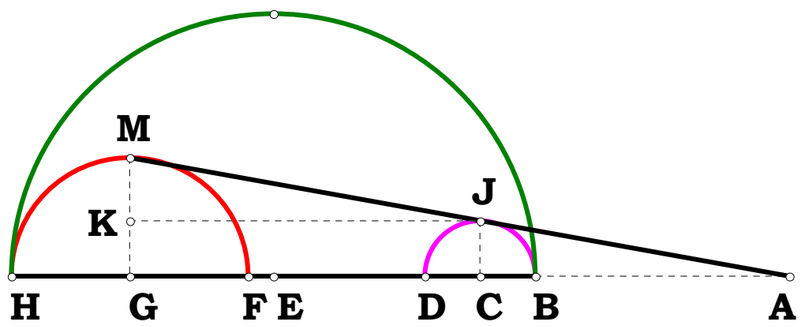
Definitions.

$BH - (N_1 + N_3 + N_2) = 0$ $BE - \frac{N_1 + N_3 + N_2}{2} = 0$ $KM - (N_2 - N_1) = 0$

$AG - \frac{N_3 \cdot N_2}{N_2 - N_1} = 0$ $AH - \frac{N_2 \cdot (N_1 - N_2 - N_3)}{N_1 - N_2} = 0$

$AB - \frac{N_1 \cdot (N_1 - N_2 + N_3)}{(N_2 - N_1)} = 0$ $AE - \frac{(N_1 - N_2)^2 + N_3 \cdot (N_1 + N_2)}{2 \cdot (N_2 - N_1)} = 0$

Midpoints and Similarity Points



What is AE given the radius of the two circles and the difference between their centers? (External Unit).

Unit.

$BH := 1$

Given.

$N_1 := 3 \quad N_3 := 4$

$N_2 := 7 \quad N_4 := 9$

091800B

Descriptions.

$$BE := \frac{BH}{2} \quad BC := BH \cdot \frac{N_1}{N_2} \quad GH := BH \cdot \frac{N_3}{N_4} \quad CG := BH - (BC + GH)$$

$$CJ := BC \quad GM := GH \quad GK := CJ \quad KM := GM - GK \quad JK := CG$$

$$AG := \frac{JK \cdot GM}{KM} \quad AH := AG + GH \quad AB := AH - BH \quad AE := AB + BE$$

Definitions.

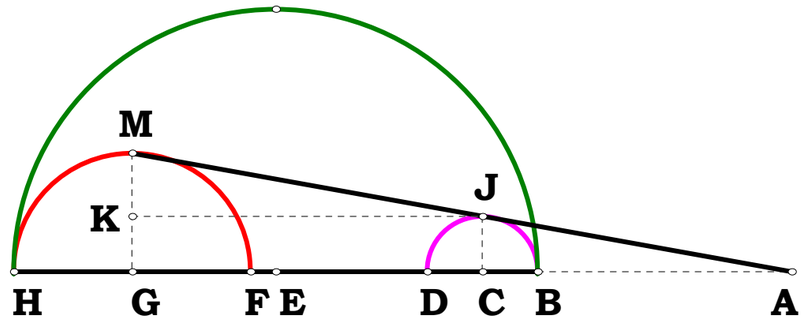
$$BE - \frac{1}{2} = 0 \quad BC - \frac{N_1}{N_2} = 0 \quad GH - \frac{N_3}{N_4} = 0 \quad CG - \frac{\left(N_2 \cdot N_4 - N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0$$

$$CJ - \frac{N_1}{N_2} = 0 \quad GM - \frac{N_3}{N_4} = 0 \quad GK - \frac{N_1}{N_2} = 0 \quad KM - \frac{\left(N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0 \quad JK - \frac{\left(N_2 \cdot N_4 - N_2 \cdot N_3 - N_1 \cdot N_4\right)}{N_2 \cdot N_4} = 0$$

$$AG - \frac{N_3 \cdot \left(N_1 \cdot N_4 + N_2 \cdot N_3 - N_2 \cdot N_4\right)}{N_4 \cdot \left(N_1 \cdot N_4 - N_2 \cdot N_3\right)} = 0 \quad AH - \frac{N_3 \cdot \left(2 \cdot N_1 - N_2\right)}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0 \quad AB - \frac{N_1 \cdot \left(2 \cdot N_3 - N_4\right)}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0$$

$$AE - \frac{4 \cdot N_1 \cdot N_3 - N_1 \cdot N_4 - N_2 \cdot N_3}{2 \cdot \left(N_1 \cdot N_4 - N_2 \cdot N_3\right)} = 0$$

Midpoints and Similarity Points



What is AE if given the difference between their extremes and the radius given as a ratio of that segment? (Internal Unit).

Unit.

Given.

$N_1 := 11.69458$ $BE := N_1$

$N_2 := 2.96916$ $BC := N_2$

000920A

Descriptions.

$BD := \frac{BE}{2}$ $CD := BD - BC$ $CH := BD$

$DH := \sqrt{BD^2 + CD^2}$ $DG := \frac{DH^2}{2BD}$ $AD := \frac{BD \cdot DG}{CD}$

$AE := AD + BD$ $AB := AE - BE$ $AC := BC + AB$

$AC - \sqrt{AB \cdot AE} = 0$

Definitions.

$BD - \frac{N_1}{2} = 0$ $CD - \left(\frac{N_1 - 2 \cdot N_2}{2}\right) = 0$ $CH - \frac{N_1}{2} = 0$

$DH - \frac{\sqrt{N_1^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2^2}}{\sqrt{2}} = 0$

$DG - \frac{N_1^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2^2}{2 \cdot N_1} = 0$

$AD - \frac{N_1^2 - 2 \cdot N_1 \cdot N_2 + 2 \cdot N_2^2}{2 \cdot (N_1 - 2 \cdot N_2)} = 0$ $AE - \frac{(N_1 - N_2)^2}{N_1 - 2 \cdot N_2} = 0$

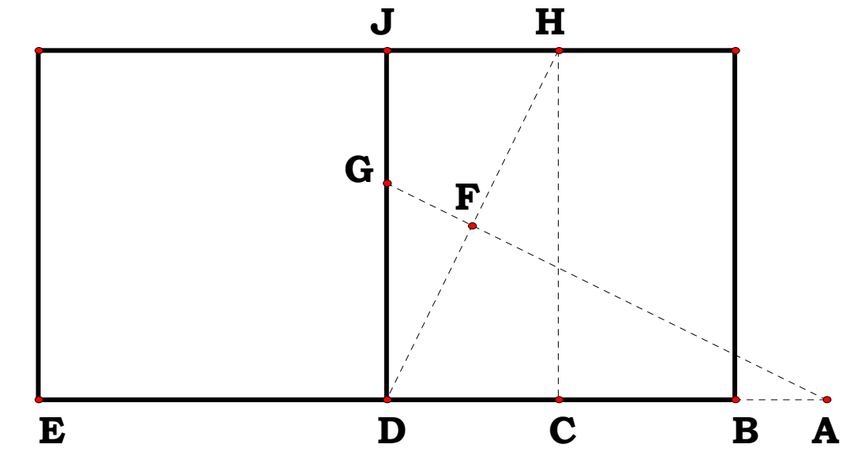
$AB - \frac{N_2^2}{N_1 - 2 \cdot N_2} = 0$ $AC - \frac{N_2 \cdot (N_1 - N_2)}{N_1 - 2 \cdot N_2} = 0$

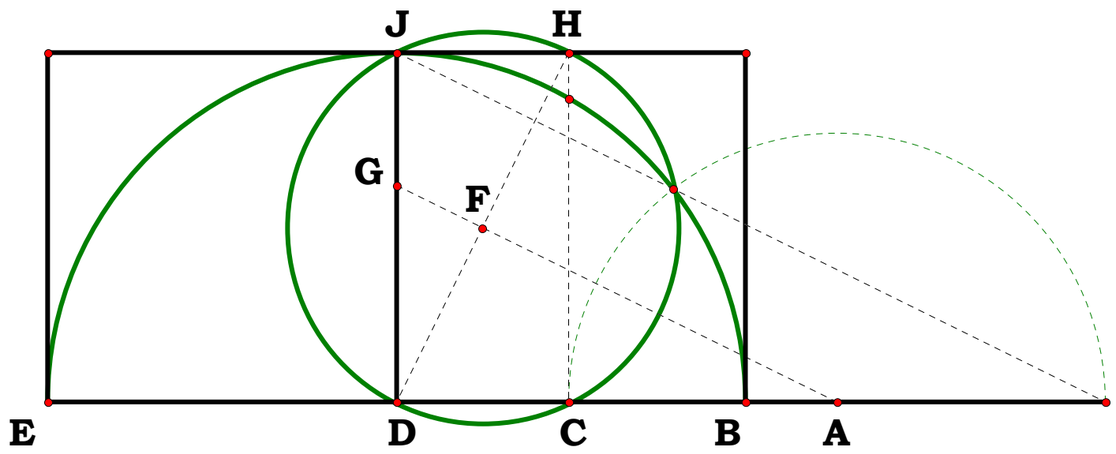
Squaring

Is AC the square root of AB x AE?

Given BC, find AB such that AB x AE is the square root.

This plate solves for the figure using a traditional square root figure.
The next two plates, which I put off doing until now, will approach the figure differently and the last might be a surprise but it was what I was pondering when I set up the original figure for solving cube roots. The last figure will actuall glue both of these figures together. Now, if one had glued them both together, one would be hard pressed to prove that cube roots are impossible in geometry as the compound figure makes the claim rather dubious.





092000BZ

Descriptions.

Unit.

$BE := 1$

Given.

$N_1 := 5$

$N_2 := 20$

$DE := \frac{BE}{2}$ $BD := DE$ $CD := BD \cdot \frac{N_1}{N_2}$

$CE := CD + DE$ $DF := \sqrt{CD^2 + BD^2}$

$DG := \frac{DF}{2}$ $AD := \frac{DF \cdot DG}{CD}$ $AE := AD + DE$

$AB := AD - BD$ $AC := AD - CD$

Definitions.

$DE - \frac{1}{2} = 0$ $BD - \frac{1}{2} = 0$ $CD - \frac{N_1}{2 \cdot N_2}$

$CE - \frac{N_1 + N_2}{2 \cdot N_2} = 0$ $DF - \frac{\sqrt{N_1^2 + N_2^2}}{2 \cdot N_2} = 0$

$DG - \frac{\sqrt{N_1^2 + N_2^2}}{4 \cdot N_2} = 0$ $AD - \frac{N_1^2 + N_2^2}{4 \cdot N_1 \cdot N_2} = 0$ $AE - \frac{(N_1 + N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$

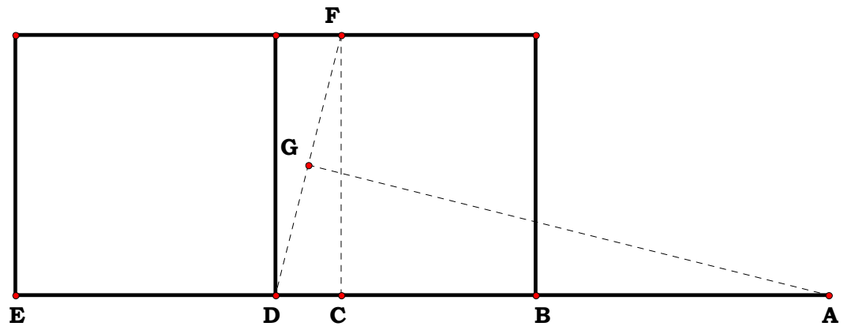
$AB - \frac{(N_1 - N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$ $AC - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$

$AB \cdot AE - \frac{(N_1 - N_2)^2}{4 \cdot N_1 \cdot N_2} \cdot \frac{(N_1 + N_2)^2}{4 \cdot N_1 \cdot N_2} = 0$ $AC - \sqrt{\frac{(N_1 + N_2)^2 \cdot (N_1 - N_2)^2}{16 \cdot N_1^2 \cdot N_2^2}} = 0$ $AC - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$

Squaring

Is AC the square root of AB x AE?

Given BC, find AB such that AB x AE is the square root.



BE = 1.00000

DE = 0.50000

CE = 0.62500

AE = 1.56250

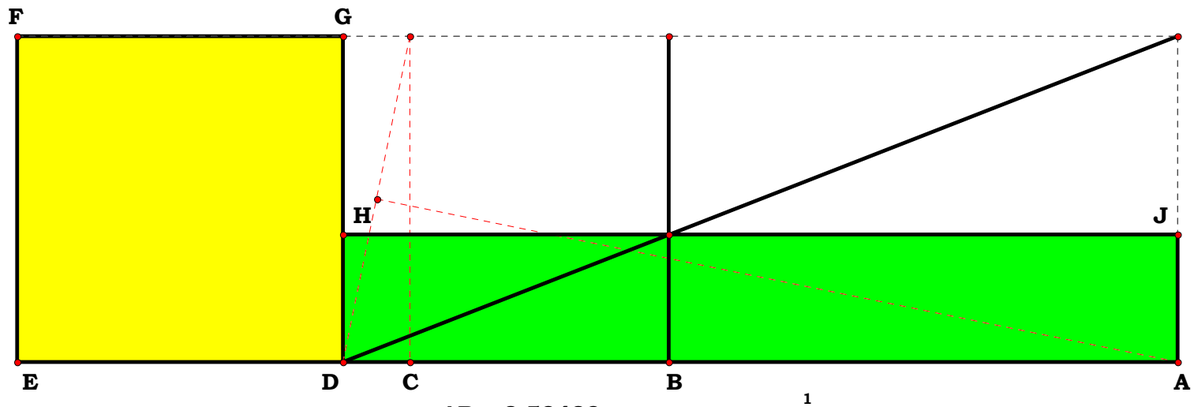
$\frac{DC}{DB} = 0.25000$

Unit.
Given.
Descriptions.
Definitions.

092000BZ

I am not going to write this up as it is obvious. One will notice that the part of the figure with the green in it is a figure in the Elements involving complements. The original two ways to express the same figure produces a cube root figure, but one which is not exactly as we would hope for.
The figure does, however, disprove the claim that cube roots cannot be done at all when it is actually a primive.

Cube Root Primitive



Area FGDE = 29.87898 cm²
Area HJAD = 29.87898 cm²

DE = 5.46617 cm
DH = 2.13559 cm
AD = 13.99098 cm

AB = 8.52482 cm
AC = 12.87900 cm
AE = 19.45715 cm

$(AB \cdot AE)^{\frac{1}{2}} - AC = 0.00000$
$AB \cdot AC \cdot AE = 2136.22303\ cm^3$

$DE \cdot DH \cdot AD = 163.32347\ cm^3$
$(AB \cdot AC \cdot AE)^{\frac{1}{3}} - AC = 0.00000$
$(DE \cdot DH \cdot AD)^{\frac{1}{3}} - DE = 0.00000$

111300

Descriptions.

Unit.

Given.

Definitions.

$N_1 := 3$ $AB := N_1$

$N_2 := 5$ $AD := N_2$

$N_3 := 1$ $DE := N_3$

For Two Right Triangles.

Given AB, DE, AD find BE, AC, CD, CE, BC.

BAD and BED are right.

$BD := \sqrt{AB^2 + AD^2}$ $BF := \frac{AB^2}{BD}$ $DG := \frac{DE^2}{BD}$ $BE := \sqrt{BD^2 - DE^2}$

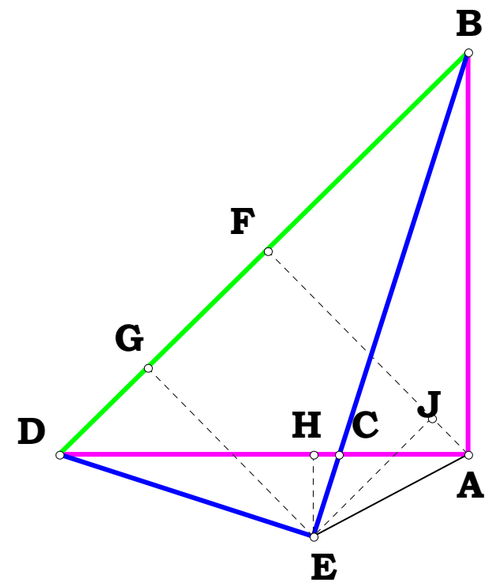
$AF := \sqrt{AB^2 - BF^2}$ $EG := \sqrt{DE^2 - DG^2}$ $FG := BD - (BF + DG)$

$EJ := FG$ $FJ := EG$ $AJ := AF - FJ$ $AE := \sqrt{EJ^2 + AJ^2}$

$S_1 := AD$ $S_2 := DE$ $S_3 := AE$ $AH := \frac{S_3^2 + S_1^2 - S_2^2}{2 \cdot S_1}$

$EH := \sqrt{AE^2 - AH^2}$ $CH := \frac{EH \cdot AH}{AB + EH}$ $AC := AH - CH$

$CE := \frac{AC \cdot DE}{AB}$ $CD := AD - AC$ $BC := BE - CE$

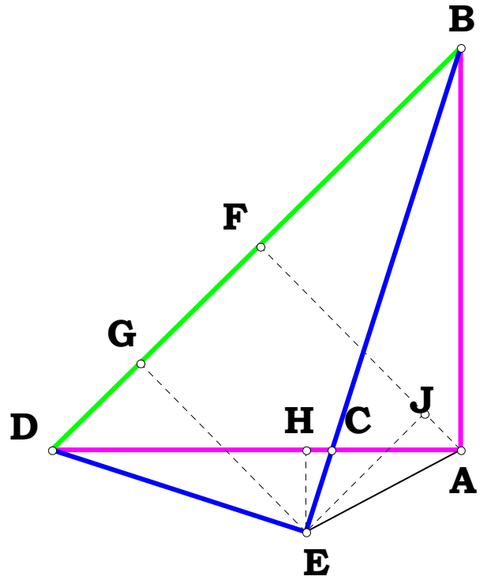


Definitions:

$$BE - \sqrt{N_1^2 + N_2^2 - N_3^2} = 0$$

$$\frac{N_1 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - AC = 0$$

$$N_2 + \frac{N_1 \cdot N_2 \cdot N_3^2 - N_2 \cdot \left(N_1^3 + N_1 \cdot N_2^2\right) + N_1^2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CD = 0$$



$$\frac{N_3 \cdot \left(N_1^2 \cdot N_2 - N_2 \cdot N_3^2 + N_2^3 - N_1 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}\right)}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}} - CE = 0$$

$$BC - \left(\sqrt{N_1^2 + N_2^2 - N_3^2} + \frac{N_2 \cdot N_3^3 - N_2^3 \cdot N_3 - N_1^2 \cdot N_2 \cdot N_3 + N_1 \cdot N_3^2 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}{N_1 \cdot N_2^2 + N_1^3 + N_3 \cdot \sqrt{N_2^4 + N_1^2 \cdot N_2^2 + N_1^2 \cdot N_3^2 - N_2^2 \cdot N_3^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot \sqrt{N_1^2 + N_2^2 - N_3^2}}}\right) = 0$$

Unit.

$A1 := 1$

Given.

$N := 8$

112700

Descriptions.

Let us suppose that we have any unit, or thing and we want to parse that thing into units, any number of units at all, we must first define the unit. In the following case, our starting unit, or thing is A to 1. Now when we parse it, we are defining smaller units. We do not want to call them fractions, or have to deal with fractions, so we rename our thing in terms of a given number of our new chosen unit, AB. This figure shows how to proceed to parse A1 into a 2N exponential series. We do this, as said, by creating a fraction, and giving the unit of that fraction the name of our new working unit. In short, we are converting base systems. We go from base 1, always to some other base. The base is named for the number of units it contains, or subsets of our given set. In this case, 8 is a subset of 1, and every member of 1, is defined in terms of 1.

$AB := \frac{A1}{N}$ AB is a member of the set A1.

$BF := \sqrt{AB \cdot (A1 - AB)}$ $AF := \sqrt{AB^2 + BF^2}$ $AD := AF$

$DH := \sqrt{AD \cdot (A1 - AD)}$ $AH := \sqrt{AD^2 + DH^2}$ $AE := AH$

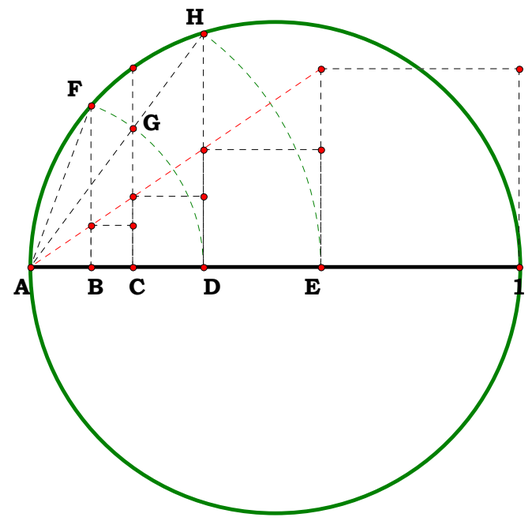
$AG := AD$ $AC := \frac{AD \cdot AD}{AH}$

Definitions.

$\frac{A1}{AB} = 8$ $\frac{A1}{AC} = 4.756828$ $\frac{A1}{AD} = 2.828427$ $\frac{A1}{AE} = 1.681793$ $C := \frac{A1}{AC}$ $D := \frac{A1}{AD}$ $E := \frac{A1}{AE}$ $N^{\frac{3}{4}} - C = 0$ $N^{\frac{2}{4}} - D = 0$ $N^{\frac{1}{4}} - E = 0$ Etc.

Any 2N root series.

This is just 112293 with lipstick and a dress. I have always felt a bit of annoyance with those who write Algebra books claiming that exponential series is a pure conceptual abstraction which has no geometric figure to demonstrate it. Apparently those writers do not even know simple geometry. I do not mind someone being ignorant, but when such words are in school books it is disinformation and misleading of students. I had never lernt geometry when I read that in a school book, however I was still amazed at the author putting words into a text which could not have possibly been true. Euclid gave his readers individual components to work with, the readers inability to combine those components together to figure our their interaction is not the fault of Euclid, it is the stupidity and lazyness of the reader.



$\frac{Unit}{XY} = 8.00000$

$x_B = 0.12500$

$x_C = 0.21022$

$x_D = 0.35355$

$x_E = 0.59460$

$\frac{Unit}{x_B} = 8.00000$

$\frac{Unit}{x_C} = 4.75683$

$\frac{Unit}{x_D} = 2.82843$

$\frac{Unit}{x_E} = 1.68179$

$B = 8.00000$ $B^{\frac{4}{4}} = 8.00000$

$C = 4.75683$ $B^{\frac{3}{4}} = 4.75683$

$D = 2.82843$ $B^{\frac{2}{4}} = 2.82843$

$E = 1.68179$ $B^{\frac{1}{4}} = 1.68179$

Now we can think of A1 as being a class or a noun, and B, C, D, E, members of that class, or its defining characteristics.

112800A

Unit.

$AE := 1 \quad JK := AE$

Given.

$N := 3 \quad EJ := N$

Descriptions.

$$HJ := \frac{JK \cdot EJ}{JK + EJ} \quad EH := EJ - HJ \quad GH := \frac{EH \cdot HJ}{EH + HJ}$$

$$EG := EH - GH \quad FG := \frac{EG \cdot GH}{EG + GH} \quad EF := EG - FG$$

$$DE := \frac{EF \cdot AE}{EF + AE} \quad AD := AE - DE \quad CD := \frac{AD \cdot DE}{AD + DE}$$

$$AC := AD - CD \quad BC := \frac{AC \cdot CD}{AC + CD} \quad AB := AC - BC$$

$M := 0 .. 3 \quad P := 0 .. 3 \quad AEAB_{M,P} := \left[\frac{N^{M+1}}{(N+1)^{M}} + 1 \right]^{P}$

Definitions.

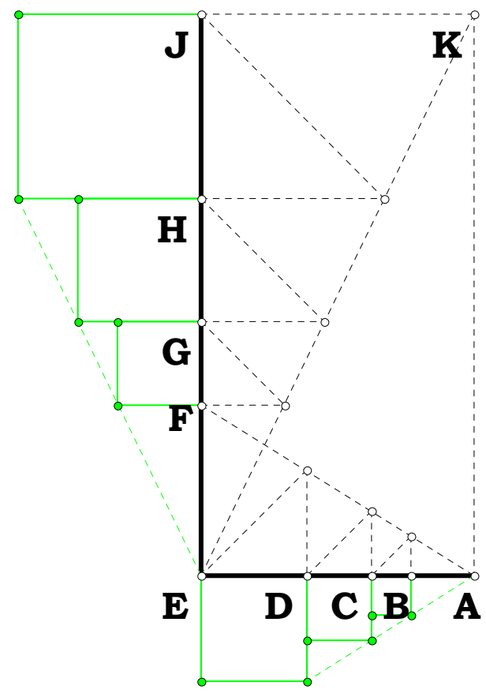
$$AEAB = \begin{pmatrix} 1 & 4 & 16 & 64 \\ 1 & 3.25 & 10.5625 & 34.328125 \\ 1 & 2.6875 & 7.222656 & 19.410889 \\ 1 & 2.265625 & 5.133057 & 11.629581 \end{pmatrix}$$

$$AEAB_{3,3} - \frac{AE}{AB} = 0 \quad AEAB_{3,2} - \frac{AE}{AC} = 0 \quad AEAB_{3,1} - \frac{AE}{AD} = 0 \quad AEAB_{3,0} - \frac{AE}{AE} = 0$$

Means On Means

Modify 02/28/98 for Mean proportionals between E and J.

As I have always found this little exercise quite useless, I have decided on a B writeup aimed at helping to explain how to use things like this in template making for geometric progression. My templates tend all to be arithmetic in expression, however, using them to construct one which effects geometric progression is quite easy. So, try to reflect on what the plate is demonstrating.



112900A

Unit.

$AC := 1$

Given.

$N_1 := 3$ $AH := N_1$

$N_2 := 12$ $CJ := N_2$

Descriptions for Division.

$$AB := \frac{AH}{(CJ + AH)} \cdot AC \quad BC := AC - AB$$

$$BD := BC \quad CG := \frac{BD \cdot AC}{AB} \quad CG = 4$$

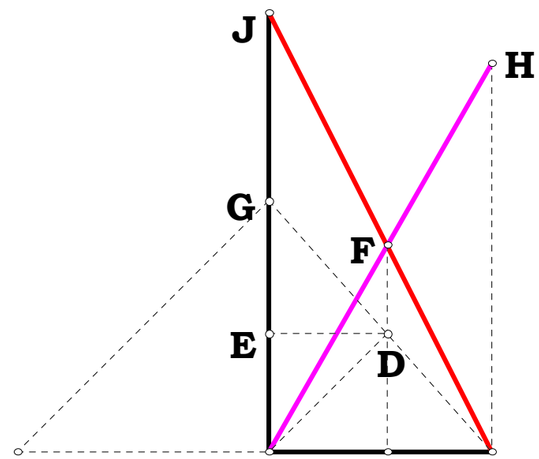
Definitions.

$$AB - \frac{N_1}{(N_2 + N_1)} = 0 \quad BC - \frac{N_2}{N_1 + N_2} = 0$$

$$BD - \frac{N_2}{N_1 + N_2} = 0 \quad CG - \frac{N_2}{N_1} = 0$$

Multiplication and Division-Line By A Line

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



112900B

Unit.

$AC := 1$

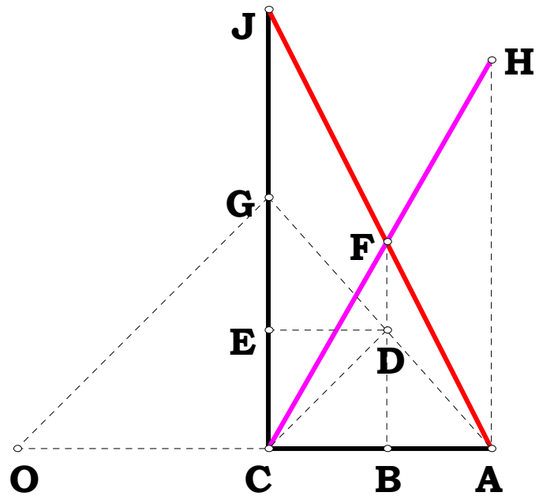
Given.

$N_1 := 5 \quad AH := N_1$

$N_2 := 7 \quad CG := N_2$

Multiplication and Division-Line By A Line

Given some unit, and two differences, multiply or divide the one difference by the other.
For Division:



Descriptions for Multiplication.

$CO := CG \qquad BD := \frac{CG \cdot AC}{AC + CO}$

$BC := BD \qquad AB := AC - BC \qquad BF := \frac{AH \cdot BC}{AC}$

$CJ := BF \cdot \frac{AC}{AB} \qquad CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$

Definitions.

$CO - N_2 = 0 \qquad BD - \frac{N_2}{N_2 + 1} = 0$

$BC - \frac{N_2}{N_2 + 1} = 0 \qquad AB - \frac{1}{N_2 + 1} = 0$

$BF - \frac{N_1 \cdot N_2}{N_2 + 1} = 0 \qquad CJ - N_1 \cdot N_2 = 0$

$CJ - N_1 \cdot N_2 = 0 \qquad CJ = 35$

120500

Descriptions.

Unit.

$N_3 := 1$ $CE := N_3$

Given.

$N_1 := 5$ $BC := N_1$

$N_2 := 25$ $AC := N_2$

$N_4 := .5$ $EF := N_4$

$BD := \frac{EF \cdot BC}{CE}$ $CF := \sqrt{CE^2 - EF^2}$

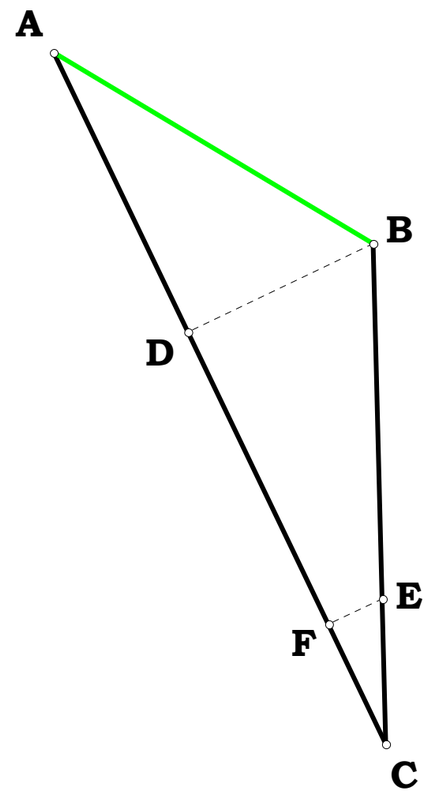
$CD := \frac{CF \cdot BC}{CE}$ $AD := AC - CD$

$AB := \sqrt{BD^2 + AD^2}$ $AB = 20.82051$

Definitions.

$$AB - \frac{\sqrt{N_1^2 \cdot N_3 + N_2^2 \cdot N_3 - 2 \cdot N_1 \cdot N_2 \cdot \sqrt{N_3^2 - N_4^2}}}{\sqrt{N_3}} = 0$$

From an observer C, the distance to star A and B are known, a reference CEF has been constructed, find the difference between the two stars.



010101

Unit.

Given.

$N_1 := 5$ $\quad AC := N_1$

$N_2 := 3$ $\quad BC := N_2$

Square Root, common segment common endpoint.

Alternate method for common segment common endpoint square root. $\sqrt{AC \cdot BC} = CD$

Descriptions.

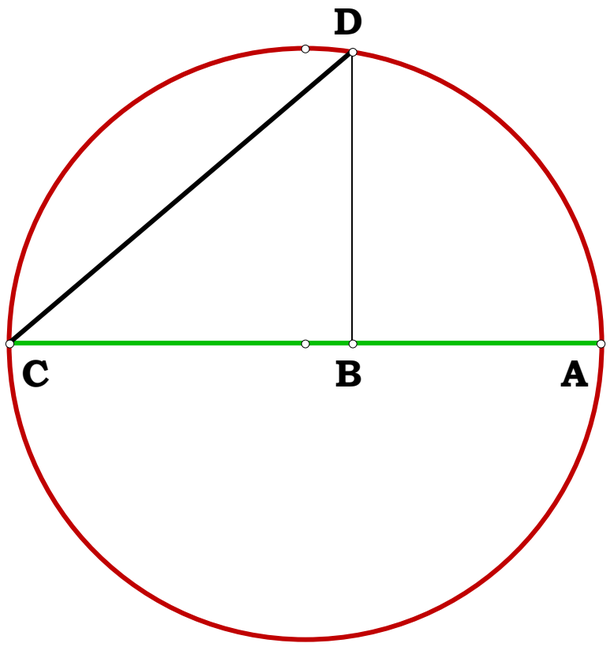
$AB := AC - BC$

$BD := \sqrt{AB \cdot BC}$

$CD := \sqrt{BD^2 + BC^2}$

Definitions.

$\sqrt{(N_1 \cdot N_2)} - CD = 0$



042101

Descriptions.

Unit.

Given.

$N_1 := 2 \quad AB := N_1$

$N_2 := 3 \quad CD := N_2$

$N_3 := 4 \quad AC := N_3$

$BC := \sqrt{AB^2 + AC^2} \quad CG := \frac{CD^2}{BC} \quad BF := \frac{AB^2}{BC}$

$BG := BC - CG \quad CF := BC - BF \quad AF := \sqrt{AB^2 - BF^2}$

$DG := \sqrt{CD^2 - CG^2} \quad FH := \frac{BG \cdot AF}{DG} \quad CH := CF + FH$

$BD := \sqrt{BG^2 + DG^2} \quad AH := \frac{BD \cdot FH}{BG} \quad BE := \frac{AH \cdot BC}{CH}$

$DE := BD - BE \quad CE := \frac{AC \cdot BC}{CH} \quad AE := AC - CE$

Definitions.

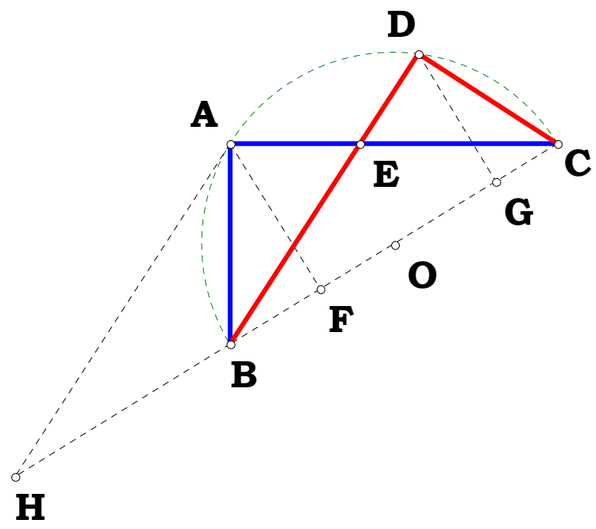
$\sqrt{N_1^2 + N_3^2} - BC = 0 \quad \frac{N_2^2}{\sqrt{N_1^2 + N_3^2}} - CG = 0$

$\frac{N_1^2}{\sqrt{N_1^2 + N_3^2}} - BF = 0 \quad \frac{\left(N_1^2 + N_3^2 - N_2^2\right)}{\sqrt{N_1^2 + N_3^2}} - BG = 0 \quad \frac{N_3^2}{\sqrt{N_1^2 + N_3^2}} - CF = 0 \quad \frac{N_1 \cdot N_3}{\sqrt{N_1^2 + N_3^2}} - AF = 0$

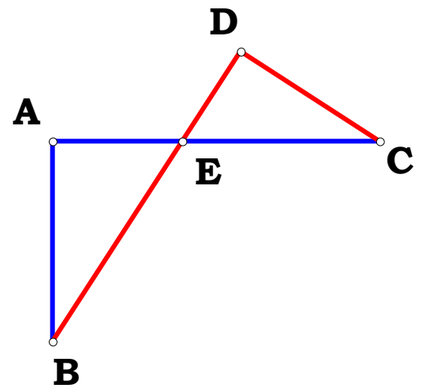
$\frac{N_2 \cdot \sqrt{N_1^2 - N_2^2 + N_3^2}}{\sqrt{N_1^2 + N_3^2}} - DG = 0 \quad \frac{N_2 \cdot \sqrt{N_1^2 - N_2^2 + N_3^2}}{\sqrt{N_1^2 + N_3^2}} \quad \frac{N_3 \cdot N_1 \cdot \left(N_1^2 + N_3^2 - N_2^2\right)}{N_2 \cdot \sqrt{\left(N_1^2 + N_3^2 - N_2^2\right) \cdot \left(N_1^2 + N_3^2\right)}} - FH = 0 \quad N_3 \cdot \frac{N_1}{N_2} - AH = 0$

$$\frac{N_3 \cdot \left[N_1 \cdot \left(N_1^2 - N_2^2 + N_3^2\right) \cdot \sqrt{N_1^2 + N_3^2} + N_2 \cdot N_3 \cdot \sqrt{\left[\left(N_1^2 + N_3^2\right) \cdot \left(N_1^2 - N_2^2 + N_3^2\right)\right]}\right]}{N_2 \cdot \sqrt{N_1^2 + N_3^2} \cdot \sqrt{\left(N_1^2 + N_3^2\right) \cdot \left(N_1^2 - N_2^2 + N_3^2\right)}} - CH = 0$$

Given AB, CD, AC and that CDB, and BAC are right angles, what are BD, AE, CE, BE, DE?



Three Given Five Taken



The Five Sought:

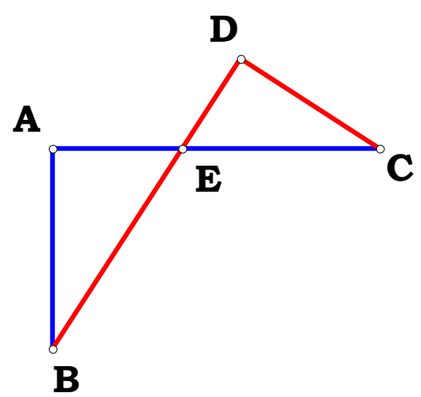
$$\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} - BD = 0$$

$$N_2 \cdot \frac{\left(\sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_3 - N_1 \cdot N_2\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - DE = 0$$

$$N_1 \cdot \frac{\left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - BE = 0$$

$$N_3 - \frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{\left(N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1\right)} - AE = 0$$

$$\frac{N_2 \cdot \left(N_1^{\,2} + N_3^{\,2}\right)}{N_3 \cdot N_2 + \sqrt{N_1^{\,2} + N_3^{\,2} - N_2^{\,2}} \cdot N_1} - CE = 0$$



042201

Descriptions.

Unit.

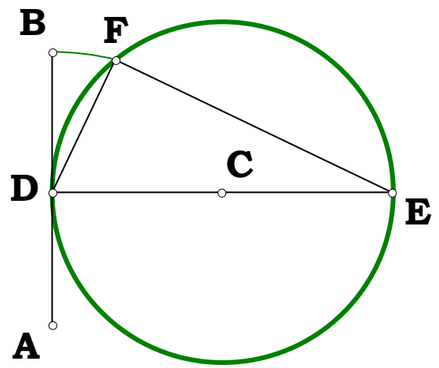
$AB := 1$

Given.

$N_1 := 2.052$

$N_2 := .62$

Given AB as unit, AD and DC, what is EF and DF?



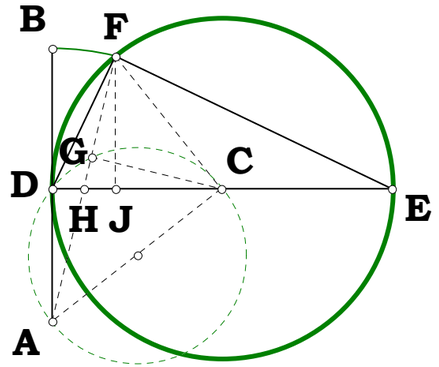
$AD := \frac{AB}{N_1}$ $CD := AB \cdot N_2$ $DE := 2CD$ $AF := AB$ $CF := CD$

$AC := \sqrt{AD^2 + CD^2}$ $AG := \frac{AC^2 + AF^2 - CF^2}{2 \cdot AF}$ $CG := \sqrt{AC^2 - AG^2}$

$DH := CD - \frac{CG \cdot \left(AD^2 + CD^2\right)}{CD \cdot CG + \sqrt{AD^2 + CD^2 - CG^2} \cdot AD}$ $AH := AD \cdot \frac{\left(AD^2 + CD^2\right)}{\left(CD \cdot CG + \sqrt{AD^2 + CD^2 - CG^2} \cdot AD\right)}$

$FH := AF - AH$ $HJ := \frac{DH \cdot FH}{AH}$ $DJ := DH + HJ$ $EJ := DE - DJ$ $FJ := \frac{AD \cdot FH}{AH}$

$EF := \sqrt{FJ^2 + EJ^2}$ $DF := \sqrt{DJ^2 + FJ^2}$



Definitions.

$\frac{AB}{N_1} - AD = 0$ $AB \cdot N_2 - CD = 0$ $(2AB) \cdot N_2 - DE = 0$ $AB \cdot \frac{\sqrt{\left(1 + N_2^2 \cdot N_1^2\right)}}{N_1} - AC = 0$

$\frac{1}{2} \cdot AB \cdot \frac{\left(1 + N_1^2\right)}{N_1^2} - AG = 0$ $AB \cdot \frac{\sqrt{\left(2 \cdot N_1^2 \cdot N_2 + N_1^2 - 1\right) \cdot \left(2 \cdot N_1^2 \cdot N_2 - N_1^2 + 1\right)}}{2 \cdot N_1^2} - CG = 0$

$AB \cdot \frac{N_2 \cdot N_1^3 + N_2 \cdot N_1 - \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4}}{N_1^3 + N_1^2 \cdot N_2 \cdot \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4} + N_1} - DH = 0$ $2 \cdot AB \cdot \frac{\left(1 + N_2^2 \cdot N_1^2\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4} + 1 + N_1^2\right)} - AH = 0$

$AB - 2 \cdot AB \cdot \frac{\left(1 + N_2^2 \cdot N_1^2\right)}{\left(N_1 \cdot N_2 \cdot \sqrt{2 \cdot N_1^2 + 4 \cdot N_1^4 \cdot N_2^2 - 1 - N_1^4} + 1 + N_1^2\right)} - FH = 0$

$$\frac{-1}{2}\cdot AB\cdot\left(\frac{6\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}\cdot N_2^{\,2}+1}{N_1^{\,2}\cdot N_2+N_1^{\,4}\cdot N_2+N_1^{\,3}\cdot N_2^{\,2}\cdot\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}-\frac{N_1^{\,4}\cdot N_2^{\,2}+3\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+1}{N_1^{\,4}\cdot N_2^{\,3}+N_1^{\,2}\cdot N_2}\right)-HJ=0$$

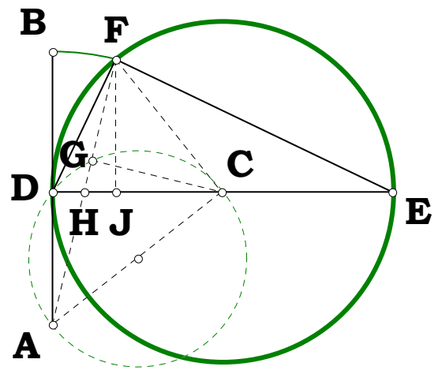
$$\frac{1}{2}\cdot AB\cdot\frac{N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-DJ=0$$

$$\frac{1}{2}\cdot AB\cdot\frac{4\cdot N_1^{\,3}\cdot N_2^{\,3}-N_1^{\,3}\cdot N_2+3\cdot N_1\cdot N_2+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}}{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}-EJ=0$$

$$\frac{-1}{2}\cdot AB\frac{\left(-N_1\cdot N_2\cdot\sqrt{2\cdot N_1^{\,2}+4\cdot N_1^{\,4}\cdot N_2^{\,2}-1-N_1^{\,4}}+1-N_1^{\,2}+2\cdot N_2^{\,2}\cdot N_1^{\,2}\right)}{\left[N_1\cdot\left(1+N_2^{\,2}\cdot N_1^{\,2}\right)\right]}-FJ=0$$

$$AB\cdot\frac{\sqrt{N_2\cdot\left[N_1\cdot N_2\cdot\left(4\cdot N_1^{\,2}\cdot N_2^{\,2}-N_1^{\,2}+3\right)+\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right]}}{\sqrt{N_1^{\,3}\cdot N_2^{\,2}+N_1}}-EF=0$$

$$AB\cdot\frac{\sqrt{\left[N_2\cdot\left(N_1^{\,3}\cdot N_2+N_1\cdot N_2-\sqrt{4\cdot N_1^{\,4}\cdot N_2^{\,2}-N_1^{\,4}+2\cdot N_1^{\,2}-1}\right)\right]}}{\sqrt{N_1\cdot\left(N_1^{\,2}\cdot N_2^{\,2}+1\right)}}-DF=0$$



042301A

Unit.

$AB := 1$

Given.

$N := .36307 \quad BC := N$

$AC := AB + BC \quad AG := \frac{AB}{2} \quad GM := \sqrt{3 \cdot AG^2} \quad CG := AC - AG$

$CN := \sqrt{CG^2 + AG^2} \quad NO := \frac{AG \cdot AB}{CN} \quad CO := CN - NO$

$CE := \frac{CN \cdot CO}{CG} \quad GE := CG - CE \quad JS := \sqrt{(AB + GE) \cdot (AB - GE)}$

$EJ := JS - GM \quad CJ := \sqrt{CE^2 + EJ^2} \quad CL := \frac{CG \cdot CE}{CJ}$

Definitions.

$AC - (1 + N) = 0 \quad AG - \frac{1}{2} = 0 \quad GM - \frac{\sqrt{3}}{2} = 0$

$CG - \frac{2 \cdot N + 1}{2} \quad CN - \frac{\sqrt{2 \cdot N^2 + 2 \cdot N + 1}}{\sqrt{2}} = 0$

$NO - \frac{\sqrt{2}}{2 \cdot \sqrt{2 \cdot N^2 + 2 \cdot N + 1}} = 0 \quad CO - \frac{\sqrt{2} \cdot N \cdot (N + 1)}{\sqrt{2 \cdot N^2 + 2 \cdot N + 1}} = 0$

$CE - \frac{2 \cdot N \cdot (N + 1)}{2 \cdot N + 1} = 0 \quad GE - \frac{1}{2 \cdot (2 \cdot N + 1)} = 0$

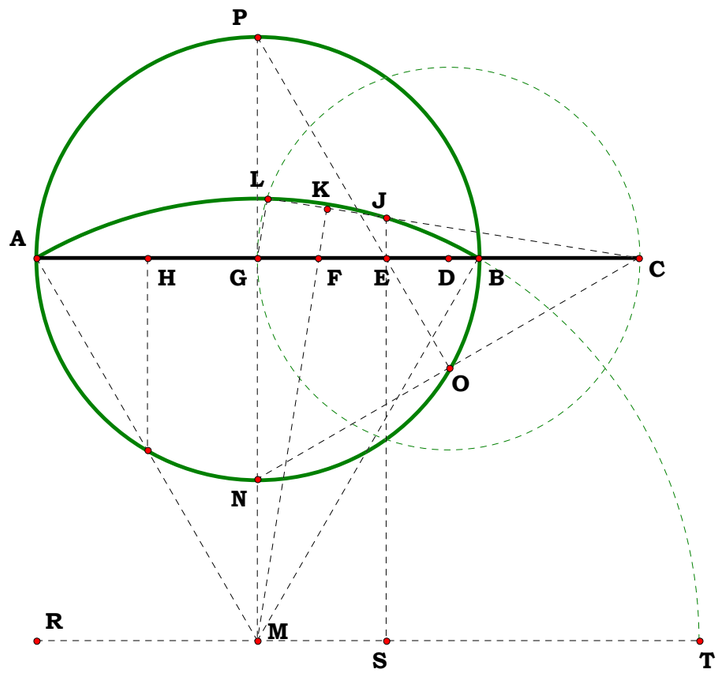
$JS - \frac{\sqrt{(4 \cdot N + 1) \cdot (4 \cdot N + 3)}}{2 \cdot (2 \cdot N + 1)} = 0 \quad EJ - \frac{\sqrt{16 \cdot N^2 + 16 \cdot N + 3} - \sqrt{3} \cdot (2 \cdot N + 1)}{4 \cdot N + 2} = 0$

$CJ - \frac{\sqrt{2 \cdot N \cdot \left(2 \cdot N^2 + 3 \cdot N + 4\right)} + 3 - \sqrt{[3 \cdot (4 \cdot N + 3) \cdot (4 \cdot N + 1)]}}{\sqrt{4 \cdot N + 2}} = 0$

$CL - \frac{2 \cdot \sqrt{2} \cdot N \cdot (N + 1) \cdot \left(N + \frac{1}{2}\right)}{\sqrt{2 \cdot N + 1} \cdot \sqrt{2 \cdot N \cdot \left(2 \cdot N^2 + 3 \cdot N + 4\right)} + 3 - \sqrt{[3 \cdot (4 \cdot N + 3) \cdot (4 \cdot N + 1)]}} = 0$

Counterpoint

When I origionally did part of this, it was a mess. I have taken it upon myself to keep the figure and title.



Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

042401A

Descriptions.

$BG := AG - AB \quad BF := \frac{BG}{2} \quad AF := AB + BF \quad AK := AF$

$FK := BF \quad AE := \frac{2AK^2 - FK^2}{2AF} \quad AJ := AE \quad JK := AK - AJ$

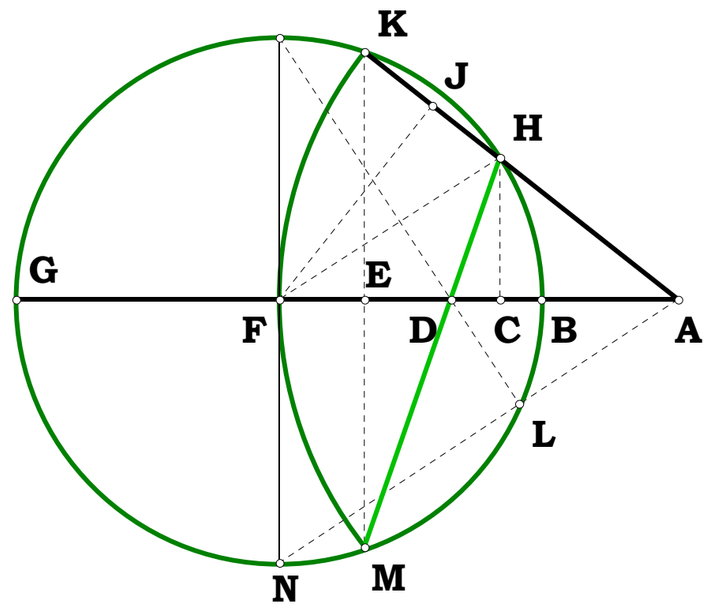
$HJ := JK \quad AH := AK - (JK + HJ) \quad AC := \frac{AE \cdot AH}{AK} \quad CE := AE - AC$

$BE := AE - AB \quad EG := BG - BE \quad EK := \sqrt{BE \cdot EG} \quad BC := AC - AB$

$CG := BG - BC \quad CH := \sqrt{BC \cdot CG} \quad DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE \quad HM := \sqrt{CE^2 + (EK + CH)^2}$

Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?



Definitions.

$$N - 1 - BG = 0 \qquad \frac{1}{2}\cdot N - \frac{1}{2} - BF = 0$$

$$\frac{1}{2} + \frac{1}{2}\cdot N - AF = 0 \qquad \frac{1}{4}\cdot\frac{\left(N^2 + 6\cdot N + 1\right)}{(1+N)} - AE = 0$$

$$\frac{1}{4}\cdot\frac{\left(1 - 2\cdot N + N^2\right)}{(1+N)} - JK = 0 \qquad 2\cdot\frac{N}{(1+N)} - AH = 0$$

$$\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^3}\cdot N - AC = 0 \qquad \frac{1}{4}\cdot\left(N^2 + 6\cdot N + 1\right)\cdot\frac{(N-1)^2}{(1+N)^3} - CE = 0$$

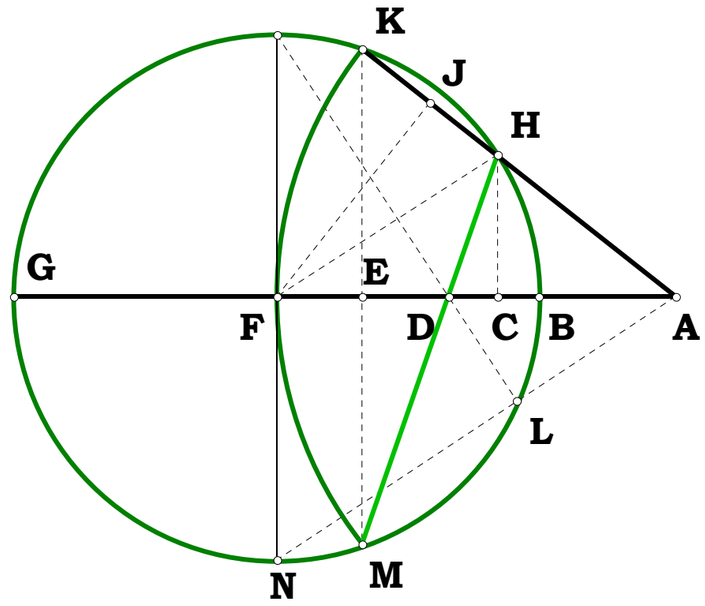
$$\frac{1}{4}\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} - BE = 0 \qquad \frac{1}{4}\cdot(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)} - EG = 0$$

$$(3\cdot N+1)\cdot\frac{(N-1)}{(1+N)^3} - BC = 0 \qquad \frac{1}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} - EK = 0$$

$$N^2\cdot(N+3)\cdot\frac{(N-1)}{(1+N)^3} - CG = 0 \qquad \sqrt{(N+3)\cdot(3\cdot N+1)}\cdot N\cdot\frac{(N-1)}{(1+N)^3} - CH = 0$$

$$\frac{1}{4}\cdot\frac{(N-1)^2}{(1+N)} - DE = 0 \qquad \frac{1}{2}\cdot\frac{(N-1)^2}{(N+1)} - DF = 0$$

$$\frac{1}{2}\cdot(N-1)\cdot\frac{\left(1 + 6\cdot N + N^2\right)}{(1+N)^2} - HM = 0$$



Unit.

$AB := 1$

Given.

$N := 5 \qquad AG := N$

042401B

Descriptions.

$BG := AG - AB \qquad BF := \frac{AB}{2} \qquad GF := BG + BF \qquad GK := GF$

$FK := BF \qquad EG := \frac{2GK^2 - FK^2}{2GF} \qquad GJ := EG \qquad JK := GK - GJ \qquad HJ := JK$

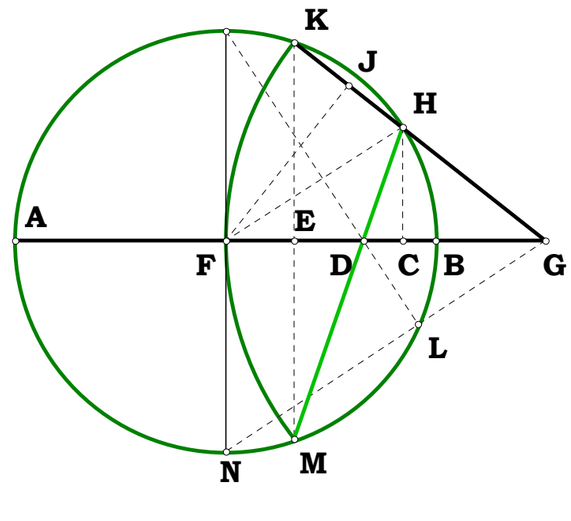
$GH := GK - (JK + HJ) \qquad CG := \frac{EG \cdot GH}{GK} \qquad CE := EG - CG$

$BE := EG - BG \qquad AE := AB - BE \qquad EK := \sqrt{BE \cdot AE} \qquad AC := AG - CG$

$BC := AB - AC \qquad CH := \sqrt{BC \cdot AC} \qquad DE := \frac{CE \cdot EK}{EK + CH}$

$DF := 2 \cdot DE \qquad HM := \sqrt{CE^2 + (EK + CH)^2}$

Does HM intersect at D? What is the Algebraic name of HM in relation to AB and AG?



Definitions.

$$BG - (N-1) = 0 \quad BF - \frac{1}{2} = 0 \quad GF - \frac{2\cdot N-1}{2} = 0 \quad GK - \frac{2\cdot N-1}{2} = 0$$

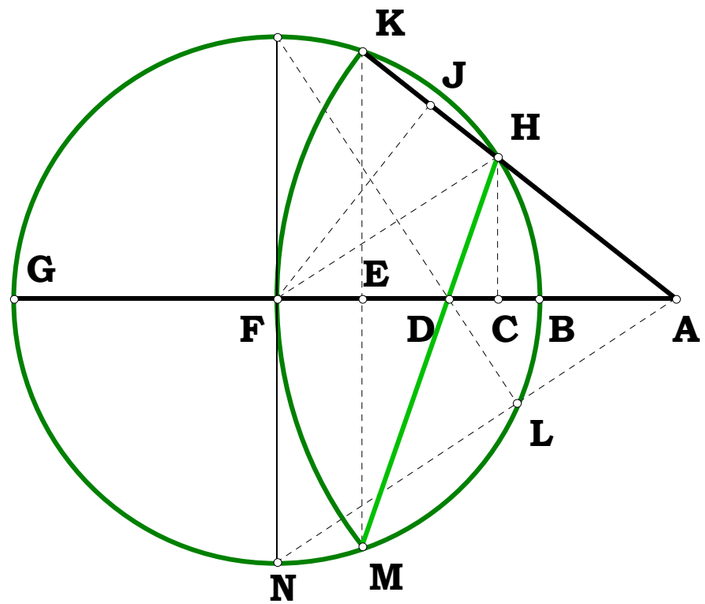
$$FK - \frac{1}{2} = 0 \quad EG - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0 \quad GJ - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)} = 0$$

$$JK - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad HJ - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad GH - \frac{2\cdot N\cdot(N-1)}{2\cdot N-1} = 0$$

$$CG - \frac{N\cdot(N-1)\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)}{(2\cdot N-1)^3} = 0 \quad CE - \frac{8\cdot N^2 - 8\cdot N + 1}{4\cdot(2\cdot N-1)^3} = 0$$

$$BE - \frac{4\cdot N-3}{4\cdot(2\cdot N-1)} = 0 \quad AE - \frac{4\cdot N-1}{4\cdot(2\cdot N-1)} = 0 \quad EK - \frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)} = 0$$

$$AC - \frac{N^2\cdot(4\cdot N-3)}{(2\cdot N-1)^3} = 0 \quad BC - \frac{(4\cdot N-1)\cdot(N-1)^2}{(2\cdot N-1)^3} = 0$$



$$CH - \frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3} = 0$$

Mathcad claims that this is solvable, however, it was spread over several pages and I do not know what to make of that; however, if it is right, then there you go for trisection.

$$DE - \frac{1}{4\cdot(2\cdot N-1)} = 0 \quad DF - \frac{2}{4\cdot(2\cdot N-1)} = 0 \quad HM - \frac{\left(8\cdot N^2 - 8\cdot N + 1\right)}{2\cdot(2\cdot N-1)^2} = 0$$

Unit.

$AB := 1$

Given.

Descriptions.

$N := 5.768$ $AJ := N$

042501A

$BJ := AJ - AB$ $BH := \frac{BJ}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $AH := AB + BH$

$AO := AH$ $AG := \sqrt{AO^2 - GO^2}$

$HQ := BH$ $AQ := AH$ $FH := \frac{HQ^2}{2 \cdot AH}$

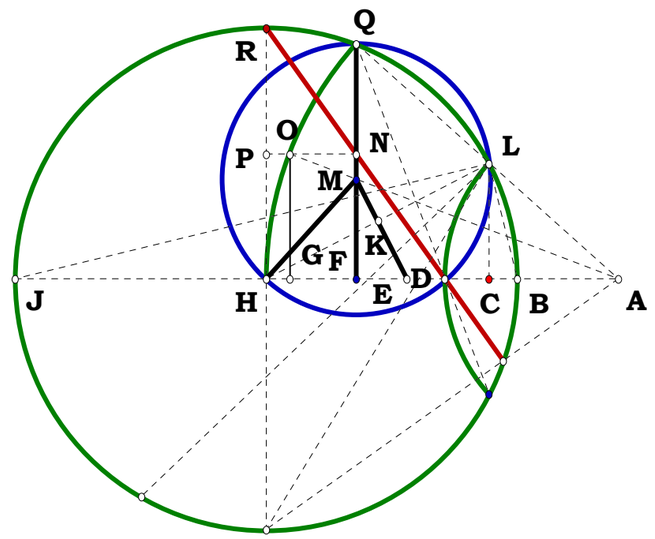
$AF := AH - FH$ $FM := \frac{GO \cdot AF}{AG}$

$HJ := BH$ $FJ := FH + HJ$ $BF := BJ - FJ$

$FQ := \sqrt{BF \cdot FJ}$ $MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$

$HM - MQ = 0$ $DH := \frac{HR^2}{AH}$ $\frac{DH}{2} - FH = 0$

What is the Algebraic name of the circle HM? Does point N divide DR in half?



Definitions.

$BJ - (N - 1) = 0$ $BH - \frac{1}{2} \cdot (N - 1) = 0$ $HP - \frac{1}{4} \cdot (N - 1) = 0$

$AH - \frac{1}{2} \cdot (1 + N) = 0$ $AG - \frac{1}{4} \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} = 0$ $\frac{1}{4} \cdot \frac{(N - 1)^2}{(1 + N)} - FH = 0$

$AF - \frac{1}{4} \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{(1 + N)} = 0$ $FM - \frac{1}{4} \cdot (N - 1) \cdot \frac{\left(1 + 6 \cdot N + N^2\right)}{\left[(1 + N) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}\right]} = 0$

$$BF - \frac{1}{4}\cdot(N+3)\cdot\frac{(N-1)}{(1+N)} = 0$$

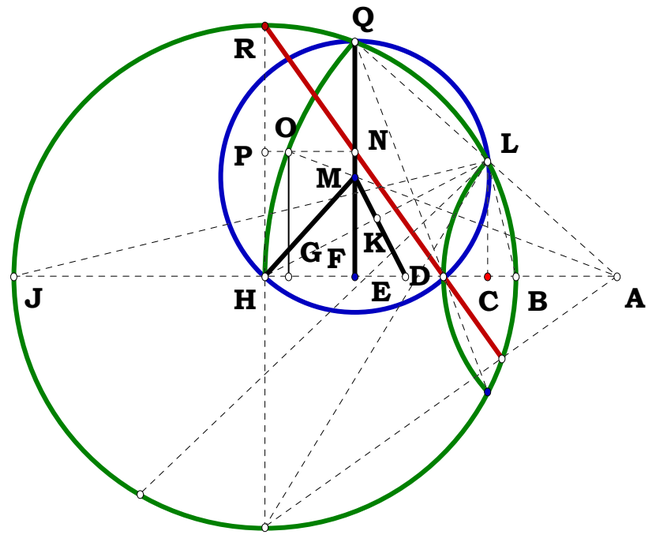
$$FJ - \frac{1}{4}\cdot(N-1)\cdot\frac{(3\cdot N+1)}{(1+N)} = 0$$

$$FQ - \frac{1}{4}\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\frac{(N-1)}{(1+N)} = 0$$

$$MQ - \frac{1}{2}\cdot(1+N)\cdot\frac{(N-1)}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$HM - \frac{1}{2}\cdot(N-1)\cdot\frac{(1+N)}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$DH - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \qquad FH - \frac{(N-1)^2}{4\cdot(N+1)} = 0$$



Unit.

$AB := 1$

Given.

Descriptions.

$N := 6$ $AJ := N$

042501B

$BJ := AJ - AB$ $BH := \frac{AB}{2}$ $HR := BH$

$HP := \frac{HR}{2}$ $GO := HP$ $JH := BJ + BH$

$JO := JH$ $JG := \sqrt{JO^2 - GO^2}$

$HQ := BH$ $JQ := JH$ $FH := \frac{HQ^2}{2 \cdot JH}$

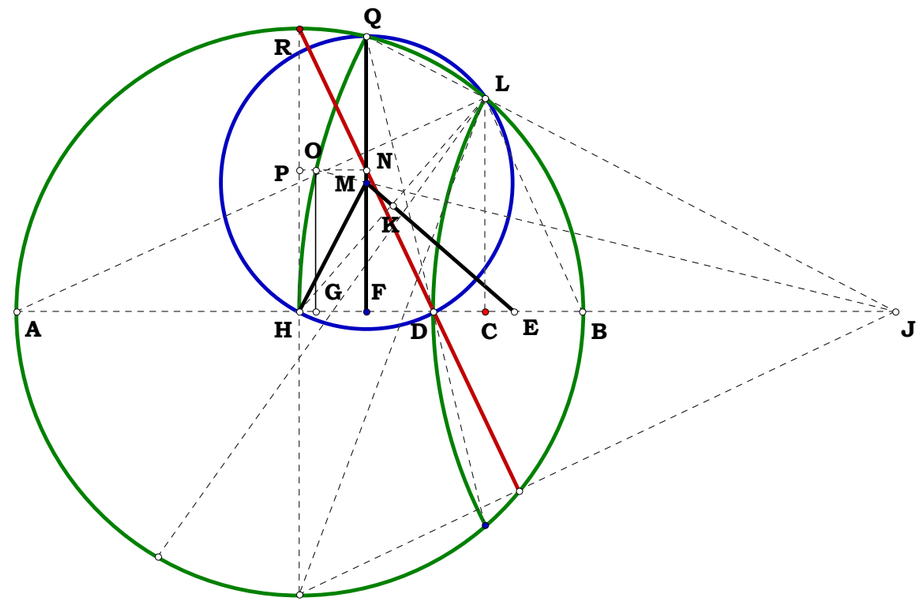
$AH := BH$ $FJ := JH - FH$ $FM := \frac{GO \cdot FJ}{JG}$

$AF := FH + AH$ $BF := AB - AF$ $FQ := \sqrt{BF \cdot AF}$

$MQ := FQ - FM$ $HM := \sqrt{FH^2 + FM^2}$

$HM - MQ = 0$ $DH := \frac{HR^2}{JH}$ $\frac{DH}{2} - FH = 0$

What is the Algebraic name of the circle HM? Does point N divide DR in half?



Definitions.

$$BJ - (N - 1) = 0 \quad BH - \frac{1}{2} = 0 \quad HR - \frac{1}{2} = 0$$

$$HP - \frac{1}{4} = 0 \quad GO - \frac{1}{4} = 0 \quad JH - \frac{2 \cdot N - 1}{2} = 0$$

$$JO - \frac{2 \cdot N - 1}{2} = 0 \quad JG - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4} = 0$$

$$HQ - \frac{1}{2} = 0 \quad JQ - \frac{2 \cdot N - 1}{2} = 0 \quad FH - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0$$

$$AH - \frac{1}{2} = 0 \quad FJ - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0$$

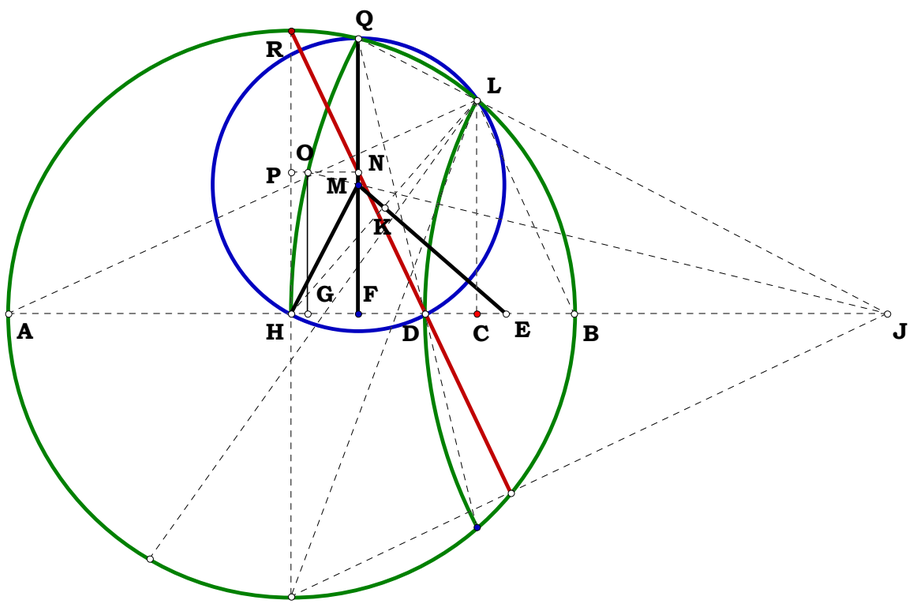
$$FM - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} \cdot (2 \cdot N - 1)} = 0$$

$$AF - \frac{4 \cdot N - 1}{4 \cdot (2 \cdot N - 1)} = 0 \quad BF - \frac{4 \cdot N - 3}{4 \cdot (2 \cdot N - 1)} = 0$$

$$FQ - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4 \cdot (2 \cdot N - 1)} = 0 \quad MQ - \frac{2 \cdot N - 1}{2 \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}} = 0$$

$$HM - \frac{(2 \cdot N - 1)}{\sqrt{4 \cdot (4 \cdot N - 1) \cdot (4 \cdot N - 3)}} = 0 \quad DH - \frac{1}{2 \cdot (2 \cdot N - 1)} = 0$$

$$HM - MQ = 0 \quad \frac{DH}{2} - FH = 0$$



042901A

Unit.

$AB := 1$

Given.

$N := 5 \qquad AD := N$

Descriptions.

$BD := AD - AB \qquad BC := \frac{BD}{2} \qquad CR := BC$

$CP := BC \qquad CD := BC \qquad AC := AB + BC \qquad CG := \frac{CP^2}{AC}$

$AG := AC - CG \qquad AV := AG \qquad CV := BC \qquad CF := \frac{CV^2 + AC^2 - AV^2}{2AC}$

$BF := BC - CF \qquad DF := CF + CD \qquad FV := \sqrt{BF \cdot DF} \qquad CE := \frac{CF \cdot CR}{CR - FV}$

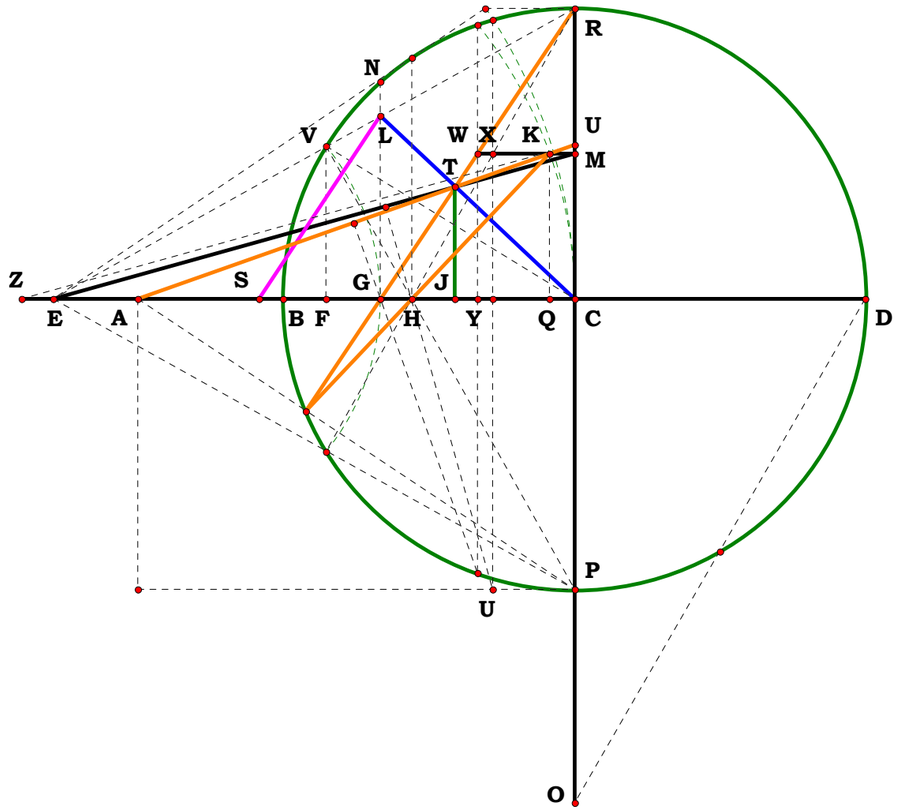
$BE := CE - BC \qquad BG := BC - CG \qquad EF := BE + BF \qquad EG := BE + BG$

$GL := \frac{FV \cdot EG}{EF} \qquad GS := \frac{CG \cdot GL}{CR} \qquad CJ := \frac{CG^2}{GS + CG}$

$JT := \frac{GL \cdot CJ}{CG}$

Four Lines To A Point

Does the difference CJ and JT each have but one Algebraic name?



$$CM := \frac{CR}{2} \qquad AK := AC \qquad KQ := CM$$

$$AQ := \sqrt{AK^2 - KQ^2} \qquad CU := \frac{KQ \cdot AC}{AQ}$$

$$CZ := \frac{CE \cdot CU}{CM} \qquad AU := \sqrt{AC^2 + CU^2}$$

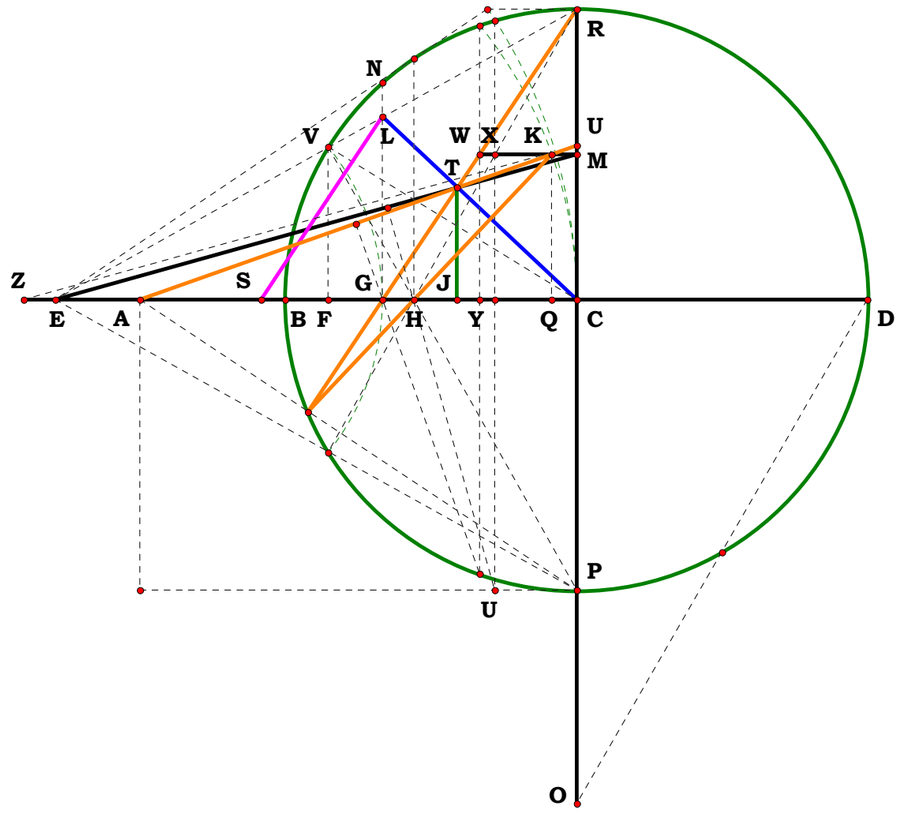
$$AZ := CZ - AC \qquad AE := CE - AC$$

$$AJ := \frac{AC \cdot AE}{AZ} \qquad JC := AC - AJ$$

$$TJ := \frac{CU \cdot AJ}{AC}$$

$$JC - CJ = 0$$

$$JT - TJ = 0$$



$$N - 5 = 0 \qquad AD - N = 0$$

Definitions.

$$BD - (N-1) = 0 \quad BC - \frac{N-1}{2} = 0 \quad CR - \frac{N-1}{2} = 0 \quad CP - \frac{N-1}{2} = 0$$

$$CD - \frac{N-1}{2} = 0 \quad AC - \frac{N+1}{2} \quad CG - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \quad AG - \frac{2\cdot N}{N+1} = 0$$

$$AV - \frac{2\cdot N}{N+1} = 0 \quad CV - \frac{N-1}{2} = 0 \quad CF - \frac{\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{2\cdot(N+1)^3} = 0$$

$$BF - \frac{(3\cdot N+1)\cdot(N-1)}{(N+1)^3} = 0 \qquad DF - \frac{N^2\cdot(N+3)\cdot(N-1)}{(N+1)^3} = 0$$

$$FV - \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} = 0$$

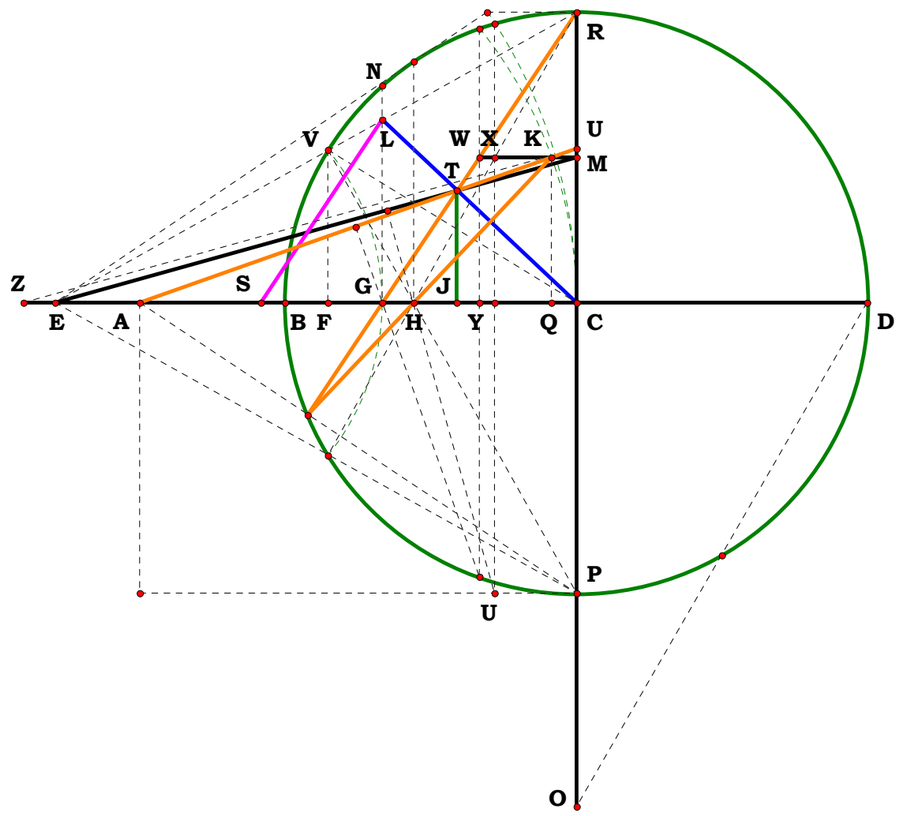
$$CE - \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]} = 0$$

$$BE - \frac{(N-1)\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-3\cdot N-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1} = 0 \qquad BG - \frac{N-1}{N+1} = 0$$

$$EF - \frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0 \qquad EG - \frac{N\cdot(N-1)^2\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$

$$GL - \frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}} = 0 \qquad GS - \frac{N\cdot(N-1)^2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{(N+1)^2\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}} = 0$$

$$CJ - \frac{N^4+2\cdot N^3-6\cdot N^2+2\cdot N+1}{14\cdot N\cdot(N+1)+2\cdot N^3+4\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+2} = 0 \qquad JT - \frac{N\cdot(N-1)\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{7\cdot N\cdot(N+1)+N^3+2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$



$$CM - \frac{N-1}{4} = 0 \qquad AK - \frac{N+1}{2} = 0$$

$$KQ - \frac{N-1}{4} = 0 \qquad AQ - \frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0$$

$$CU - \frac{(N+1)\cdot(N-1)}{2\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$CZ - \frac{(N-1)^2\cdot(N+1)\cdot\left(N^2+4\cdot N+1\right)}{\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]} = 0$$

$$AU - \frac{(N+1)^2}{\sqrt{(N+3)\cdot(3\cdot N+1)}} = 0$$

$$AZ - \frac{\left[\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^3+3\cdot N^2+3\cdot N+1\right)-2\cdot N\cdot\left(N^3+5\cdot N^2+4\cdot N+5\right)-2\right]\cdot(N+1)}{4\cdot N\cdot(N+3)\cdot(3\cdot N+1)-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(2\cdot N^3+6\cdot N^2+6\cdot N+2\right)} = 0$$

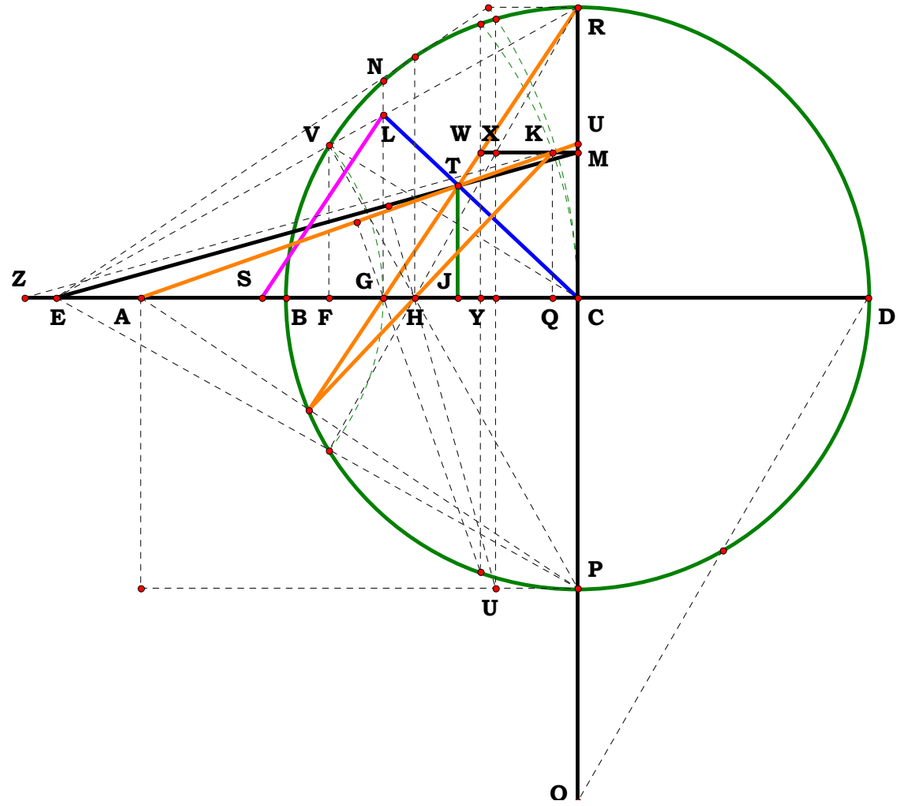
$$AE - \frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^2+N\right)-\left(N+6\cdot N^2+N^3\right)}{3\cdot N+3\cdot N^2+N^3-2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$

$$AJ - \frac{3\cdot N-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[N\cdot(N+1)\cdot\left(N^4+14\cdot N^3+34\cdot N^2+14\cdot N+1\right)\right]+N^2\cdot\left(3\cdot N^5+28\cdot N^4+117\cdot N^3+216\cdot N^2+117\cdot N+28\right)}{2\cdot N\cdot\left(N^6+11\cdot N^5+41\cdot N^4+75\cdot N^3+75\cdot N^2+41\cdot N+11\right)-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[N^2\cdot\left(N^4+10\cdot N^3+35\cdot N^2+36\cdot N+35\right)+10\cdot N+1\right]+2} = 0$$

$$JC - \frac{18\cdot N-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(N^7+9\cdot N^6+15\cdot N^5-25\cdot N^4-25\cdot N^3+15\cdot N^2+9\cdot N+1\right)+48\cdot N^2-2\cdot N^3-132\cdot N^4-2\cdot N^5+48\cdot N^6+18\cdot N^7+2\cdot N^8+2}{44\cdot N+164\cdot N^2+300\cdot N^3+300\cdot N^4+164\cdot N^5+44\cdot N^6+4\cdot N^7-\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(2\cdot N^6+20\cdot N^5+70\cdot N^4+72\cdot N^3+70\cdot N^2+20\cdot N+2\right)+4} = 0$$

$$TJ - \frac{99\cdot N^5-25\cdot N^2-89\cdot N^3-99\cdot N^4-3\cdot N+89\cdot N^6+25\cdot N^7+3\cdot N^8+\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left(33\cdot N^3-14\cdot N^6-33\cdot N^5-N^7+14\cdot N^2+N\right)}{2\cdot(N+1)^5\cdot\left(N^2+6\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}-208\cdot N^2-488\cdot N^3-570\cdot N^4-488\cdot N^5-208\cdot N^6-40\cdot N^7-3\cdot N^8-40\cdot N-3} = 0$$

$$TJ - JT = 0 \qquad JT - \frac{N\cdot(N-1)\cdot\left(N+\sqrt{3\cdot N^2+10\cdot N+3}+1\right)}{7\cdot N\cdot(N+1)+N^3+2\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot N+1} = 0$$



Unit.

$AB := 1$

Given.

$N := 1.22457 \qquad AD := N$

042901B

Descriptions.

Four Lines To A Point

Does the difference CJ and JT each have but one Algebraic name?

$$BD := AD - AB \qquad BC := \frac{AB}{2} \qquad CR := BC$$

$$CP := BC \qquad AC := BC \qquad CD := AD - AC \qquad CG := \frac{CP^2}{CD}$$

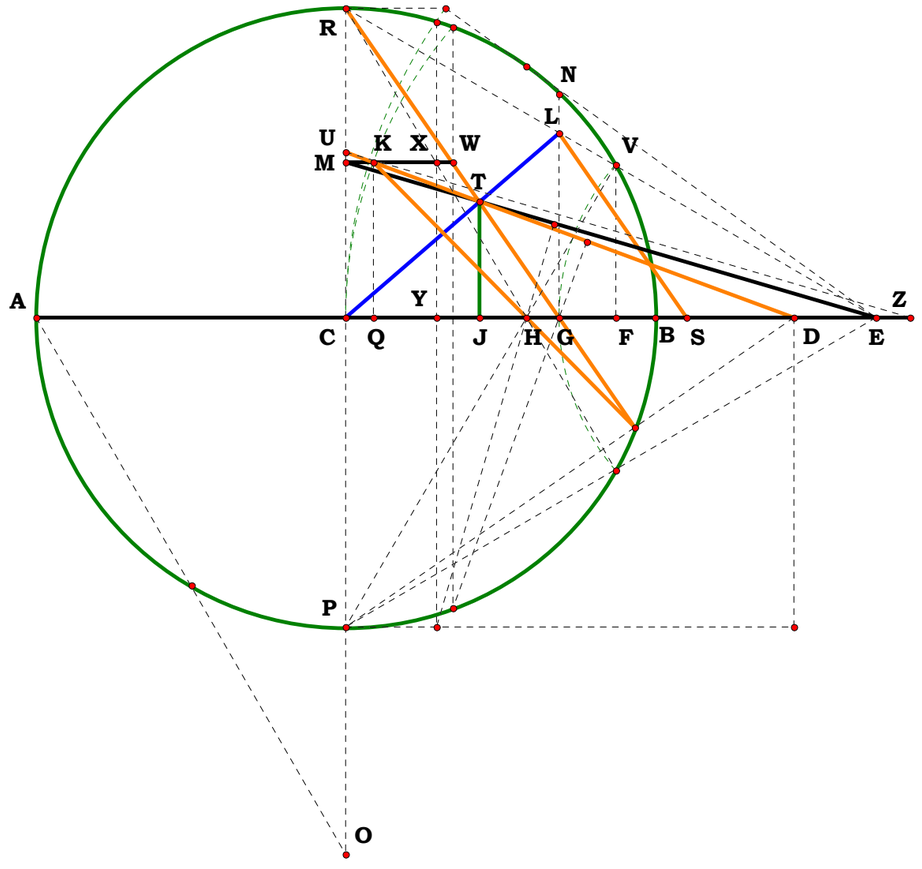
$$DG := CD - CG \qquad DV := DG \qquad CV := BC \qquad CF := \frac{CV^2 + CD^2 - DV^2}{2CD}$$

$$BF := BC - CF \qquad AF := CF + AC \qquad FV := \sqrt{BF \cdot AF} \qquad CE := \frac{CF \cdot CR}{CR - FV}$$

$$BE := CE - BC \qquad BG := BC - CG \qquad EF := BE + BF \qquad EG := BE + BG$$

$$GL := \frac{FV \cdot EG}{EF} \qquad GS := \frac{CG \cdot GL}{CR} \qquad CJ := \frac{CG^2}{GS + CG}$$

$$JT := \frac{GL \cdot CJ}{CG}$$



$$CM := \frac{CR}{2} \qquad DK := CD \qquad KQ := CM$$

$$DQ := \sqrt{DK^2 - KQ^2} \qquad CU := \frac{KQ \cdot CD}{DQ}$$

$$CZ := \frac{CE \cdot CU}{CM} \qquad DU := \sqrt{CD^2 + CU^2}$$

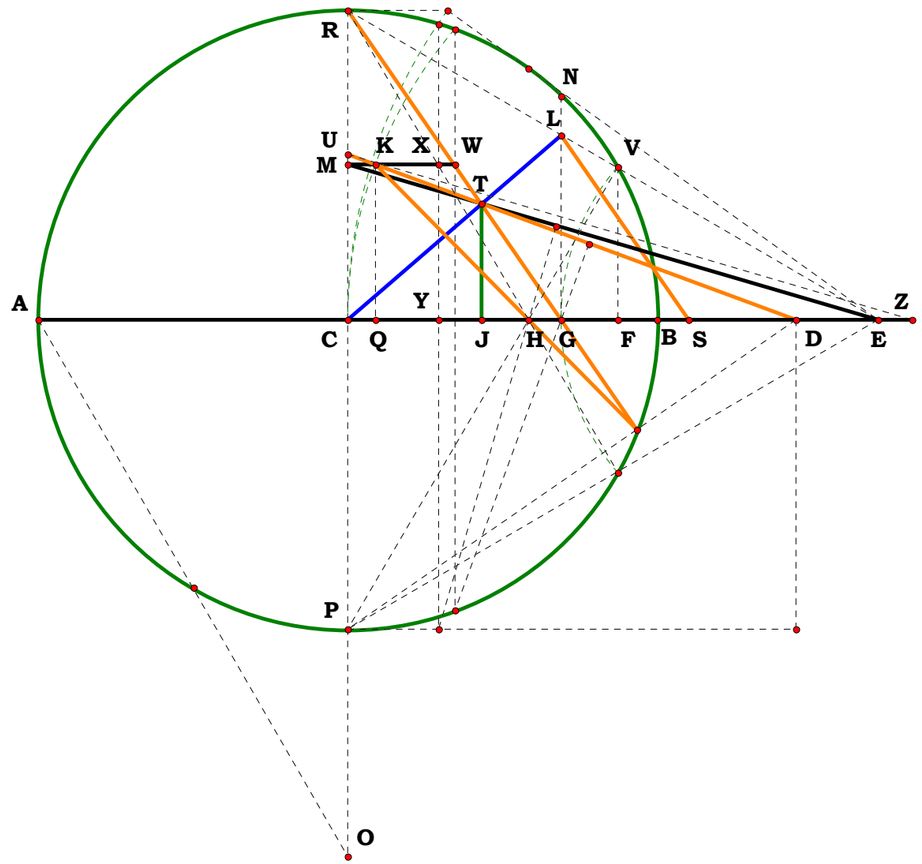
$$DZ := CZ - CD \qquad DE := CE - CD$$

$$DJ := \frac{CD \cdot DE}{DZ} \qquad JC := CD - DJ$$

$$TJ := \frac{CU \cdot DJ}{CD}$$

$$JC - CJ = 0$$

$$JT - TJ = 0$$



Definitions.

$$BD-(N-1)=0 \quad BC-\frac{1}{2}=0 \quad CR-\frac{1}{2}=0 \quad CP-\frac{1}{2}=0 \quad AC-\frac{1}{2}=0 \quad CD-\frac{2\cdot N-1}{2}=0$$

$$CG-\frac{1}{2\cdot(2\cdot N-1)}=0 \quad DG-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \quad DV-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \quad CV-\frac{1}{2}=0$$

$$CF-\frac{6\cdot N^2-6\cdot N+1}{2\cdot(2\cdot N-1)^3}=0 \quad BF-\frac{(4\cdot N-1)\cdot(N-1)^2}{(2\cdot N-1)^3}=0 \quad AF-\frac{N^2\cdot(4\cdot N-3)}{(2\cdot N-1)^3}$$

$$FV-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3}=0 \quad CE-\frac{6\cdot N^2-6\cdot N+1}{\left(4\cdot N-4\cdot N^2\right)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}+2\cdot(2\cdot N-1)^3}=0$$

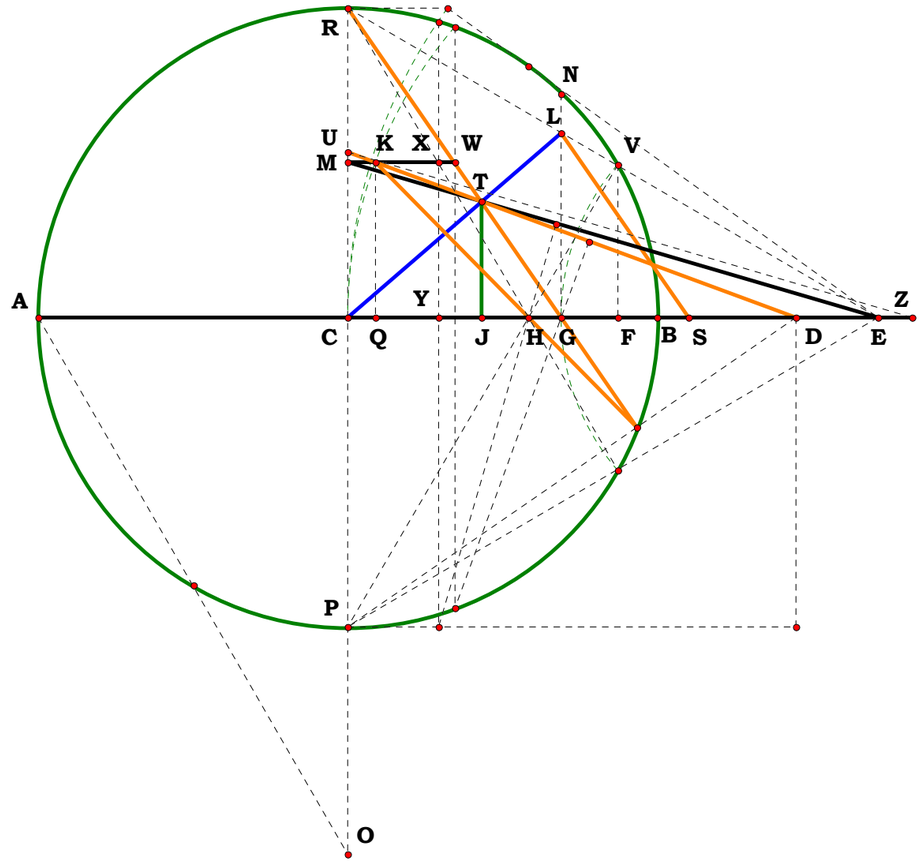
$$BE-\frac{(N-1)\cdot\left(5\cdot N+N\cdot\sqrt{16\cdot N^2-16\cdot N+3}-4\cdot N^2-1\right)}{\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+8\cdot N^3-12\cdot N^2+6\cdot N-1}=0 \quad BG-\frac{N-1}{2\cdot N-1}=0$$

$$EF-\frac{\sqrt{16\cdot N^2-16\cdot N+3}\cdot N\cdot(N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)}{(2\cdot N-1)^3\cdot\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+(2\cdot N-1)^3\cdot\left[(2\cdot N-1)^3\right]}=0$$

$$EG-\frac{N\cdot(N-1)\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)\cdot\left[\left(8\cdot N^3-12\cdot N^2+6\cdot N-1\right)+\left(2\cdot N-2\cdot N^2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}\right]}=0 \quad GL-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)\cdot\left(6\cdot N^2-6\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0$$

$$GS-\frac{N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{(2\cdot N-1)^2\cdot\left(6\cdot N^2-6\cdot N+1\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0 \quad CJ-\frac{\sqrt{16\cdot N^2-16\cdot N+3}\cdot\left(6\cdot N^2-6\cdot N+1\right)}{\left(4\cdot N^2-4\cdot N\right)\cdot\left(\sqrt{16\cdot N^2-16\cdot N+3}\right)^2+\left(32\cdot N^3-48\cdot N^2+20\cdot N-2\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}}=0$$

$$JT-\frac{N\cdot(N-1)\cdot\left(2\cdot N+\sqrt{16\cdot N^2-16\cdot N+3}-1\right)}{\left(2\cdot N^2-2\cdot N\right)\cdot\sqrt{16\cdot N^2-16\cdot N+3}+(2\cdot N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0$$



$$CM - \frac{1}{4} = 0 \qquad DK - \frac{2\cdot N - 1}{2} = 0 \qquad KQ - \frac{1}{4} = 0$$

$$DQ - \frac{\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)}}{4} = 0 \qquad CU - \frac{2\cdot N - 1}{2\cdot\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)}} = 0$$

$$CZ - \frac{(2\cdot N - 1)\cdot\left(6\cdot N^2 - 6\cdot N + 1\right)}{(2\cdot N - 1)^3\cdot\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)} + (4\cdot N - 1)\cdot(4\cdot N - 3)\cdot\left(2\cdot N - 2\cdot N^2\right)} = 0$$

$$DU - \frac{(2\cdot N - 1)^2}{\sqrt{(4\cdot N - 1)\cdot(4\cdot N - 3)}} = 0$$

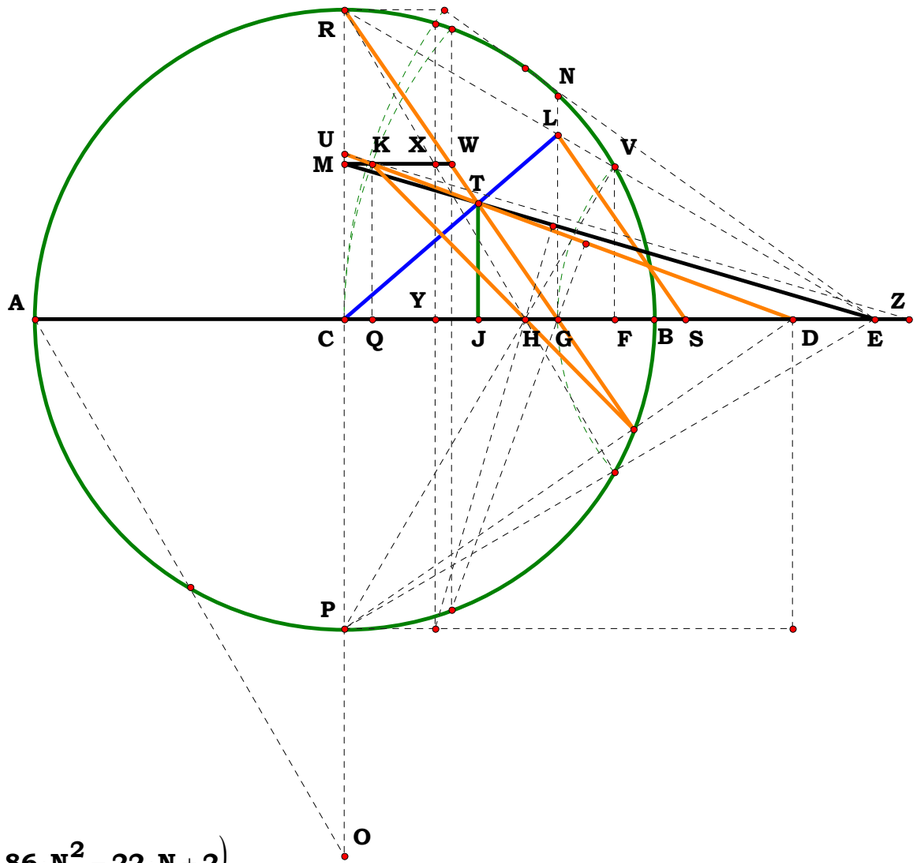
$$DZ - \frac{2\cdot N\cdot\left(32\cdot N^4 - 80\cdot N^3 + 82\cdot N^2 - 43\cdot N + 11\right) - (2\cdot N - 1)^4\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - 2}{2\cdot(2\cdot N - 1)^3\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - 4\cdot N\cdot(N - 1)\cdot(4\cdot N - 3)\cdot(4\cdot N - 1)} = 0$$

$$DE - \frac{N\cdot(N - 1)\cdot(2\cdot N - 1)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - N\cdot(N - 1)\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)}{\left(2\cdot N - 2\cdot N^2\right)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} + 8\cdot N^3 - 12\cdot N^2 + 6\cdot N - 1} = 0$$

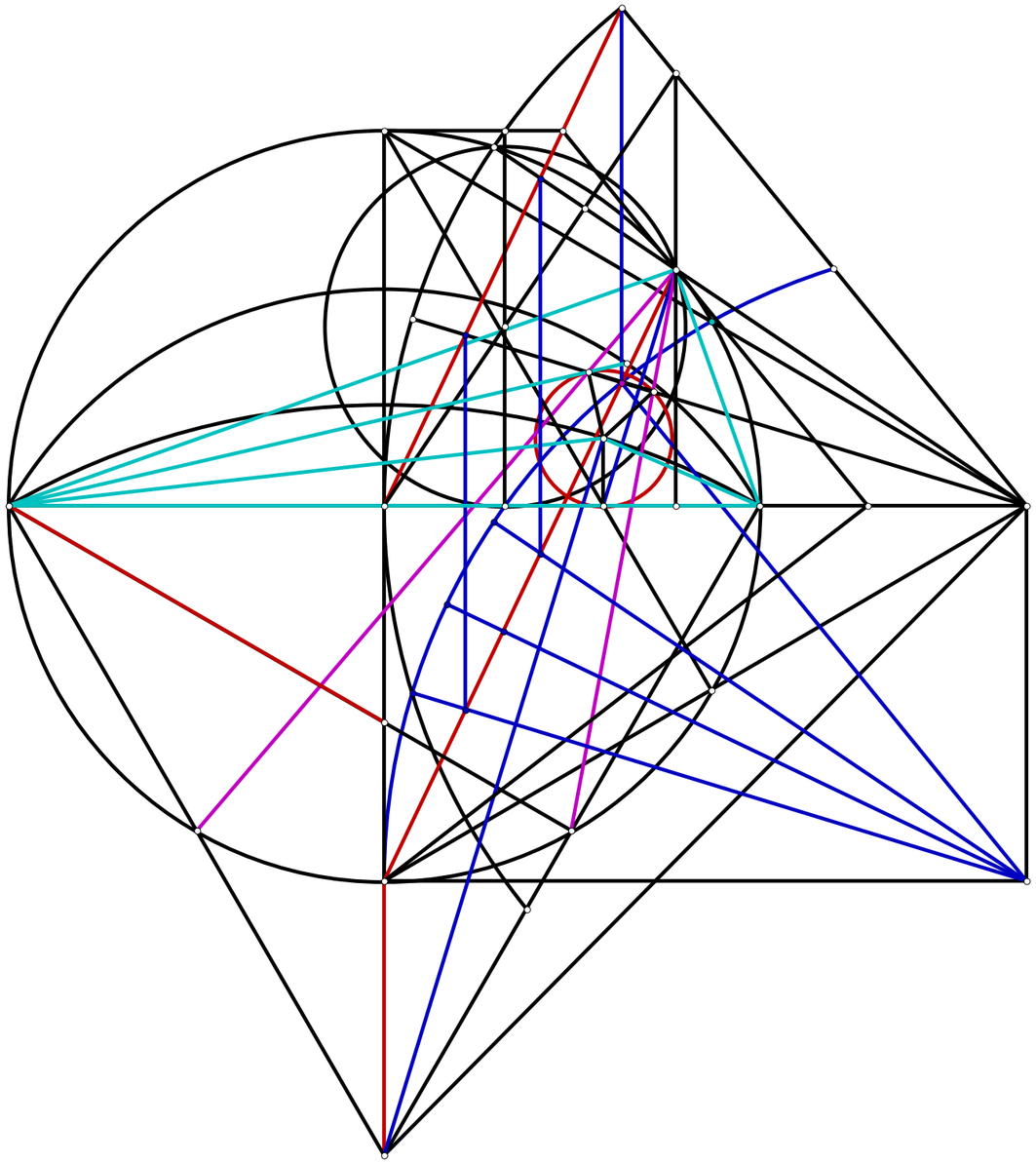
$$JC - \frac{\left(6\cdot N^2 - 6\cdot N + 1\right)\cdot\left(16\cdot N^3 - 24\cdot N^2 + 10\cdot N - 1\right)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - \left(6\cdot N^2 - 6\cdot N + 1\right)\cdot\left(64\cdot N^4 - 128\cdot N^3 + 86\cdot N^2 - 22\cdot N + 2\right)}{\left[8\cdot N\cdot(N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 53\cdot N^2 - 21\cdot N + 4\right) + 2\right]\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} + -4\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot(2\cdot N - 1)^5} = 0$$

$$DJ - \frac{N\cdot(N - 1)\cdot(2\cdot N - 1)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3}\cdot\left(64\cdot N^4 - 128\cdot N^3 + 82\cdot N^2 - 18\cdot N + 1\right) - N\cdot(N - 1)\cdot(4\cdot N - 3)\cdot(4\cdot N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 42\cdot N^2 - 10\cdot N + 1\right)}{\sqrt{16\cdot N^2 - 16\cdot N + 3}\cdot\left(128\cdot N^6 - 384\cdot N^5 + 468\cdot N^4 - 296\cdot N^3 + 100\cdot N^2 - 16\cdot N + 1\right) - (2\cdot N - 1)^5\cdot\left(16\cdot N^2 - 16\cdot N + 2\right)} = 0$$

$$TJ - \frac{N\cdot(N - 1)\cdot(4\cdot N - 3)\cdot(4\cdot N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 42\cdot N^2 - 10\cdot N + 1\right) - N\cdot(N - 1)\cdot(2\cdot N - 1)\cdot\left(64\cdot N^4 - 128\cdot N^3 + 82\cdot N^2 - 18\cdot N + 1\right)\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3}}{2\cdot\left(8\cdot N^2 - 8\cdot N + 1\right)\cdot(2\cdot N - 1)^5\cdot\sqrt{16\cdot N^2 - 16\cdot N + 3} - (4\cdot N - 3)\cdot(4\cdot N - 1)\cdot\left[4\cdot N\cdot(N - 1)\cdot\left(32\cdot N^4 - 64\cdot N^3 + 53\cdot N^2 - 21\cdot N + 4\right) + 1\right]} = 0$$



Just an Ilustration.



Unit.

$AB := 1$

Given.

$N := 4$ $AE := N$

Just some Algebraic Names

050601A

Descriptions.

$BE := AE - AB$ $BO := \frac{BE}{2}$ $AO := AB + BO$ $AJ := AO$ $JO := BO$

$GO := \frac{JO}{2}$ $AG := \sqrt{AO^2 - GO^2}$ $AP := \frac{AG^2}{AO}$ $OP := AO - AP$ $NO := 2 \cdot OP$

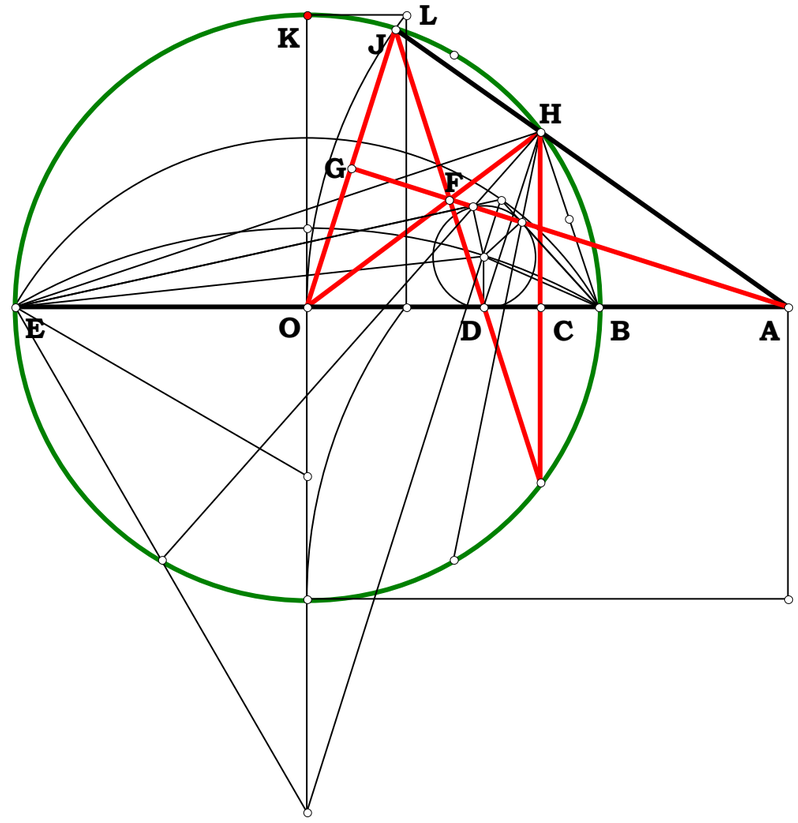
$AN := AO - NO$ $JM := NO$ $HO := BO$ $HJ := 2 \cdot JM$ $AH := AJ - HJ$

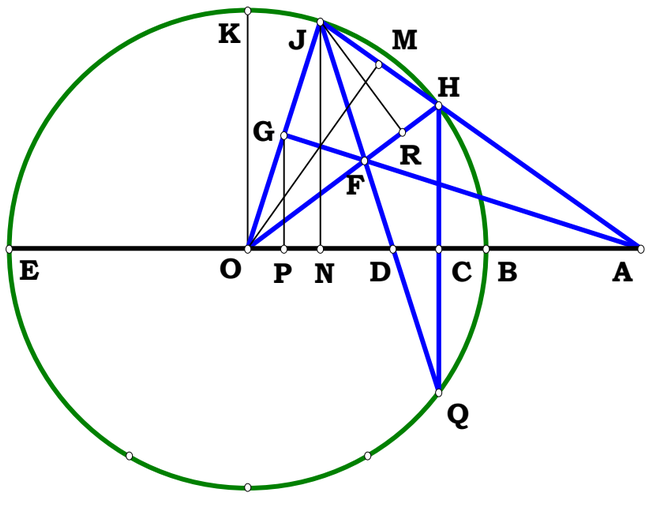
$AC := \frac{AN \cdot AH}{AJ}$ $CH := \sqrt{AH^2 - AC^2}$ $HQ := 2 \cdot CH$ $CN := AN - AC$

$JN := \frac{CH \cdot AJ}{AH}$ $CQ := CH$ $JQ := \sqrt{(CQ + JN)^2 + CN^2}$ $OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO}$

$JR := \sqrt{JO^2 - OR^2}$ $FO := \frac{JO \cdot GO}{OR}$ $FJ := FO$ $DQ := \frac{JQ \cdot CQ}{CQ + JN}$ $DF := JQ - (DQ + FJ)$

$FH := HO - FO$ $FG := \frac{JR \cdot GO}{OR}$ $AF := AG - FG$





Definitions.

$$AE - N = 0 \quad BE - (N-1) = 0 \quad BO - \frac{N-1}{2} = 0 \quad AO - \frac{N+1}{2} = 0 \quad AG - \frac{\sqrt{(N+3)\cdot(3\cdot N+1)}}{4} = 0$$

$$AP - \frac{(3\cdot N+1)\cdot(N+3)}{8\cdot(N+1)} = 0 \quad OP - \frac{(N-1)^2}{8\cdot(N+1)} = 0 \quad NO - \frac{(N-1)^2}{4\cdot(N+1)} = 0 \quad AN - \frac{N^2+6\cdot N+1}{4\cdot(N+1)} = 0$$

$$HJ - \frac{(N-1)^2}{2\cdot(N+1)} = 0 \quad AH - \frac{2\cdot N}{N+1} = 0 \quad AC - \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} = 0 \quad CH - \frac{N\cdot(N-1)\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{(N+1)^3} = 0$$

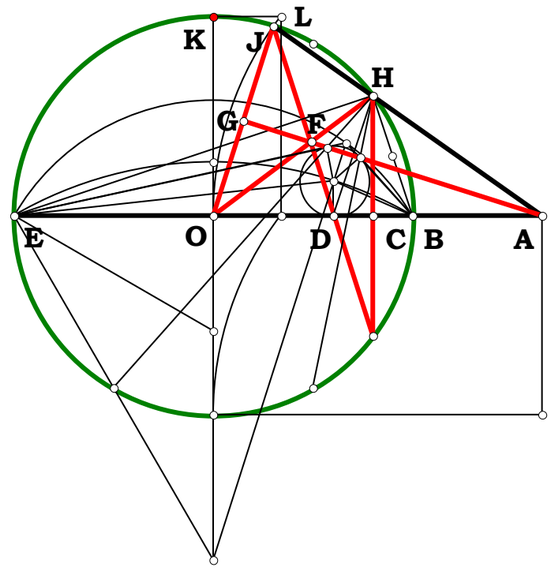
$$HQ - \frac{2\cdot N\cdot(N-1)\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{(N+1)^3} = 0 \quad CN - \frac{(N-1)^2\cdot\left(N^2+6\cdot N+1\right)}{4\cdot(N+1)^3} = 0 \quad JN - \frac{\sqrt{N+3}\cdot\sqrt{3\cdot N+1}\cdot(N-1)}{4\cdot(N+1)} = 0$$

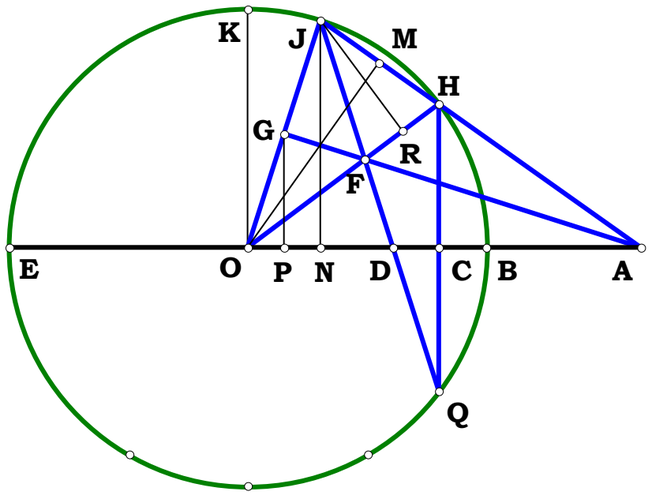
$$JQ - \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} = 0 \quad OR - \frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{4\cdot(N+1)^2} = 0 \quad JR - \frac{(N-1)^2\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{4\cdot(N+1)^2} = 0$$

$$FO - \frac{(N-1)\cdot(N+1)^2}{2\cdot\left(N^2+6\cdot N+1\right)} = 0 \quad DQ - \frac{2\cdot N\cdot(N-1)}{(N+1)^2} = 0 \quad DF - \frac{2\cdot N\cdot(N-1)}{N^2+6\cdot N+1} = 0 \quad FH - \frac{2\cdot N\cdot(N-1)}{N^2+6\cdot N+1} = 0$$

$$FG - \frac{(N-1)^2\cdot\sqrt{N+3}\cdot\sqrt{3\cdot N+1}}{4\cdot\left(N^2+6\cdot N+1\right)} = 0 \quad GO - \frac{N-1}{4} = 0$$

$$AF - \frac{\left(2\cdot N-N^2-1\right)\cdot\sqrt{3\cdot N+1}\cdot\sqrt{N+3}+\left(N^2+6\cdot N+1\right)\cdot\sqrt{3\cdot N^2+10\cdot N+3}}{4\cdot\left(N^2+6\cdot N+1\right)} = 0$$





Unit.

$AB := 1$

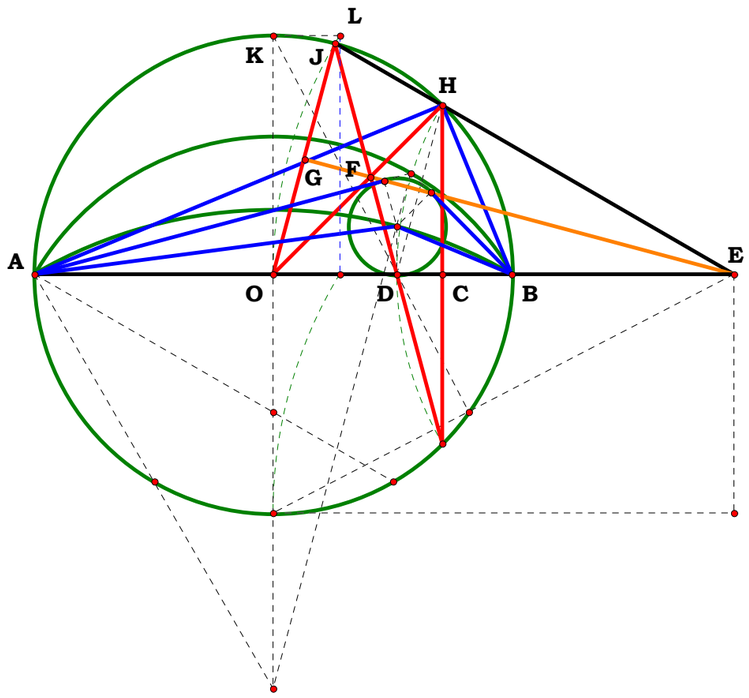
Given.

$N := 1.46595 \quad AE := N$

050601B

Descriptions.

Just some Algebraic Names



$BE := AE - AB \quad BO := \frac{AB}{2} \quad EO := BE + BO \quad EJ := EO$

$JO := BO \quad GO := \frac{JO}{2} \quad EG := \sqrt{EO^2 - GO^2}$

$EP := \frac{EG^2}{EO} \quad OP := EO - EP \quad NO := 2 \cdot OP$

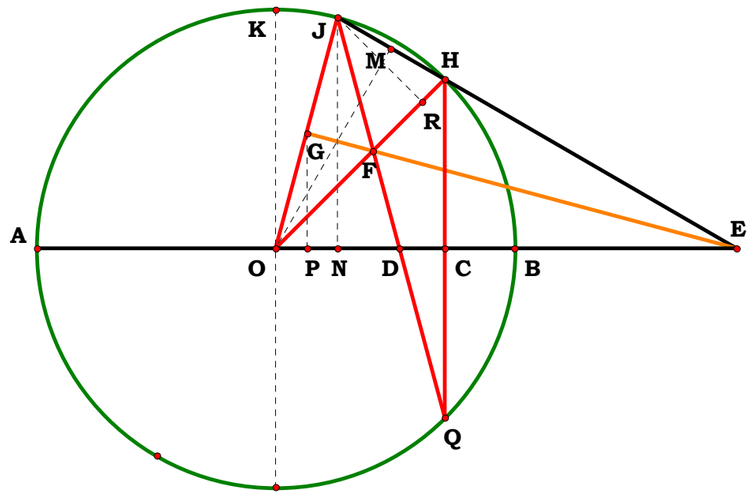
$EN := EO - NO \quad JM := NO \quad HO := BO \quad HJ := 2 \cdot JM \quad EH := EJ - HJ$

$EC := \frac{EN \cdot EH}{EJ} \quad CH := \sqrt{EH^2 - EC^2} \quad HQ := 2 \cdot CH \quad CN := EN - EC$

$JN := \frac{CH \cdot EJ}{EH} \quad CQ := CH \quad JQ := \sqrt{(CQ + JN)^2 + CN^2} \quad OR := \frac{JO^2 + HO^2 - HJ^2}{2 \cdot HO}$

$JR := \sqrt{JO^2 - OR^2} \quad FO := \frac{JO \cdot GO}{OR} \quad FJ := FO \quad DQ := \frac{JQ \cdot CQ}{CQ + JN}$

$DF := JQ - (DQ + FJ) \quad FH := HO - FO \quad FG := \frac{JR \cdot GO}{OR} \quad EF := EG - FG$



Definitions.

$$BE-(N-1)=0 \quad BO-\frac{1}{2}=0 \quad EO-\frac{2\cdot N-1}{2}=0 \quad EJ-\frac{2\cdot N-1}{2}=0$$

$$JO-\frac{1}{2}=0 \quad GO-\frac{1}{4}=0 \quad EG-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4}=0$$

$$EP-\frac{(4\cdot N-1)\cdot(4\cdot N-3)}{8\cdot(2\cdot N-1)}=0 \quad OP-\frac{1}{8\cdot(2\cdot N-1)}=0 \quad NO-\frac{1}{4\cdot(2\cdot N-1)}=0$$

$$EN-\frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)}=0 \quad JM-\frac{1}{4\cdot(2\cdot N-1)}=0 \quad HO-\frac{1}{2}=0 \quad HJ-\frac{1}{2\cdot(2\cdot N-1)}=0$$

$$EH-\frac{2\cdot N\cdot(N-1)}{2\cdot N-1}=0 \quad EC-\frac{N\cdot(N-1)\cdot\left(8\cdot N^2-8\cdot N+1\right)}{(2\cdot N-1)^3}=0 \quad CH-\frac{N\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot(N-1)}{(2\cdot N-1)^3}=0$$

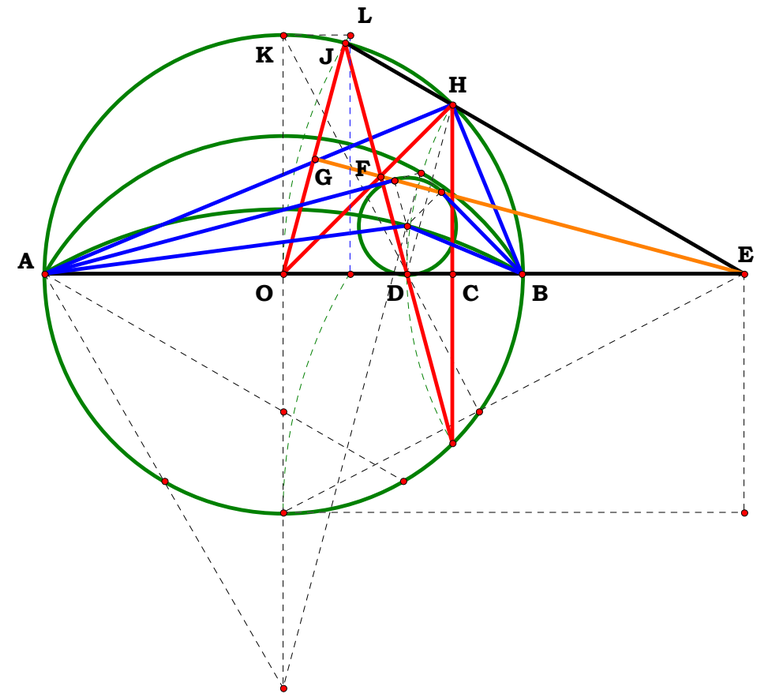
$$HQ-\frac{2\cdot N\cdot(N-1)\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{(2\cdot N-1)^3}=0 \quad CN-\frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)^3}=0 \quad JN-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)}=0$$

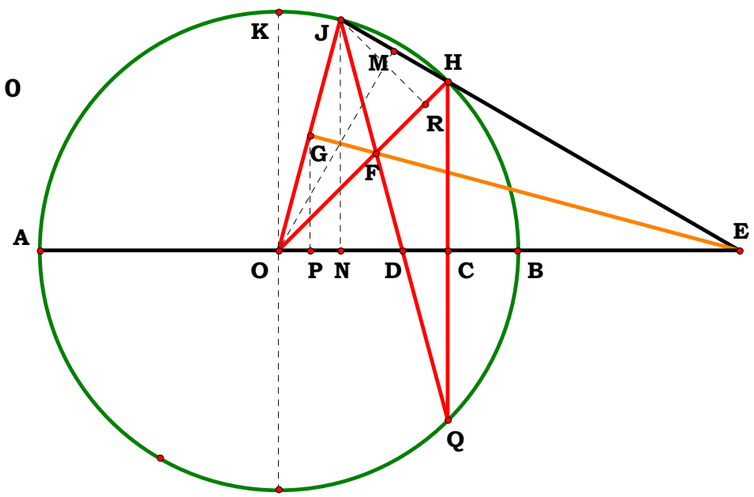
$$CQ-\frac{N\cdot\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}\cdot(N-1)}{(2\cdot N-1)^3}=0 \quad JQ-\frac{\left(8\cdot N^2-8\cdot N+1\right)}{2\cdot(2\cdot N-1)^2}=0 \quad OR-\frac{8\cdot N^2-8\cdot N+1}{4\cdot(2\cdot N-1)^2}=0$$

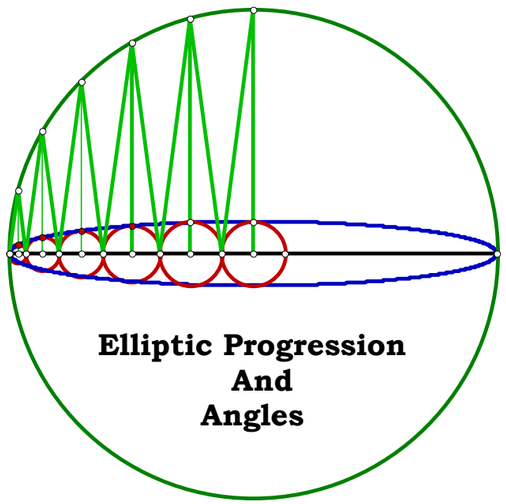
$$JR-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot(2\cdot N-1)^2}=0 \quad FO-\frac{(2\cdot N-1)^2}{2\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0 \quad FJ-\frac{HO}{4\cdot\left(-HJ^2+HO^2+JO^2\right)}=0$$

$$DQ-\frac{2\cdot N\cdot(N-1)}{(2\cdot N-1)^2}=0 \quad DF-\frac{2\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1}=0 \quad FH-\frac{2\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1}=0$$

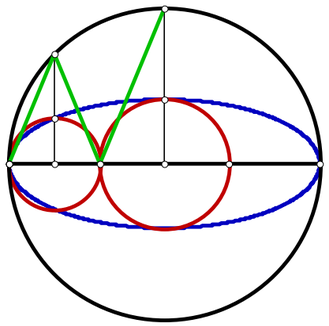
$$FG-\frac{\sqrt{(4\cdot N-1)\cdot(4\cdot N-3)}}{4\cdot\left(8\cdot N^2-8\cdot N+1\right)}=0 \quad EF-\frac{2\cdot\sqrt{16\cdot N^2-16\cdot N+3}\cdot N\cdot(N-1)}{8\cdot N^2-8\cdot N+1}=0$$



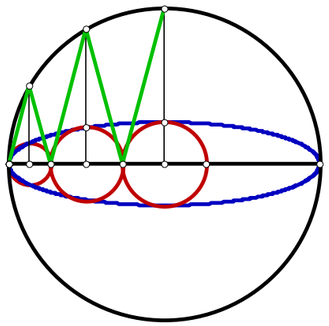




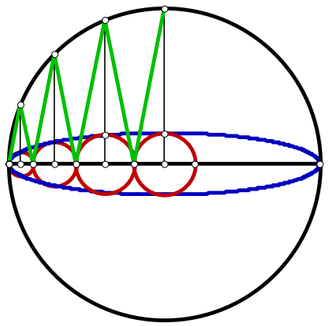
$\frac{m\angle CAAW}{m\angle ARCN} = 2.000$



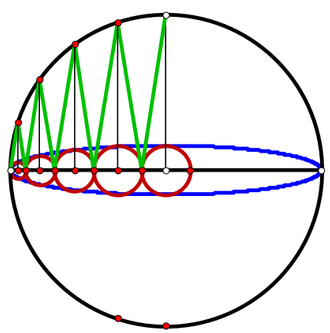
$\frac{m\angle CAAW}{m\angle ARCN} = 3.000$



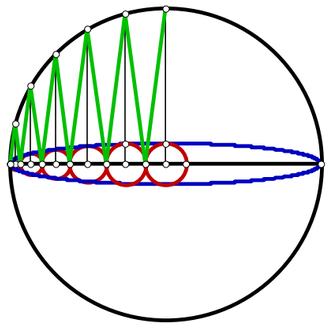
$\frac{m\angle CAAW}{m\angle ARCN} = 4.000$



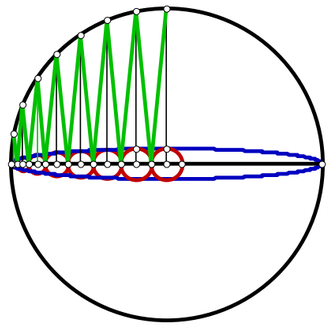
$\frac{m\angle CAAW}{m\angle BCA} = 5.000$



$\frac{m\angle CAAW}{m\angle ARCN} = 6.000$



$\frac{m\angle CAAW}{m\angle ARCN} = 8.000$



Unit.

$AB := 1$

Given.

$N_1 := \frac{AB}{10}$

$N_2 := 8$

050701 EP

Descriptions.

$AQ := N_1 \cdot N_2$ $\quad BQ := AB - AQ$ $\quad DQ := \sqrt{BQ \cdot AQ}$

$AO := \frac{AB}{2}$ $\quad AY := \sqrt{2 \cdot AO^2}$ $\quad OQ := AQ - AO$

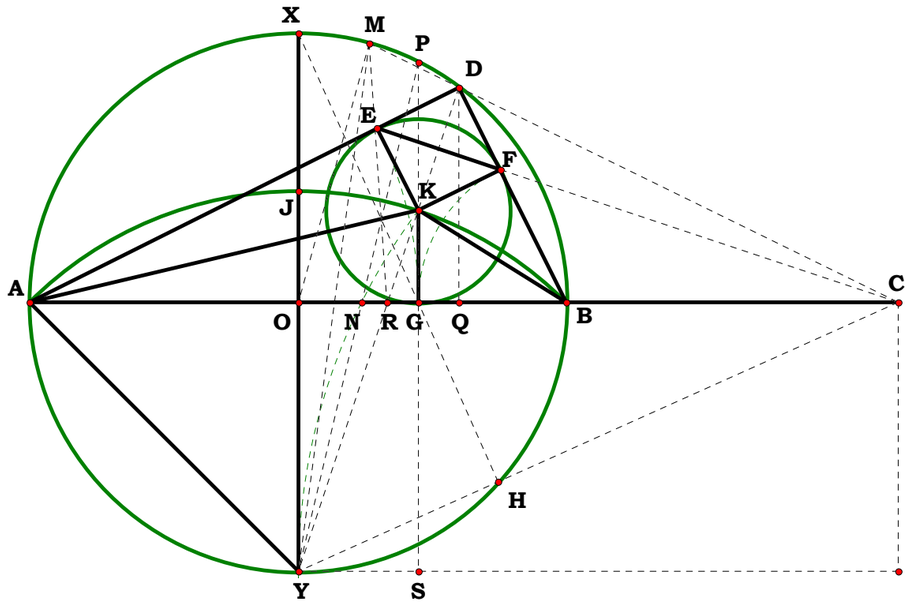
$DY := \sqrt{(AO + DQ)^2 + OQ^2}$ $\quad OR := \frac{OQ \cdot AO}{AO + DQ}$

$QR := OQ - OR$ $\quad KS := \frac{AY \cdot (DQ + AO)}{DY}$

$GK := KS - AO$ $\quad GR := \frac{QR \cdot GK}{DQ}$ $\quad GO := GR + OR$

$CO := \frac{AO^2}{GO}$ $\quad BC := CO - AO$ $\quad AC := AB + BC$

What is an Angle?



$AC = 1.61803$

$N_2 = 8.00000$

$N_1 = 10.00000$

$$\frac{\left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right) + \sqrt{1 + 2 \cdot \sqrt{\left(\frac{1}{N_1}\right) \cdot N_2 - \frac{1}{N_1}^2 \cdot N_2^2}}}{2 \cdot \left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right)} = 1.61803$$

$$AC - \frac{\left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right) + \sqrt{1 + 2 \cdot \sqrt{\left(\frac{1}{N_1}\right) \cdot N_2 - \frac{1}{N_1}^2 \cdot N_2^2}}}{2 \cdot \left(2 \cdot \left(\frac{1}{N_1}\right) \cdot N_2 - 1\right)} = 0.00000$$

Definitions.

$AQ - N_1 \cdot N_2 = 0$ $\quad BQ - \left(1 - N_1 \cdot N_2\right) = 0$ $\quad DQ - \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} = 0$

$AO - \frac{1}{2} = 0$ $\quad AY - \frac{1}{\sqrt{2}} = 0$ $\quad OQ - \frac{2 \cdot N_1 \cdot N_2 - 1}{2} = 0$ $\quad DY - \frac{\sqrt{4 \cdot \sqrt{-N_1 \cdot N_2 \cdot \left(N_1 \cdot N_2 - 1\right)} + 2}}{2} = 0$ $\quad OR - \frac{2 \cdot N_1 \cdot N_2 - 1}{2 \cdot \left(2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1\right)} = 0$

$QR - \frac{\sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)}{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} = 0$ $\quad KS - \frac{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}{2 \cdot \sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}} = 0$ $\quad GK - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1}{2} = 0$

$GR - \frac{\left(\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1\right) \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)}{2 \cdot \left(2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1\right)}$ $\quad GO - \frac{N_1 \cdot N_2 - \frac{1}{2}}{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}} = 0$ $\quad CO - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1}}{2 \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)} = 0$

$BC - \frac{\sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 2 \cdot N_1 \cdot N_2 + 1}{4 \cdot N_1 \cdot N_2 - 2} = 0$ $\quad AC - \frac{2 \cdot N_1 \cdot N_2 + \sqrt{2 \cdot \sqrt{N_1 \cdot N_2 - N_1^2 \cdot N_2^2} + 1} - 1}{2 \cdot \left(2 \cdot N_1 \cdot N_2 - 1\right)} = 0$ $\quad AC - \frac{N_2 + \sqrt{5} \cdot \sqrt{\sqrt{10 \cdot N_2 - N_2^2} + 5} - 5}{2 \cdot N_2 - 10} = 0$

Unit.

$AB := 1$

0507013
Descriptions.

$AC := \frac{AB}{2}$ $BF := \sqrt{2 \cdot AC^2}$

$BG := \frac{BF}{2}$ $CH := AC$ $CG := BG$

$GH := CH - CG$ $BH := \sqrt{BG^2 + GH^2}$

$BD := \frac{BH^2}{AB}$ $CE := \frac{AB - (4 \cdot BD)}{2}$

Definitions.

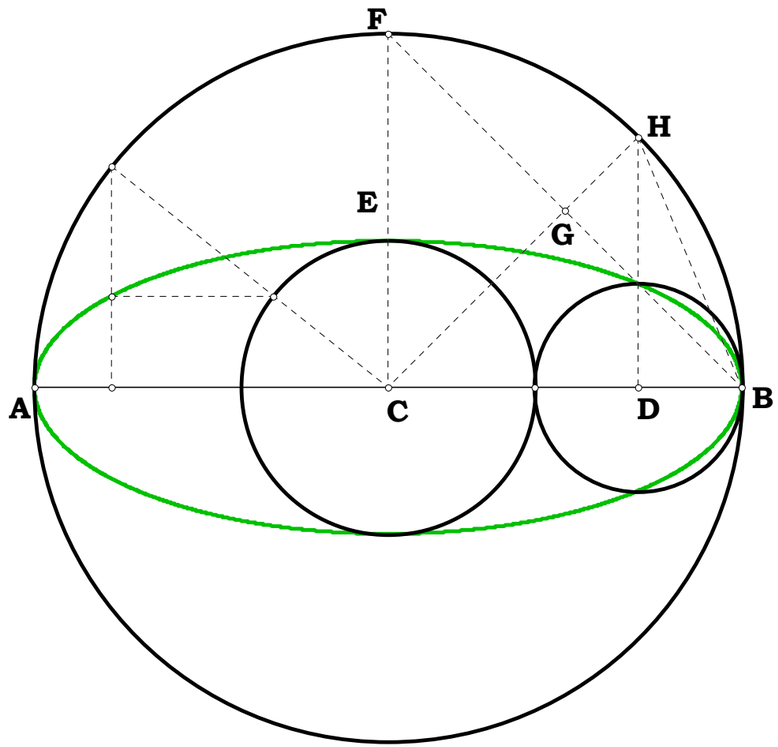
$AC - \frac{1}{2} = 0$ $BF - \frac{\sqrt{2}}{2} = 0$

$BG - \frac{\sqrt{2}}{4} = 0$ $CH - \frac{1}{2} = 0$ $CG - \frac{\sqrt{2}}{4} = 0$

$GH - \frac{2 - \sqrt{2}}{4} = 0$ $BH - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0$

$BD - \frac{2 - \sqrt{2}}{4} = 0$ $CE - \frac{\sqrt{2} - 1}{2} = 0$

Angles by Ellipse:



Unit.

$AN := 1$

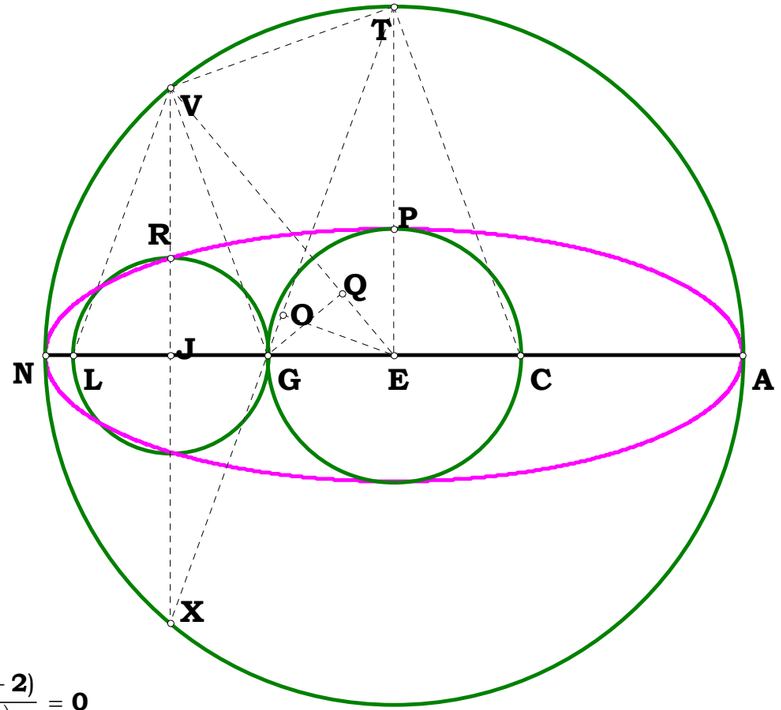
Given.

$N := 5.52$

0507012

Elliptic Progression Outtake Two

Angles TEV and EVJ equals CTG.



Descriptions.

$$EN := \frac{AN}{2} \quad ET := EN \quad EV := EN \quad EG := \frac{AN}{N} \quad EP := EG$$

$$GT := \sqrt{EG^2 + ET^2} \quad GO := \frac{EG^2}{GT} \quad GX := GT - 2 \cdot GO \quad JX := \frac{ET \cdot GX}{GT}$$

$$GJ := \frac{EG \cdot GX}{GT} \quad \frac{ET}{EP} - \frac{JX}{GJ} = 0 \quad JV := JX \quad EJ := EG + GJ$$

$$TV := \sqrt{ET^2 - 2 \cdot ET \cdot JV + JV^2 + EJ^2} \quad EQ := \frac{EJ \cdot EG}{EV} \quad GQ := \frac{JV \cdot EQ}{EJ}$$

$$GQ - GJ = 0 \quad \frac{ET}{TV} - \frac{GT}{2 \cdot EG} = 0 \quad TV - \frac{2 \cdot AN}{\sqrt{N^2 + 4}} = 0$$

Definitions.

$$EN - \frac{1}{2} = 0 \quad ET - \frac{1}{2} = 0 \quad EV - \frac{1}{2} = 0 \quad EG - \frac{1}{N} = 0 \quad EP - \frac{1}{N} = 0$$

$$GT - \frac{\sqrt{N^2 + 4}}{2 \cdot N} = 0 \quad GO - \frac{2}{N \cdot \sqrt{N^2 + 4}} = 0 \quad GX - \frac{(N-2) \cdot (N+2)}{2 \cdot N \cdot \sqrt{N^2 + 4}} = 0 \quad JX - \frac{(N-2) \cdot (N+2)}{2 \cdot (N^2 + 4)} = 0$$

$$GJ - \frac{(N-2) \cdot (N+2)}{N \cdot (N^2 + 4)} = 0 \quad \frac{ET}{EP} - \frac{JX}{GJ} = 0 \quad JV - \frac{(N-2) \cdot (N+2)}{2 \cdot (N^2 + 4)} = 0 \quad EJ - \frac{2 \cdot N}{N^2 + 4} = 0$$

$$TV - \frac{2}{\sqrt{N^2 + 4}} = 0 \quad EQ - \frac{4}{N^2 + 4} = 0 \quad GQ - \frac{(N-2) \cdot (N+2)}{N \cdot (N^2 + 4)} = 0$$

Unit.

$AL := 1$

0507013

Descriptions.

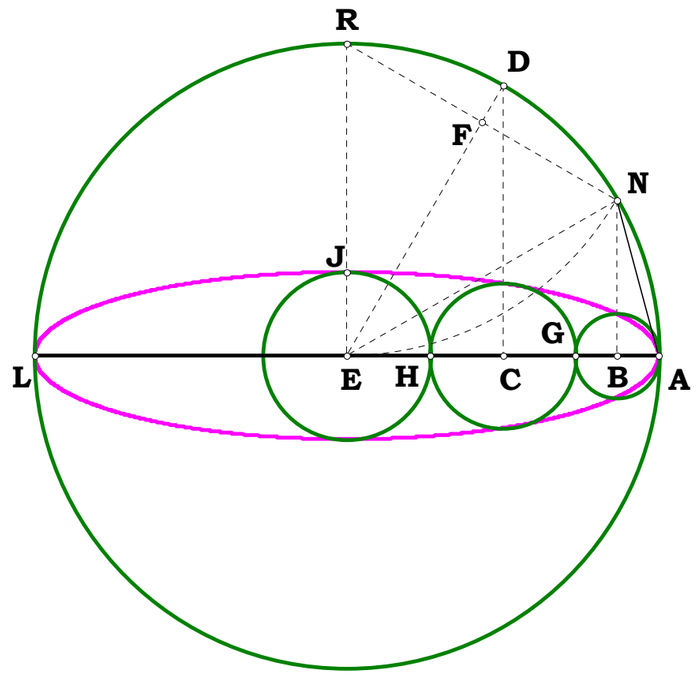
$AE := \frac{AL}{2}$ $ER := AE$ $NR := ER$ $FN := \frac{NR}{2}$ $EN := AE$

$EF := \sqrt{EN^2 - FN^2}$ $DE := AE$ $DF := DE - EF$ $DN := \sqrt{DF^2 + FN^2}$

$AN := DN$ $AB := \frac{AN^2}{AL}$ $EL := AE$ $BL := AL - AB$ $BN := \sqrt{AN^2 - AB^2}$

$EJ := \frac{BN \cdot EL}{BL}$ $GH := AE - (EJ + 2 \cdot AB)$

Trisection:



Definitions.

$FN - \frac{1}{4} = 0$ $EF - \frac{\sqrt{3}}{4} = 0$ $DF - \frac{2 - \sqrt{3}}{4} = 0$ $AN - \frac{\sqrt{2} \cdot (\sqrt{3} - 1)}{4} = 0$

$\frac{2 - \sqrt{3}}{4} - AB = 0$ $\frac{\sqrt{3} + 2}{4} - BL = 0$ $\frac{1}{4} - BN = 0$ $\frac{1}{2 \cdot \sqrt{3} + 4} - EJ = 0$ $\frac{\sqrt{3}}{2 \cdot \sqrt{3} + 4} - GH = 0$

Unit.

$AC := 1$

Given.

050701A

Descriptions.

$CP := \sqrt{2AC^2}$ $AP := AC$ $EP := CP$

$CE := \sqrt{AC^2 + (AP + EP)^2}$

$EK := CE$ $EH := CE$ $AE := \sqrt{CE^2 - AC^2}$

$AH := EH - AE$ $AT := AH$ $EU := \frac{AT \cdot EK}{AC}$

$\frac{EK}{EU} = 5.027339$ $1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}} = 5.027339$

Definitions.

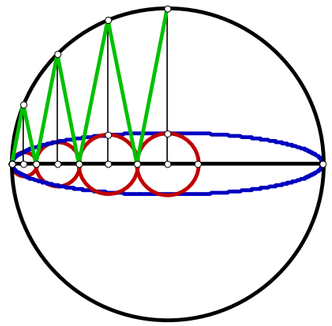
$\frac{EK}{EU} - \left(1 + \sqrt{2} + \sqrt{2} \cdot \sqrt{2 + \sqrt{2}}\right) = 0$

$CP - \sqrt{2} = 0$ $EP - \sqrt{2} = 0$ $CE - \sqrt{2} \cdot \sqrt{2 + \sqrt{2}} = 0$

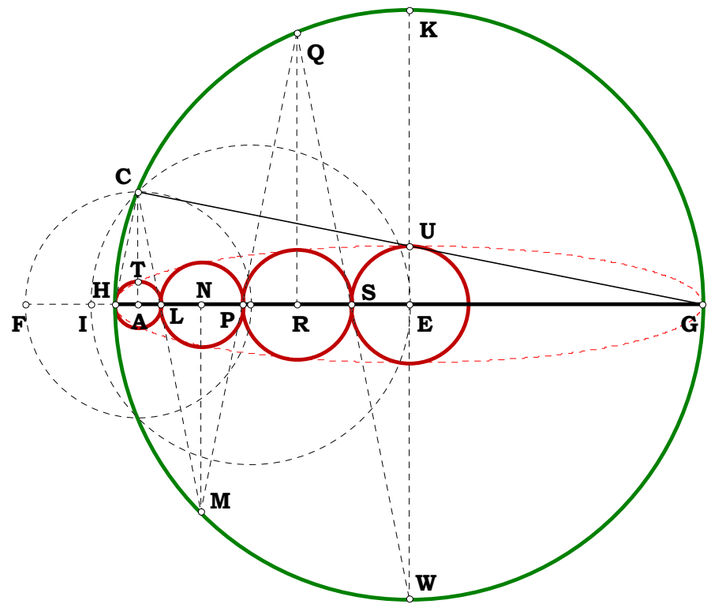
$AE - \left(\sqrt{2} + 1\right) = 0$ $AH - \left(\sqrt{2} \cdot \sqrt{2 + \sqrt{2}} - 1 - \sqrt{2}\right) = 0$

$EU - \left(2 - \sqrt{\sqrt{2} + 2}\right) \cdot \left(\sqrt{2} + 2\right) = 0$

$\frac{m\angle CAAW}{m\angle ARCN} = 4.000$



An Elliptic Progression takes place on a finite length of line. An Elliptic Progression may be defined in terms of a number of diameters of smaller circles, each defined by the same angle from the circumferance of the larger circle, from the center of a circle to its perimeter. When the sum of the number of those diameters minus one half the starting diameter are equal to the radius of the larger circle, the angle that defined the smaller circles will divide the larger circle evenly and the same number of times as the total number of smaller circles.



Unit.

$AL := 1$

0507013B

Descriptions.

$AE := \frac{AL}{2}$ $AR := \sqrt{2 \cdot AE^2}$

$AM := \frac{AR}{2}$ $EO := AE$ $EM := AM$

$MO := EO - EM$ $AO := \sqrt{AM^2 + MO^2}$

$AY := \frac{AO}{2}$ $EY := \sqrt{AE^2 - AY^2}$

$EN := AE$ $NY := EN - EY$ $AN := \sqrt{AY^2 + NY^2}$

$AP := \frac{AN^2}{AL}$ $NP := \sqrt{AN^2 - AP^2}$ $ES := \frac{NP \cdot AE}{AL - AP}$

Definitions.

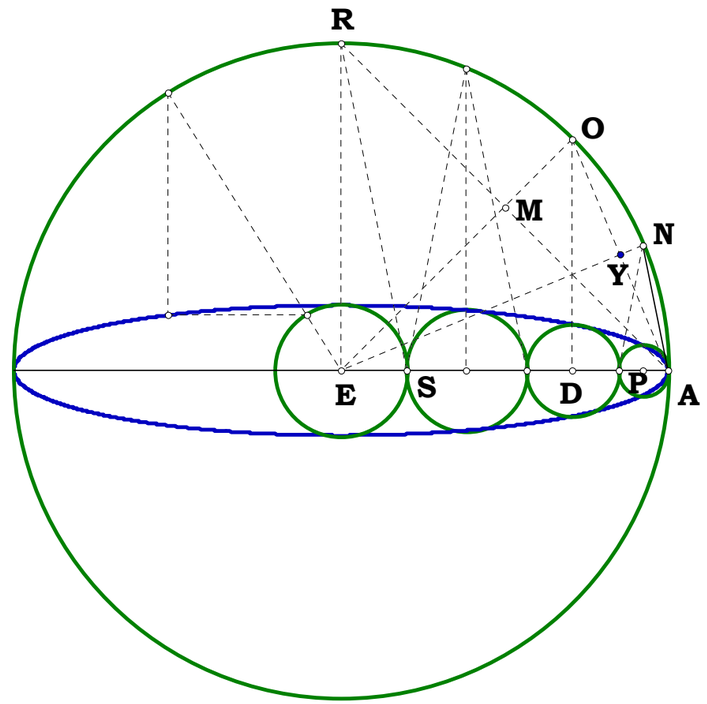
$AR - \frac{\sqrt{2}}{2} = 0$ $AM - \frac{\sqrt{2}}{4} = 0$ $MO - \frac{2 - \sqrt{2}}{4} = 0$ $AO - \frac{\sqrt{2 - \sqrt{2}}}{2} = 0$

$AY - \frac{\sqrt{2 - \sqrt{2}}}{4} = 0$ $EY - \frac{\sqrt{\sqrt{2} + 2}}{4} = 0$ $NY - \frac{2 - \sqrt{\sqrt{2} + 2}}{4} = 0$

$AN - \frac{\sqrt{2 - \sqrt{\sqrt{2} + 2}}}{2} = 0$ $AP - \frac{2 - \sqrt{2 + \sqrt{2}}}{4} = 0$ $NP - \frac{\sqrt{2 - \sqrt{2}}}{4} = 0$

$ES - \frac{\sqrt{2 - \sqrt{2}}}{2 \cdot \left(\sqrt{\sqrt{2} + 2} + 2\right)} = 0$

Quadsection:



Unit.

$AG := 1$

0507014

Descriptions.

$AE := \frac{AG}{2}$ $AC := \frac{AE}{2}$ $CG := AG - AC$ $CJ := \sqrt{AC \cdot CG}$

$EL := AE$ $CE := AC$ $JL := \sqrt{EL^2 - 2 \cdot EL \cdot CJ + CJ^2 + CE^2}$

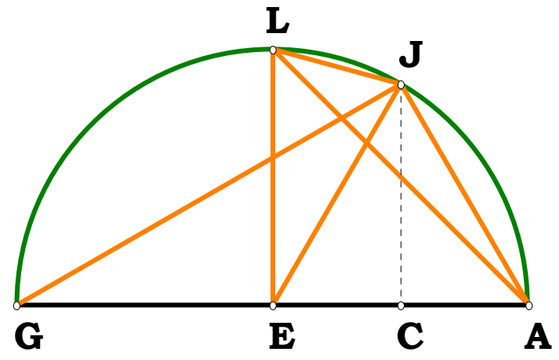
$AJ := \sqrt{AC^2 + CJ^2}$ $GJ := \sqrt{CG^2 + CJ^2}$ $AL := \sqrt{AE^2 + EL^2}$

Definitions.

$AE - \frac{1}{2} = 0$ $AC - \frac{1}{4} = 0$ $CG - \left(1 - \frac{1}{4}\right) = 0$ $CJ - \frac{1}{4} \cdot \sqrt{3} = 0$

$JL - \left(\frac{1}{4} \cdot \sqrt{6} - \frac{1}{4} \cdot \sqrt{2}\right) = 0$ $AJ - \frac{1}{2} = 0$ $GJ - \frac{1}{2} \cdot \sqrt{3} = 0$ $AL - \frac{1}{2} \cdot \sqrt{2} = 0$

Outtake Four: Some Names



Unit.

$AL := 1$

0507013

Descriptions.

Alternate Method: Pentasection Or Irrational Rationals

Irrational means the inability of a grammar to provide a name for a given thing. Many things are then irrational in Arithmetic due to the principles of the naming convention. Algebraic naming solves the problem by incorporating operands as part of a name. Algebra provides a degree of rationality then that is not achievable by Arithmetic. The following are some Algebraic names.

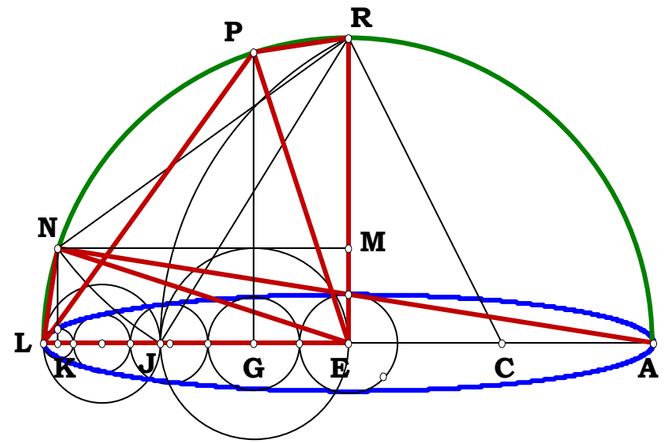
$$AE := \frac{AL}{2} \qquad AC := \frac{AE}{2} \qquad CE := AC \qquad ER := AE \qquad CR := \sqrt{CE^2 + ER^2} \qquad CJ := CR$$

$$EJ := CJ - CE \qquad JR := \sqrt{EJ^2 + ER^2} \qquad NR := JR \qquad EN := AE \qquad EM := \frac{EN^2 + ER^2 - NR^2}{2 \cdot ER}$$

$$KN := EM \qquad EK := \sqrt{EN^2 - KN^2} \qquad EL := AE \qquad KL := EL - EK \qquad LN := \sqrt{KL^2 + KN^2}$$

$$EG := \frac{EJ}{2} \qquad GL := EL - EG \qquad AG := AE + EG \qquad GP := \sqrt{AG \cdot GL}$$

$$PR := \sqrt{ER^2 - 2 \cdot ER \cdot GP + GP^2 + EG^2} \qquad PR - LN = 0 \qquad AN := \sqrt{AL^2 - LN^2}$$



Definitions:

$$AE - \frac{1}{2} = 0 \qquad AC - \frac{1}{4} = 0 \qquad CR - \frac{1}{4} \cdot \sqrt{5} = 0 \qquad EJ - \left(\frac{\sqrt{5}}{4} - \frac{1}{4} \right) = 0$$

$$JR - \frac{\sqrt{2} \cdot \sqrt{5 - \sqrt{5}}}{4} = 0 \qquad EM - \left(\frac{\sqrt{5}}{8} - \frac{1}{8} \right) = 0 \qquad EK - \frac{1}{2} \cdot \sqrt{\frac{\sqrt{5}}{8} + \frac{5}{8}} = 0$$

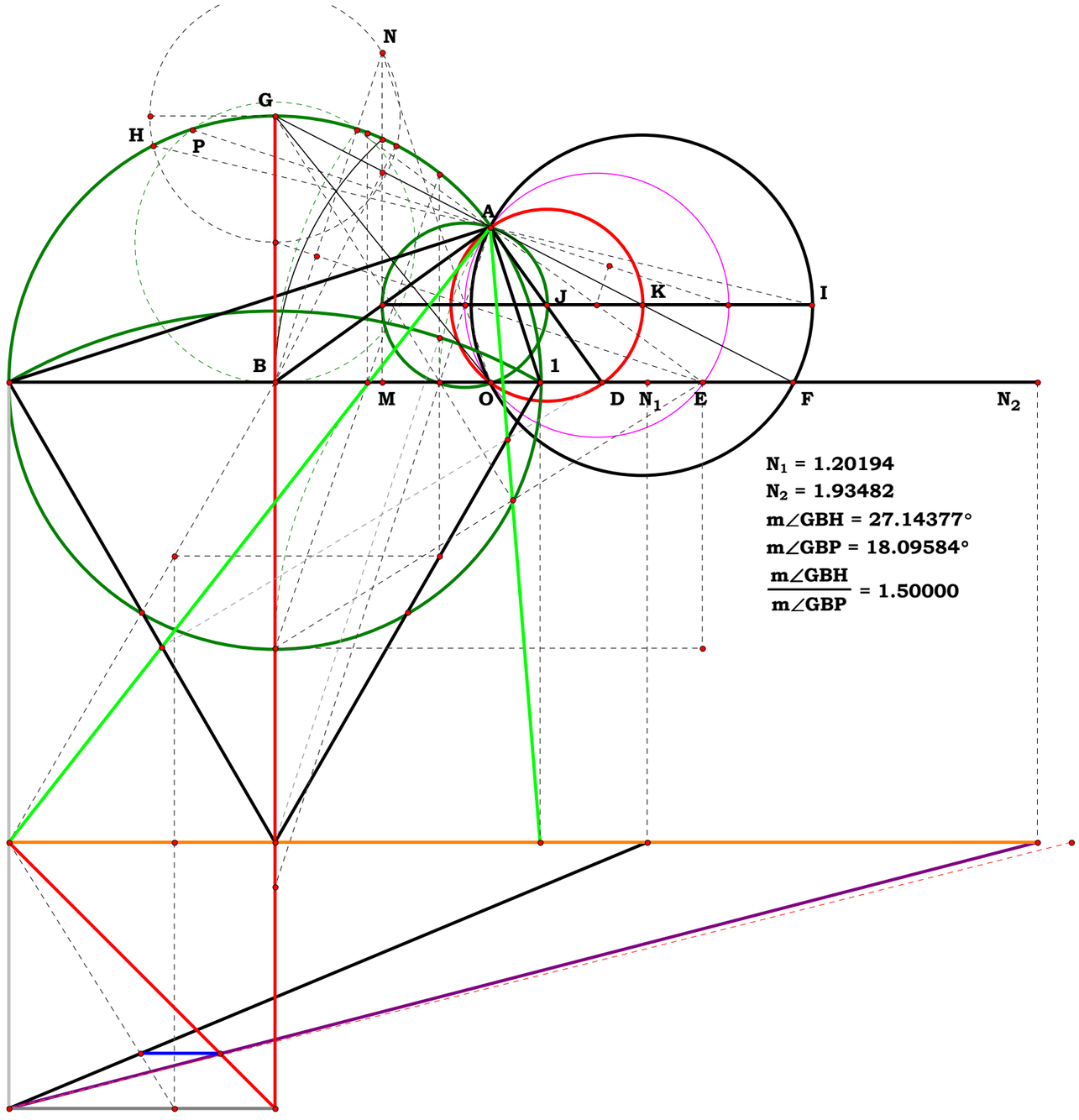
$$KL - \left(\frac{1}{2} - \frac{\sqrt{2} \cdot \sqrt{\sqrt{5} + 5}}{8} \right) = 0 \qquad LN - \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0 \qquad EG - \left(\frac{-1}{8} + \frac{1}{8} \cdot \sqrt{5} \right) = 0$$

$$GL - \left(\frac{5}{8} - \frac{1}{8} \cdot \sqrt{5} \right) = 0 \qquad AG - \left(\frac{3}{8} + \frac{1}{8} \cdot \sqrt{5} \right) = 0 \qquad GP - \frac{1}{8} \cdot \sqrt{10 + 2 \cdot \sqrt{5}} = 0$$

$$PR - \frac{1}{4} \cdot \sqrt{8 - 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0 \qquad AN - \frac{1}{4} \cdot \sqrt{8 + 2 \cdot \sqrt{10 + 2 \cdot \sqrt{5}}} = 0$$

051101 Round Tuits

I had intended to someday write this figure up, it is chock full of fixed intersections which are part of the figure, you just have to find them. Here it is, some eighteen years later. I know that there are a lot of circles in it, but in order to writhe this figure up, I need a round tuit.



Unit.

$AB := 1$

Given.

$N_1 := 1.20194$

$N_2 := 1.93482$

Descriptions.

$AS := \frac{N_1 + N_2}{2 \cdot N_2}$ $AS = 0.810608$ $ST := \sqrt{AS \cdot (AB - AS)}$ $AO := \frac{AB}{2}$

$OQ := AO$ $OP := AO$ $OS := AS - AO$ $PS := \sqrt{OS^2 + OP^2}$ $CO := \frac{OP^2}{OS}$

$AC := CO + AO$ $CW := CO$ $OW := AO$ $OX := \frac{CO^2 + OW^2 - CW^2}{2 \cdot CO}$

$FW := 2 \cdot OX$ $CF := CW - FW$ $CI := \frac{CF}{2}$ $FO := AO$ $EO := \frac{CO^2 + FO^2 - CF^2}{2 \cdot CO}$

$BO := AO$ $BE := BO - EO$ $AE := AO + EO$ $EF := \sqrt{AE \cdot BE}$ $CE := CF$

$DF := \frac{FO \cdot EF}{EO}$ $FH := \frac{DF}{2}$ $DE := \frac{EF \cdot DF}{FO}$ $DO := EO + DE$ $GH := \frac{DO}{2}$

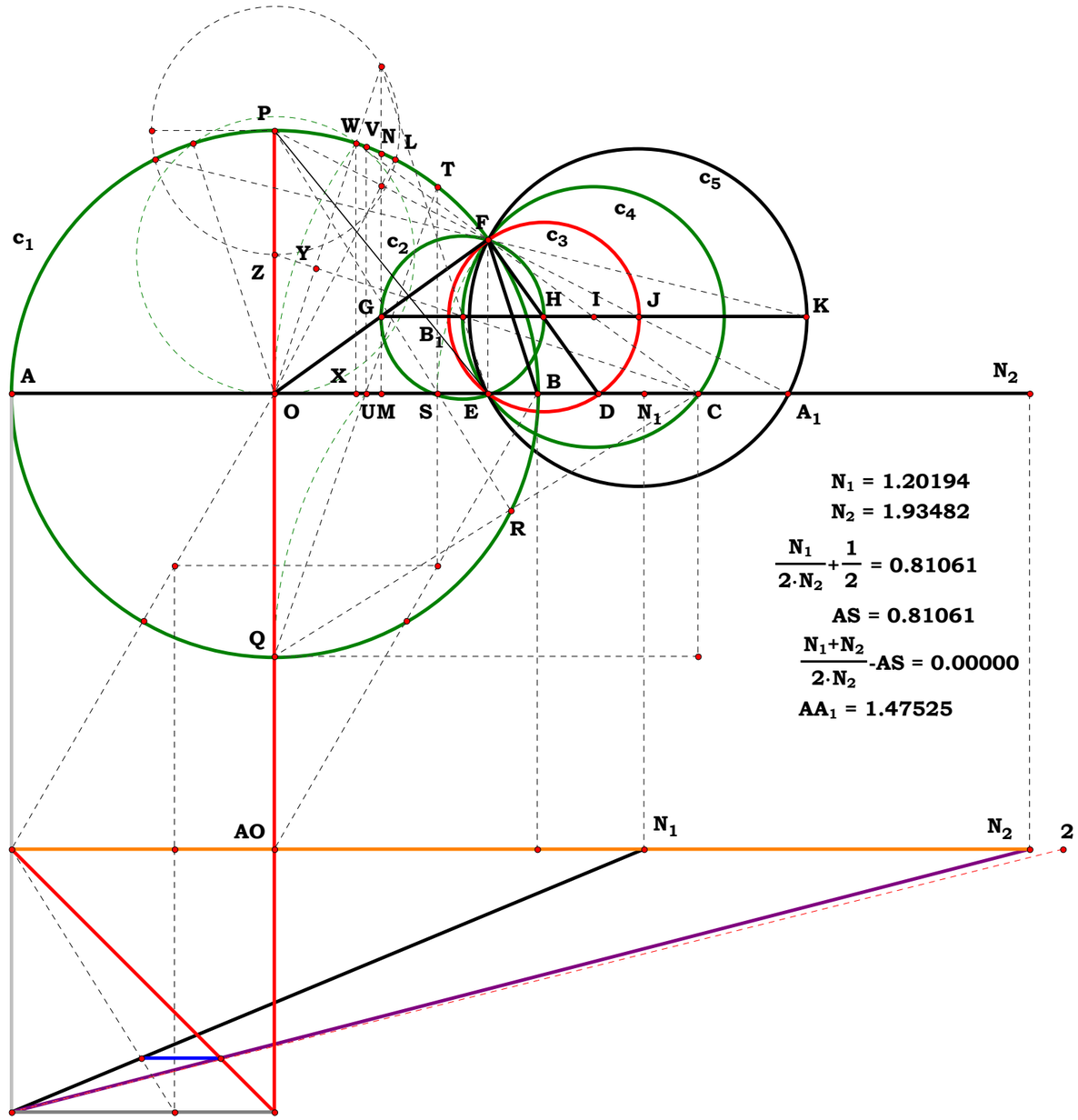
$GB_1 := \frac{GH}{2}$ $AA_1 := \frac{EO \cdot OP}{OP - EF} + AO$ $AA_1 = 1.475247$ $OA_1 := AA_1 - AO$

$PA_1 := \sqrt{OA_1^2 + OP^2}$ $EA_1 := OA_1 - EO$ $FA_1 := \frac{PA_1 \cdot EF}{OP}$ $FJ := \frac{FA_1}{2}$

$c_1 := AO$ $c_2 := GB_1$ $c_3 := FH$ $c_4 := CI$ $c_5 := FJ$

$c_1 = 0.5$ $c_2 = 0.153949$ $c_3 = 0.179725$

$c_4 = 0.247133$ $c_5 = 0.319863$



$c_1 = 0.5$ $c_2 = 0.153949$ $c_3 = 0.179725$

$c_4 = 0.247133$ $c_5 = 0.319863$

Definitions.

$$AS - \frac{N_1 + N_2}{2 \cdot N_2} = 0 \quad ST - \frac{\sqrt{(N_1 + N_2) \cdot (N_2 - N_1)}}{2 \cdot N_2} = 0 \quad AO - \frac{1}{2} = 0$$

$$OQ - \frac{1}{2} = 0 \quad OP - \frac{1}{2} = 0 \quad OS - \frac{N_1}{2 \cdot N_2} = 0 \quad PS - \frac{\sqrt{N_1^2 + N_2^2}}{2 \cdot N_2} = 0$$

$$CO - \frac{N_2}{2 \cdot N_1} = 0 \quad AC - \frac{N_1 + N_2}{2 \cdot N_1} = 0 \quad CW - \frac{N_2}{2 \cdot N_1} = 0$$

$$OW - \frac{1}{2} = 0 \quad OX - \frac{N_1}{4 \cdot N_2} = 0 \quad FW - \frac{N_1}{2 \cdot N_2} = 0$$

$$CF - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{2 \cdot N_1 \cdot N_2} = 0 \quad CI - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{4 \cdot N_1 \cdot N_2} = 0$$

$$FO - \frac{1}{2} = 0 \quad EO - \frac{N_1 \cdot (3 \cdot N_2^2 - N_1^2)}{4 \cdot N_2^3} = 0 \quad BO - \frac{1}{2} = 0$$

$$BE - \frac{(N_1 + 2 \cdot N_2) \cdot (N_1 - N_2)^2}{4 \cdot N_2^3} = 0 \quad AE - \frac{(2 \cdot N_2 - N_1) \cdot (N_1 + N_2)^2}{4 \cdot N_2^3} = 0$$

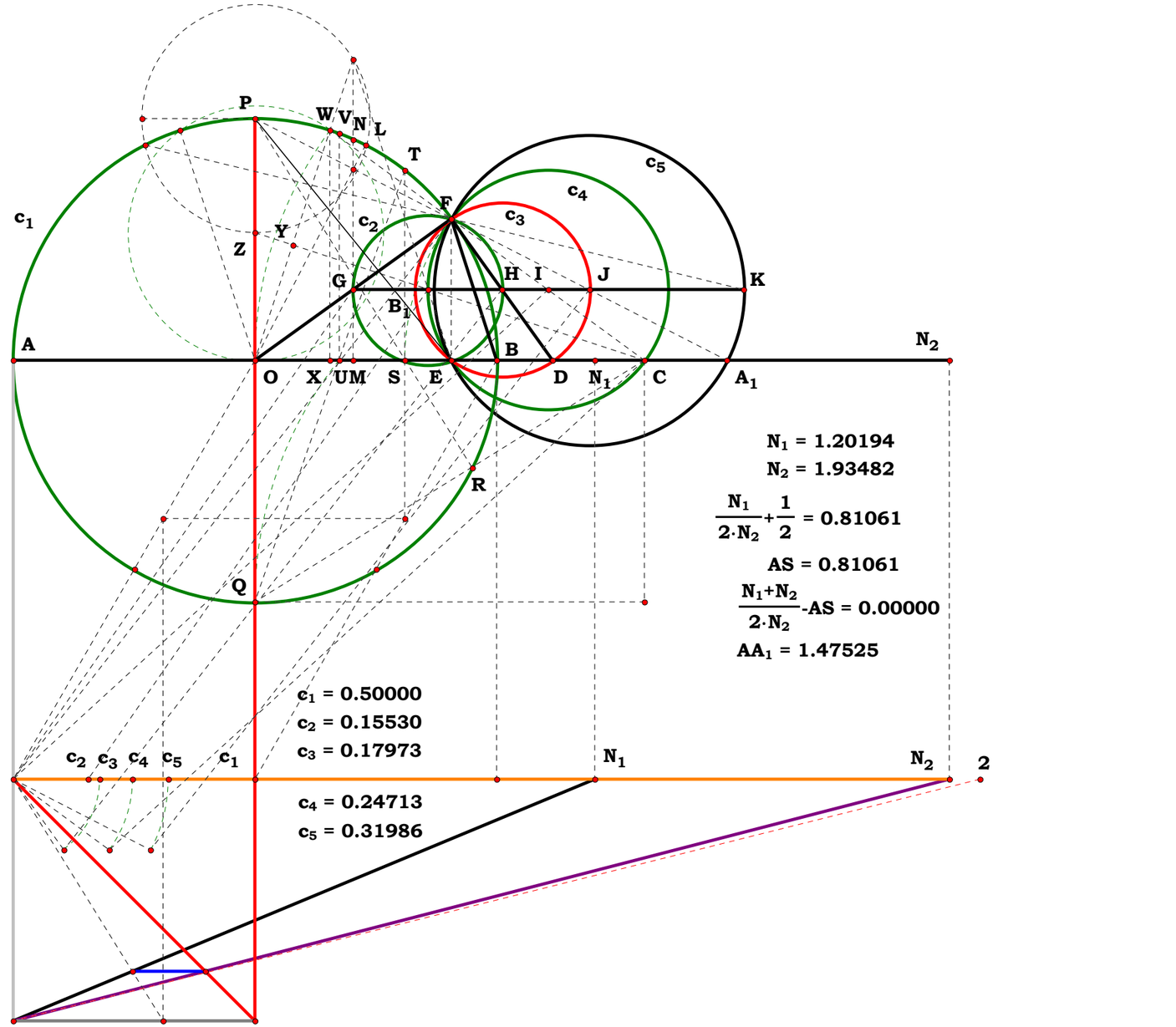
$$EF - \frac{(N_1 + N_2) \cdot (N_2 - N_1) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)}}{4 \cdot N_2^3} = 0$$

$$CE - \frac{(N_2 - N_1) \cdot (N_1 + N_2)}{2 \cdot N_1 \cdot N_2} = 0$$

$$DF - \frac{(N_1 + N_2) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)} \cdot (N_1 - N_2)}{2 \cdot N_1 \cdot (N_1^2 - 3 \cdot N_2^2)} = 0$$

$$FH - \frac{(N_1 + N_2) \cdot \sqrt{(2 \cdot N_2 - N_1) \cdot (N_1 + 2 \cdot N_2)} \cdot (N_1 - N_2)}{4 \cdot N_1 \cdot (N_1^2 - 3 \cdot N_2^2)} = 0$$

$$DE - \frac{(N_1 + N_2)^2 \cdot (N_1 - N_2)^2 \cdot (N_1 - 2 \cdot N_2) \cdot (N_1 + 2 \cdot N_2)}{4 \cdot N_1 \cdot N_2^3 \cdot (N_1^2 - 3 \cdot N_2^2)} = 0$$



$$DO - \left[\frac{N_2^3}{N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)}\right] = 0 \qquad GH - \frac{N_2^3}{2\cdot N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)} = 0$$

$$GB_1 - \frac{N_2^3}{4\cdot N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)} = 0$$

$$AA_1 - \frac{\left(N_1+N_2\right)\cdot\left[N_1\cdot\sqrt{4\cdot N_2^2 - N_1^2} - N_2\cdot\sqrt{4\cdot N_2^2 - N_1^2} - \left(N_1 - 2\cdot N_2\right)\cdot\left(N_1+N_2\right)\right]}{2\cdot\left(N_1^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} - N_2^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} + 2\cdot N_2^3\right)} = 0$$

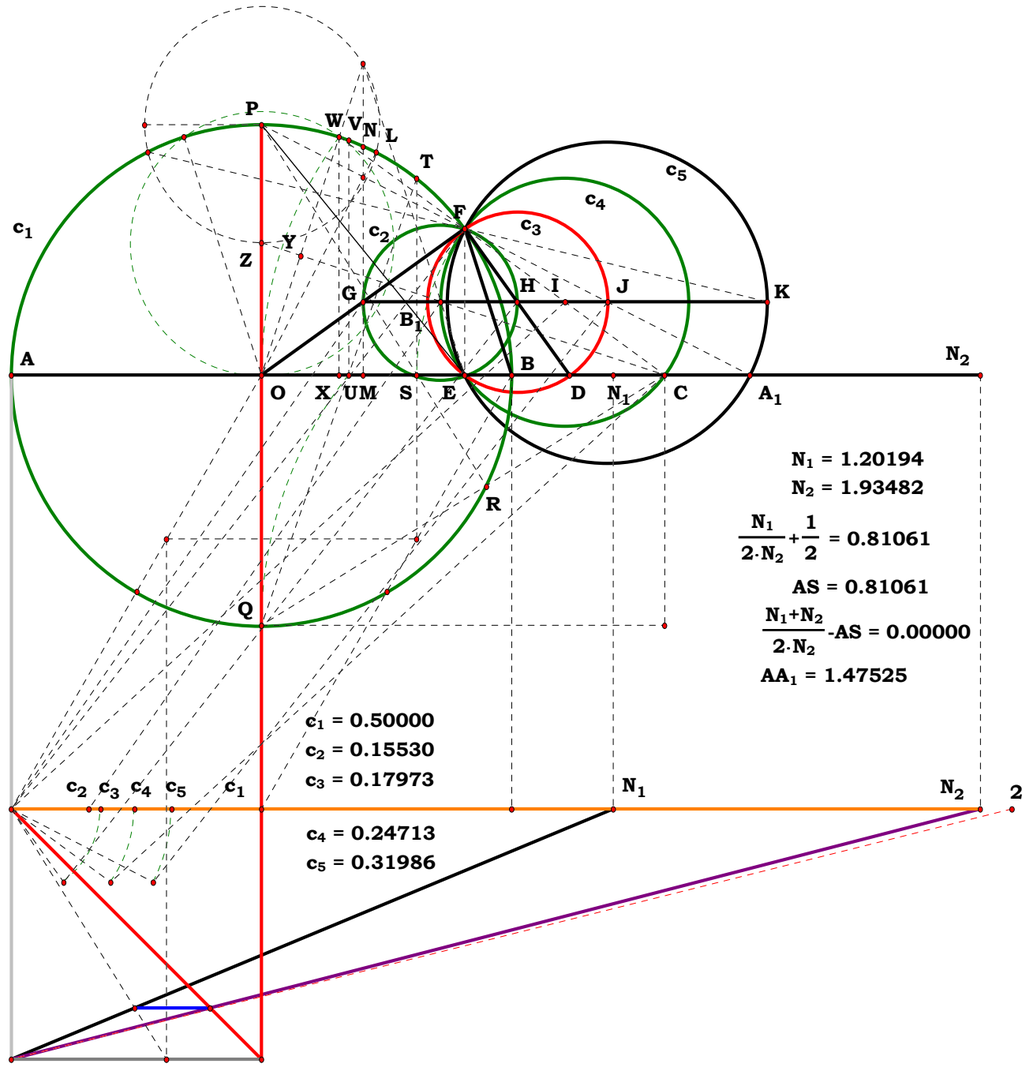
$$OA_1 - \frac{N_1\cdot\left(3\cdot N_2^2 - N_1^2\right)}{2\cdot\left(N_1^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} - N_2^2\cdot\sqrt{4\cdot N_2^2 - N_1^2} + 2\cdot N_2^3\right)} = 0$$

$$PA_1 - \frac{\sqrt{N_2^3\cdot\left[\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right) + 2\cdot N_2^3\right]}}{\sqrt{6\cdot N_1^4\cdot N_2^2 - N_1^6 - 9\cdot N_1^2\cdot N_2^4 + 8\cdot N_2^6 + 4\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot N_2^3\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right)}} = 0$$

$$EA_1 - \frac{\sqrt{4\cdot N_2^2 - N_1^2}\cdot N_1\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right)\cdot\left(N_1^2 - 3\cdot N_2^2\right)}{4\cdot N_2^3\cdot\left[\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\cdot\left(N_1+N_2\right) + 2\cdot N_2^3\right]} = 0$$

$$FA_1 - \frac{\left(N_1+N_2\right)\cdot\sqrt{N_2^3\cdot\left[2\cdot N_2^3 + \left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\right]}\cdot\sqrt{-\left(N_1 - 2\cdot N_2\right)\cdot\left(N_1 + 2\cdot N_2\right)\cdot\left(N_2 - N_1\right)}}{2\cdot N_2^3\cdot\sqrt{6\cdot N_1^4\cdot N_2^2 - N_1^6 - 9\cdot N_1^2\cdot N_2^4 + 8\cdot N_2^6 + 4\cdot N_2^3\cdot\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)}} = 0$$

$$FJ - \frac{\left(N_1+N_2\right)\cdot\sqrt{N_2^3\cdot\left[2\cdot N_2^3 + \left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)\right]}\cdot\sqrt{-\left(N_1 - 2\cdot N_2\right)\cdot\left(N_1 + 2\cdot N_2\right)\cdot\left(N_2 - N_1\right)}}{4\cdot N_2^3\cdot\sqrt{6\cdot N_1^4\cdot N_2^2 - N_1^6 - 9\cdot N_1^2\cdot N_2^4 + 8\cdot N_2^6 + 4\cdot N_2^3\cdot\left(N_1+N_2\right)\cdot\sqrt{4\cdot N_2^2 - N_1^2}\cdot\left(N_1 - N_2\right)}} = 0$$



Unit.

$BG := 1$

Given.

$N := 9 \quad AG := N$

051301

Descriptions.

$AB := AG - BG \quad BF := \frac{BG}{2} \quad AF := AB + BF$

$AN := AF \quad AK := AN \quad FK := BF$

$AE := \frac{AK^2 + AF^2 - FK^2}{2 \cdot AF} \quad AI := AE \quad IK := AK - AI$

$HI := IK \quad AH := AK - (HI + IK) \quad AC := \frac{AE \cdot AH}{AK}$

$BC := AC - AB \quad BE := AE - AB$

Definitions.

$N - 1 - AB = 0 \quad \frac{1}{2} - BF = 0 \quad \frac{1}{2} \cdot (2 \cdot N - 1) - AF = 0$

$\frac{1}{4} \cdot \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)} - AE = 0 \quad \frac{1}{4} \cdot \frac{1}{(2 \cdot N - 1)} - IK = 0$

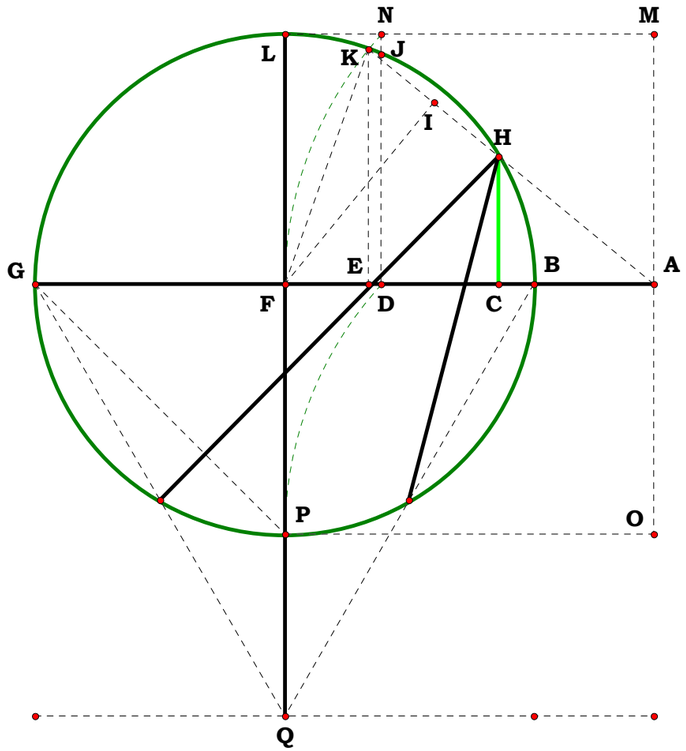
$2 \cdot N \cdot \frac{(N - 1)}{(2 \cdot N - 1)} - AH = 0 \quad \frac{\left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} \cdot N \cdot (N - 1) - AC = 0$

$(N - 1)^2 \cdot \frac{(4 \cdot N - 1)}{(2 \cdot N - 1)^3} - BC = 0 \quad \frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

$\frac{1}{4} \cdot \frac{(-3 + 4 \cdot N)}{(2 \cdot N - 1)} - BE = 0$

On Trisection

For any given trisection what is the Algebraic names of BC and BE taking BG as unit?



Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

051401

Descriptions.

$BG := AG - AB \quad BO := \frac{BG}{2} \quad NO := BO \quad AO := AB + BO \quad AM := AO$

$AF := \frac{AM^2 + AO^2 - NO^2}{2 \cdot AO} \quad AK := AF \quad KM := AM - AF \quad JK := KM$

$AJ := AM - (JK + KM) \quad AD := \frac{AF \cdot AJ}{AM} \quad BD := AD - AB \quad DG := BG - BD$

$DJ := \sqrt{BD \cdot DG} \quad OT := DJ \quad OP := BO \quad DO := BO - BD \quad PT := OP - OT$

$OU := \frac{DO \cdot OP}{PT} \quad BU := OU - BO \quad AU := BU - AB \quad DU := AU + AD$

$PU := \sqrt{OU^2 + OP^2} \quad JU := \frac{PU \cdot DU}{OU} \quad JP := PU - JU \quad AP := \sqrt{AO^2 + OP^2}$

$AS := \frac{AO^2}{AP} \quad PS := AP - AS \quad HS := PS \quad AH := AP - (PS + HS) \quad CH := \frac{OP \cdot AH}{AP}$

$AC := \frac{AO \cdot AH}{AP} \quad BC := AC - AB \quad CU := BU + BC \quad HU := \sqrt{CU^2 + CH^2}$

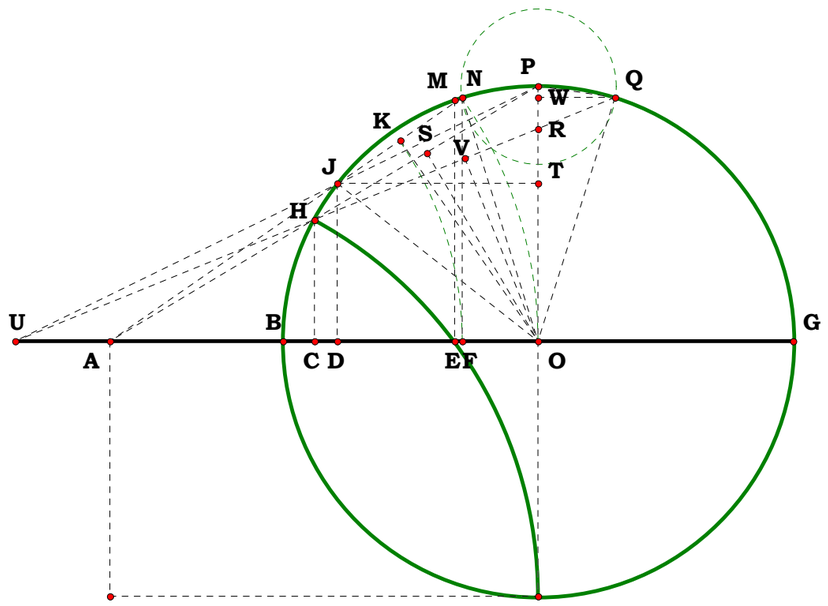
$UV := \frac{CU \cdot OU}{HU} \quad HV := UV - HU \quad QV := HV \quad QU := HU + (HV + QV)$

$OR := \frac{CH \cdot OU}{CU} \quad RU := \frac{HU \cdot OU}{CU} \quad QR := QU - RU \quad RW := \frac{CH \cdot QR}{HU}$

$QW := \frac{CU \cdot QR}{HU} \quad PW := OP - (OR + RW) \quad PQ := \sqrt{PW^2 + QW^2} \quad GU := BO + OU$

For any given QLX, XLZ is 1/3 of that angle. What are the Algebraic names in this figure for the cords QX and XZ?

On Trisection



Definitions.

$$BG-(N-1)=0 \quad BO-\frac{N-1}{2}=0 \quad NO-\frac{N-1}{2}=0 \quad AO-\frac{N+1}{2}=0 \quad AM-\frac{N+1}{2}=0$$

$$AF-\frac{N^2+6\cdot N+1}{4\cdot(N+1)}=0 \quad AK-\frac{N^2+6\cdot N+1}{4\cdot(N+1)}=0 \quad KM-\frac{(N-1)^2}{4\cdot(N+1)}=0 \quad JK-\frac{(N-1)^2}{4\cdot(N+1)}=0$$

$$AJ-\frac{2\cdot N}{N+1}=0 \quad AD-\frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3}=0 \quad BD-\frac{(3\cdot N+1)\cdot(N-1)}{(N+1)^3}=0 \quad DG-\frac{N^2\cdot(N+3)\cdot(N-1)}{(N+1)^3}=0$$

$$DJ-\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}=0 \quad OT-\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3}=0 \quad OP-\frac{N-1}{2}=0$$

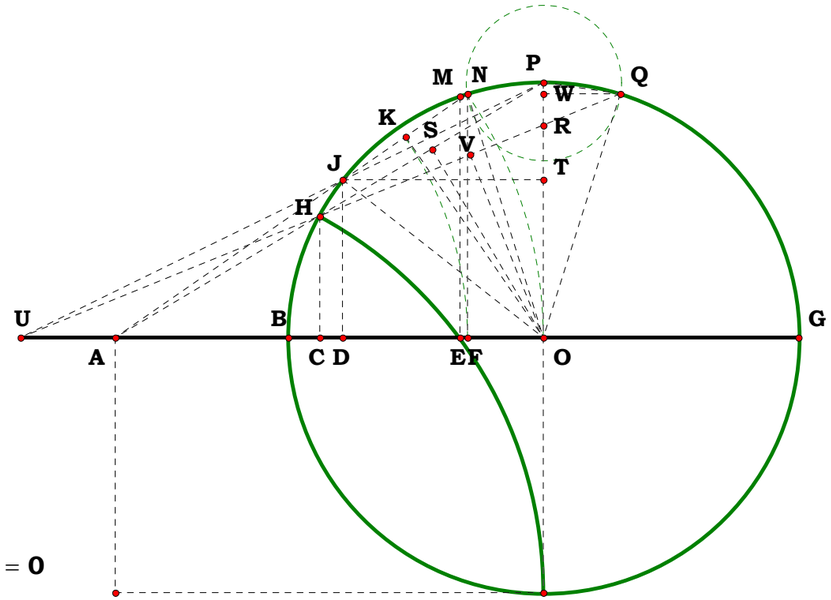
$$DO-\frac{\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{2\cdot(N+1)^3}=0 \quad PT-\frac{(N-1)\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}{2\cdot(N+1)^3}=0$$

$$OU-\frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}=0 \quad BU-\frac{(N-1)\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-3\cdot N-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}=0$$

$$AU-\frac{N\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-6\cdot N-N^2+\sqrt{3\cdot N^2+10\cdot N+3}-1\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}=0 \quad DU-\frac{\sqrt{3\cdot N^2+10\cdot N+3}\cdot N\cdot\left(N^2+4\cdot N+1\right)\cdot(N-1)^2}{(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}=0$$

$$PU-\frac{\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}}{\sqrt{2\cdot\left[N^6+6\cdot N^5+3\cdot N\cdot\left(9\cdot N^3+20\cdot N^2+9\cdot N+2\right)+1\right]-8\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}}}=0$$

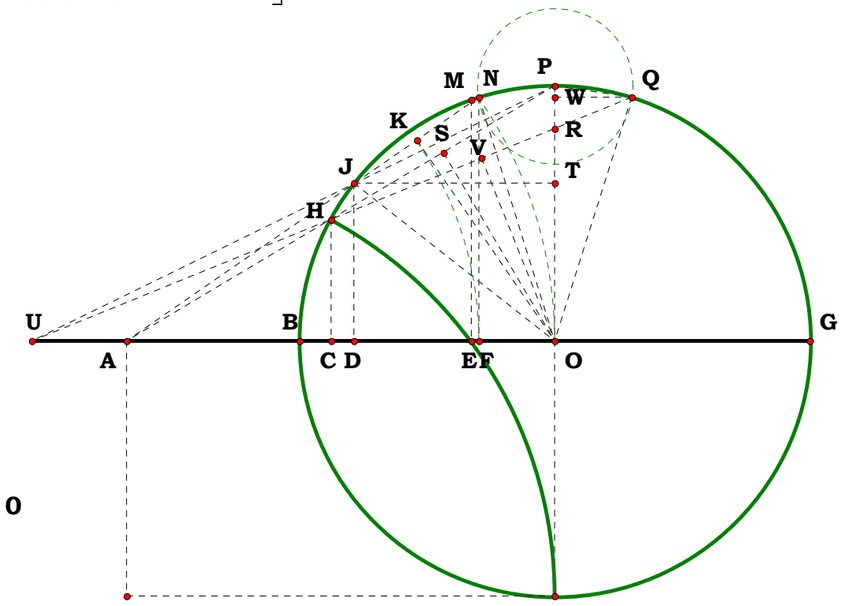
$$JU-\frac{\sqrt{2}\cdot N\cdot\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}\cdot\sqrt{3\cdot N^2+10\cdot N+3}\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}{(N+1)^3\cdot\sqrt{6\cdot N^5+N^6+3\cdot N\cdot\left(9\cdot N^3+20\cdot N^2+9\cdot N+2\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}+1}\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}=0$$



$$JP - \left[\frac{\sqrt{2}\cdot\sqrt{(N-1)^2\cdot(N+1)^3\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}{2\cdot(N+1)^3\cdot\sqrt{\left(N^6+6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}}} \right] = 0$$

$$AP - \frac{\sqrt{N^2+1}}{\sqrt{2}} = 0 \qquad AS - \frac{\sqrt{2}\cdot(N+1)^2}{4\cdot\sqrt{N^2+1}} = 0 \qquad PS - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0 \qquad HS - \frac{\sqrt{2}\cdot(N-1)^2}{4\cdot\sqrt{N^2+1}} = 0$$

$$AH - \frac{\sqrt{2}\cdot N}{\sqrt{N^2+1}} = 0 \qquad CH - \frac{N\cdot(N-1)}{N^2+1} = 0 \qquad AC - \frac{N\cdot(N+1)}{N^2+1} = 0 \qquad BC - \frac{N-1}{N^2+1} = 0$$



$$CU - \frac{N\cdot(N-1)^2\cdot\left(N\cdot\sqrt{3\cdot N^2+10\cdot N+3}-2\cdot N+\sqrt{3\cdot N^2+10\cdot N+3}\right)}{\left(N^2+1\right)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)} = 0$$

$$HU - \frac{2\cdot N\cdot(N-1)\cdot\sqrt{(N+1)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1\right)}}{\sqrt{\left(N^2+1\right)\cdot\left[N^6+\left(6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}} = 0$$

$$UV - \frac{(N-1)^3\cdot\sqrt{\left(N^2+1\right)\cdot\left[\left(N^6+6\cdot N^5+27\cdot N^4+60\cdot N^3+27\cdot N^2+6\cdot N+1\right)-4\cdot N\cdot(N+1)^3\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}\cdot\left[(N+1)\cdot\sqrt{3\cdot N^2+10\cdot N+3}-2\cdot N\right]\cdot\left(N^2+4\cdot N+1\right)}{4\cdot\sqrt{(N+1)\cdot\left[(N+1)^3-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}\right]}\cdot\left(N^2+1\right)\cdot\left(3\cdot N-2\cdot N\cdot\sqrt{10\cdot N+3\cdot N^2+3}+3\cdot N^2+N^3+1\right)^2} = 0$$

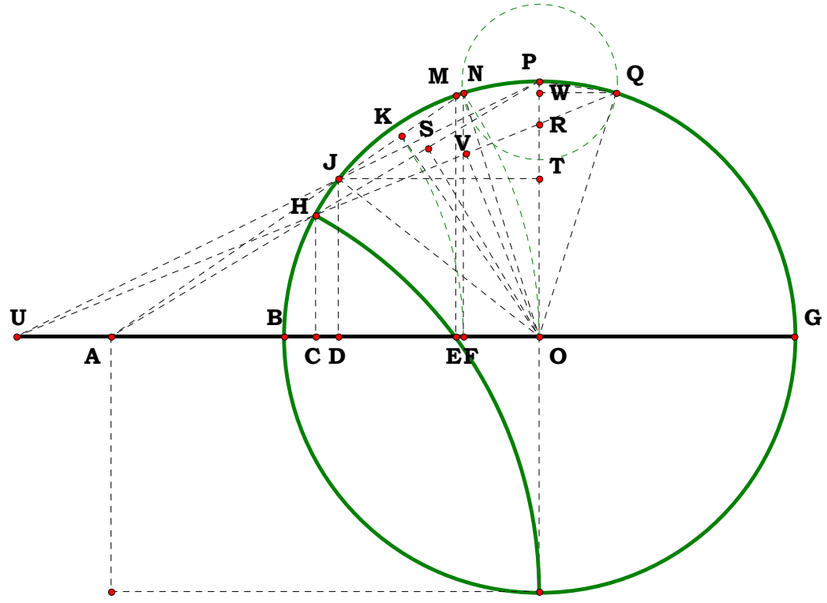
$$HV-(UV-HU)=0$$

$$QV-HV=0 \qquad QU-[HU+(HV+QV)]=0$$

$$OR-\frac{CH\cdot OU}{CU}=0 \qquad RU-\frac{HU\cdot OU}{CU}=0$$

$$QR-(QU-RU)=0 \qquad RW-\frac{CH\cdot QR}{HU}=0 \qquad QW-\frac{CU\cdot QR}{HU}=0$$

$$PW-[OP-(OR+RW)]=0$$



$$PQ-\frac{(N-1)\cdot\sqrt{14\cdot N+32\cdot N^2+14\cdot N^3+2\cdot N^4}-(N+1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}\cdot\left(N^2+6\cdot N+1\right)+2}{2\cdot\sqrt{(N+1)\cdot\left[3\cdot N-2\cdot N\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}+3\cdot N^2+N^3+1\right]}}=0$$

$$GU-\left[\frac{N\cdot(N-1)\cdot\left(3\cdot N+N^2-\sqrt{3\cdot N^2+10\cdot N+3}\right)}{3\cdot N-2\cdot N\cdot\sqrt{3\cdot N^2+10\cdot N+3}+3\cdot N^2+N^3+1}\right]=0$$

Unit.

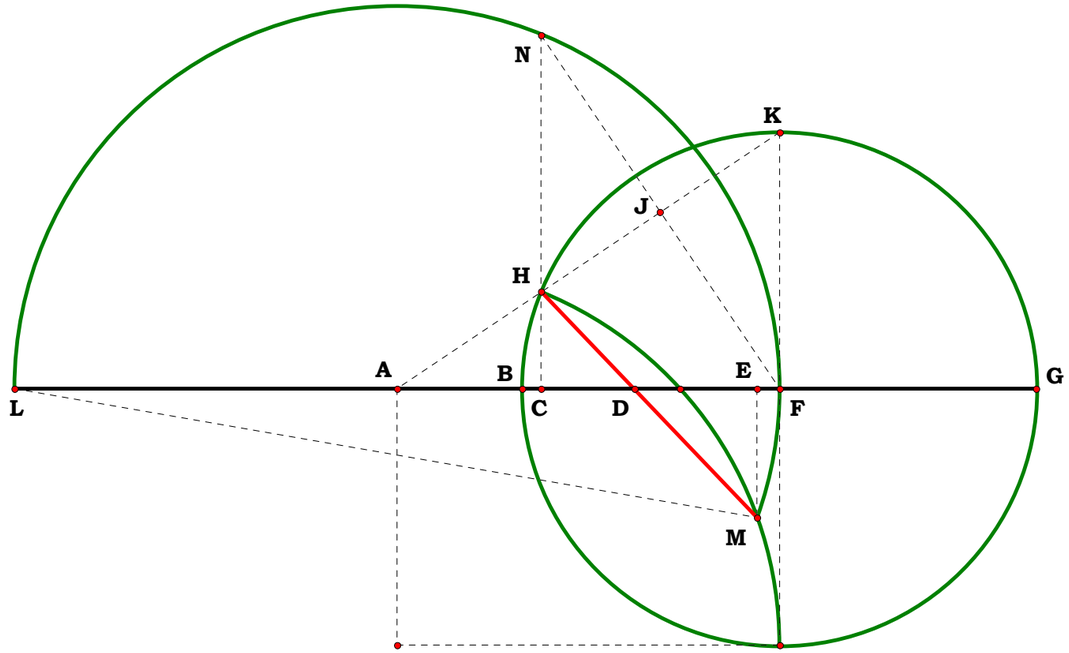
$AB := 1$

Given.

$N := 5 \quad AG := N$

Segment DF And HM

Given AB and AG, what is HM and DF?



052201A

Descriptions.

$BG := AG - AB \quad BF := \frac{BG}{2} \quad FK := BF \quad AF := AB + BF$

$AK := \sqrt{AF^2 + FK^2} \quad AJ := \frac{AF^2}{AK} \quad JK := AK - AJ \quad HJ := JK$

$AH := AK - (JK + HJ) \quad AC := \frac{AF \cdot AH}{AK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot AF$

$EF := \frac{FL - \sqrt{FL^2 - 4 \cdot EM^2}}{2} \quad AE := AF - EF \quad CE := AE - AC \quad CH := \frac{FK \cdot AH}{AK}$

$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$

Definitions.

$BG - (N - 1) = 0 \quad BF - \frac{N - 1}{2} = 0 \quad FK - \frac{N - 1}{2} = 0 \quad AF - \frac{N + 1}{2} = 0$

$AK - \frac{\sqrt{N^2 + 1}}{\sqrt{2}} = 0 \quad AJ - \frac{\sqrt{2} \cdot (N + 1)^2}{4 \cdot \sqrt{N^2 + 1}} = 0 \quad JK - \frac{\sqrt{2} \cdot (N - 1)^2}{4 \cdot \sqrt{N^2 + 1}} = 0 \quad HJ - \frac{\sqrt{2} \cdot (N - 1)^2}{4 \cdot \sqrt{N^2 + 1}} = 0 \quad AH - \frac{\sqrt{2} \cdot N}{\sqrt{N^2 + 1}} = 0 \quad AC - \frac{N \cdot (N + 1)}{N^2 + 1} = 0$

$EM - \frac{N - 1}{4} = 0 \quad FL - (N + 1) \quad EF - \frac{2 \cdot N - \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 2}{4} = 0 \quad AE - \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{4} = 0 \quad CE - \frac{\left(N^2 + 1\right) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - 4 \cdot N \cdot (N + 1)}{4 \cdot \left(N^2 + 1\right)} = 0$

$CH - \frac{N \cdot (N - 1)}{N^2 + 1} = 0 \quad HM - \frac{\sqrt{(N + 1) \cdot \left(3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1\right)}}{2 \cdot \sqrt{N^2 + 1}} = 0 \quad DE - \frac{\left(N^2 + 1\right) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} - \left(4 \cdot N^2 + 4 \cdot N\right)}{4 \cdot \left(N^2 + 4 \cdot N + 1\right)} = 0$

$DF - \frac{3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1}{2 \cdot \left(N^2 + 4 \cdot N + 1\right)} = 0$

Unit.

$AB := 1$

Given.

$N := 5 \quad AG := N$

052201B

Descriptions.

$$BG := AG - AB \quad BF := \frac{AB}{2} \quad FK := BF \quad GF := AG - BF$$

$$GK := \sqrt{GF^2 + FK^2} \quad GJ := \frac{GF^2}{GK} \quad JK := GK - GJ \quad HJ := JK$$

$$GH := GK - (JK + HJ) \quad CG := \frac{GF \cdot GH}{GK} \quad EM := \frac{BF}{2} \quad FL := 2 \cdot GF$$

$$EF := \frac{FL - \sqrt{FL^2 - 4 \cdot EM^2}}{2} \quad GE := GF - EF \quad CE := GE - CG \quad CH := \frac{FK \cdot GH}{GK}$$

$$HM := \sqrt{(EM + CH)^2 + CE^2} \quad DE := \frac{CE \cdot EM}{EM + CH} \quad DF := DE + EF$$

Definitions.

$$BG - (N - 1) = 0 \quad BF - \frac{1}{2} = 0 \quad FK - \frac{1}{2} = 0 \quad GF - \frac{2 \cdot N - 1}{2} = 0$$

$$GK - \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{\sqrt{2}} = 0 \quad GJ - \frac{\sqrt{2} \cdot (2 \cdot N - 1)^2}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad JK - \frac{\sqrt{2}}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0$$

$$HJ - \frac{\sqrt{2}}{4 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad GH - \frac{\sqrt{2} \cdot N \cdot (N - 1)}{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad CG - \frac{N \cdot (N - 1) \cdot (2 \cdot N - 1)}{2 \cdot N^2 - 2 \cdot N + 1} = 0$$

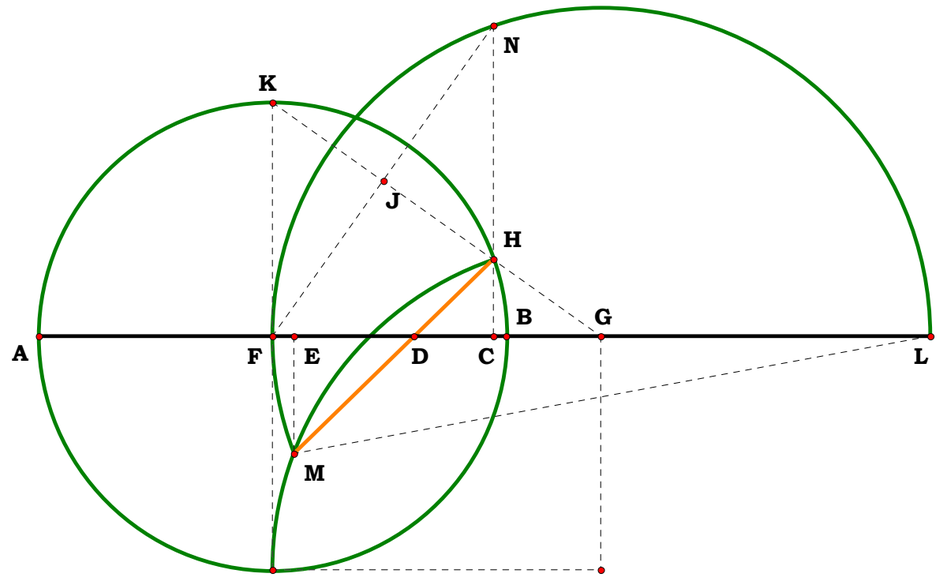
$$EM - \frac{1}{4} = 0 \quad FL - (2 \cdot N - 1) = 0 \quad EF - \frac{4 \cdot N - \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} - 2}{4} = 0 \quad GE - \frac{\sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4} = 0 \quad CE - \frac{\left(2 \cdot N^2 - 2 \cdot N + 1\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} - 4 \cdot N \cdot (2 \cdot N - 1) \cdot (N - 1)}{4 \cdot \left(2 \cdot N^2 - 2 \cdot N + 1\right)} = 0$$

$$CH - \frac{N \cdot (N - 1)}{2 \cdot N^2 - 2 \cdot N + 1} = 0 \quad HM - \frac{\sqrt{(2 \cdot N - 1) \cdot \left[\left(2 \cdot N - 2 \cdot N^2\right) \cdot \sqrt{16 \cdot N^2 - 16 \cdot N + 3} + (2 \cdot N - 1)^3\right]}}{2 \cdot \sqrt{2 \cdot N^2 - 2 \cdot N + 1}} = 0 \quad DE - \frac{4 \cdot N \cdot (2 \cdot N - 1) \cdot (1 - N) - \left(2 \cdot N - 2 \cdot N^2 - 1\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{4 \cdot \left(6 \cdot N^2 - 6 \cdot N + 1\right)} = 0$$

$$DF - \frac{\left(2 \cdot N - 2 \cdot N^2\right) \cdot \sqrt{16 \cdot N^2 - 16 \cdot N + 3} + (2 \cdot N - 1)^3}{2 \cdot \left(6 \cdot N^2 - 6 \cdot N + 1\right)} = 0$$

Segment DF And HM

Given AB and AG, what is HM and DF?



Unit.

$AB := 1$

Point of Intersection

Given.

$N := 5 \qquad AH := N$

052701A

Descriptions.

$BH := AH - AB \qquad BG := \frac{BH}{2} \qquad GN := BG \qquad GM := BG$

$GP := BG \qquad AG := AB + BG \qquad AM := AG \qquad FG := \frac{GM^2 + AG^2 - AM^2}{2 \cdot AG}$

$AF := AG - FG \qquad AL := AF \qquad LM := AM - AL \qquad KL := LM$

$AK := AM - (LM + KL) \qquad AC := \frac{AF \cdot AK}{AM} \qquad CK := \sqrt{AK^2 - AC^2}$

$CG := AG - AC \qquad DG := \frac{CG \cdot GP}{(GP + CK)}$

Definitions.

$BH - (N - 1) = 0 \qquad BG - \frac{(N-1)}{2} = 0 \qquad GN - \frac{(N-1)}{2} = 0 \qquad GM - \frac{(N-1)}{2} = 0$

$GP - \frac{(N-1)}{2} = 0 \qquad AG - \frac{N+1}{2} = 0 \qquad AM - \frac{N+1}{2} = 0 \qquad FG - \frac{(N-1)^2}{4 \cdot (N+1)} = 0$

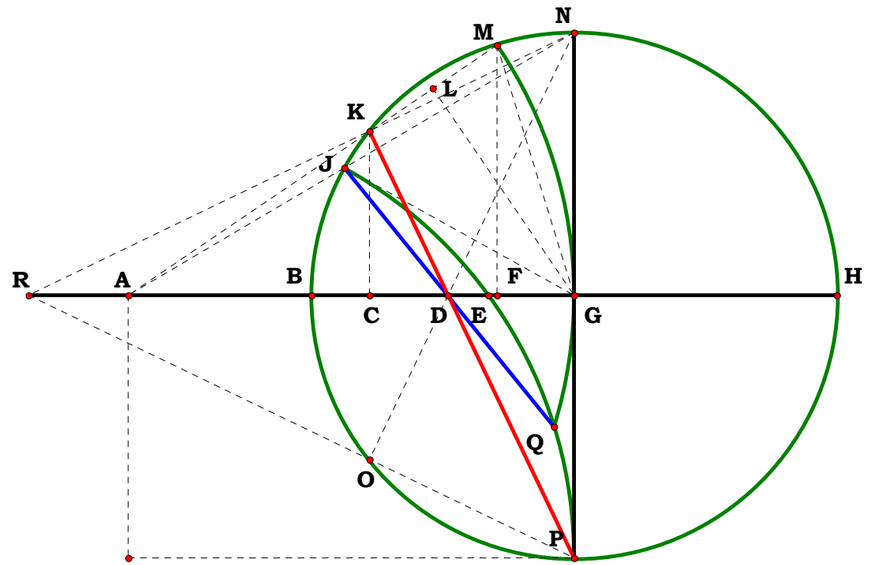
$AF - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad AL - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N+1)} = 0 \qquad LM - \frac{(N-1)^2}{4 \cdot (N+1)} = 0$

$KL - \frac{(N-1)^2}{4 \cdot (N+1)} = 0 \qquad AK - \frac{2 \cdot N}{N+1} = 0 \qquad AC - \frac{N \cdot \left(N^2 + 6 \cdot N + 1\right)}{(N+1)^3} = 0$

$CK - \frac{N \cdot (N-1) \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}}{(N+1)^3} = 0 \qquad CG - \frac{\left(N^2 + 4 \cdot N + 1\right) \cdot (N-1)^2}{2 \cdot (N+1)^3} = 0$

$DG - \frac{(N-1)^2 \cdot \left(N^2 + 4 \cdot N + 1\right)}{2 \cdot \left[(N+1)^3 + 2 \cdot N \cdot \sqrt{(N+3) \cdot (3 \cdot N + 1)}\right]} = 0$

Do KP and JQ intersect at D?



This is the other way to look at the figure I work on for trisection.

$m\angle ABC = 9.59307°$

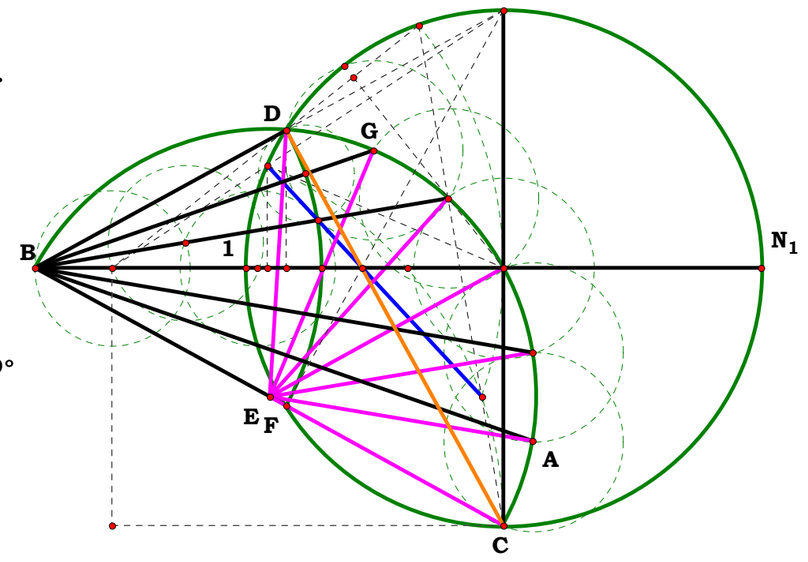
$m\angle DBF = 57.55844°$

$\frac{m\angle DBF}{m\angle ABC} = 6.00000$

$m\angle DEC = 115.11689°$

$m\angle DEG = 19.18615°$

$\frac{m\angle DEC}{m\angle DEG} = 6.00000$



Unit.

$AB := 1$

Given.

$N := 1.26810 \quad AH := N$

052701A

Descriptions.

$BH := AH - AB \quad BG := \frac{AB}{2} \quad GN := BG \quad GM := BG$

$GP := BG \quad GH := AH - BG \quad HM := GH \quad FG := \frac{GM^2 + GH^2 - HM^2}{2 \cdot GH}$

$FH := GH - FG \quad HL := FH \quad LM := HM - HL \quad KL := LM$

$HK := HM - (LM + KL) \quad CH := \frac{FH \cdot HK}{HM} \quad CK := \sqrt{HK^2 - CH^2}$

$CG := GH - CH \quad DG := \frac{CG \cdot GP}{(GP + CK)}$

Definitions.

$BH - (N - 1) = 0 \quad BG - \frac{1}{2} = 0 \quad GN - \frac{1}{2} = 0 \quad GM - \frac{1}{2} = 0$

$GP - \frac{1}{2} = 0 \quad GH - \frac{2 \cdot N - 1}{2} = 0 \quad HM - \frac{2 \cdot N - 1}{2} = 0$

$FG - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad FH - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0 \quad HL - \frac{8 \cdot N^2 - 8 \cdot N + 1}{4 \cdot (2 \cdot N - 1)} = 0$

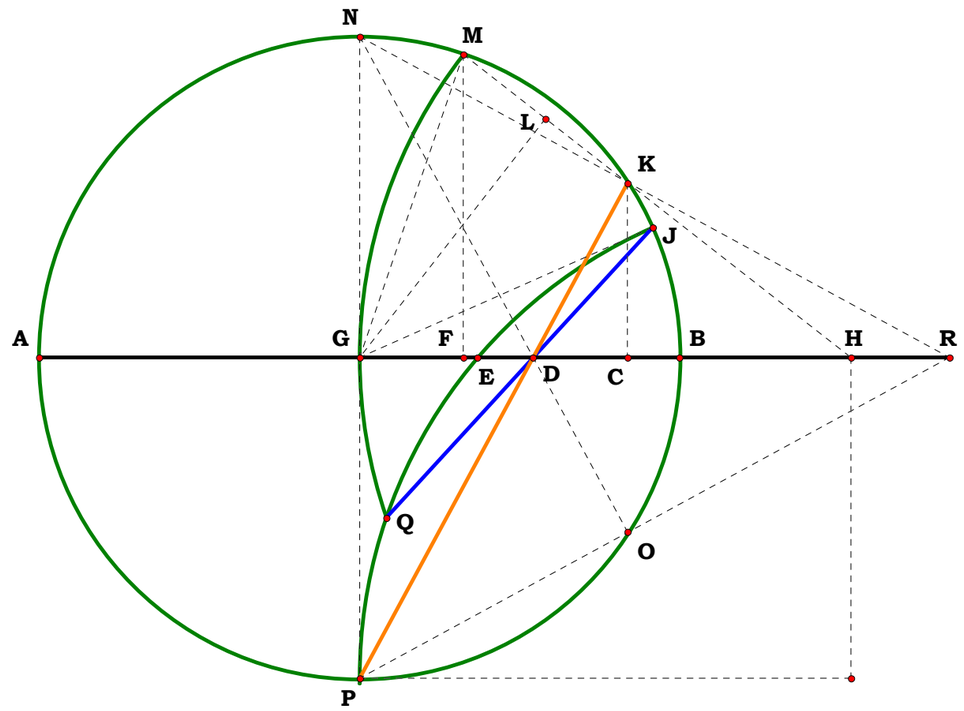
$LM - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad KL - \frac{1}{4 \cdot (2 \cdot N - 1)} = 0 \quad HK - \frac{2 \cdot N \cdot (N - 1)}{2 \cdot N - 1} = 0$

$CH - \frac{N \cdot (N - 1) \cdot \left(8 \cdot N^2 - 8 \cdot N + 1\right)}{(2 \cdot N - 1)^3} = 0 \quad CK - \frac{N \cdot (N - 1) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)}}{(2 \cdot N - 1)^3} = 0$

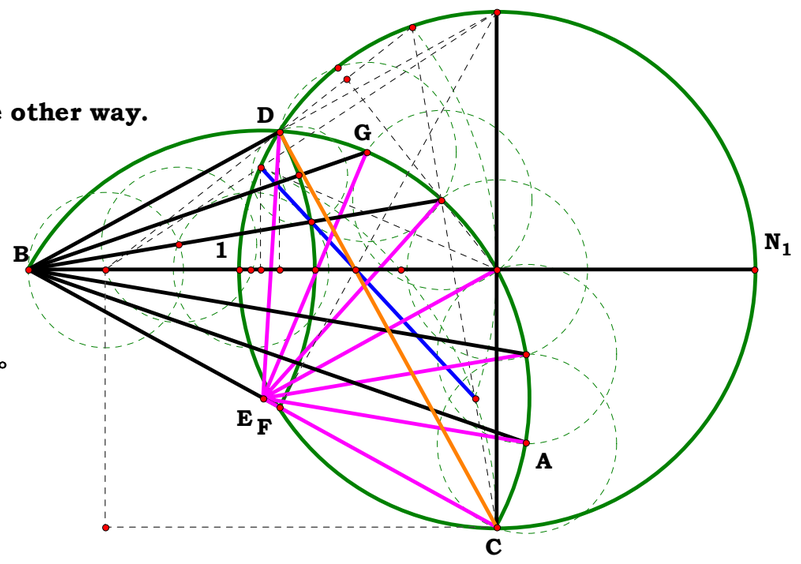
$CG - \frac{6 \cdot N^2 - 6 \cdot N + 1}{2 \cdot (2 \cdot N - 1)^3} = 0 \quad DG - \frac{6 \cdot N^2 - 6 \cdot N + 1}{2 \cdot \left[\left(2 \cdot N^2 - 2 \cdot N\right) \cdot \sqrt{(4 \cdot N - 1) \cdot (4 \cdot N - 3)} + (2 \cdot N - 1)^3\right]} = 0$

Point of Intersection

Do KP and JQ intersect at D?



No, I am not drawing this the other way.



$m\angle ABC = 9.59307°$

$m\angle DBF = 57.55844°$

$\frac{m\angle DBF}{m\angle ABC} = 6.00000$

$m\angle DEC = 115.11689°$

$m\angle DEG = 19.18615°$

$\frac{m\angle DEC}{m\angle DEG} = 6.00000$

Unit.

$AB := 1$

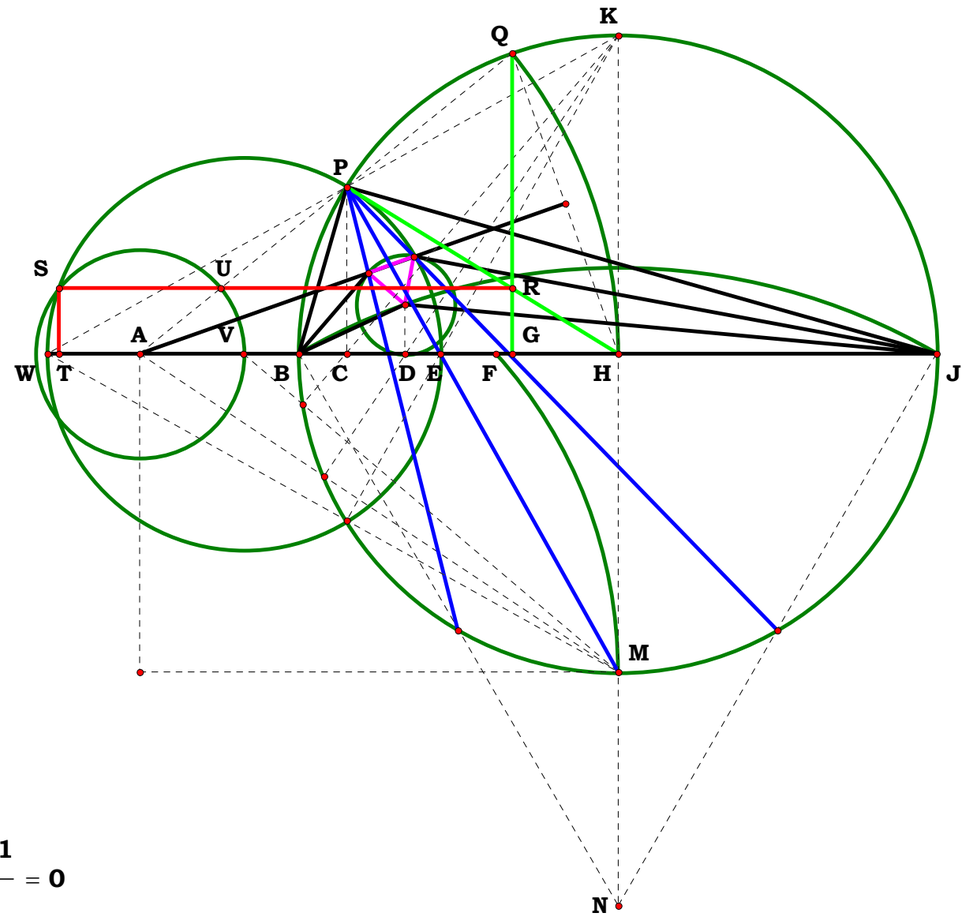
Given.

$N_1 := 5 \qquad AJ := N_1$

0530011A

What is AV and ST?

Given any angle BHP, the unit which defines it will lay between W and V.



Descriptions.

$BJ := AJ - AB \qquad BH := \frac{BJ}{2} \qquad AH := AB + BH \qquad AQ := AH$

$PQ := \frac{BH^2}{AQ} \qquad HM := BH \qquad AP := AQ - PQ \qquad AC := \frac{AP^2 + AH^2 - BH^2}{2 \cdot AH}$

$CP := \sqrt{AP^2 - AC^2} \qquad HK := BH \qquad CH := AH - AC \qquad CK := \sqrt{CH^2 + HK^2}$

$HV := \frac{HK \cdot HM}{CH} \qquad AV := AH - HV \qquad ST := \frac{CP \cdot AV}{AP}$

Definitions.

$AJ - N_1 = 0 \qquad BJ - (N_1 - 1) = 0 \qquad BH - \frac{N_1 - 1}{2} = 0 \qquad AH - \frac{N_1 + 1}{2} = 0$

$AQ - \frac{N_1 + 1}{2} = 0 \qquad PQ - \frac{(N_1 - 1)^2}{2 \cdot (N_1 + 1)} = 0 \qquad HM - \frac{N_1 - 1}{2} = 0 \qquad AP - \frac{2 \cdot N_1}{N_1 + 1} = 0$

$AC - \frac{N_1 \cdot (N_1^2 + 6 \cdot N_1 + 1)}{(N_1 + 1)^3} = 0 \qquad CP - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)}}{(N_1 + 1)^3} = 0 \qquad HK - \frac{N_1 - 1}{2} = 0$

$CH - \frac{(N_1^2 + 4 \cdot N_1 + 1) \cdot (N_1 - 1)^2}{2 \cdot (N_1 + 1)^3} = 0 \qquad CK - \frac{(N_1 - 1) \cdot \sqrt{N_1^6 + 6 \cdot N_1^5 + 9 \cdot N_1^4 + 9 \cdot N_1^2 + 6 \cdot N_1 + 1}}{\sqrt{2} \cdot (N_1 + 1)^3} = 0$

$HV - \frac{(N_1 + 1)^3}{2 \cdot (N_1^2 + 4 \cdot N_1 + 1)} = 0 \qquad AV - \frac{N_1 \cdot (N_1 + 1)}{N_1^2 + 4 \cdot N_1 + 1} = 0 \qquad ST - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)}}{2 \cdot (N_1 + 1) \cdot (N_1^2 + 4 \cdot N_1 + 1)} = 0 \qquad AV - \frac{(AB + BJ) \cdot (AB + BJ + AB)}{AB^2 + 2 \cdot AB \cdot BJ + 4 \cdot AB + BJ^2 + 4 \cdot BJ + AB} = 0$

Unit.

$AB := 1$

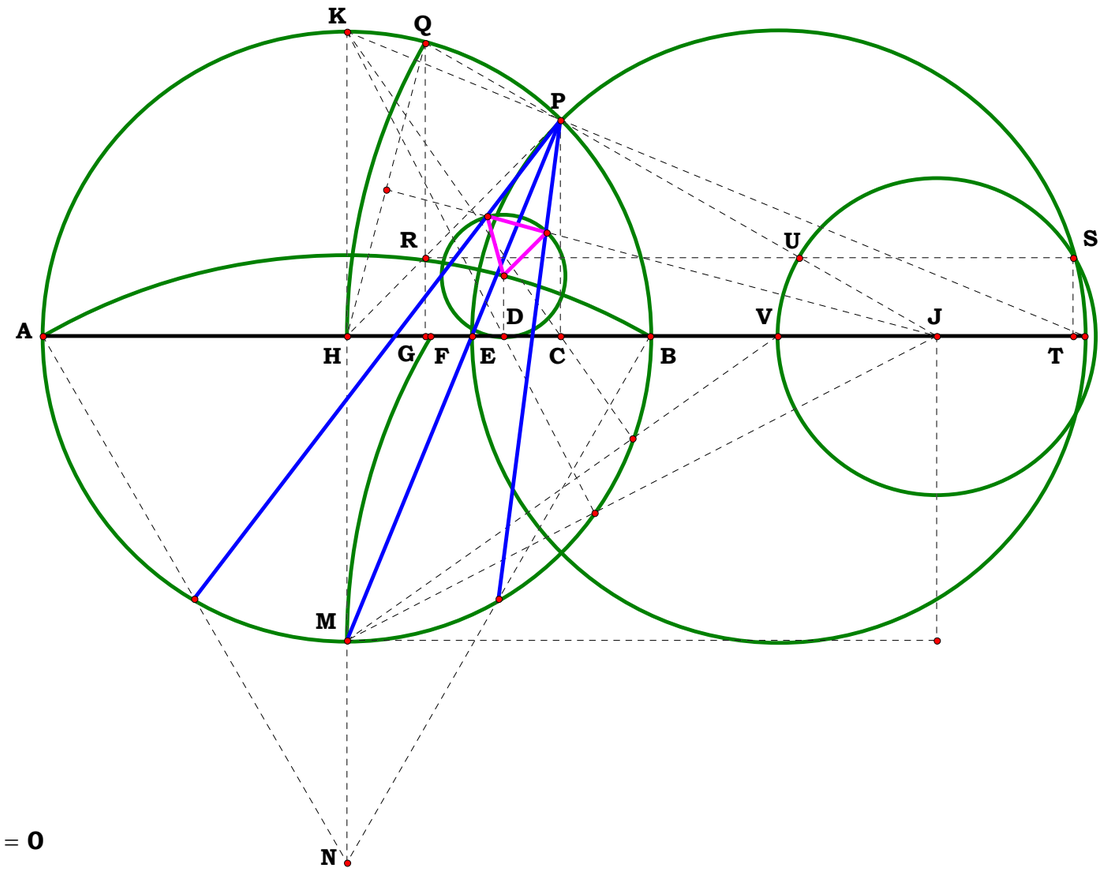
Given.

$N_1 := 1.47027$ $AJ := N_1$

0530011B

What is AV and ST?

Given any angle BHP, the unit which defines it will lay between W and V.



Descriptions.

$BJ := AJ - AB$ $BH := \frac{AB}{2}$ $JH := AJ - BH$ $JQ := JH$

$PQ := \frac{BH^2}{JQ}$ $HM := BH$ $JP := JQ - PQ$ $CJ := \frac{JP^2 + JH^2 - BH^2}{2 \cdot JH}$

$CP := \sqrt{JP^2 - CJ^2}$ $HK := BH$ $CH := JH - CJ$ $CK := \sqrt{CH^2 + HK^2}$

$HV := \frac{HK \cdot HM}{CH}$ $JV := JH - HV$ $ST := \frac{CP \cdot JV}{JP}$

Definitions.

$AJ - N_1 = 0$ $BJ - (N_1 - 1) = 0$ $BH - \frac{1}{2} = 0$ $JH - \frac{2 \cdot N_1 - 1}{2}$

$JQ - \frac{2 \cdot N_1 - 1}{2} = 0$ $PQ - \frac{1}{2 \cdot (2 \cdot N_1 - 1)} = 0$ $HM - \frac{1}{2} = 0$ $JP - \frac{2 \cdot N_1 \cdot (N_1 - 1)}{2 \cdot N_1 - 1} = 0$

$CJ - \frac{N_1 \cdot (N_1 - 1) \cdot (8 \cdot N_1^2 - 8 \cdot N_1 + 1)}{(2 \cdot N_1 - 1)^3} = 0$ $CP - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(4 \cdot N_1 - 1) \cdot (4 \cdot N_1 - 3)}}{(2 \cdot N_1 - 1)^3} = 0$

$HK - \frac{1}{2} = 0$ $CH - \frac{6 \cdot N_1^2 - 6 \cdot N_1 + 1}{2 \cdot (2 \cdot N_1 - 1)^3} = 0$ $CK - \frac{\sqrt{2 \cdot N_1 \cdot (N_1 - 1) \cdot [16 \cdot N_1^3 \cdot (N_1 - 2) + 37 \cdot N_1^2 - 21 \cdot N_1 + 6] + 1}}{\sqrt{2} \cdot (2 \cdot N_1 - 1)^3} = 0$

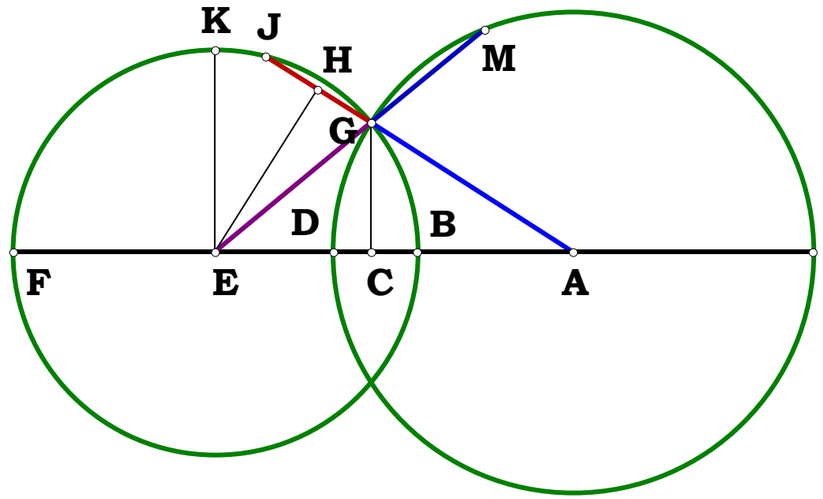
$HV - \frac{(2 \cdot N_1 - 1)^3}{2 \cdot (6 \cdot N_1^2 - 6 \cdot N_1 + 1)} = 0$ $JV - \frac{N_1 \cdot (2 \cdot N_1 - 1) \cdot (N_1 - 1)}{6 \cdot N_1^2 - 6 \cdot N_1 + 1} = 0$ $ST - \frac{N_1 \cdot (N_1 - 1) \cdot \sqrt{(4 \cdot N_1 - 1) \cdot (4 \cdot N_1 - 3)}}{2 \cdot (2 \cdot N_1 - 1) \cdot (6 \cdot N_1^2 - 6 \cdot N_1 + 1)} = 0$

Unit.
Given.
Descriptions.
Definitions.

060101

A Small Extrapolation

Given AE, AG, and EG, what is the Algebraic name of the segment GJ?



$S_1 := 6.00604$ $S_2 := 4.02167$ $S_3 := 3.38667$

$AE := S_1$ $AG := S_2$ $EG := S_3$

$$AC := \frac{AG^2 + AE^2 - EG^2}{2AE}$$

$$AH := \frac{AC \cdot AE}{AG}$$ $GH := AH - AG$

$HJ := GH$ $GJ := GH + HJ$

Some Algebraic Names:

$$\frac{S_2^2 + S_1^2 - S_3^2}{2S_1} - AC = 0$$

$$\frac{S_1^2 + S_2^2 - S_3^2}{2S_2} - AH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{2S_2} - GH = 0$$

$$\frac{S_1^2 - S_2^2 - S_3^2}{S_2} - GJ = 0$$

Unit.

$AB := 1$

Given.

$N := 5.727 \quad AG := N$

060201A

Descriptions.

$BG := AG - AB \qquad BF := \frac{BG}{2} \qquad AF := AB + BF$

$AE := \sqrt{AB \cdot AG} \qquad BE := AE - AB$

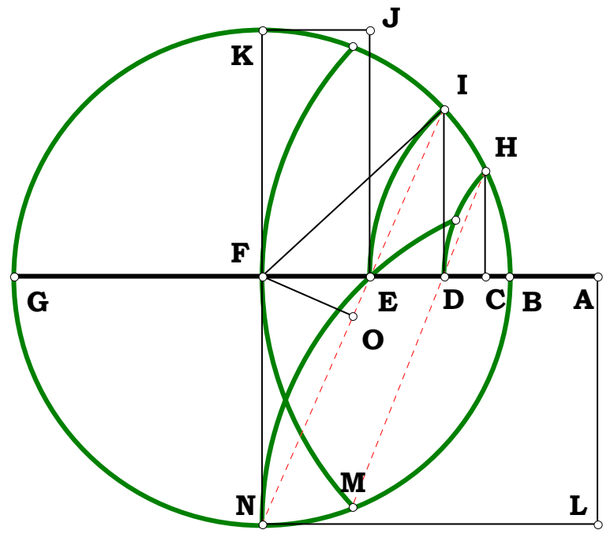
$FN := BF \qquad EF := BF - BE \qquad EN := \sqrt{FN^2 + EF^2}$

$NO := \frac{FN^2}{EN} \qquad NI := 2 \cdot NO \qquad EI := NI - EN$

$DE := \frac{EF \cdot EI}{EN}$

Units From Both Sides

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.



Definitions.

$BG - (N - 1) = 0 \qquad BF - \frac{N - 1}{2} = 0 \qquad AF - \frac{N + 1}{2} = 0$

$AE - \sqrt{N} = 0 \qquad BE - \left(\sqrt{N} - 1\right) = 0 \qquad FN - \frac{N - 1}{2} = 0$

$EF - \frac{\left(\sqrt{N} - 1\right)^2}{2} = 0 \qquad EN - \frac{\sqrt{2} \cdot \sqrt{N + 1} \cdot \left(\sqrt{N} - 1\right)}{2} = 0 \qquad NO - \frac{\sqrt{2} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)^2}{4 \cdot \sqrt{N + 1}} = 0$

$NI - \frac{\sqrt{2} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)^2}{2 \cdot \sqrt{N + 1}} = 0 \qquad EI - \frac{\sqrt{2} \cdot \sqrt{N} \cdot \left(\sqrt{N} - 1\right)}{\sqrt{N + 1}} = 0 \qquad DE - \frac{\sqrt{N} \cdot \left(\sqrt{N} - 1\right)^2}{N + 1} = 0$

060201B

Unit.

$BG := 1$

Given.

$N := 2$

Descriptions.

$$BF := \frac{BG}{2} \qquad BE := \frac{BF}{N} \qquad EF := BF - BE$$

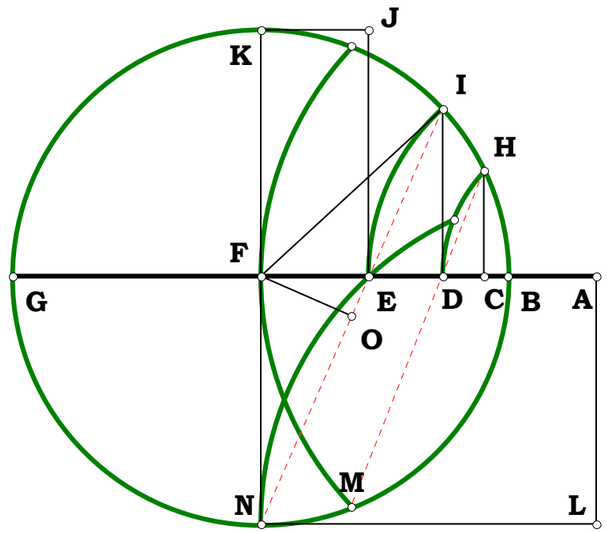
$$FN := BF \qquad EN := \sqrt{EF^2 + FN^2} \qquad NP := \frac{EN}{2}$$

$$LN := \frac{EN \cdot NP}{EF} \qquad AF := LN \qquad AB := AF - BF$$

$AB = 0.125$

Units From Both Sides

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.



Definitions.

$$\frac{BG}{2} - BF = 0 \qquad \frac{BG}{(2 \cdot N)} - BE = 0 \qquad \frac{BG}{2} \cdot \frac{(N-1)}{N} - EF = 0$$

$$\frac{BG}{2} \cdot \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{N} - EN = 0 \qquad \frac{BG}{4} \cdot \frac{\sqrt{2 \cdot N^2 - 2 \cdot N + 1}}{N} - NP = 0$$

$$\frac{BG}{4} \cdot \frac{\left(2 \cdot N^2 - 2 \cdot N + 1\right)}{[N \cdot (N-1)]} - LN = 0 \qquad \frac{BG}{4 \cdot N \cdot (N-1)} - AB = 0$$

060201C

Unit.

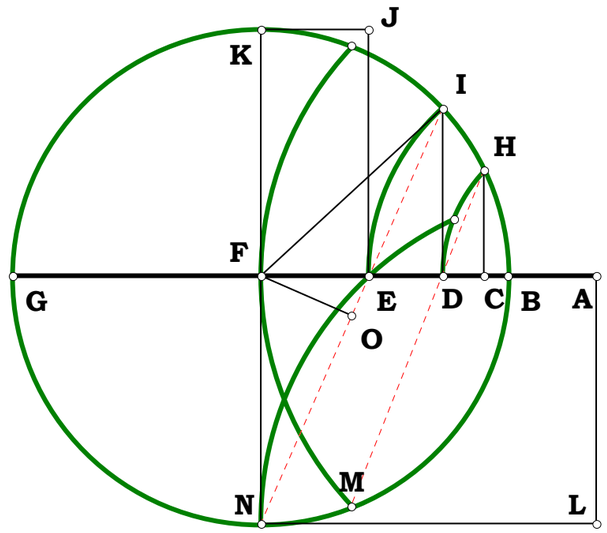
$BG := 1$

Given.

$N := 2$

Units From Both Sides

Start with AB as unit and find. . . . then start with . . . . as unit and find AB.



Descriptions.

$BF := \frac{BG}{2}$ $BD := \frac{BF}{N}$ $DG := BG - BD$

$DI := \sqrt{BD \cdot DG}$ $DF := BF - BD$ $FN := BF$

$EF := \frac{DF \cdot FN}{FN + DI}$ $EN := \sqrt{EF^2 + FN^2}$ $NP := \frac{EN}{2}$

$LN := \frac{EN \cdot NP}{EF}$ $AF := LN$ $AB := AF - BF$

$AB = 0.5$

Definitions.

$BD - \frac{1}{2} \cdot \frac{BG}{N} = 0$ $DG - \frac{1}{2} \cdot BG \cdot \frac{(2 \cdot N - 1)}{N} = 0$ $DI - \frac{1}{(2 \cdot N)} \cdot BG \cdot \sqrt{2 \cdot N - 1} = 0$

$DF - \frac{1}{2} \cdot BG \cdot \frac{(N - 1)}{N} = 0$ $EF - \frac{1}{2} \cdot BG \cdot \frac{(N - 1)}{\left(N + \sqrt{2 \cdot N - 1}\right)} = 0$ $NP - \frac{1}{4} \cdot BG \cdot \sqrt{2} \cdot \sqrt{\frac{N}{\left(N + \sqrt{2 \cdot N - 1}\right)}} = 0$

$EN - \frac{1}{2} \cdot BG \cdot \sqrt{2} \cdot \sqrt{\frac{N}{\left(N + \sqrt{2 \cdot N - 1}\right)}} = 0$ $LN - \frac{1}{2} \cdot BG \cdot \frac{N}{(N - 1)} = 0$ $AB - \frac{1}{2} \cdot \frac{BG}{(N - 1)} = 0$

Unit.

$BE := 1$

Given.

$N := 4$

060301

Descriptions.

$BD := \frac{BE}{2}$ $BC := \frac{BE}{N}$ $CE := BE - BC$

$CG := \sqrt{BC \cdot CE}$ $CD := BD - BC$ $AC := \frac{CG^2}{CD}$

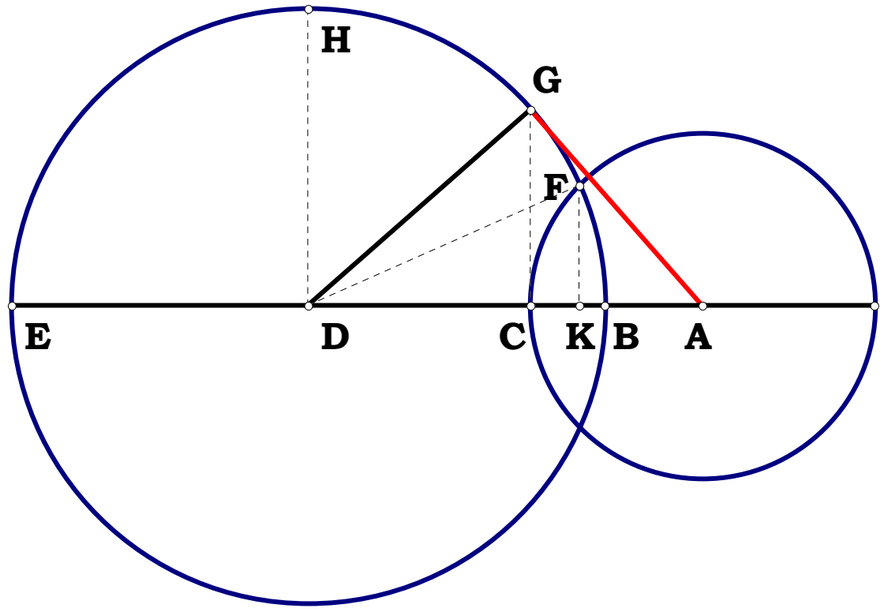
$AF := AC$ $AD := AC + CD$ $DF := BD$

$DK := \frac{DF^2 + AD^2 - AF^2}{2AD}$ $BK := BD - DK$

$CK := BC - BK$

Isolating A Problem

If one is given point F, then finding point G would lead straightway to the solution. How is BK related to BC?



Definitions.

$\frac{N-1}{N} - CE = 0$ $\frac{\sqrt{N-1}}{N} - CG = 0$ $\frac{N-2}{2 \cdot N} - CD = 0$

$\frac{2 \cdot (N-1)}{N \cdot (N-2)} - AC = 0$ $\frac{N}{2 \cdot (N-2)} - AD = 0$ $\frac{(N-2) \cdot \left(N^2 + 2 \cdot N - 2\right)}{2 \cdot N^3} - DK = 0$

$\frac{3 \cdot N - 2}{N^3} - BK = 0$ $\frac{(N-1) \cdot (N-2)}{N^3} - CK = 0$ $\frac{BK}{BC} - \frac{(3 \cdot N - 2)}{N^2} = 0$

Unit.

$BE := 1$

Given.

$N := 4$

060301

Descriptions.

$BD := \frac{BE}{2}$ $BC := \frac{BD}{N}$ $AC := \frac{1}{2} \cdot \frac{BE}{N} \cdot \frac{(2 \cdot N - 1)}{(N - 1)}$

$CM := \frac{1}{4} \cdot BE \cdot (2 \cdot N - 1) \cdot \frac{(N - 1)}{N^3}$ $AM := AC - CM$

$AF := AC$ $FM := \sqrt{AF^2 - AM^2}$ $AD := \frac{1}{2} \cdot BE \cdot \frac{N}{(N - 1)}$

$FK := \frac{AD^2 - AC^2 - BD^2}{AC}$ $FK = 0.375$

Definitions.

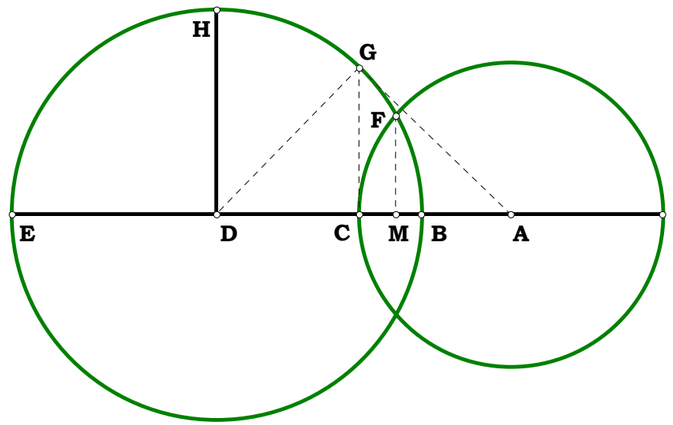
$BD - \frac{1}{2} = 0$ $BC - \frac{1}{2 \cdot N} = 0$ $AC - \frac{2 \cdot N - 1}{2 \cdot N \cdot (N - 1)} = 0$

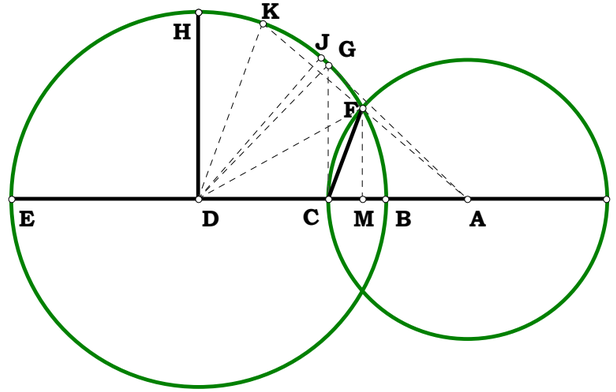
$CM - \frac{(N - 1) \cdot (2 \cdot N - 1)}{4 \cdot N^3} = 0$ $AM - \frac{(2 \cdot N - 1) \cdot \left(N^2 + 2 \cdot N - 1\right)}{4 \cdot N^3 \cdot (N - 1)} = 0$

$AF - \frac{2 \cdot N - 1}{2 \cdot N \cdot (N - 1)} = 0$ $FM - \frac{\sqrt{(N + 1) \cdot (3 \cdot N - 1)} \cdot (2 \cdot N - 1)}{4 \cdot N^3} = 0$

$AD - \frac{N}{2 \cdot (N - 1)} = 0$ $FK - \frac{N - 1}{2 \cdot N} = 0$

For any point C on BD, FCG is 1/3 of the angle FDH, will the Algebraic Name for DM remain constant if one 'steps back to it' from D?





061001

Given.

$N_1 := 5$

$N_2 := 3$

Descriptions.

For any $N_1 \cdot N_2$ what is DG?

$AF := N_1 \cdot N_2 \quad BF := AF - N_2 \quad BE := \frac{BF}{2}$

$EK := BE \quad AE := N_2 + BE \quad DE := \frac{EK^2}{AE}$

$EF := BE \quad FM := BF \quad EM := \sqrt{FM^2 - EF^2}$

$GM := FM \quad GQ := DE \quad MQ := \sqrt{GM^2 - GQ^2}$

$EQ := MQ - EM \quad DG := EQ$

Definitions.

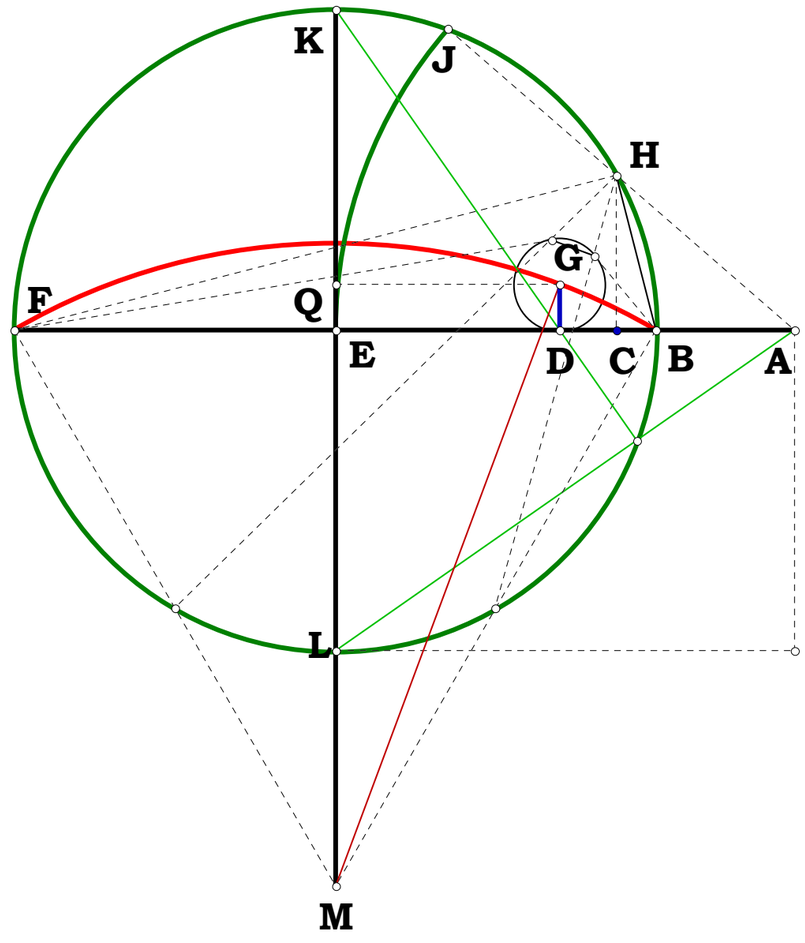
$N_2 \cdot (N_1 - 1) - BF = 0 \qquad \frac{N_2 \cdot (N_1 - 1)}{2} - BE = 0$

$\frac{1}{2} \cdot N_2 \cdot (N_1 + 1) - AE = 0 \qquad \frac{1}{2} \cdot N_2 \cdot \frac{(N_1 - 1)^2}{(N_1 + 1)} - DE = 0$

$\frac{1}{2} \cdot \sqrt{3} \cdot N_2 \cdot (N_1 - 1) - EM = 0 \qquad \frac{N_2 \cdot \left[\sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)} \cdot (N_1 - 1)\right]}{2 \cdot (N_1 + 1)} - MQ = 0$

$\frac{N_2 \cdot (N_1 - 1) \cdot \left[\sqrt{(N_1 + 3) \cdot (3 \cdot N_1 + 1)} - \sqrt{3} - \sqrt{3} \cdot N_1\right]}{2 \cdot (N_1 + 1)} - DG = 0$

For Any $N_1 \cdot N_2$



082601

Unit.

$AF := 1$

Given.

$N_1 := 4$

$N_2 := 3$

Descriptions.

$AC := \frac{AF}{2}$ $CJ := AC$ $BC := \frac{AC}{N_1}$

$AE := \frac{AF}{N_2}$ $EF := AF - AE$ $EH := \sqrt{AE \cdot EF}$

$EG := \frac{BC \cdot EH}{CJ}$ $CE := AE - AC$ $CG := \sqrt{CE^2 + EG^2}$

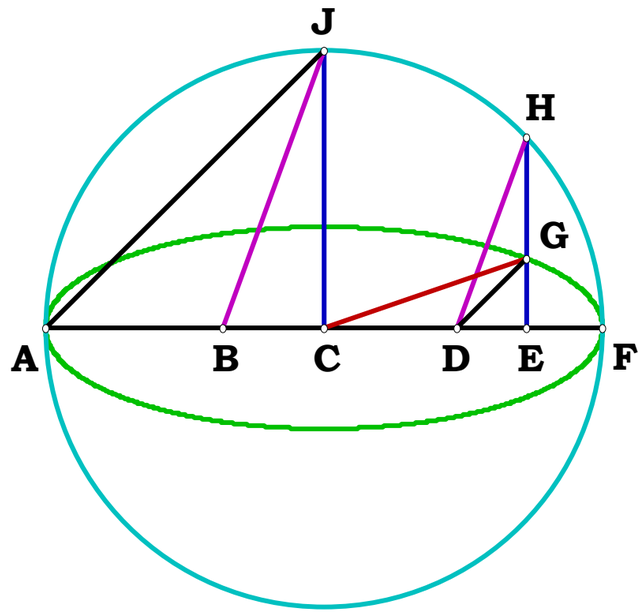
Definitions.

$BC - \frac{1}{(2 \cdot N_1)} = 0$ $AE - \frac{1}{N_2} = 0$ $EF - \left(1 - \frac{1}{N_2}\right) = 0$

$EH - \frac{\sqrt{N_2 - 1}}{N_2} = 0$ $EG - \frac{\sqrt{N_2 - 1}}{N_1 \cdot N_2} = 0$

$CE - \left(\frac{1}{N_2} - \frac{1}{2}\right) = 0$ $CG - \frac{1}{2} \cdot \frac{\sqrt{(N_2 - 2)^2 \cdot N_1^2 + 4 \cdot (N_2 - 1)}}{(N_1 \cdot N_2)} = 0$

Elipse By Parallels



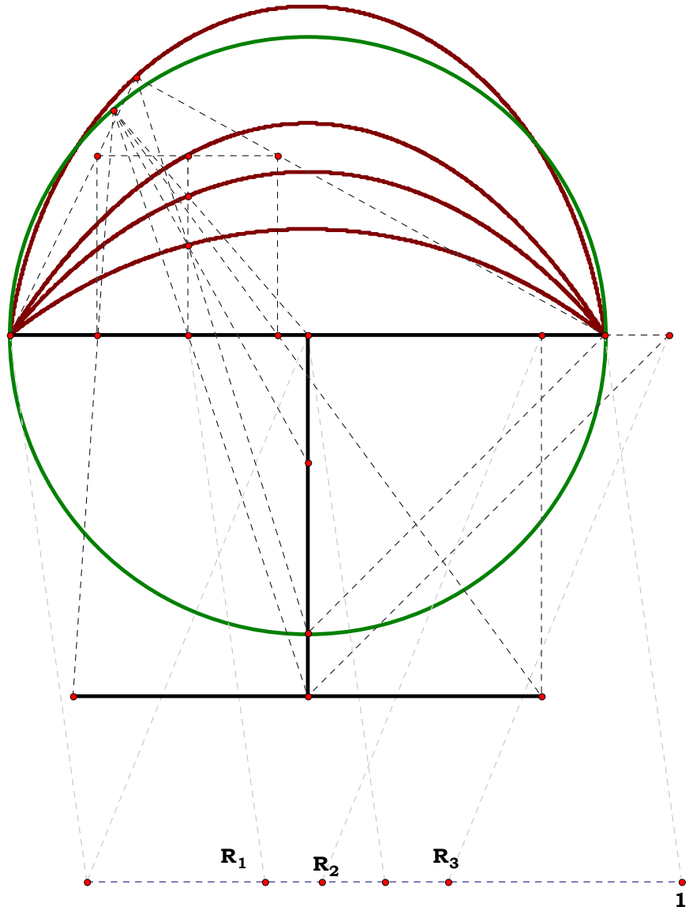
102201

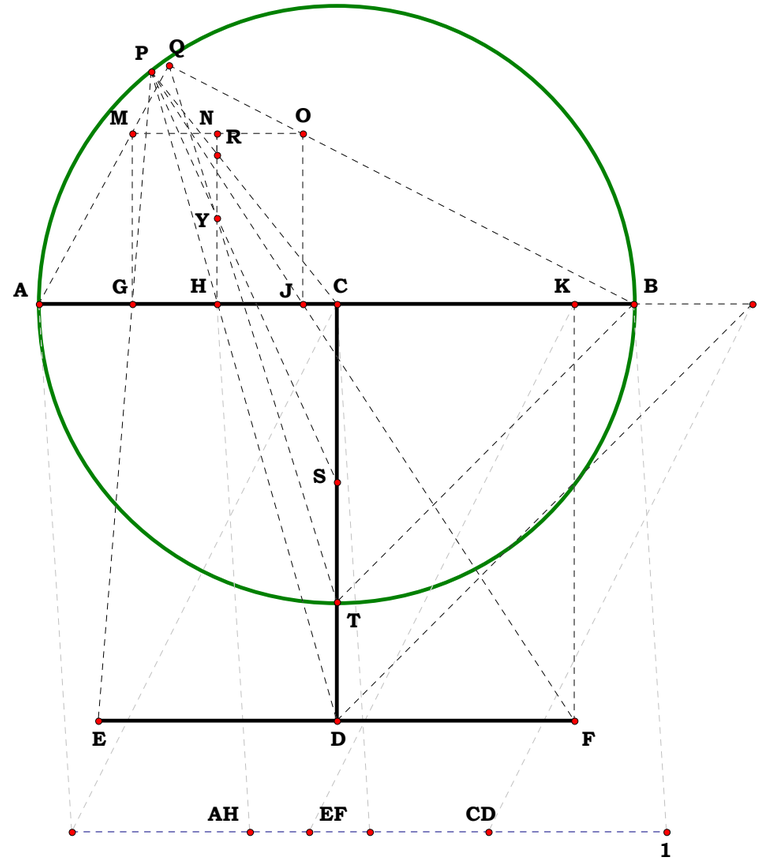
I had sketched this out over 18 years ago and have put off writing it up for some very good reasons, the most prominate is because of the way I wanted to write it up. Normally I aim for between one and four variables for this novel; this one I need six. For the work Basic Analog Mathematics that number of variables is not unusual, but here it is. For BAM, I set my limit on 8, which is twice that I set for this work.

I am so motivated to write this up that when I got to it in this Delian Quest revision, I put this project aside and did the projects OTOH (On The Other Hand), Alice Innocent Plays and Conducts Bach, and Sergio Vosh Goshen Inkscapes Durer, which took me over a month to do. I was curious about the state of ABC notation and Vector Graphics which have been on my mind for some 20 years now.

I also made some new Windows 95 and 98 virtual boxes which have all the software, and more, that I started these projects with.

Four Curves and Procrastination.





Unit = 1.00000
AH = 0.30000
N_1 = 6.00000
N_2 = 20.00000

EF = 0.80000
N_3 = 8.00000
N_4 = 10.00000

CD = 0.70000
N_5 = 7.00000
N_6 = 10.00000

I am not even going to go through all of this, just the major portion.

Unit.

$AB := 1$

Given.

$N_1 := 6 \quad N_2 := 20$

$N_3 := 8 \quad N_4 := 10$

$N_5 := 7 \quad N_6 := 11$

Descriptions.

$$AH := \frac{N_1}{N_2} \quad EF := \frac{N_3}{N_4} \quad CD := \frac{N_5}{N_6} \quad AC := \frac{AB}{2}$$

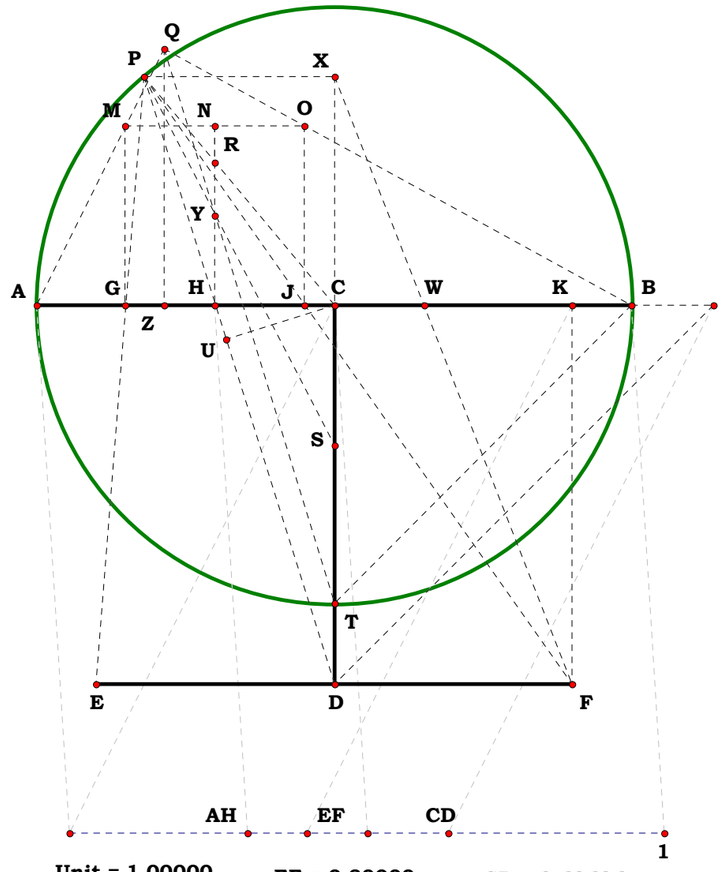
$$CP := AC \quad CH := \sqrt{(AC - AH)^2} \quad DH := \sqrt{CD^2 + CH^2}$$

$$CU := \frac{CD \cdot CH}{DH} \quad PU := \sqrt{CP^2 - CU^2} \quad HU := \frac{CH^2}{DH}$$

$$HP := PU - HU \quad DP := DH + HP \quad DX := \frac{CD \cdot DP}{DH}$$

$$DF := \frac{EF}{2} \quad CX := DX - CD \quad CW := \frac{DF \cdot CX}{DX}$$

$GJ := 2 \cdot CW$ **Etc.**



Unit = 1.00000
AH = 0.30000
N_1 = 6.00000
N_2 = 20.00000

EF = 0.80000
N_3 = 8.00000
N_4 = 10.00000

CD = 0.63636
N_5 = 7.00000
N_6 = 11.00000

Definitions.

$$AH - \frac{N_1}{N_2} = 0 \quad EF - \frac{N_3}{N_4} = 0 \quad CD - \frac{N_5}{N_6} = 0 \quad AC - \frac{1}{2}$$

$$CP - \frac{1}{2} = 0 \quad CH - \frac{\sqrt{(2 \cdot N_1 - N_2)^2}}{2 \cdot N_2} = 0 \quad DH - \frac{\sqrt{(4 \cdot N_5^2 + N_6^2) \cdot N_2^2 + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}}{2 \cdot N_2 \cdot N_6} = 0$$

$$CU - \frac{N_5 \cdot \sqrt{(N_2 - 2 \cdot N_1)^2}}{\sqrt{N_2^2 \cdot (4 \cdot N_5^2 + N_6^2) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

$$PU - \frac{\sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}}{2 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2}} = 0 \quad HU - \frac{N_6 \cdot (2 \cdot N_1 - N_2)^2}{2 \cdot N_2 \cdot \sqrt{N_2^2 \cdot (4 \cdot N_5^2 + N_6^2) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

$$HP - \frac{N_2 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)} - N_6 \cdot (2 \cdot N_1 - N_2)^2}{2 \cdot N_2 \cdot \sqrt{N_2^2 \cdot (4 \cdot N_5^2 + N_6^2) + 4 \cdot N_1 \cdot N_6^2 \cdot (N_1 - N_2)}} = 0$$

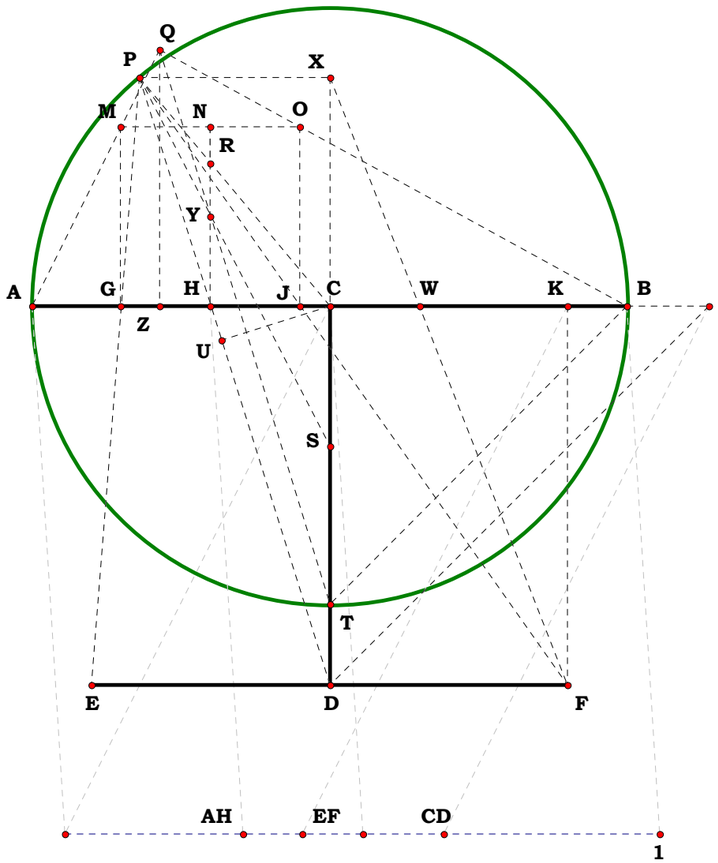
$$DP - \frac{4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}}{2 \cdot N_6 \cdot \sqrt{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2}} = 0$$

$$DX - \frac{N_2 \cdot N_5 \cdot \left[4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}\right]}{N_6 \cdot \left[(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2\right]} = 0$$

$$DF - \frac{N_3}{2 \cdot N_4} = 0 \qquad CX - \frac{\left[\sqrt{N_6^2 \cdot (2 \cdot N_1 - N_2)^2 - 16 \cdot N_5^2 \cdot N_1 \cdot (N_1 - N_2)} \cdot N_2 - N_6 \cdot (2 \cdot N_1 - N_2)^2\right] \cdot N_5}{(2 \cdot N_1 - N_2)^2 \cdot N_6^2 + 4 \cdot N_2^2 \cdot N_5^2} = 0$$

$$CW - \frac{N_3 \cdot N_6 \cdot \left[\sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)} \cdot N_2 - N_6 \cdot (2 \cdot N_1 - N_2)^2\right]}{2 \cdot N_2 \cdot N_4 \cdot \left[4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}\right]} = 0$$

$$GJ - \frac{N_3 \cdot N_6 \cdot \left[\sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)} \cdot N_2 - N_6 \cdot (2 \cdot N_1 - N_2)^2\right]}{N_2 \cdot N_4 \cdot \left[4 \cdot N_2 \cdot N_5^2 + N_6 \cdot \sqrt{N_6^2 \cdot (N_2 - 2 \cdot N_1)^2 - 16 \cdot N_1 \cdot N_5^2 \cdot (N_1 - N_2)}\right]} = 0$$



Unit = 1.00000
AH = 0.30000
N_1 = 6.00000
N_2 = 20.00000

EF = 0.80000
N_3 = 8.00000
N_4 = 10.00000

CD = 0.63636
N_5 = 7.00000
N_6 = 11.00000

Just Another Proof Of Paper

Unit.

$AB := 1$

Given.

$N := 7 \qquad AF := N$

010202

Descriptions.

$BF := AF - AB \qquad BE := \frac{BF}{2} \qquad AE := BE + AB$

$AJ := AE \qquad EJ := BE \qquad Ea := \frac{EJ^2 + AE^2 - AJ^2}{2 \cdot AE}$

$Gb := Ea \qquad GJ := 2 \cdot Gb \qquad AG := AJ - GJ$

$Aa := AE - Ea \qquad AU := \frac{Aa \cdot AG}{AJ}$

$Ja := \sqrt{AJ^2 - Aa^2} \qquad GU := \frac{Ja \cdot AG}{AJ}$

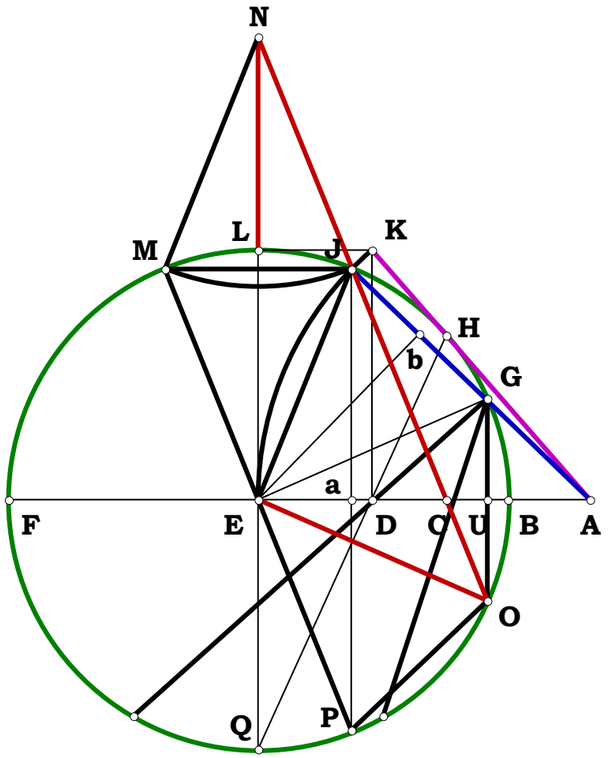
$Ua := Aa - AU \qquad JO := \sqrt{Ua^2 + (GU + Ja)^2}$

$JN := \frac{JO \cdot Ea}{Ua} \qquad JN - BE = 0$

From 4/29/94 $\qquad OP := \sqrt{Ja^2 - 2 \cdot Ja \cdot GU + GU^2 + Ua^2}$

$OP - 2 \cdot Ea = 0 \qquad NO := JO + JN \qquad EU := Ua + Ea$

$EN := \sqrt{NO^2 - EU^2} \qquad EL := BE \qquad LN := EN - EL$



Definitions.

$$N - 1 - BF = 0 \qquad \frac{N-1}{2} - BE = 0 \qquad \frac{N+1}{2} - AE = 0$$

$$\frac{(N-1)^2}{4\cdot(N+1)} - Ea = 0 \qquad \frac{(N-1)^2}{2\cdot(N+1)} - GJ = 0 \qquad \frac{N^2+6\cdot N+1}{4\cdot(N+1)} - Aa = 0$$

$$\frac{2\cdot N}{N+1} - AG = 0 \qquad \frac{N\cdot\left(N^2+6\cdot N+1\right)}{(N+1)^3} - AU = 0 \qquad \frac{(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{4\cdot(N+1)} - Ja = 0$$

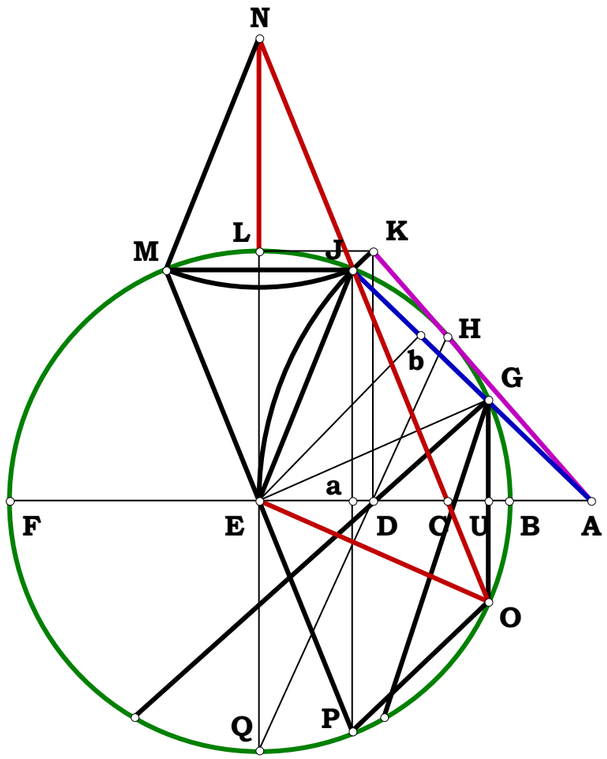
$$\frac{N\cdot(N-1)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{(N+1)^3} - GU = 0 \qquad \frac{\left(N^2+6\cdot N+1\right)\cdot(N-1)^2}{4\cdot(N+1)^3} - Ua = 0$$

$$\frac{(N-1)\cdot\left(N^2+6\cdot N+1\right)}{2\cdot(N+1)^2} - JO = 0 \qquad \frac{N-1}{2} - JN = 0 \qquad \frac{(N-1)^2}{2\cdot(N+1)} - OP = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)}{(N+1)^2} - NO = 0 \qquad \frac{(N-1)^2\cdot\left(N^2+4\cdot N+1\right)}{2\cdot(N+1)^3} - EU = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - EN = 0$$

$$\frac{(N-1)\cdot\left(N^2+4\cdot N+1\right)\cdot\sqrt{(N+3)\cdot(3\cdot N+1)}}{2\cdot(N+1)^3} - \frac{N-1}{2} - LN = 0$$



062002A

Unit.

$AC := 1$

Given.

$N_1 := 4 \quad AF := N_1$

$N_2 := 3 \quad CE := N_2$

Descriptions.

$FH := AF \quad EH := CE \quad AD := CE \quad DF := AF - AD$

$EJ := CE \quad FJ := AF \quad DE := AC \quad EF := \sqrt{DF^2 + DE^2}$

$EG := \frac{EJ^2 + EF^2 - FJ^2}{2 \cdot EF} \quad EM := \frac{DF \cdot EG}{EF} \quad CM := CE + EM$

$GM := \frac{DE \cdot EG}{EF} \quad CK := GM \quad GK := CM \quad BK := \frac{DF \cdot GK}{DE}$

$BC := CK + BK \quad BC - \frac{AC}{2} = 0$

Definitions.

$FH - N_1 = 0 \quad EH - N_2 = 0 \quad AD - N_2 = 0$

$DF - (N_1 - N_2) = 0 \quad EJ - N_2 = 0 \quad FJ - N_1 = 0$

$DE - 1 = 0 \quad EF - \sqrt{(N_1 - N_2)^2 + 1} = 0$

$EG - \frac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot \sqrt{(N_1 - N_2)^2 + 1}} = 0 \quad EM - \frac{[1 - 2 \cdot N_2 \cdot (N_1 - N_2)] \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

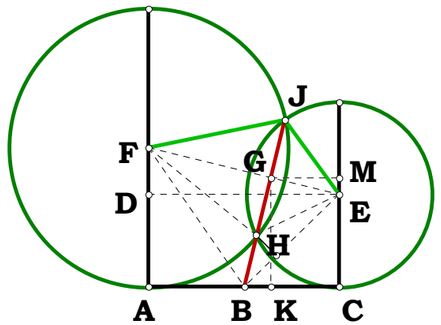
$CM - \frac{N_1 + N_2}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0 \quad GM - \frac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

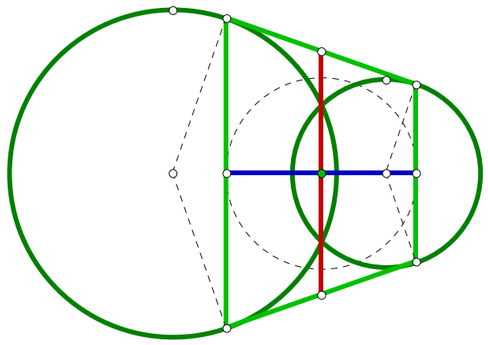
$CK - \frac{1 - 2 \cdot N_2 \cdot (N_1 - N_2)}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0 \quad GK - \frac{N_1 + N_2}{2 \cdot [(N_1 - N_2)^2 + 1]} = 0$

$BK - \frac{(N_1 + N_2) \cdot (N_1 - N_2)}{2 \cdot (N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2 + 1)} = 0 \quad BC - \frac{1}{2} = 0$

Easy Power-Line

For any two intersecting circles, the power-line BJ intersects their common tangents AC at midpoint.





Unit.

$AB := 1$

Given.

$N_1 := 7 \quad N_2 := 19$

$N_3 := 15 \quad N_4 := 17$

062002B

Descriptions.

$$AE := \frac{N_1}{N_2} \quad BF := \frac{N_3}{N_4} \quad BN := AE$$

$$FN := BF - BN \quad EF := \sqrt{AB^2 + FN^2}$$

$$EH := \frac{EF^2 + AE^2 - BF^2}{2 \cdot EF} \quad AO := \frac{AB \cdot EH}{EF}$$

$$EM := \frac{FN \cdot EH}{EF} \quad HO := AE + EM$$

$$KO := \frac{FN \cdot HO}{AB} \quad AB - (AO + KO) = 0.5$$

Definitions.

$$AE - \frac{N_1}{N_2} = 0 \quad BF - \frac{N_3}{N_4} = 0 \quad BN - \frac{N_1}{N_2} = 0$$

$$FN - \left(\frac{N_3}{N_4} - \frac{N_1}{N_2}\right) = 0 \quad FN - \frac{N_2 \cdot N_3 - N_1 \cdot N_4}{N_2 \cdot N_4} = 0$$

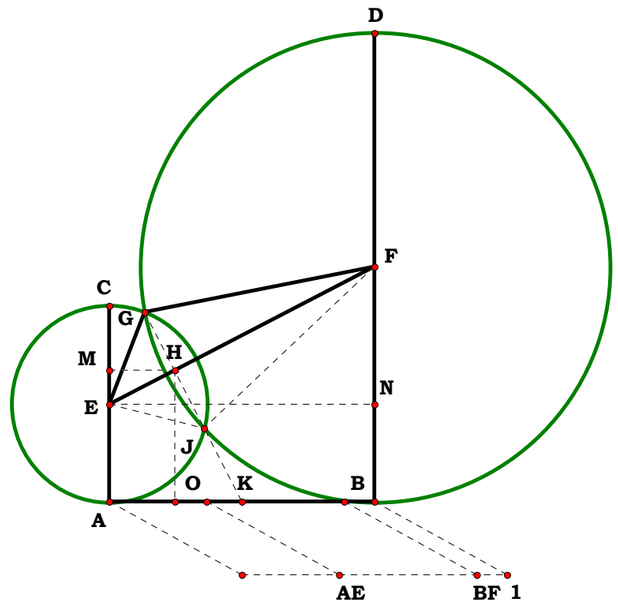
$$EF - \frac{\sqrt{\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)}}{N_2 \cdot N_4} = 0$$

$$AO - \frac{N_4 \cdot \left[\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4 - 2 \cdot N_1 \cdot N_2 \cdot N_3\right]}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$HO - \frac{N_2 \cdot N_4 \cdot \left(N_1 \cdot N_4 + N_2 \cdot N_3\right)}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

On the Other Hand

Have you ever pondered a figure in terms not of the particulars but of the concept of parallel lines? What does parallel mean? Between any two objects there is one, and only one, difference. If this is not true, what does it mean for existence itself? Is it not a self-referential fallacy? And since this is obvious, what does it say for so called non-Euclidean Geometers? If one cannot master the first prnciple of reasoning, can one ever know when they are speaking and thinking gibberish?



Unit = 1.00000
AB = 1.00000
AE = 0.36842
N_1 = 7.00000
N_2 = 19.00000

BF = 0.88235
N_3 = 15.00000
N_4 = 17.00000

Between any two objects there is one, and only one, difference such that the operations equitable to both, become half the difference between them and this is called their power.

$$EH - \frac{\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4^2 - 2 \cdot N_1 \cdot N_2 \cdot N_3 \cdot N_4}{2 \cdot N_2 \cdot N_4 \cdot \sqrt{N_4^2 \cdot \left(N_1^2 + N_2^2\right) + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)}} = 0$$

$$EM - \frac{\left(N_2 \cdot N_3 - N_1 \cdot N_4\right) \cdot \left[\left(2 \cdot N_1^2 + N_2^2\right) \cdot N_4 - 2 \cdot N_1 \cdot N_2 \cdot N_3\right]}{2 \cdot N_2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0$$

$$KO - \frac{N_2^2 \cdot N_3^2 - \left(N_1 \cdot N_4\right)^2}{2 \cdot \left[\left(N_1^2 + N_2^2\right) \cdot N_4^2 + N_2 \cdot N_3 \cdot \left(N_2 \cdot N_3 - 2 \cdot N_1 \cdot N_4\right)\right]} = 0 \quad AB - \frac{1}{2} = 0.5$$

071902

Descriptions.

Unit.

$AX := 1$

Given.

$N_1 := 9 \quad N_2 := 12$

$N_3 := 4 \quad N_4 := 11$

$$AB := \frac{N_1}{N_2} \quad AG := \frac{N_3}{N_4} \quad BG := \sqrt{AB^2 + AG^2}$$

I do not memorize equations; every time I have to use an equation from Pythagorus Revisited, I have to look it up.

$$BD := \frac{BG^2 + AB^2 - BG^2}{2 \cdot BG}$$ Or again; $$BD_1 := \frac{AB^2}{2 \cdot BG}$$

$$BE := \frac{AB \cdot BD}{BG} \quad DE := \frac{AG \cdot BE}{AB} \quad EF := \frac{AG \cdot DE}{AB}$$

$$BF := BE + EF \quad BF - \frac{AB}{2} = 0$$

Definitions.

$$AB - \frac{N_1}{N_2} = 0 \quad AG - \frac{N_3}{N_4} = 0$$

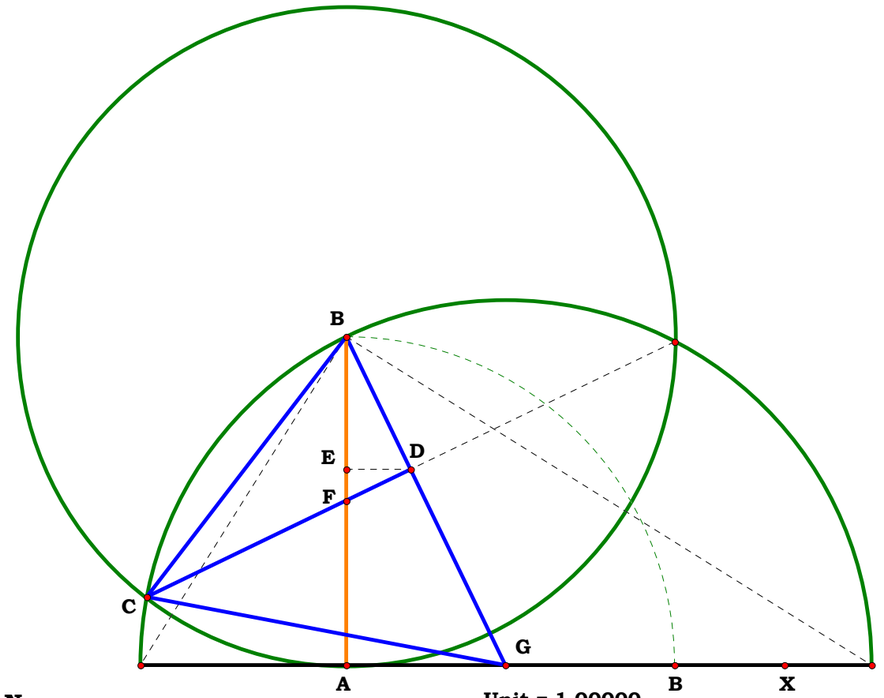
$$BG - \frac{\sqrt{N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2}}{N_2 \cdot N_4} = 0 \quad BD - \frac{N_1^2 \cdot N_4}{2 \cdot N_2 \cdot \sqrt{N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2}} = 0$$

$$BE - \frac{N_1^3 \cdot N_4^2}{2 \cdot N_2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0 \quad DE - \frac{N_1^2 \cdot N_3 \cdot N_4}{2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0$$

$$EF - \frac{N_1 \cdot N_2 \cdot N_3^2}{2 \cdot \left(N_1^2 \cdot N_4^2 + N_2^2 \cdot N_3^2\right)} = 0 \quad BF - \frac{N_1}{2 \cdot N_2} = 0$$

On Linear Division

If G were at A, then one would have the simple textbook method, however, if one took any point G on the perpendicular to AB, the results would be the same.



Unit = 1.00000
x_X = 1.00000
AB = 0.75000
N_1 = 9.00000
N_2 = 12.00000

AG = 0.36364
N_3 = 4.00000
N_4 = 11.00000

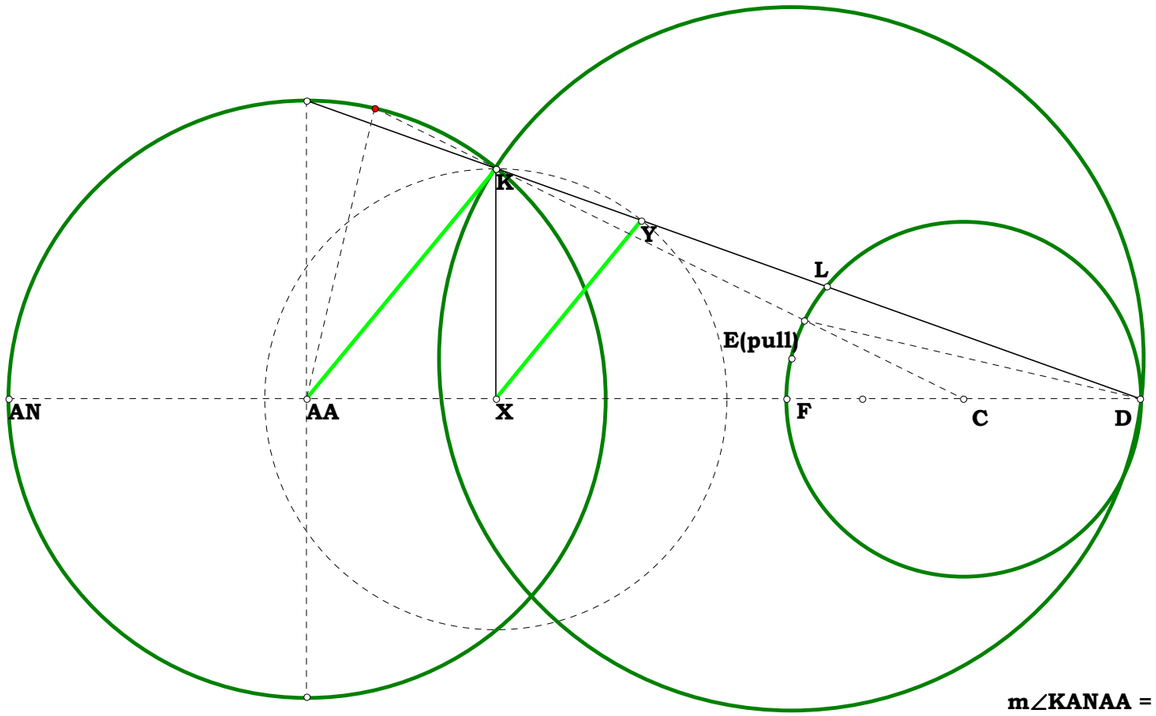
Unit.
090902
Given.
Plate 1

Parcing Project for 090902

Descriptions.

Parce the figure for project write-ups

Definitions.



m∠LCF = 39.346°
m∠LCE(pull) = 13.115°
$$\frac{m\angle LCF}{m\angle LCE(pull)} = 3.000$$
m∠E(pull)CF = 26.231°

m∠KANAA = 25.327°
m∠E(pull)CF = 26.231°
m∠YXF = 50.654°
$$\frac{m\angle YXF}{m\angle KANAA} = 2.000$$

First Plate

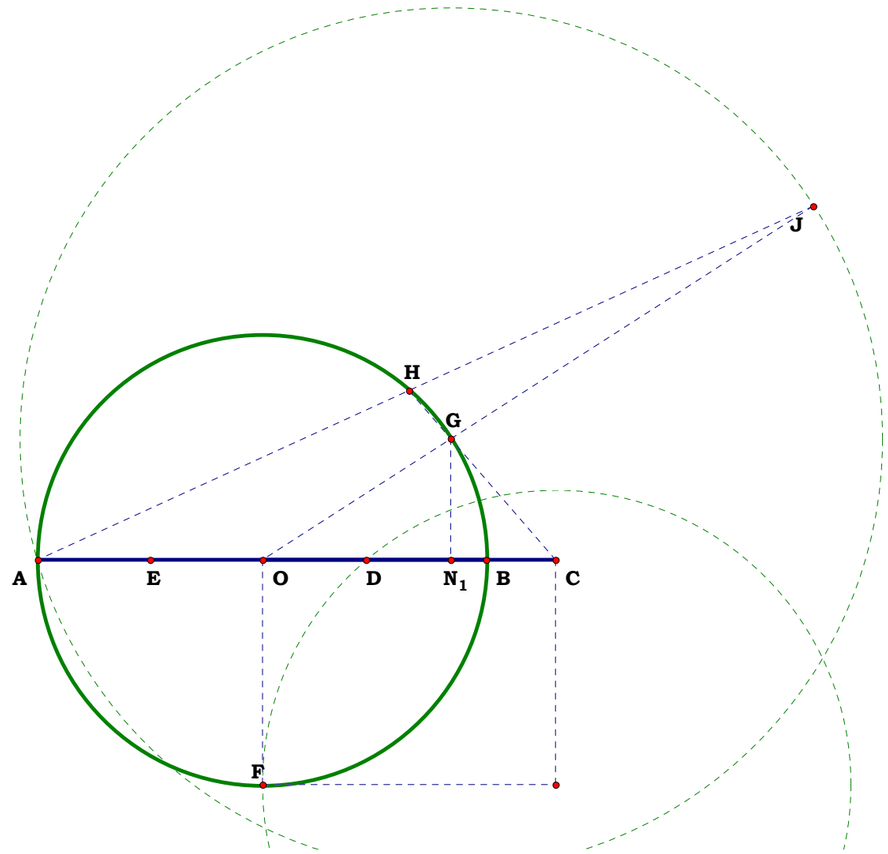
N_1 ranges from B to E producing trisection over the entire circle and $\sqrt{AC \cdot BC} = \blacksquare \cdot CD$ is also constant over the entire range even though CB goes from after AB to before it. Write it up.

Unit = 1.00000
XY = 0.89443
X = 17.88861
Y = 20.00000

$m\angle HOB = 49.02855°$
$m\angle HOG = 16.34285°$
$\frac{m\angle HOB}{m\angle HOG} = 3.00000$

CB = 1.16071 cm
CA = 8.68005 cm

$\sqrt{CB \cdot CA} = 3.17412$ cm
CD = 3.17412 cm
$\sqrt{CB \cdot CA} - CD = 0.00000$ cm



Unit.
Given.

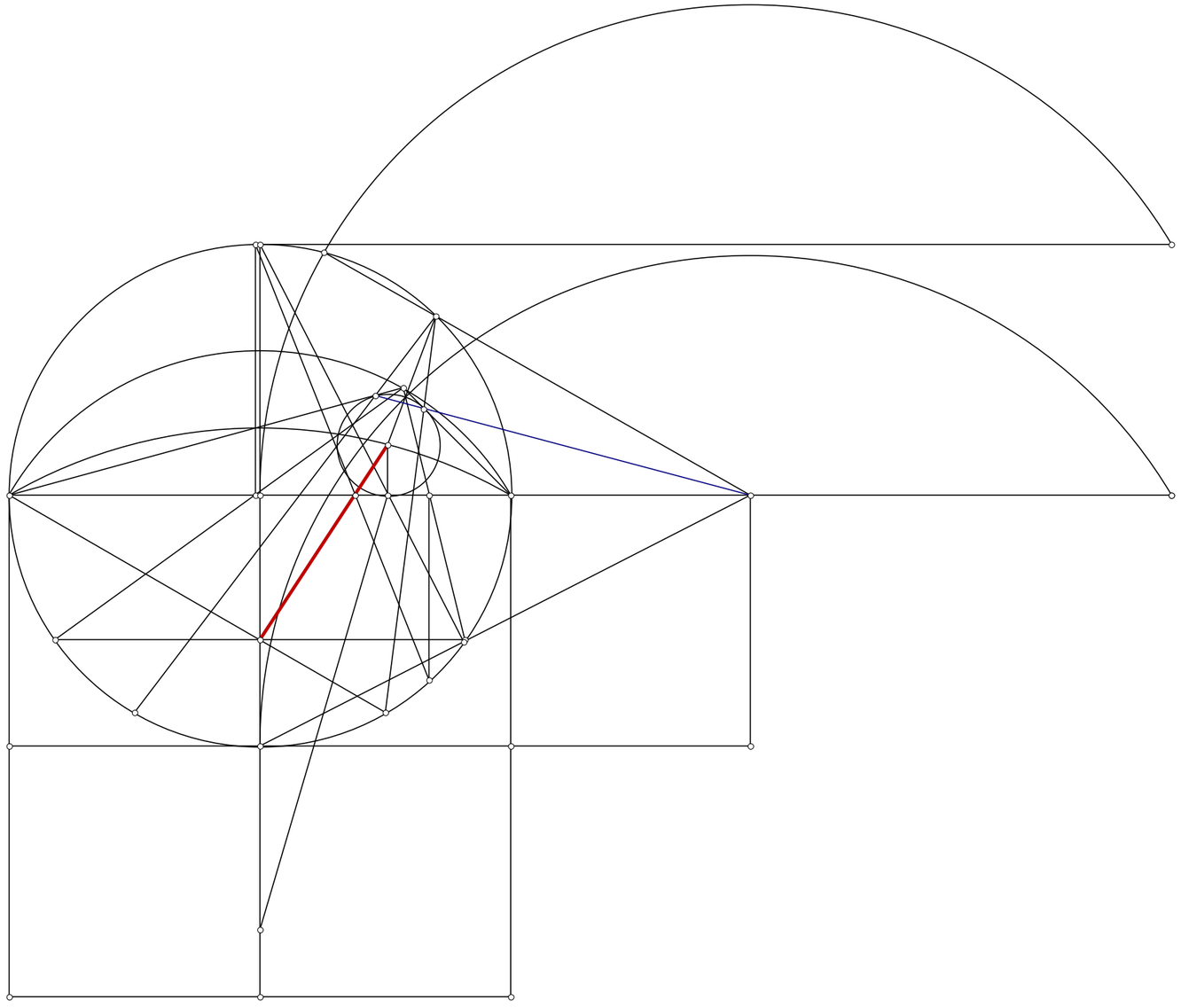
Project 092102

Name the segement in red.

092102

Descriptions.

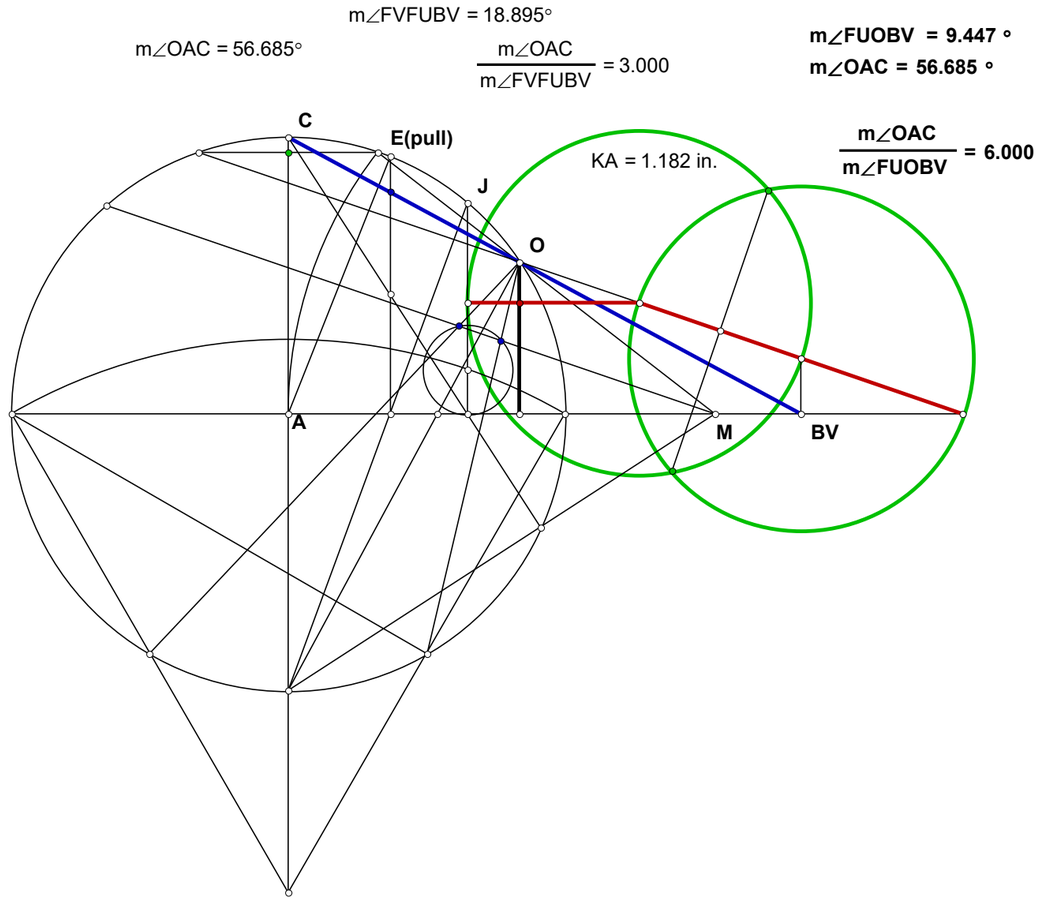
Definitions.



Unit.
Given.

Parcing project for 092202

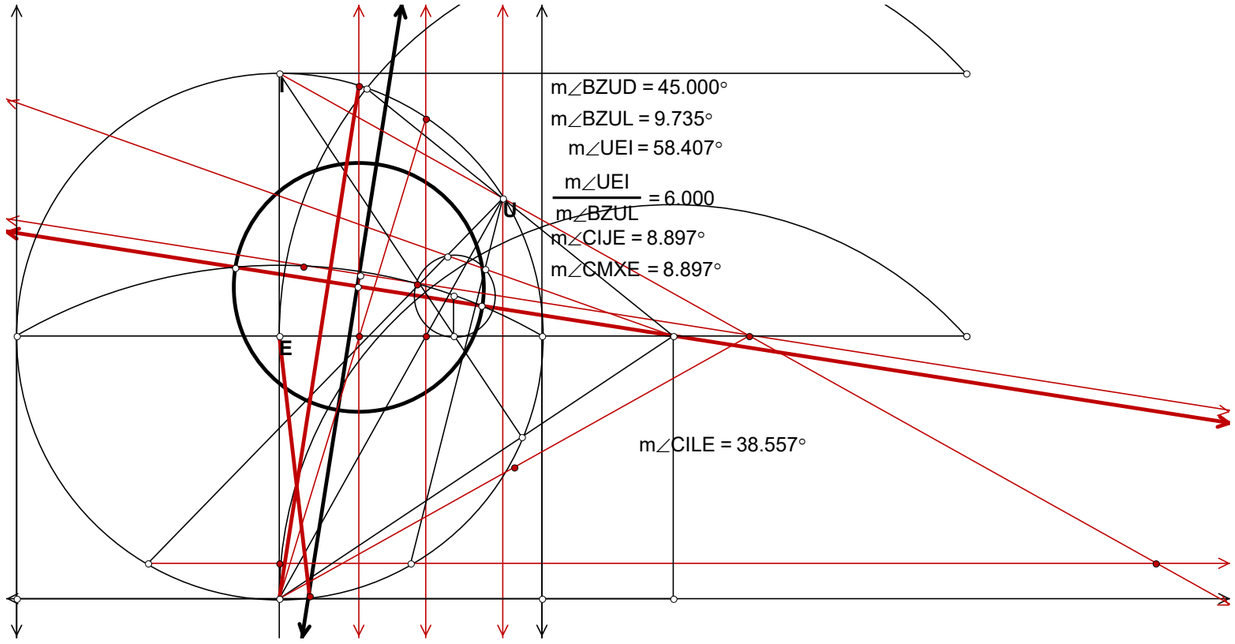
Descriptions.
Definitions.



Unit.
Given.

Parcing project for 092402

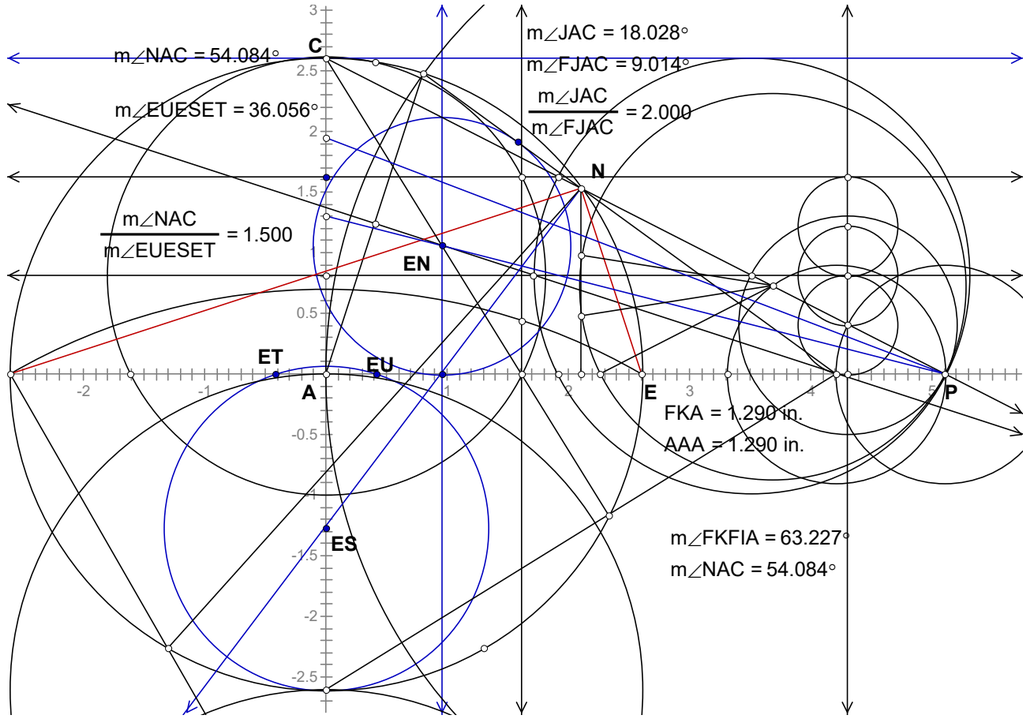
Descriptions.
Definitions.



Unit.
Given.

Parcing project for 093002

Descriptions.
Definitions.



Unit.
Given.

Parcing project 100402

Descriptions.
Definitions.

$$m\angle DBE = 31.487°$$

$$\frac{m\angle DBE}{m\angle CMBNB} = 1.333$$

$$m\angle DBE = 31.487°$$

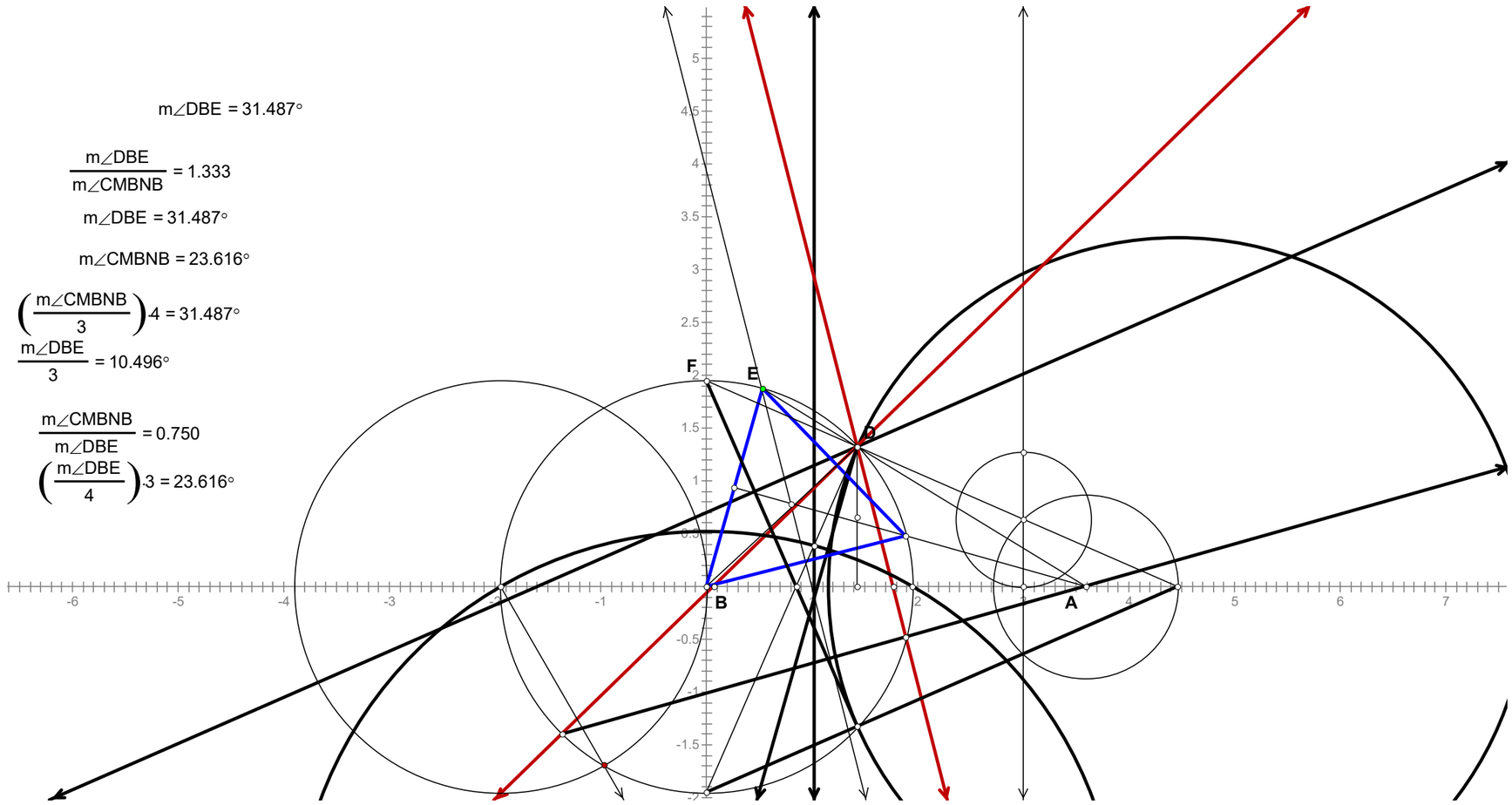
$$m\angle CMBNB = 23.616°$$

$$\left(\frac{m\angle CMBNB}{3}\right)\cdot 4 = 31.487°$$

$$\frac{m\angle DBE}{3} = 10.496°$$

$$\frac{m\angle CMBNB}{m\angle DBE} = 0.750$$

$$\left(\frac{m\angle DBE}{4}\right)\cdot 3 = 23.616°$$

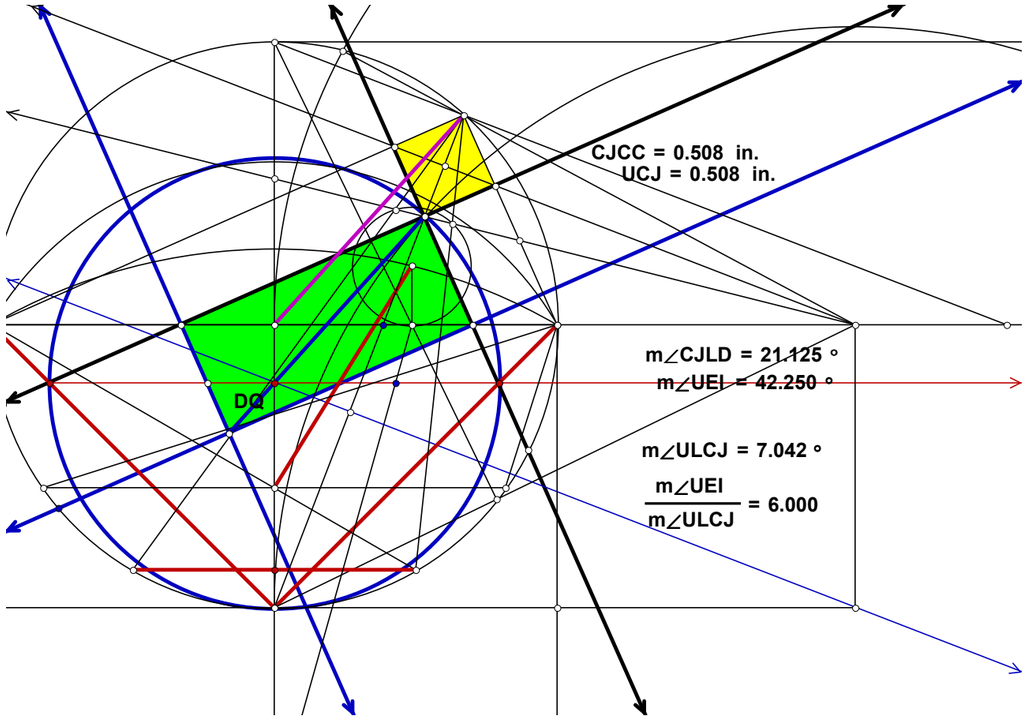


Unit.
Given.

Descriptions.
Definitions.

Parcing project 101402

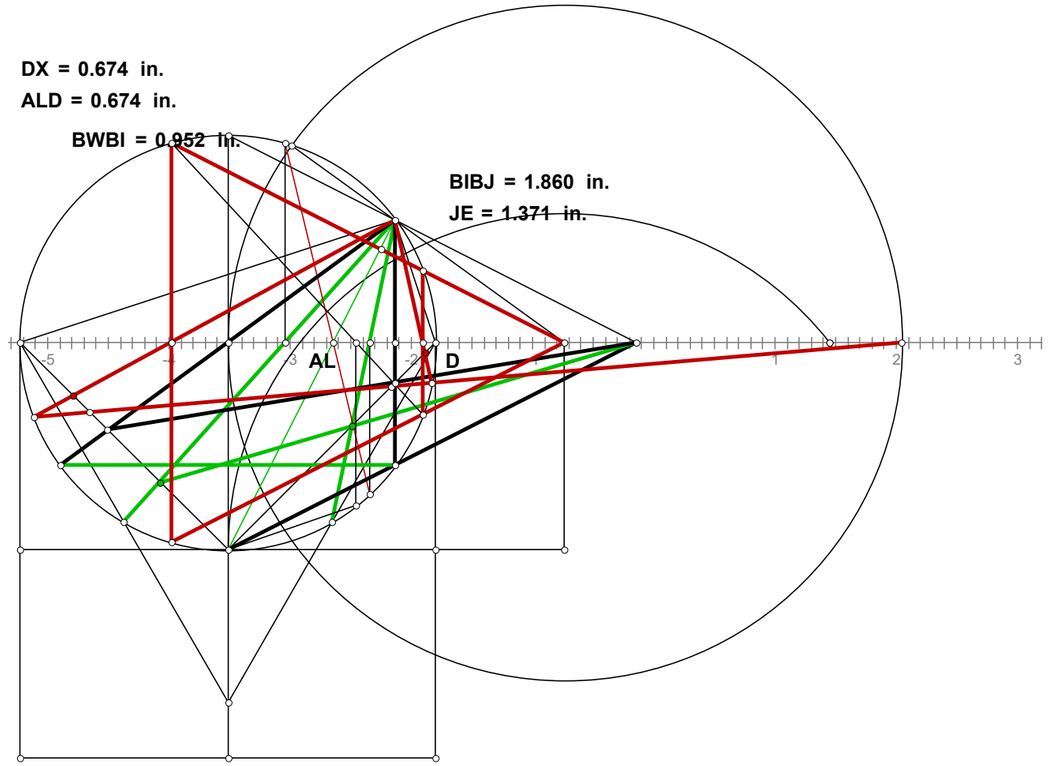
Square, rectangle and complements.



Unit.
Given.

Descriptions.
Definitions.

Parcing project for 111402



021603

Descriptions.

Unit.

$AD := 1$

Given.

$N := 1.5$

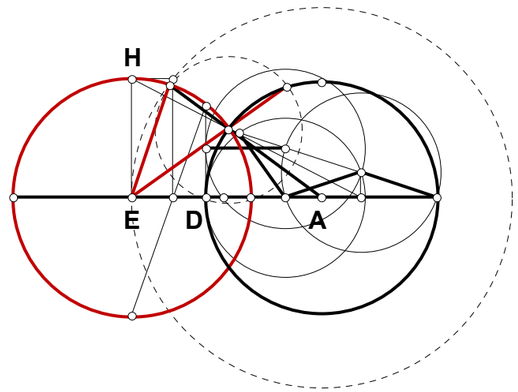
$$AB := \frac{AD}{2}$$

$$BD := AD - AB \qquad BC := \frac{BD}{N}$$

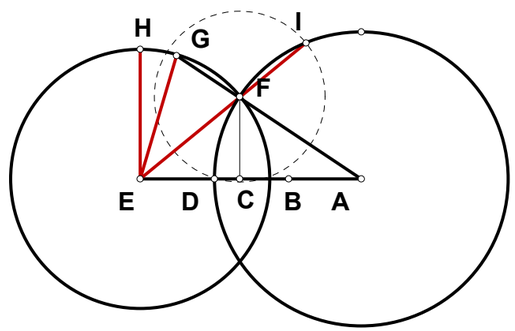
$$CD := BD - BC \qquad AC := AB + BC$$

$$AF := AD \qquad CF := \sqrt{AF^2 - AC^2}$$

$$FI := \sqrt{CD^2 + CF^2}$$



From AD project a trisection to EH. Make AD the unit.



$$DI := 2 \cdot CF \qquad AI := AD$$

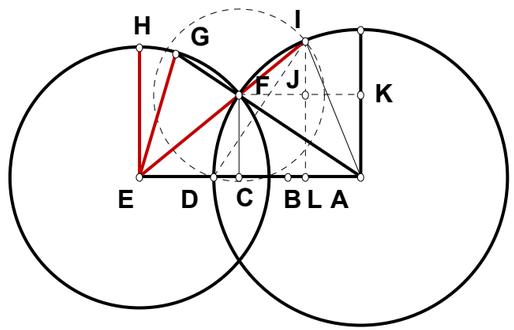
$$AL := \frac{\left(2AD^2 - DI^2\right)}{2 \cdot AD}$$

$$FJ := AC - AL \qquad IL := \sqrt{AI^2 - AL^2}$$

$$IJ := IL - CF \qquad EL := \frac{FJ \cdot IL}{IJ}$$

$$AE := AL + EL \qquad EI := \frac{FI \cdot IL}{IJ}$$

$$EF := EI - FI$$



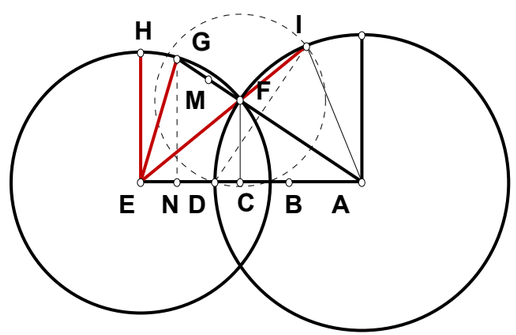
$$FG := \frac{EF^2 + AF^2 - AE^2}{-AF}$$

$$AG := AF + FG$$

$$AN := \frac{AC \cdot AG}{AF}$$

$$EN := AE - AN \qquad FM := \frac{FG}{2}$$

$$FM - EN = 0$$



Definitions. **In the following definitions, one should remember, N is not something which is a part of the figure, it is an assertion you make of the figure.**

$$AB - \frac{1}{2} = 0 \qquad BD - (N-1) = 0 \qquad BC - \frac{N-1}{N} = 0 \qquad CD - \frac{(N-1)^2}{N} = 0$$

$$AC - \frac{3 \cdot N - 2}{2 \cdot N} = 0 \qquad AF - 1 = 0 \qquad CF - \frac{\sqrt{(2-N) \cdot (5 \cdot N - 2)}}{2 \cdot N} = 0$$

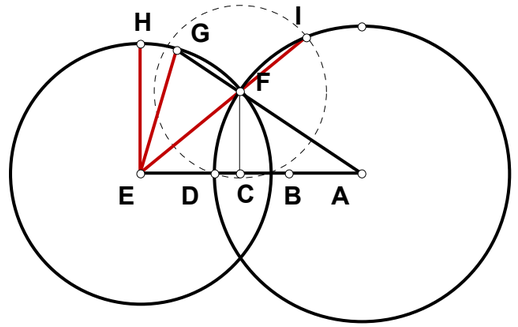
$$FI - \frac{\sqrt{4 \cdot N^3 - 16 \cdot N^2 + 19 \cdot N - 4}}{2 \cdot \sqrt{N}} = 0 \qquad DI - \frac{\sqrt{(2-N) \cdot (5 \cdot N - 2)}}{N} = 0$$

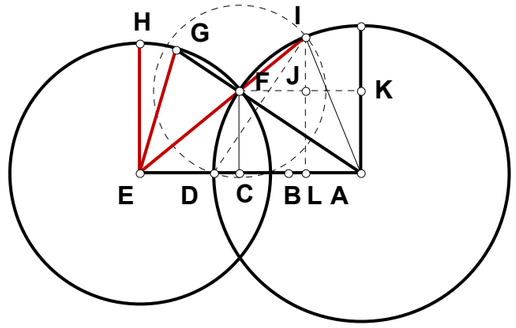
$$AI - 1 = 0 \qquad AL - \frac{7 \cdot N^2 - 12 \cdot N + 4}{2 \cdot N^2} = 0 \qquad FJ - \frac{(2 \cdot N - 1) \cdot (2 - N)}{N^2} = 0$$

$$IL - \frac{\sqrt{(5 \cdot N - 2) \cdot (2 - N)} \cdot (3 \cdot N - 2)}{2 \cdot N^2} = 0 \qquad IJ - \frac{\sqrt{12 \cdot N - 5 \cdot N^2 - 4} \cdot (N-1)}{N^2} = 0$$

$$EL - \frac{(2-N) \cdot (2 \cdot N - 1) \cdot (3 \cdot N - 2) \cdot \sqrt{(2-N) \cdot (5 \cdot N - 2)}}{2 \cdot N^2 \cdot (N-1) \cdot \sqrt{12 \cdot N - 5 \cdot N^2 - 4}} = 0 \qquad AE - \frac{N}{2 \cdot (N-1)} = 0$$

$$EI - \frac{(3 \cdot N - 2) \cdot \sqrt{-(N-2) \cdot (5 \cdot N - 2)} \cdot \sqrt{4 \cdot N^3 - 16 \cdot N^2 + 19 \cdot N - 4}}{4 \cdot \sqrt{N} \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right) \cdot \sqrt{12 \cdot N - 5 \cdot N^2 - 4}} = 0 \qquad EF - \frac{\sqrt{N} \cdot \sqrt{4 \cdot N^3 - 16 \cdot N^2 + 19 \cdot N - 4}}{4 \cdot \left(\sqrt{N} - 1\right) \cdot \left(\sqrt{N} + 1\right)} = 0$$



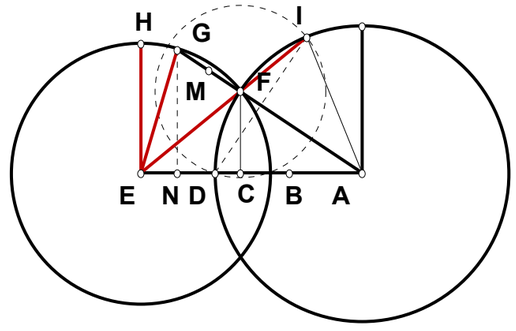


$$FG - \frac{\left(16\cdot N^3 - 4\cdot N^4 - 31\cdot N^2 + 36\cdot N - 16\right)}{16\cdot (N-1)^2} = 0$$

$$AG - \frac{N\cdot (1-2\cdot N)\cdot \left(2\cdot N^2 - 7\cdot N + 4\right)}{16\cdot (N-1)^2} = 0$$

$$AN - \frac{(1-2\cdot N)\cdot (3\cdot N-2)\cdot \left(2\cdot N^2 - 7\cdot N + 4\right)}{32\cdot (N-1)^2} = 0$$

$$EN - \frac{12\cdot N^4 - 56\cdot N^3 + 93\cdot N^2 - 58\cdot N + 8}{32\cdot (N-1)^2} = 0$$



$$FM - \frac{16\cdot N^3 - 4\cdot N^4 - 31\cdot N^2 + 36\cdot N - 16}{32\cdot (N-1)^2} = 0$$

$$FM - EN = 0$$

$$\frac{(2\cdot N-3)\cdot (1-2\cdot N)\cdot \left(2\cdot N^2 - 5\cdot N + 4\right)}{16\cdot (N-1)^2} = 0 \qquad 16\cdot (N-1)^2 = 4 \qquad (2\cdot N-3)\cdot (1-2\cdot N)\cdot \left(2\cdot N^2 - 5\cdot N + 4\right) = 0$$

$$36\cdot N^3 - 8\cdot N^4 - 62\cdot N^2 + 47\cdot N - 12$$

$$36\cdot N^3 - 8\cdot N^4 - 62\cdot N^2 + 47\cdot N - 12 = 0$$

Solve for N.

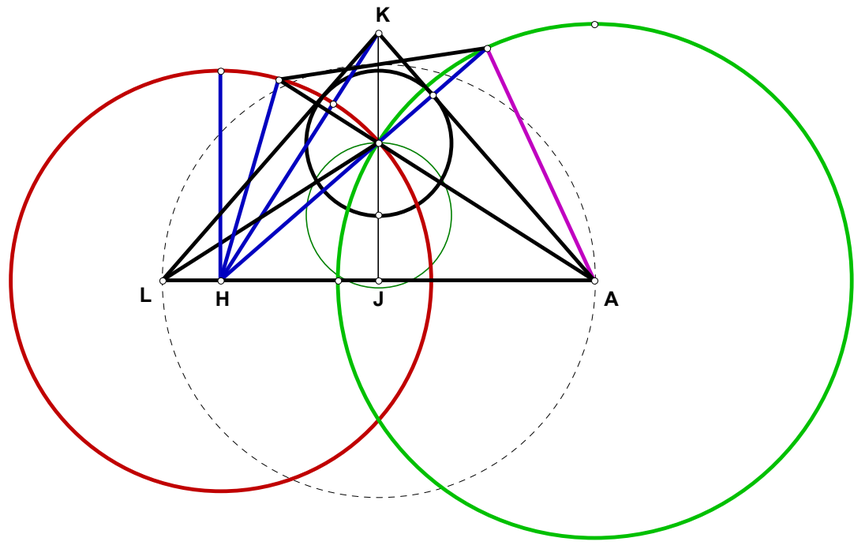
$$\begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ \frac{5}{4} + \frac{\sqrt{7}\cdot i}{4} \\ \frac{5}{4} - \frac{\sqrt{7}\cdot i}{4} \end{pmatrix}$$

Two of these solutions are not even part of the grammar. If one finds anyone who defends them, as them, show me how complete induction and deduction of a unit cannot produce anything more than a recursion of that unit.

Unit.
Given.

Parcing project for 021903

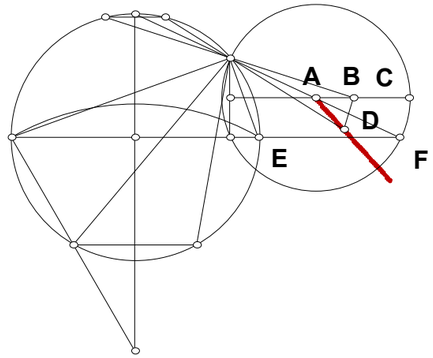
Descriptions.
Definitions.



022803

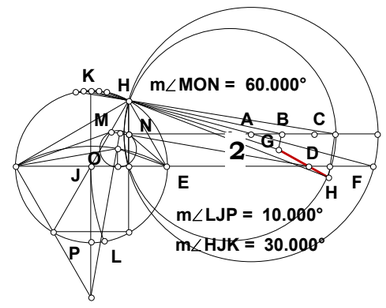
If D were between EF, then it would be the point sought for angle trisection. When it is moved between A and C the locus AD is formed. This locus is not straight, but it is fairly straight.

Fair Pencil Construction

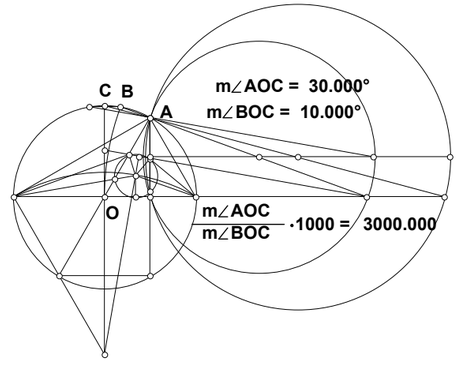


Even if a large segment is used from two points on AD, say GH for an intercept, this drawing program claims that I have attained a trisection for any angle.

If a very small segment is taken, as below, tolerance is beyond the drawing program. What does it look like using Algebra? Trisection to within millionths, in some circumstances, may be tolerable.



As the different points of intersection is not actually viewable on the finer figure, the rougher figure will be used for drawings, but the equations will refer to the finer.



Descriptions.

Unit.

$AC := 1$

Given.

$N_1 := 2$

$N_2 := 4$

$AO := \frac{AC}{2}$

$AB := \frac{AC}{N_2}$ $BC := AC - AB$

$BE := \sqrt{AB \cdot BC}$ $DE := \frac{BE}{2}$

$BO := AO - AB$ $GO := AO$

$DL := \frac{BO \cdot DE}{GO - BE}$ $EL := \sqrt{DL^2 + DE^2}$

$LN := EL$ $LM := \frac{LN}{N_1}$ $DM := DL + LM$ $EM := \sqrt{DE^2 + DM^2}$ $EP := \frac{EM \cdot BE}{DE}$

$BP := \frac{DM \cdot BE}{DE}$ $PO := BP + BO$

$EH := \frac{AO^2 + EP^2 - PO^2}{-EP}$

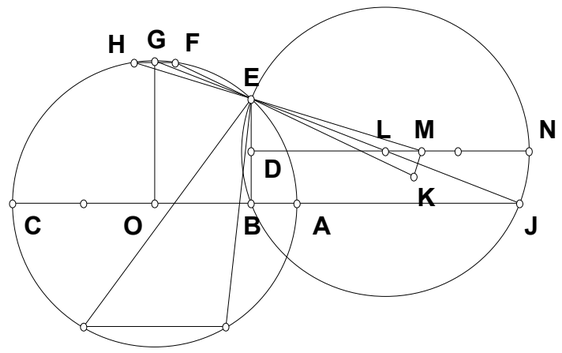
$HP := EP + EH$ $PR := \frac{BP \cdot HP}{EP}$

$OR := PR - PO$ $BQ := BO - OR$

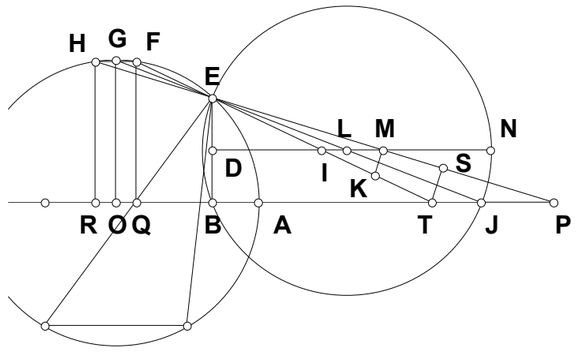
$HR := \frac{BE \cdot HP}{EP}$ $BT := \frac{BQ \cdot BE}{HR - BE}$

$ET := \sqrt{BE^2 + BT^2}$ $PT := BP - BT$ $PS := \frac{PT^2 + EP^2 - ET^2}{2 \cdot EP}$ $ES := EP - PS$ $ST := \sqrt{ET^2 - ES^2}$

$KM := \frac{ST \cdot EM}{ES}$ $EI := \frac{ET}{2}$ $EK := \frac{ET \cdot EM}{ES}$ $OT := BT + BO$



4



5

$$IW := EI DI := \sqrt{EI^2 - DE^2} \qquad DW := DI + IW$$

$$EW := \sqrt{DE^2 + DW^2} \qquad EZ := \frac{EW \cdot BE}{DE} \qquad BZ := \frac{DW \cdot BE}{DE}$$

$$OZ := BZ + BO \qquad TW := \sqrt{ET^2 - EW^2}$$

$$EX := \frac{AO^2 + EZ^2 - OZ^2}{-EZ} \qquad XZ := EZ + EX$$

$$Za := \frac{BZ \cdot XZ}{EZ} \qquad Oa := Za - OZ \qquad Bb := BO - Oa$$

$$Xa := \frac{BE \cdot XZ}{EZ} \qquad BU := \frac{Bb \cdot BE}{Xa - BE} \qquad EU := \sqrt{BE^2 +]UZ\hat{}} := BZ - BU \qquad Zc := \frac{UZ^2 + EZ^2 - EU^2}{2 \cdot EZ}$$

$$Uc := \sqrt{UZ^2 - Zc^2} \qquad VW := \frac{Uc \cdot EW}{EZ - Zc} \qquad OU := BU + BO \qquad IM := DM - DI \quad IK := EK - EI$$

$$Ig := \frac{IK^2 + IM^2 - KM^2}{2 \cdot IM} \qquad EV := \frac{EU \cdot EW}{EZ - Zc} \qquad TU := OT - OU \quad UV := EV - EU$$

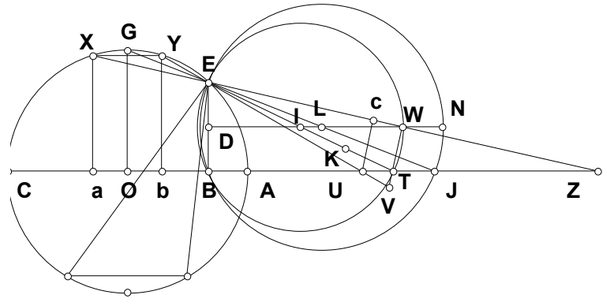
$$TV := VW - TW \qquad Uh := \frac{UV^2 + TU^2 - TV^2}{2 \cdot TU} \qquad Kg := \sqrt{IK^2 - Ig^2}$$

$$Ke := DE - Kg \qquad Vh := \sqrt{UV^2 - Uh^2} \qquad Dg := DI + Ig \qquad Bh := BU + Uh$$

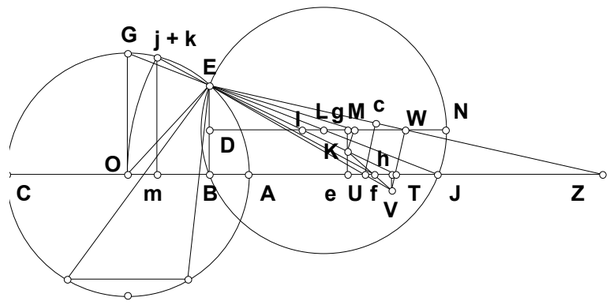
$$eh := Bh - Dg \qquad ef := \frac{eh \cdot Ke}{(Ke + Vh)} \qquad Bf := Dg + ef \qquad Of := Bf + BO$$

$$Om := \frac{AO^2}{2 \cdot Of} \quad Ej := \frac{AO^2}{Of} \qquad Om - \frac{Ej}{2} = 0 \qquad Ef := \sqrt{BE^2 + Bf^2} \qquad Ek := \frac{AO^2 + Ef^2 - Of^2}{-Ef}$$

$$Om - \frac{Ek}{2} = -2.639125 \times 10^{-12} \qquad \frac{Ek}{Ej} = 1$$



6



7

And so the Trisection is accurate to within a few decimal places. 30 degree angle shown.

Unit.

Given.

$\Delta := 22$

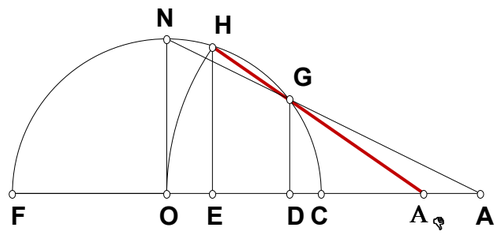
$\delta := 0 .. \Delta$

$N := 2$

030503

Descriptions.

The Gravitating Answer.



How many itterations to go beyond 15 decimal places precision in trisection? The itteration is from AO where GH determines a new A.

$CF := 1 \quad CO := \frac{CF}{2}$

$CD := \frac{CO}{N} \quad DO := CO - CD$

$DF := DO + C \, DG := \sqrt{CD \cdot DF}$

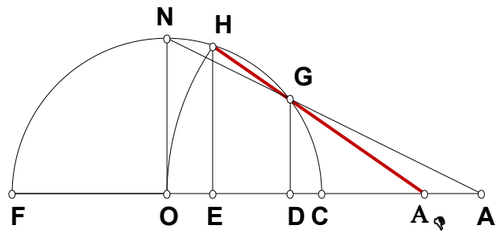
$AD_0 := \frac{DO \cdot DG}{CO - DG} \quad AO_0 := AD_0 + DO \quad EO_0 := \frac{CO^2}{2 \cdot (AD_0 + DO)} \quad DE_0 := DO - EO_0$

$EH_0 := \sqrt{(CO + EO_0) \cdot (CO - EO_0)}$

$$\begin{pmatrix} AD_{\delta+1} \\ AO_{\delta+1} \\ EO_{\delta+1} \\ DE_{\delta+1} \\ EF_{\delta+1} \\ EH_{\delta+1} \end{pmatrix} := \begin{bmatrix} \frac{DE_\delta \cdot DG}{EH_\delta - DG} \\ \frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \\ \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ DO - \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ CO + \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \\ \sqrt{\left[CO - \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right] \cdot \left[CO + \frac{CO^2}{2 \cdot \left(\frac{DE_\delta \cdot DG}{EH_\delta - DG} + DO \right)} \right]} \end{bmatrix}$$

$$AG := \sqrt{DG^2 + (AD_\Delta)^2}$$

$$GH := \frac{CO^2 + AG^2 - (AO_\Delta)^2}{-AG}$$



$$EO_\Delta - \frac{GH}{2} = 0$$

The displayed precision is for 15 decimal places. Trisection is beyond that. Since the physical world is quantitized, physical trisection is possible.

$$EO_\delta - \frac{GH}{2} =$$

	$\delta =$
-0.019836790725685	0
-0.004498504834066	1
-0.001019750164038	2
-0.000231158851096	3
-0.000052399460482	4
-0.000011877993414	5
-0.000002692522516	6
-0.000000610345304	7
-0.000000138354048	8
-0.000000031362317	9
-0.000000007109260	10
-0.000000001611539	11
-0.000000000365306	12
...	...

One can see how rapidly each recursion increases precision. And so, for any required precision, one can trisect an angle grearter than that, relatively rapidly--especially if one combine yesterdays plate with today's.

030803

Descriptions.

$$DO := \frac{DF}{2} \qquad Dx := \frac{DO}{N}$$

$$Ox := DO - Dx \qquad Fx := DF - Dx$$

$$Gx := \sqrt{Dx \cdot Fx} \qquad AO := \frac{Ox \cdot DO}{DO - Gx}$$

$$HO := DO \qquad LO := \frac{HO}{2}$$

$$Ex := \frac{Ox \cdot Gx}{Gx + DO} \qquad Ob := \frac{LO^2}{AO}$$

$$Lb := \sqrt{LO^2 - Ob^2} \qquad ab := \frac{Ex \cdot Lb}{Gx}$$

$$Aa := AO + ab - Ob$$

$$Ax := AO - Ox \qquad AE := Ax + Ex$$

$$Kc := \frac{Lb \cdot AE}{Aa} \qquad Cc := \frac{AO \cdot Kc}{DO}$$

$$Ec := \frac{ab \cdot AE}{Aa} \qquad CE := Cc + Ec \qquad AC := AE - CE$$

$$AB := \frac{AC}{2}$$

Unit.

$DF := 1$

Given.

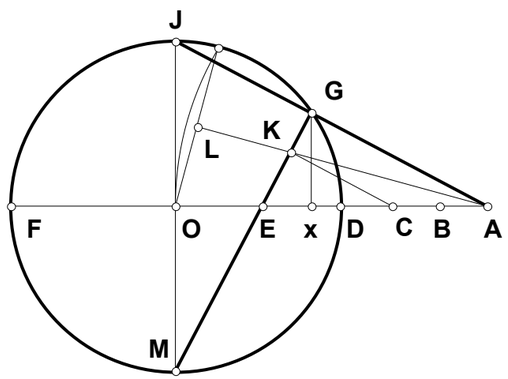
$N := 2$

$\Delta := 22$

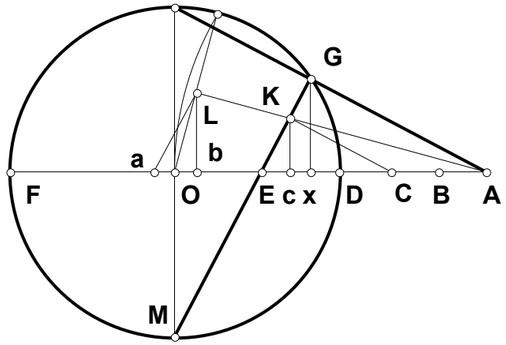
$\delta := 0 .. \Delta$

Nothing Saved

Is anything saved by starting from a much more precise point for itteration demonstrated in 0305?



J

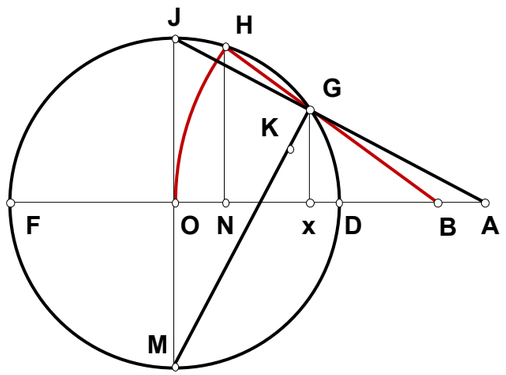


$$BO_0 := AO - AB$$

$$NO_0 := \frac{DO^2}{2\,BO_0}$$

$$Nx_0 := Ox - NO_0$$

$$HN_0 := \sqrt{(DO + NO_0)\cdot(DO - NO_0)}$$



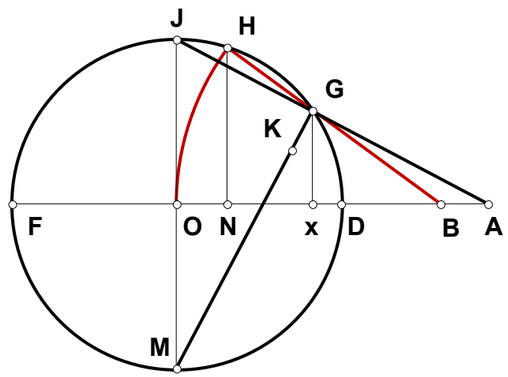
$$\begin{pmatrix} BO_{\delta+1} \\ NO_{\delta+1} \\ Nx_{\delta+1} \\ HN_{\delta+1} \end{pmatrix} := \begin{bmatrix} \frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta \\ \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ Ox - \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)} \\ \sqrt{\left[DO + \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]\cdot\left[DO - \frac{DO^2}{2\left(\frac{Nx_\delta \cdot HN_\delta}{HN_\delta - Gx} + NO_\delta\right)}\right]} \end{bmatrix}$$

$$Bx_{\Delta} := BO_{\Delta} - Ox$$

$$BG := \sqrt{Gx^2 + (Bx_{\Delta})^2}$$

$$GH := \frac{DO^2 + BG^2 - (BO_{\Delta})^2}{-BG}$$

$$NO_{\Delta} - \frac{GH}{2} = 0$$



$NO_{\delta} - \frac{GH}{2} =$

-0.000733072475044
-0.000166174134206
-0.000037668619212
-0.000008538782769
-0.000001935584914
-0.000000438761479
-0.000000099459153
-0.000000022545559
-0.000000005110663
-0.000000001158493
-0.000000000262609
-0.000000000059529
-0.000000000013494
...

$\delta =$

0
1
2
3
4
5
6
7
8
9
10
11
12
...

Although one starts off in a more precise spot, not much in the way of steps for 15 decimal place precision is saved. The steps are a waste of time.

031503

Descriptions.

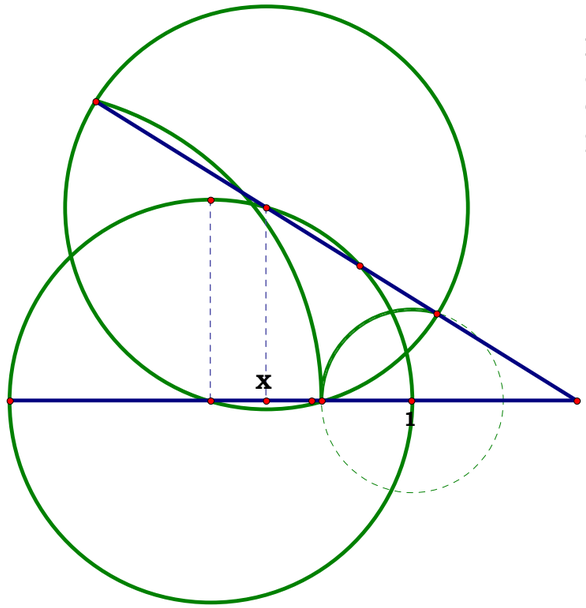
This figure may appear to many to be very counter intuitive, so I will show the construction step by step. I think way back then is the only other time I drew this. I have lost count on how many ways one can actually draw a figure demonstrating trisection. Why the so called intellectuals still claim it is impossible is way beyond me and my talent.

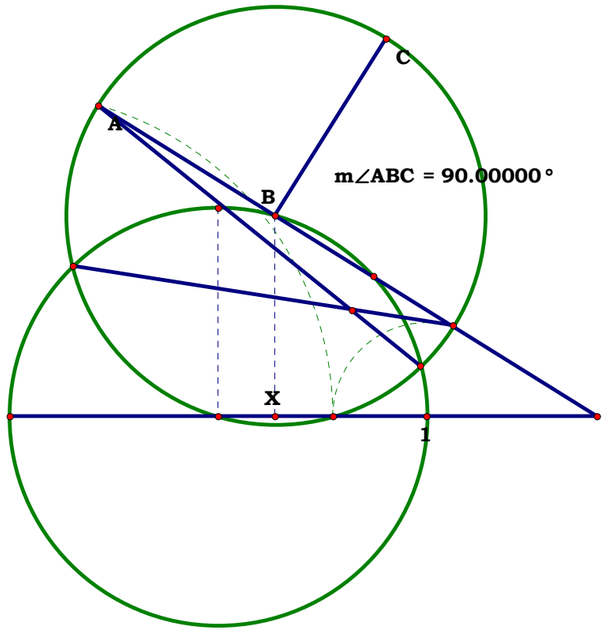
X will range between the center and first half of that segment giving one 30 working degrees.

From that point I will draw the second circle to the center of the first which will produce a second point of intersection with the diameter. From that point I will draw two more circles to produce the line that terminates on the base like. I immediately have my square root point of the figure and I have the point

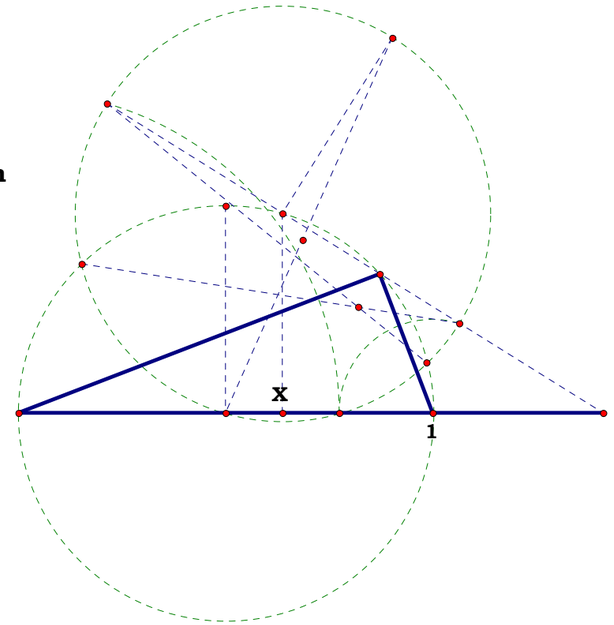
I will construct two lines with opposing endpoints which will produce a point which is suspended in the air it seems.

Trisection and Square Roots

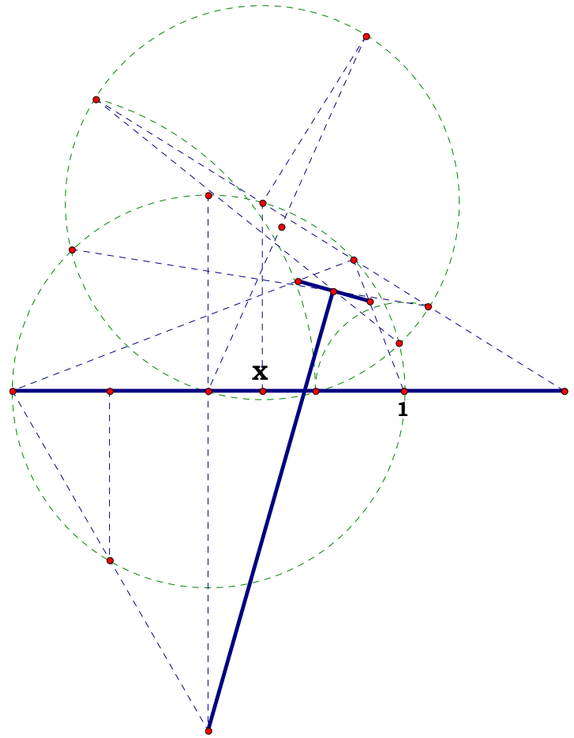




I will use the second point of intersetion of top line to construct a right triangle. One might say a tent for my hanging point.

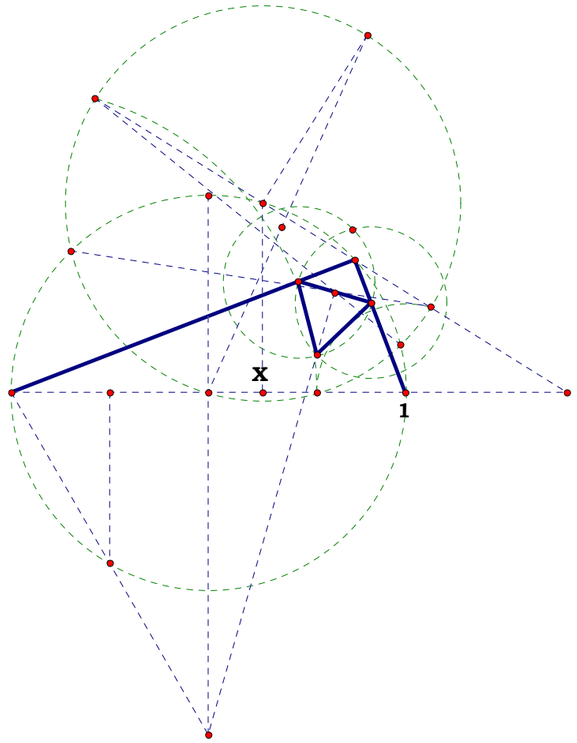


From the point we draw a 60 degree arc from I am going to make a segment with our floating point and form a perpendicular limited by our tent so that we have a very skinny long T.



Descriptions.

With that skinny long T, we are going to make an equilateral triangle.

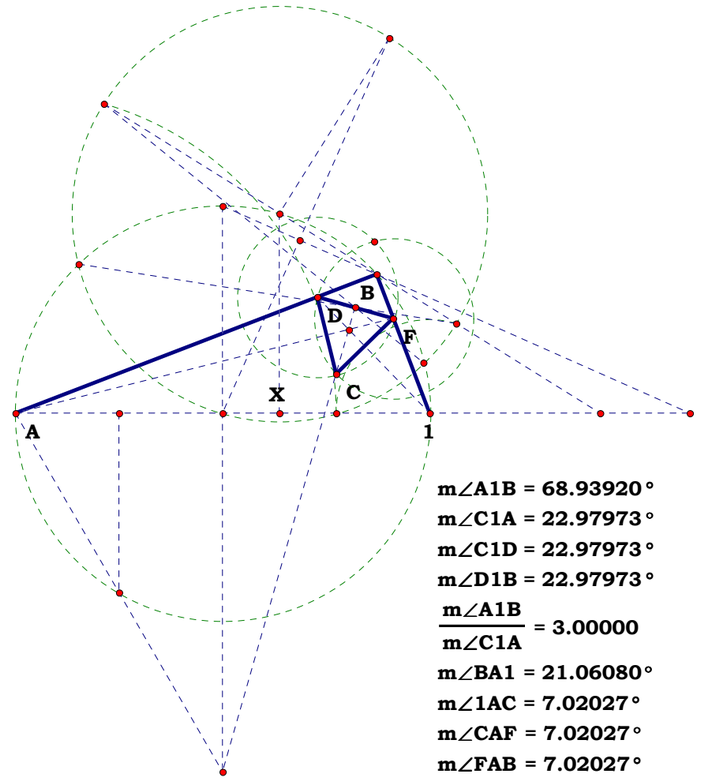


That equilateral triangle is the triangle in a figure that demonstrates angle trisection.

And as you have seen for yourself, it just fell out of the sky.

I will not make another plate with all the dressing. One may say, this is just lunch to go.

And so, angle trisection is actually a well documented part of geometry, all one has to do is find the write ups at the end of their instrument of choice.



$m\angle A1B = 68.93920°$
$m\angle C1A = 22.97973°$
$m\angle C1D = 22.97973°$
$m\angle D1B = 22.97973°$
$\frac{m\angle A1B}{m\angle C1A} = 3.00000$
$m\angle BA1 = 21.06080°$
$m\angle 1AC = 7.02027°$
$m\angle CAF = 7.02027°$
$m\angle FAB = 7.02027°$
$\frac{m\angle BA1}{m\angle 1AC} = 3.00000$

031503

Descriptions.

Unit.

$BO := 1$

Given.

$Y := 20$

$X := 11$

$AB := 2 \cdot BO$ $NO := \frac{X}{2 \cdot Y}$ $EO := BO$ $AO := BO$

$FO := \frac{EO}{2}$ $GO := \frac{NO}{2}$ $FG := \sqrt{FO^2 - GO^2}$

$CO := \frac{FO^2}{GO}$ $CO = 1.818182$ $AC := AO + CO$

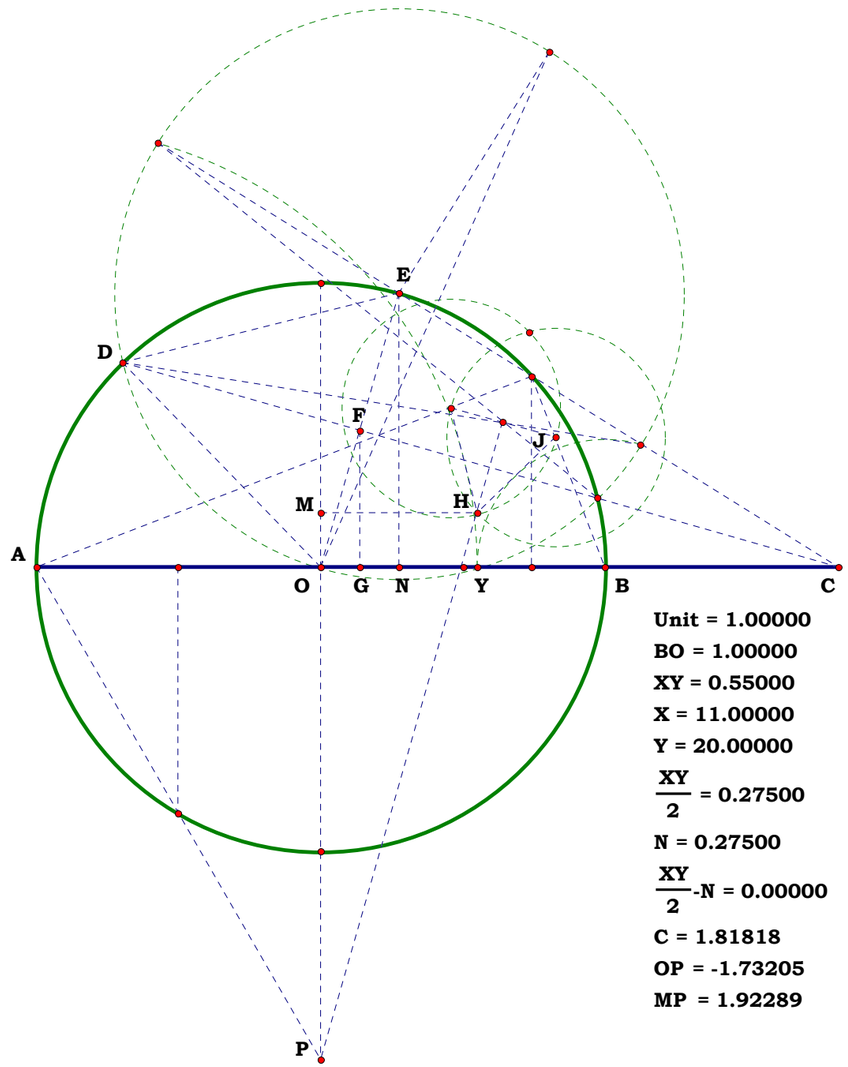
$OY := 2 \cdot NO$ $HP := AB$ $OP := \sqrt{AB^2 - BO^2}$

$OP = 1.732051$ $MP := \sqrt{AB^2 - OY^2}$ $MP = 1.922888$

$MO := MP - OP$ $MO = 0.190838$ $HJ := 2 \cdot MO$

$CY := CO - OY$ $\frac{AC}{CY} = 2.222222$ $AY := AO + OY$

$\frac{AC}{OY} = 5.123967$ $\frac{AB}{AY} = 1.290323$ etc.



Unit = 1.00000
BO = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000
$\frac{XY}{2} = 0.27500$
N = 0.27500
$\frac{XY}{2}$-N = 0.00000
C = 1.81818
OP = -1.73205
MP = 1.92289

Definitions.

$AB - 2 = 0$ $NO - \frac{X}{2 \cdot Y} = 0$ $EO - 1 = 0$ $AO - 1 = 0$ $FO - \frac{1}{2} = 0$

$GO - \frac{X}{4 \cdot Y} = 0$ $FG - \frac{\sqrt{4 \cdot Y^2 - X^2}}{4 \cdot Y} = 0$ $CO - \frac{Y}{X} = 0$ $AC - \frac{X + Y}{X} = 0$

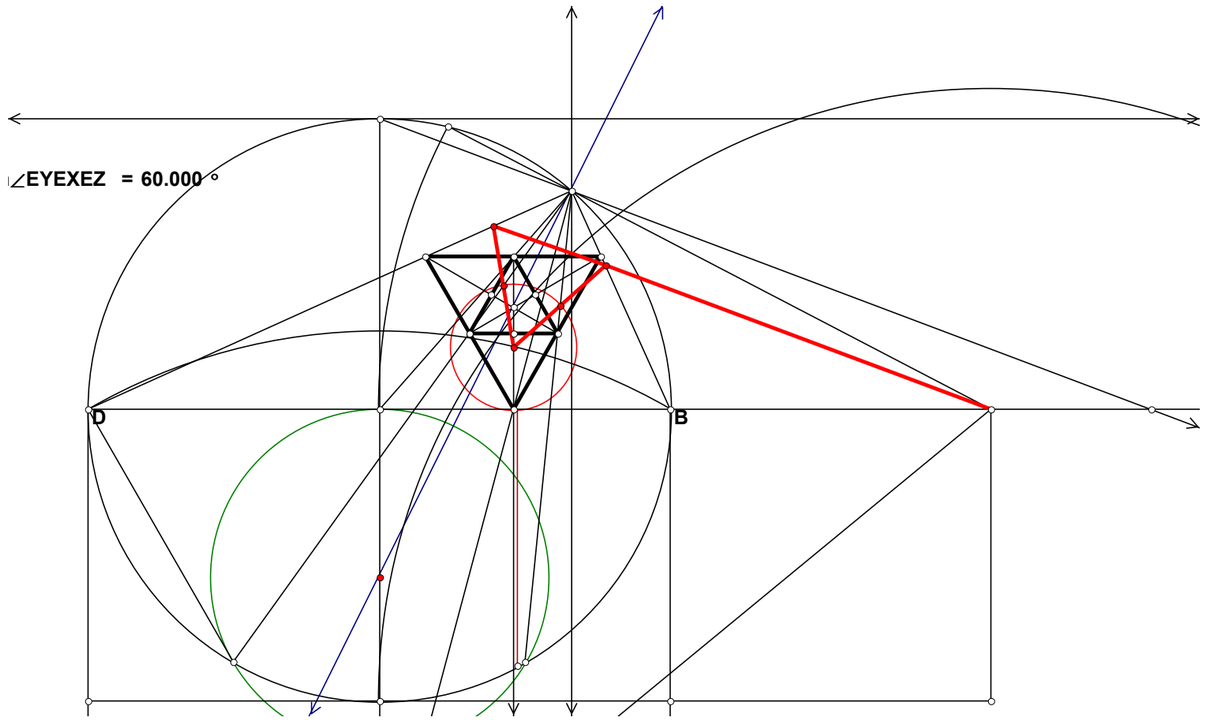
$OY - \frac{X}{Y} = 0$ $HP - 2 = 0$ $OP - \sqrt{3} = 0$ $MP - \frac{\sqrt{4 \cdot Y^2 - X^2}}{Y} = 0$ $MO - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0$ $HJ - 2 \cdot \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y} = 0$

$CY - \frac{Y^2 - X^2}{X \cdot Y} = 0$ $AY - \frac{X + Y}{Y} = 0$ $\frac{AC}{CY} - \frac{Y}{(Y - X)} = 0$ $\frac{AC}{OY} - \frac{Y \cdot (X + Y)}{X^2} = 0$ $\frac{AB}{AY} - \frac{2 \cdot Y}{X + Y} = 0$

Unit.
Given.

Descriptions.
Definitions.

Parcing project for 032003



Unit.

$AB := 1$

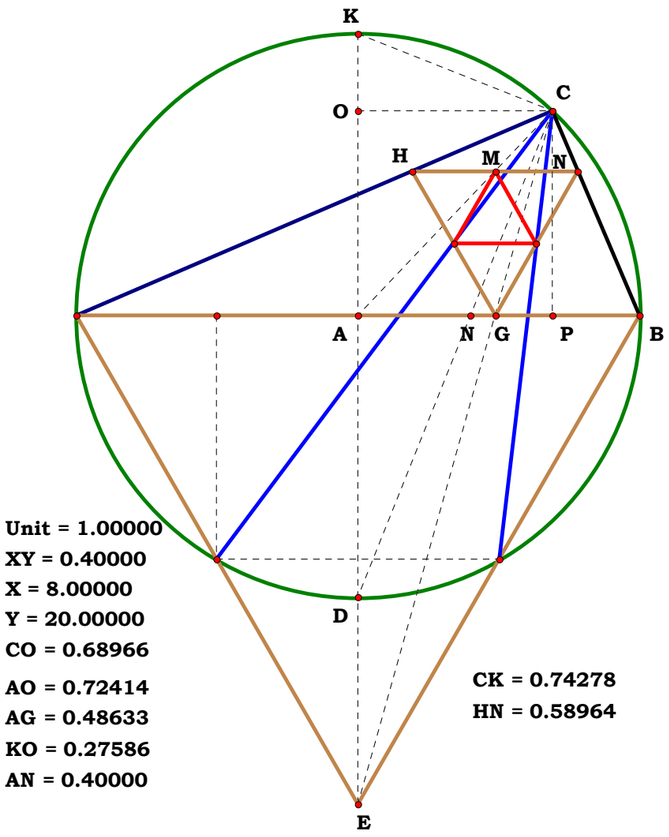
Given.

$Y := 20$

$X := 8$

032303

Figure in a Figure.



Descriptions.

$AN := \frac{X}{Y}$ $AD := AB$ $DN := \sqrt{AD^2 + AN^2}$ $CK := \frac{AN \cdot 2 \cdot AD}{DN}$

$KO := \frac{(AN \cdot CK)}{DN}$ $CO := \frac{AD \cdot KO}{AN}$ $AP := CO$ $AO := AB - KO$

$AK := \sqrt{3}$ $HN := \frac{2 \cdot AB \cdot AO}{AK + AO}$ $HN = 0.589644$

$AG := \frac{CO \cdot AK}{AK + AO}$ $AG = 0.48633$ etc.

Definitions.

$AN - \frac{X}{Y} = 0$ $AD - 1 = 0$ $DN - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$ $CK - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$

$KO - \frac{2 \cdot X^2}{X^2 + Y^2} = 0$ $CO - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $AP - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$

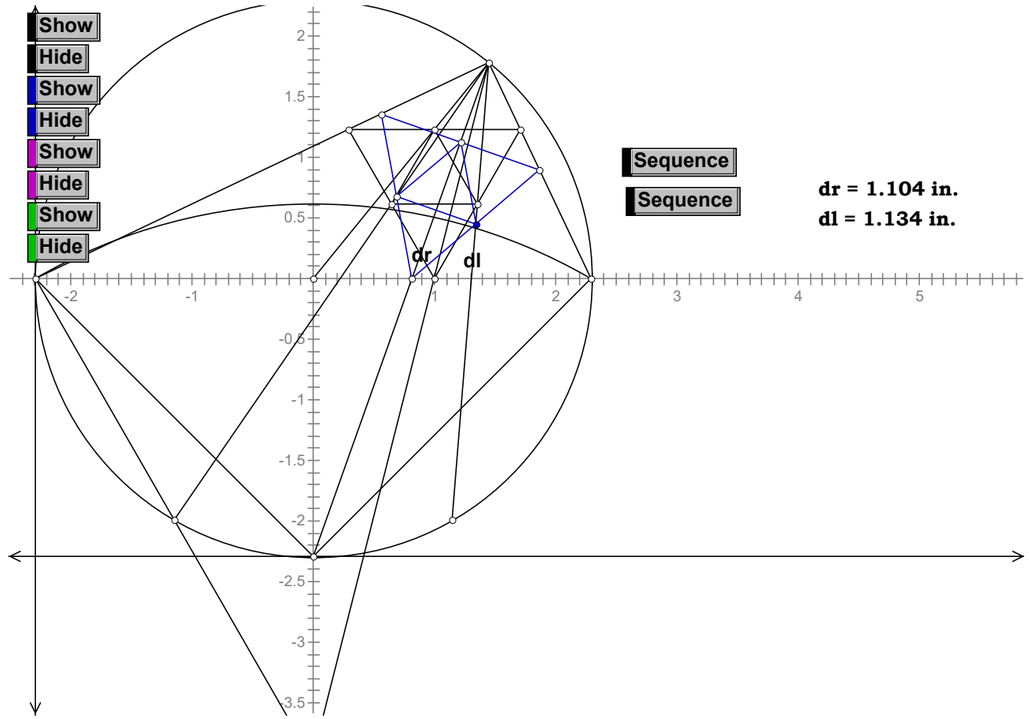
$AO - \frac{Y^2 - X^2}{X^2 + Y^2} = 0$ $AK - \sqrt{3} = 0$ $HN - \frac{2 \cdot (Y^2 - X^2)}{(X^2 + Y^2) \cdot (\sqrt{3} - 1) + 2 \cdot Y^2} = 0$

$AG - \frac{2 \cdot \sqrt{3} \cdot X \cdot Y}{(X^2 + Y^2) \cdot (\sqrt{3} - 1) + 2 \cdot Y^2} = 0$ etc.

Unit.
Given.

Descriptions.
Definitions.

Parcing project for 032403

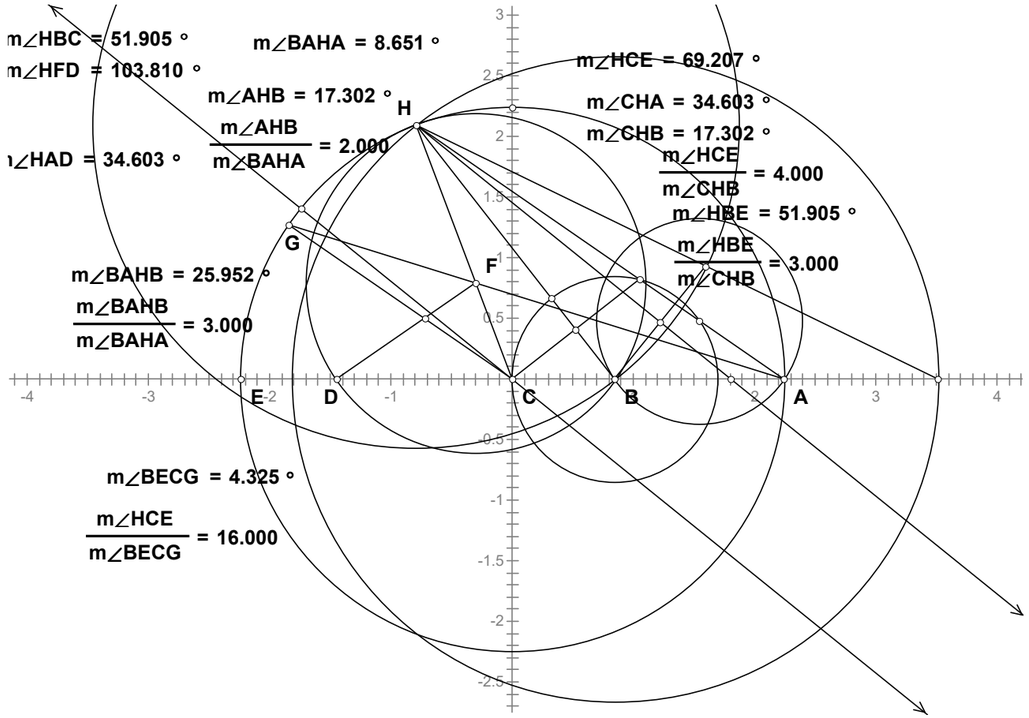


dr = 1.104 in.
dl = 1.134 in.

Unit.
Given.

Descriptions.
Definitions.

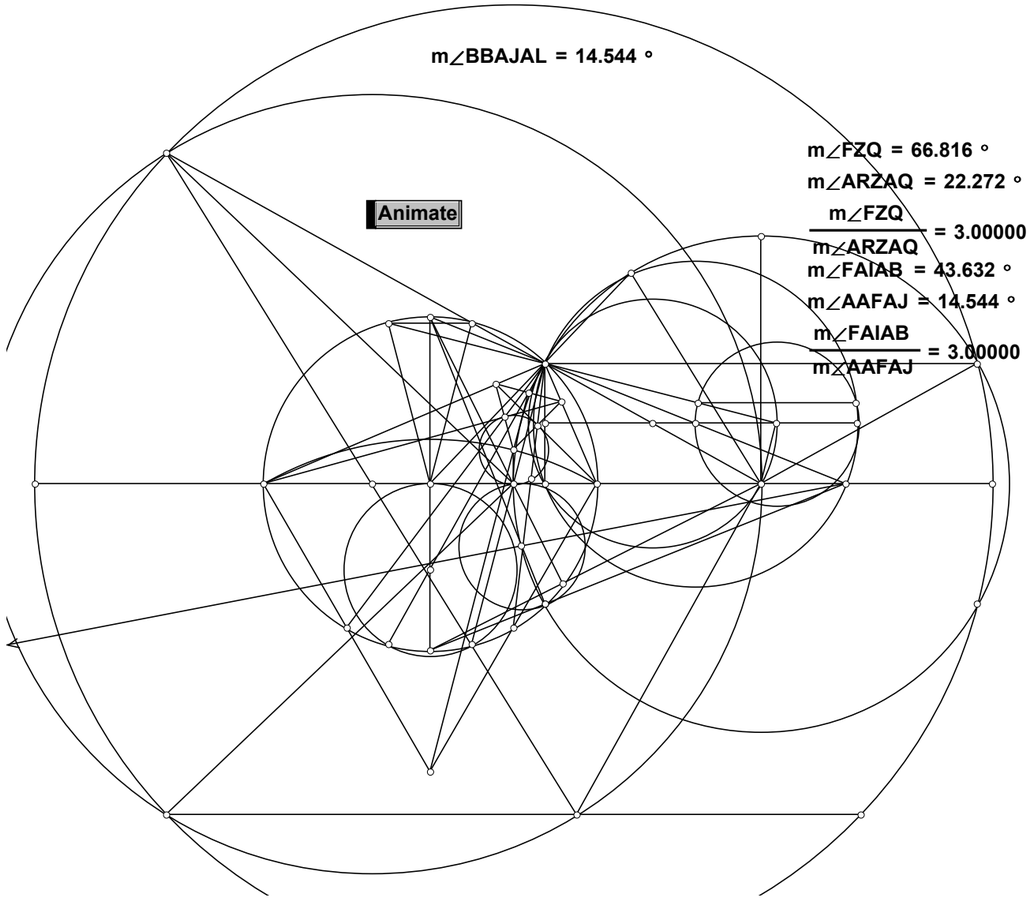
Parcing project for 032703



Unit.
Given.

Descriptions.
Definitions.

Animation writeup 040903



Unit.
Given.
Descriptions.
Definitions.

Lardner was wholly unaware that the problem actually descibes an ellipse which means that it is not indeterminate at all.

An Indeterminate Problem Reduced To An Equation

Page 5 of *A Treatise on Algebraic Geometry* by Rev. Dionysius Lardner, 1831 **072903**

Given the base AB, and the sum of the sides (AC and BC) of a triangle, to find the vertex (C).
Let AB = *a*, AC = *y*, and CB = *x*, and the the excess of the sum of the sides above the base be *d*.

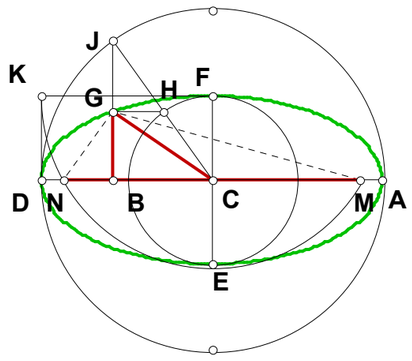
$\therefore\ y + x = a + b.$

Any values of *y* and *x*, which fulfill the conditions of this equation, represent the sides of the triangle, whose vertex solves the problem.

Perhaps this problem is indeterminate is because the author did not have a clue that a point (C) is not a magnitude. How does one find a non-magnitude from magnitues? Only by establishing a co-ordinate system.

09/11/97 The Ellipse

Given that the major axis is AD and the minor axis EF, derive the formula for the radius CG, the height BG, and the foci axis MN.



$N_1 := 3 \quad N_2 := 4$

$AD := 1 \quad EF := \frac{AD}{N_1} \quad AB := \frac{AD}{N_2}$

$BD := AD - AB \quad BJ := \sqrt{AB \cdot BD}$

$AC := \frac{AD}{2} \quad BC := AC - AB \quad CH := \frac{EF}{2}$

$CJ := AC \quad BG := \frac{BJ \cdot CH}{CJ}$

$CG := \sqrt{BG^2 + BC^2} \quad MN := 2 \cdot \sqrt{\left(\frac{AD}{2}\right)^2 - CH^2}$

$$\frac{\sqrt{4 \cdot N_2 - 4 + N_2^2 \cdot N_1^2 - 4 \cdot N_2 \cdot N_1^2 + 4 \cdot N_1^2}}{2 \cdot N_1 \cdot N_2} - CG = 0 \qquad \frac{\sqrt{N_2 - 1}}{(N_2 \cdot N_1)} - BG = 0 \qquad \frac{\sqrt{(N_1^2 - 1)}}{N_1} - MN = 0$$

Unit.

$AD := 1$

Given.

02

031604A

Descriptions.

$AK := AD \quad AJ := AD \quad AH := \frac{\sqrt{2 \cdot AD^2}}{2}$

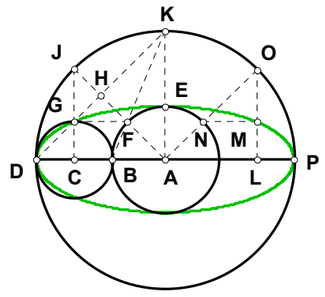
$HJ := AJ - AH \quad AC := AH$

$CD := AD - AC \quad FJ := 2 \cdot CD$

$AF := AJ - FJ \quad AB := AF$

$DK := \sqrt{2 \cdot AK^2} \quad AK + AB - DK = 0$

$\frac{AB}{CD} = 1.414214 \quad \frac{DK}{AD} = 1.414214 \sqrt{2} = 1.414214$

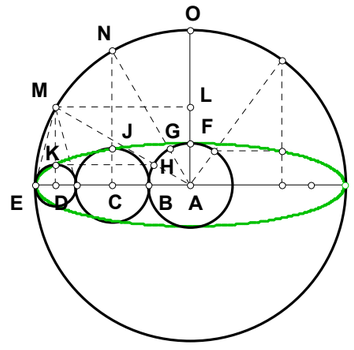


Unit.

$AE := 1$

Given.

03



031604B

Descriptions.

$AO := AE \quad AL := \frac{AO}{2}$

$AM := AE \quad ML := \sqrt{AM^2 - AL^2}$

$AD := ML \quad DE := AE - AD$

$DK := DE \quad DM := AL \quad AH := \frac{AM \cdot DK}{DM}$

$\frac{AH}{DE} = 2$

031704

Unit.

$AE := 1 \quad AG := AE$

Given.

$N := 3 \quad AC := N$

Descriptions.

$DE := AE \cdot 2 \quad CJ := AE$

$CE := \sqrt{AE^2 + AC^2}$

$CG := AE \quad FG := \frac{DE \cdot CG}{CE}$

Definitions.

$JL := FG \quad JM := \frac{JL^2}{2 \cdot CJ}$

$CM := CJ - JM \quad LM := \sqrt{JL^2 - JM^2} \quad NO := AC$

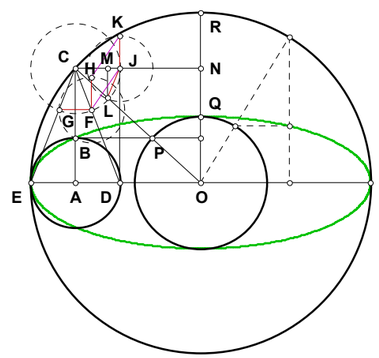
$CN := \frac{CM \cdot NO}{LM} \quad AO := CN \quad EO := AO + AE \quad OP := \frac{AE^2}{LM}$

$Major := EO \quad Minor := OP$

$Major - \frac{N^2 + 1}{2} = 0 \quad Minor - \frac{N^2 + 1}{2 \cdot N} = 0$

$\frac{Major}{Minor} - N = 0$

Given AE, AG, AC
find the ellipse



On given any AE, AC find the diameter of the Circle.

$Major = 5$

$Minor = 1.666667$

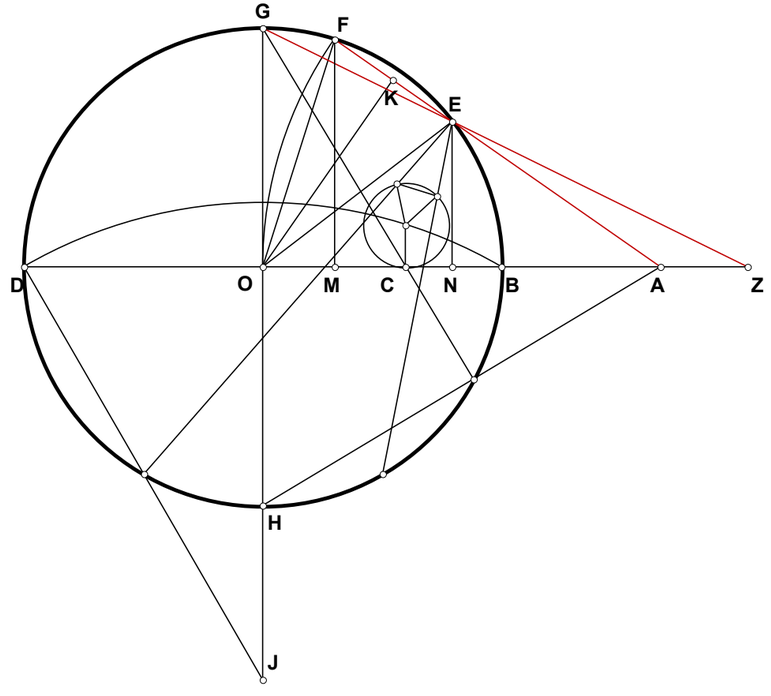
Unit.

$AB := 1$

Given.

$N := 4$

032004



Descriptions.

$AD := N \quad BD := AD - AB \quad BO := \frac{BD}{2} \quad AO := AB + BO$

$FO := BO \quad AF := AO \quad FK := \frac{FO^2}{2 \cdot AF} \quad EF := 2 \cdot FK$

$MO := FK \quad AM := AO - MO \quad FM := \sqrt{AF^2 - AM^2} \quad GO := BO$

$AE := AF - EF \quad EN := \frac{FM \cdot AE}{AF} \quad AN := \frac{AM \cdot AE}{AF} \quad NO := AO - AN$

$ZO := \frac{NO \cdot GO}{GO - EN} \quad AZ := ZO - AO \quad ZN := ZO - NO \quad GZ := \sqrt{ZO^2 + GO^2}$

$EG := \frac{GZ \cdot NO}{ZO} \quad EG = 1.323879$

Definitions.

$AD - N = 0 \quad BD - (N - 1) = 0 \quad BO - \frac{N - 1}{2} = 0 \quad AO - \frac{N + 1}{2} = 0 \quad FO - \frac{N - 1}{2} = 0 \quad AF - \frac{N + 1}{2} = 0 \quad FK - \frac{(N - 1)^2}{4 \cdot (N + 1)} = 0 \quad EF - \frac{(N - 1)^2}{2 \cdot (N + 1)} = 0$

$MO - \frac{(N - 1)^2}{4 \cdot (N + 1)} = 0 \quad AM - \frac{N^2 + 6 \cdot N + 1}{4 \cdot (N + 1)} = 0 \quad FM - \frac{\sqrt{(N + 3) \cdot (3 \cdot N + 1)} \cdot (N - 1)}{4 \cdot (N + 1)} = 0 \quad GO - \frac{N - 1}{2} = 0 \quad AE - \frac{2 \cdot N}{N + 1} = 0 \quad EN - \frac{N \cdot (N - 1) \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)}}{(N + 1)^3} = 0$

$AN - \frac{N \cdot (N^2 + 6 \cdot N + 1)}{(N + 1)^3} = 0 \quad NO - \frac{(N^2 + 4 \cdot N + 1) \cdot (N - 1)^2}{2 \cdot (N + 1)^3} = 0 \quad ZO - \frac{(N - 1)^2 \cdot (N^2 + 4 \cdot N + 1)}{2 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]} = 0 \quad AZ - \frac{N \cdot \left[(N + 1) \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} - (N^2 + 6 \cdot N + 1)\right]}{3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1} = 0$

$ZN - \frac{\sqrt{3 \cdot N^2 + 10 \cdot N + 3} \cdot N \cdot (N^2 + 4 \cdot N + 1) \cdot (N - 1)^2}{(N + 1)^3 \cdot \left(3 \cdot N - 2 \cdot N \cdot \sqrt{3 \cdot N^2 + 10 \cdot N + 3} + 3 \cdot N^2 + N^3 + 1\right)} = 0 \quad GZ - \frac{\sqrt{(N - 1)^2 \cdot (N + 1)^3 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{[(N + 3) \cdot (3 \cdot N + 1)]} + 3 \cdot N^2 + N^3 + 1\right]}}{\sqrt{2 \cdot \left[N^6 + \left[27 \cdot (N^4 + N^2) + 6 \cdot (N^5 + N) + 60 \cdot N^3 + 1\right] - 4 \cdot N \cdot (N + 1)^3 \cdot \sqrt{[(N + 3) \cdot (3 \cdot N + 1)]}\right]}} = 0$

$EG - \frac{\sqrt{2} \cdot \sqrt{(N - 1)^2 \cdot (N + 1)^3 \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]} \cdot \left[3 \cdot N - 2 \cdot N \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 3 \cdot N^2 + N^3 + 1\right]}{2 \cdot (N + 1)^3 \cdot \sqrt{27 \cdot (N^4 + N^2) + \left[6 \cdot (N^5 + N) + 60 \cdot N^3 + N^6 - 4 \cdot N \cdot (N + 1)^3 \cdot \sqrt{(N + 3) \cdot (3 \cdot N + 1)} + 1\right]}} = 0$

032304

Descriptions.

Unit.

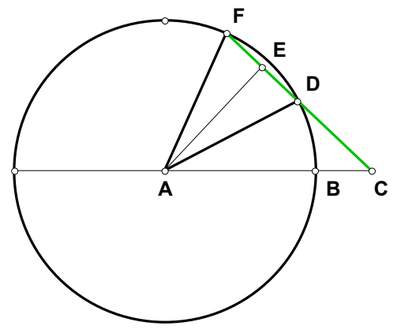
Given.

$N_1 := 1.708$ $AC := N_1$

$N_2 := 1.649$ $CF := N_2$

$N_3 := 1.24$ $AF := N_3$

Given AC, AB and either point of contact, D or F from any C, what is the lenght of the cord DF cut off by a line from any C?



$$DF_1 := \frac{N_3^2 + N_2^2 - N_1^2}{N_2}$$

$CD := CF - DF_1$ $N_4 := CD$

$$DF_2 := \frac{N_3^2 + N_4^2 - N_1^2}{N_4}$$

$DF_1 = 0.812333$ $DF_2 = -0.812333$ $DF_1 + DF_2 = 0$

Unit.

$CE := 1$

Given.

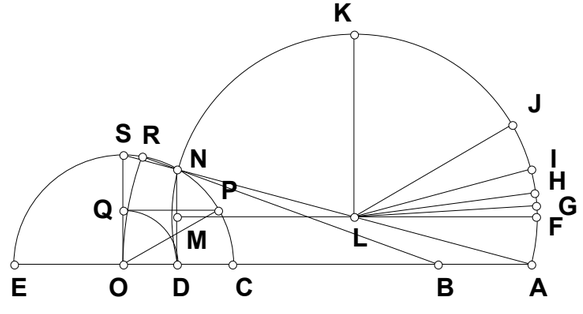
$N := 4$

Definitions.

0405043

Descriptions.

All this extra work and I have lost accuracy from 022803!



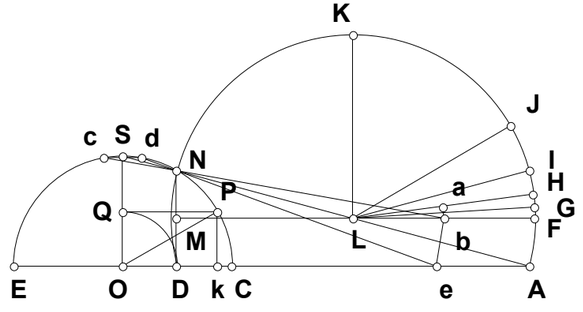
$CO := \frac{CE}{2}$ $CD := \frac{CE}{N}$

$DO := CO - CD$

$DN := \sqrt{(CO + DO) \cdot (CO - DO)}$

$AD := \frac{DO \cdot DN}{CO - DN}$ $DM := \frac{DN}{2}$

$AN := \sqrt{AD^2 + DN^2}$ $HL := \frac{AN}{2}$ $LA := \frac{HL}{2}$ $Ok := \sqrt{CO^2 - DO^2}$ $Ck := CO - Ok$



$Fm := \frac{Ck \cdot HL}{CO}$ $Jm := \frac{DO \cdot HL}{CO}$

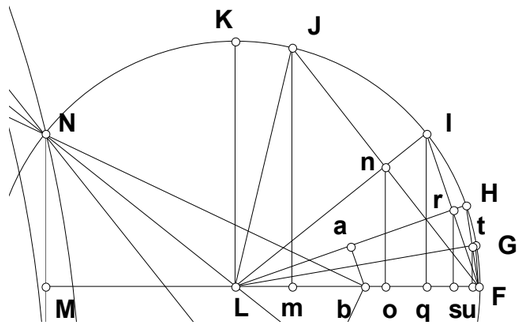
$JF := \sqrt{Fm^2 + Jm^2}$ $Fn := \frac{JF}{2}$

$Fo := \frac{Fm \cdot Fn}{JF}$ $Lo := HL - Fo$

$no := \frac{Jm}{2}$ $Ln := \sqrt{no^2 + Lo^2}$

$Iq := \frac{no \cdot HL}{Ln}$ $Lq := \frac{Lo \cdot HL}{Ln}$

$Fq := HL - Lq$ $FI := \sqrt{Iq^2 + Fq^2}$ $Fr := \frac{FI}{2}$



$Fs := \frac{Fq \cdot Fr}{FI}$ $Ls := HL - Fs$

$Lr := \sqrt{HL^2 - Fr^2}$ $Lb := \frac{HL^2}{2 \cdot Lr}$ $rs := \frac{Iq}{2}$

$$Lv := \frac{Ls \cdot HL}{Lr} \qquad Hv := \frac{rs \cdot HL}{Lr}$$

$$Lv := \frac{Ls \cdot HL}{Lr} \qquad Fv := HL - Lv$$

$$FH := \sqrt{Hv^2 + Fv^2} \qquad Ft := \frac{FH}{2}$$

$$Lf := \frac{HL}{2} \qquad Fu := \frac{Fv \cdot Ft}{FH} \qquad tu := \frac{Hv}{2}$$

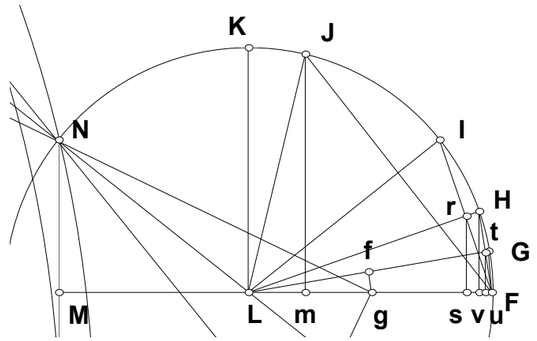
$$Lu := HL - Fu \qquad Lt := \sqrt{Lu^2 + tu^2}$$

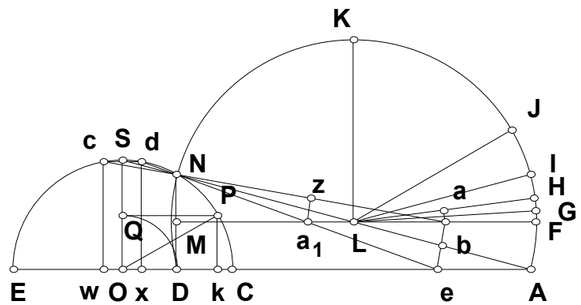
$$Lg := \frac{Lt \cdot Lf}{Lu} \qquad MN := \frac{DN}{2} \qquad NL := \frac{AN}{2}$$

$$ML := \sqrt{NL^2 - MN^2} \qquad Mb := ML + Lb$$

$$Nb := \sqrt{MN^2 + Mb^2}$$

$$Nc := \frac{CO^2 + (2 \cdot Nb)^2 - (2 \cdot Mb + DO)^2}{-2 \cdot Nb}$$





$$cw := \frac{MN \cdot (Nb + Nc)}{Nb} + MN \qquad Dw := \frac{2 \cdot Mb \cdot (2 \cdot Nb + Nc)}{2 \cdot Nb} - 2 \cdot Mb$$

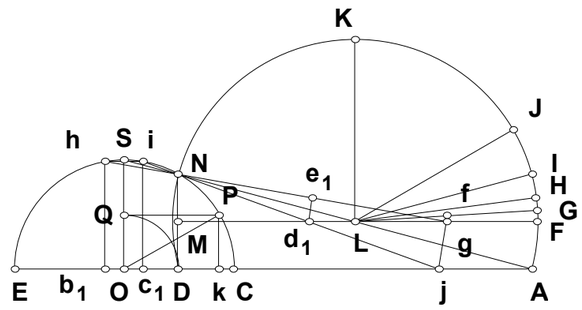
$$Ow := Dw - DO \qquad Dx := Dw - 2 \cdot Ow \qquad Ma_1 := \frac{Dx \cdot DN}{2 \cdot (cw - DN)}$$

$$Na_1 := \sqrt{MN^2 + (Ma_1)^2} \qquad ba_1 := Mb - Ma_1$$

$$Nz := \frac{(Na_1)^2 + Nb^2 - (ba_1)^2}{2 \cdot Nb} \qquad za_1 := \sqrt{(Na_1)^2 - Nz^2} \qquad be := \frac{za_1 \cdot Nb}{Nz}$$

$$Mg := ML + Lg \qquad Ng := \sqrt{MN^2 + Mg^2}$$

$$Nh := \frac{CO^2 + (2 \cdot Ng)^2 - (2 \cdot Mg + DO)^2}{-2 \cdot Ng}$$



$$hb_1 := \frac{MN \cdot (Ng + Nh)}{Ng} + MN \qquad Db_1 := \frac{2 \cdot Mg \cdot (2 \cdot Ng + Nh)}{2 \cdot Ng} - 2 \cdot Mg$$

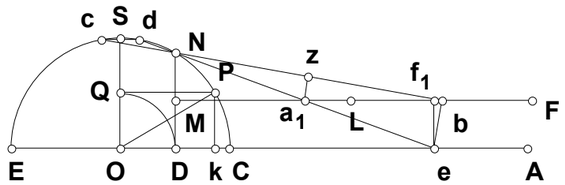
$$Ob_1 := Db_1 - DO \qquad Dc_1 := Db_1 - 2 \cdot Ob_1 \qquad Md_1 := \frac{Dc_1 \cdot DN}{2 \cdot (hb_1 - DN)}$$

$$Nd_1 := \sqrt{MN^2 + (Md_1)^2} \qquad gd_1 := Mg - Md_1 \qquad Ne_1 := \frac{(Nd_1)^2 + Ng^2 - (gd_1)^2}{2 \cdot Ng}$$

$$ed_1 := \sqrt{(Nd_1)^2 - (Ne_1)^2} \qquad gj := \frac{ed_1 \cdot Ng}{Ne_1} \qquad Ne := \frac{Na_1 \cdot Nb}{Nz}$$

$$ea_1 := Ne - Na_1$$

$$bf_1 := \frac{be^2 + ba_1^2 - ea_1^2}{2 \cdot ba_1}$$

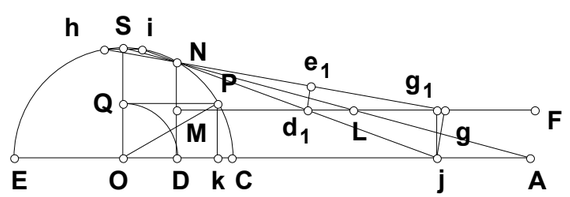


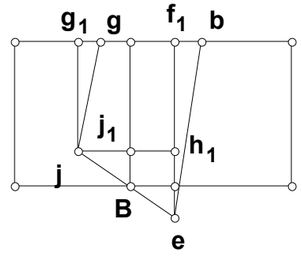
$$ef_1 := \sqrt{be^2 - bf_1^2}$$

$$Nj := \frac{Nd_1 \cdot Ng}{Ne_1} \qquad jd_1 := Nj - Nd_1$$

$$gg_1 := \frac{gj^2 + gd_1^2 - jd_1^2}{2 \cdot gd_1}$$

$$jg_1 := \sqrt{gj^2 - gg_1^2}$$





$$Mg_1 := Mg - gg_1 \qquad Mf_1 := Mb - bf_1$$

$$jh_1 := Mf_1 - Mg_1 \qquad eh_1 := ef_1 - jg_1$$

$$ej := \sqrt{jh_1^{\,2} + eh_1^{\,2}} \qquad Bj_1 := DM - jg_1$$

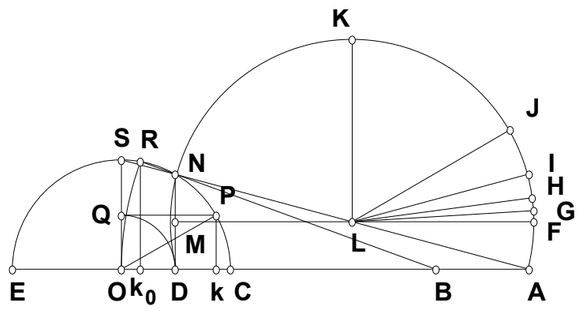
$$jj_1 := \frac{jh_1 \cdot Bj_1}{eh_1} \qquad BD := Mg_1 + jj_1 \qquad BO := BD + DO$$

$$BN := \sqrt{DN^2 + BD^2}$$

$$NR := \frac{CO^2 + BN^2 - BO^2}{-BN}$$

$$Bk_0 := \frac{BD \cdot (BN + NR)}{BN}$$

$$Ok_0 := BO - Bk_0$$



$$NR - 2 \cdot Ok_0 = -0.000000000019581$$

Compared to 022803

$$\frac{19591}{2639} = 7.423645$$

$$Om - \frac{Ek}{2} = -0.000000000002639$$

The Ellipse

041904A

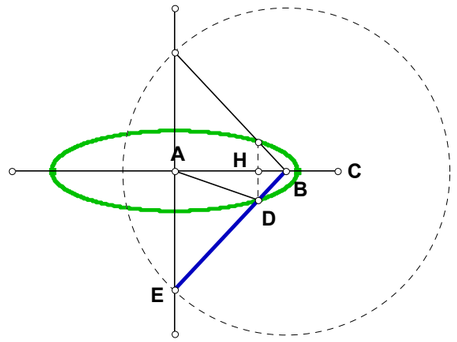
I once worked on a project I called Eloi, which was about the different ways to construct an ellipse, the different equations one would have to use to make that figure, and the different ways one could solve for those ellipses. And this entertained me until I started thinking about the figure 8 locus. One can draw the figure eight as a locus between a straight line and a cirlce.

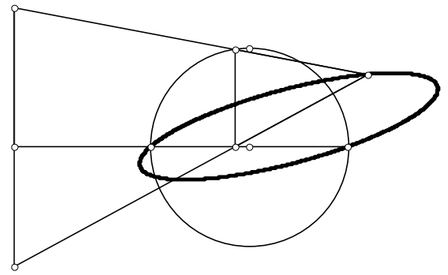
All of this applies to science and mechanics when one is writing up equations to the motion of objects; How do you comprehend what you are seeing?

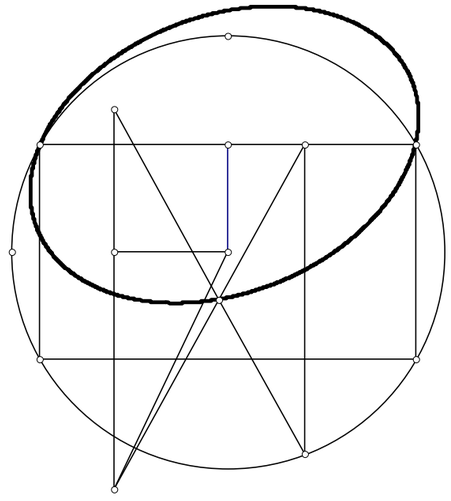
This plate series is called the Straight Line Ellipse because it reduces the ellipse to a single linear action between X and Y axies. In other words, we draw what some would claim to be a trig function, which is actually a linear function, one just doess not see it. When every possible grammar is effected by complete induction and deduction of a unit, one should keep the unit in mind instead of obfuscating it with particular names which factually do not apply. One is always in danger of claiming that there are many different mathematics, yet the same single unit makes them all; this amounts to a thing is different from itself, and if one is stupid, it produces the modern mathematician.

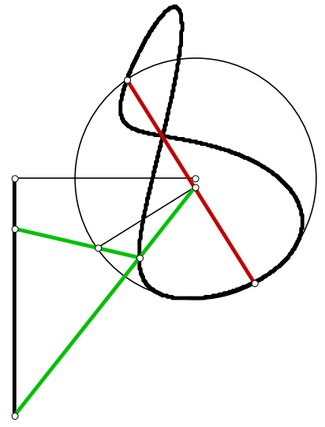
As one can see by the last figure, the major and minor axis do not determine the resulting figure, meaning the shape is independent of the axis, but when it is shaped, or created is. The resulting figure, a photon, or a burst of energy, or one can say a pulse, is the product of a simple tic, toc, tic of two objects, an oscilation. At a certain point they interact, and release an elliptical signiture, or temporarily existing third object. This means one can spend a lot of time claiming that mechanics is wave mechanics, or quantum mechanics, or that these names both miss the point; how you write an object up does not create a new grammar, unless you are really stupid. Grammar is not a theory, and if you are teaching theory, you may as well elect a Pope.

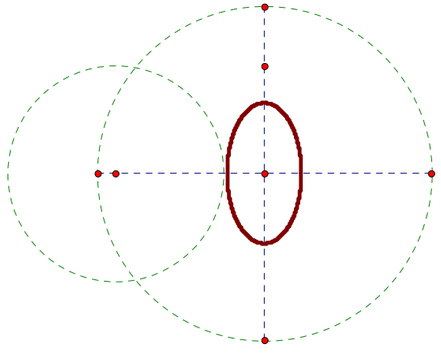
What is more important, we start to get back to fundamentals in linguistic fact, between any two limits is one, and only one relative difference, which is not only called a unit, but even observed by Plato. We can obfuscate it, like all of the other ways we can produce an ellipse, but those figures do not express the fundamental ellipse.











041904B

Unit.

$AC := 1$

Given.

$R_1 := 3$

$R_2 := 2$

Descriptions.

$BE := AC \qquad BD := \frac{BE}{R_1}$

$AB := \frac{AC}{R_2} \qquad AE := \sqrt{BE^2 - AB^2}$

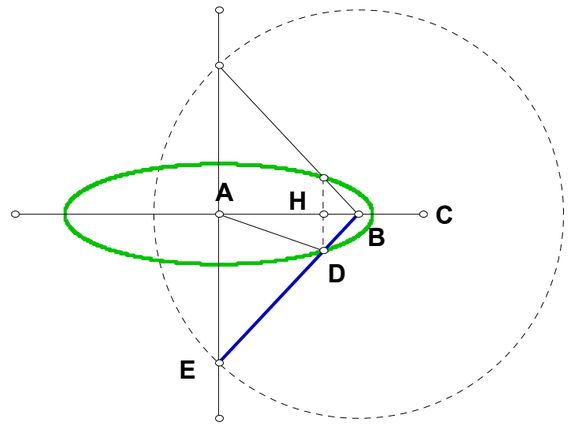
$BH := \frac{AB \cdot BD}{BE} \qquad DH := \frac{AE \cdot BD}{BE}$

$AH := AB - BH \qquad AD := \sqrt{AH^2 + DH^2}$

Definitions.

$$AD - \frac{\sqrt{R_1^2 - 2 \cdot R_1 + R_2^2}}{R_1 \cdot R_2} = 0$$

Straight Line Ellipse: Cardinal



Unit.

$AC := 1$ $BE := AC$

Given.

$N_1 := .5$ $AB := N_1$

$N_2 := .3$ $BD := N_2$

041904C

Descriptions.

$$AE := \sqrt{BE^2 - AB^2}$$

$$BH := \frac{AB \cdot BD}{BE} \quad DH := \frac{AE \cdot BD}{BE}$$

$$AH := AB - BH \quad AD := \sqrt{AH^2 + DH^2}$$

Definitions.

$$AE - \sqrt{(1 - N_1) \cdot (N_1 + 1)} = 0$$

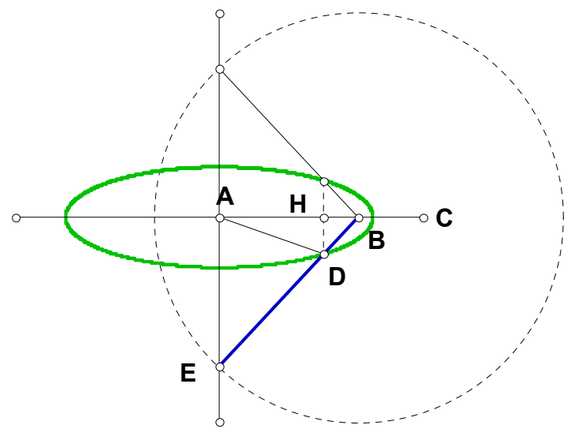
$$BH - N_1 \cdot N_2 = 0$$

$$DH - \sqrt{(1 - N_1) \cdot (N_1 + 1)} \cdot N_2 = 0$$

$$AH - (N_1 - N_1 \cdot N_2) = 0$$

$$AD - \sqrt{N_1^2 - 2 \cdot N_1^2 \cdot N_2 + N_2^2} = 0$$

Straight Line Ellipse: Ordinal



Unit.

$AC := 1$

Given.

$N_1 := 3$

031405

Descriptions.

$AD := N_1 \qquad DV := AC$

$AV := \sqrt{AD^2 + DV^2} \qquad AF := 2 \cdot AC$

$VX := AF - AD \qquad AY := \frac{AV \cdot AC}{AC - VX}$

$AG := \frac{AD \cdot AY}{AV} \qquad BC := \frac{AC \cdot AC}{AC + AD}$

$CG := AG - AC \qquad BG := BC + CG$

$BE := \frac{BG}{2} \qquad CE := BE - BC \qquad ES := BE$

$CS := \sqrt{ES^2 - CE^2} \qquad CR := \frac{DV \cdot AC}{AD}$

$EU := \frac{ES \cdot CR}{CS} \qquad EZ := EU$

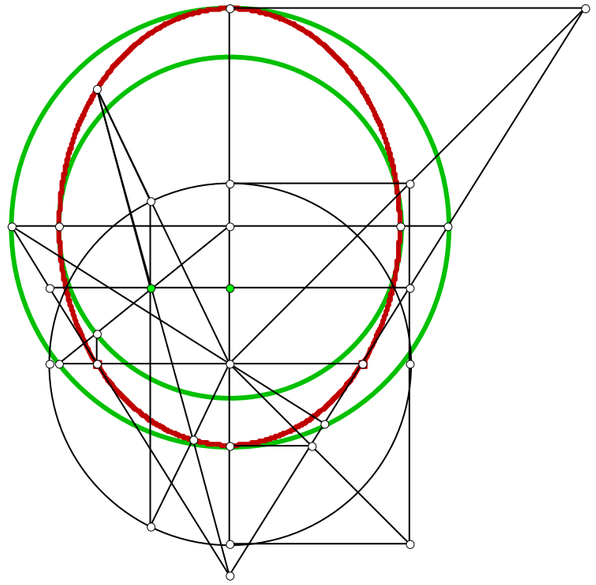
Definitions.

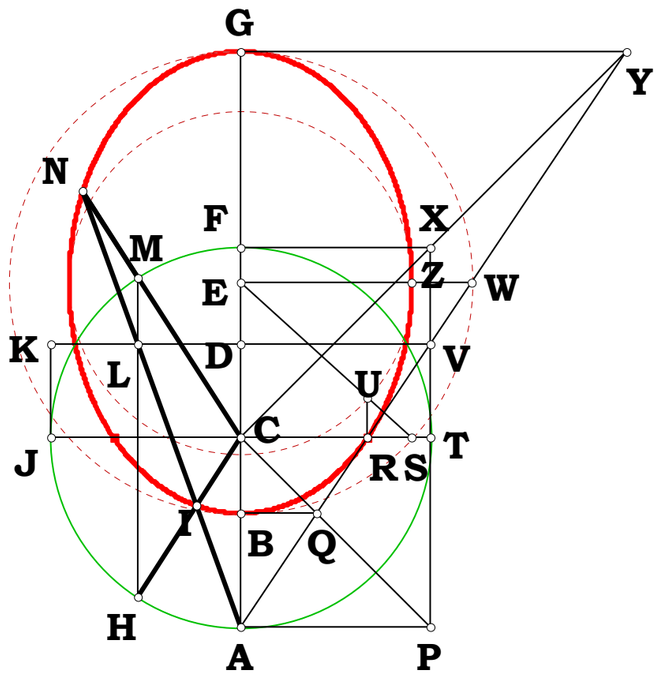
$$EZ - \frac{\sqrt{N_1{}^2 - 1}}{(N_1 - 1) \cdot (N_1 + 1)} = 0$$

$$BG - \frac{2 \cdot N_1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$$

Another Ellipse

The locus formed by N and I as determined by L provides an ellipse. Provide an Algebraic name for the Major and Minor Axis.

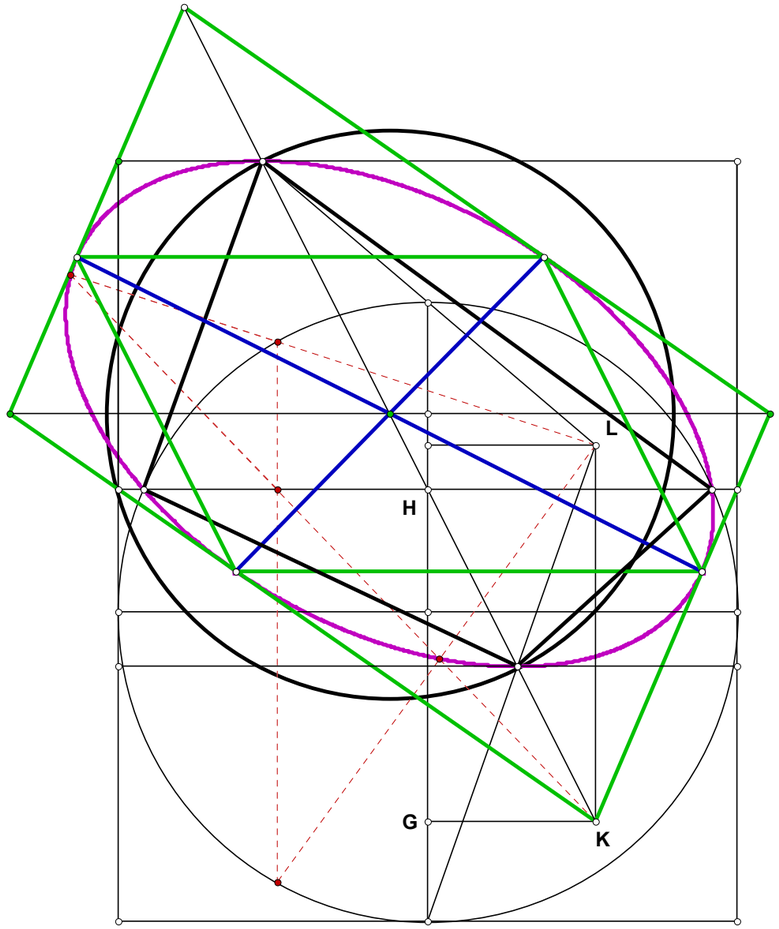




Unit.
Given.

Descriptions.
Definitions.

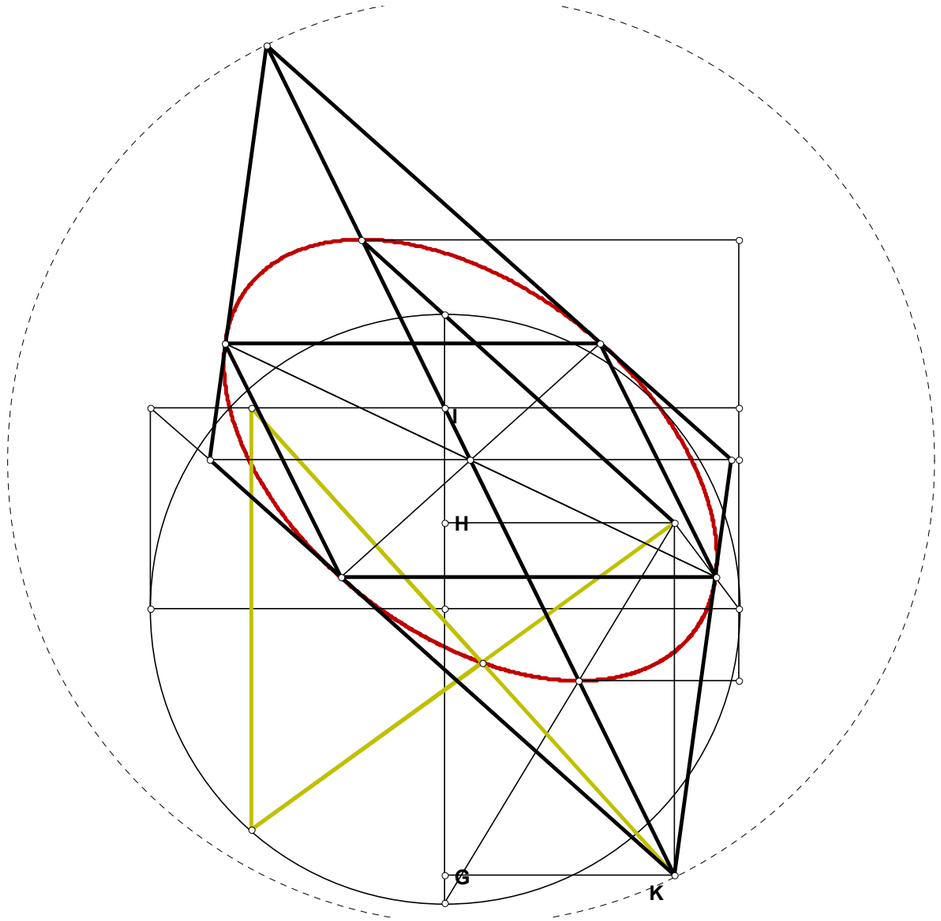
Parcing project for 031605



Unit.
Given.

Descriptions.
Definitions.

Parcing project for 031705



0932305

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := 2 \quad BE := N_1$

$N_2 := .5 \quad BD := N_2$

$MN := 2 \cdot AB$

$DE := BE - BD \qquad AG := AB$

$EP := \frac{AG \cdot BE}{AB} \qquad EN := AB$

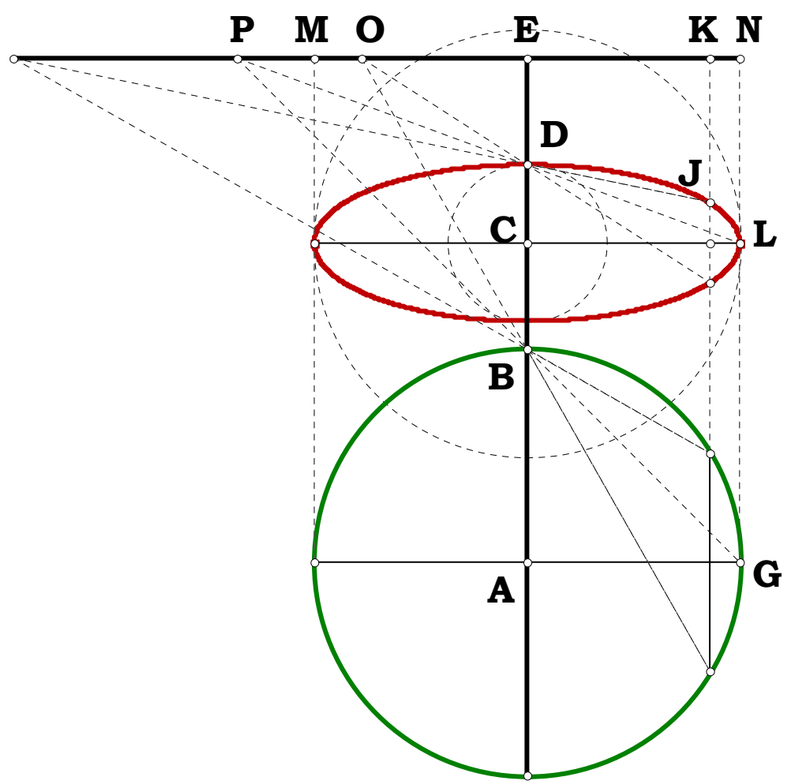
$NL := \frac{DE \cdot (EP + EN)}{EP}$

$CE := NL \qquad CD := CE - DE$

Definitions.

$\frac{(N_1 - N_2)}{N_1} - CD = 0$

An Ellipse

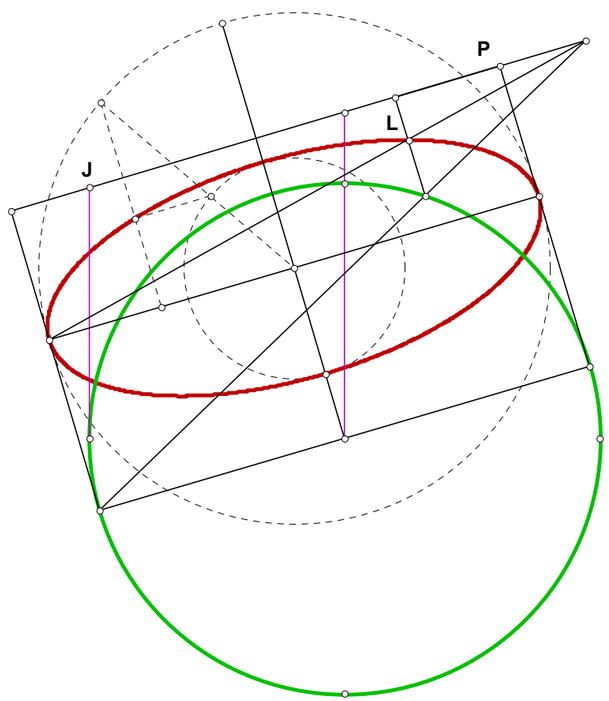


Unit.
Given.

Parcing project for 032405

Descriptions.
Definitions.

032405a.gsp



032905

Descriptions.

Unit.

Given.

$N_1 := 1.9167$ $AC := N_1$

$N_2 := .3244$ $CD := N_2$

$N_3 := .437$ $GI := N_3$

$CE := CD$ $CG := \sqrt{CE \cdot AC}$ $GH := 2 \cdot CG$

$CI := CG - GI$ $AD := AC - CD$ $AI := \sqrt{AC^2 + CI^2}$ $AM := \frac{AD}{2}$

$DM := AM$ $DU := \frac{CI \cdot AD}{AI}$ $AU := \frac{AC \cdot AD}{AI}$ $IU := AI - AU$ $AL := \frac{DU \cdot AI}{IU}$

$IL := \sqrt{AI^2 + AL^2}$ $DI := \sqrt{DU^2 + IU^2}$ $DL := IL - DI$ $DV := \frac{CD \cdot DL}{DI}$

$MV := DV - DM$ $AV := AM - MV$ $LV := \sqrt{AL^2 - AV^2}$ $MT := LV$

$AG := \sqrt{CG^2 + AC^2}$ $DW := \frac{CG \cdot AD}{AG}$ $AW := \frac{AC \cdot AD}{AG}$ $GW := AG - AW$

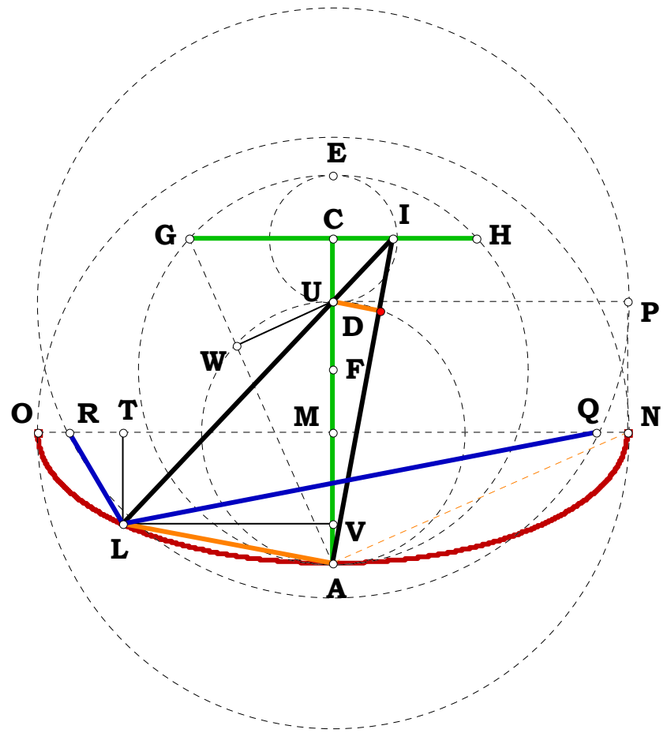
$AN := \frac{DW \cdot AG}{GW}$ $MN := \sqrt{AN^2 - AM^2}$ $ON := 2 \cdot MN$

Definitions.

$$ON - \sqrt{(N_2 - N_1)^2 \cdot \frac{N_1}{N_2}} = 0$$

Elipse Projected From a Perpendicular.

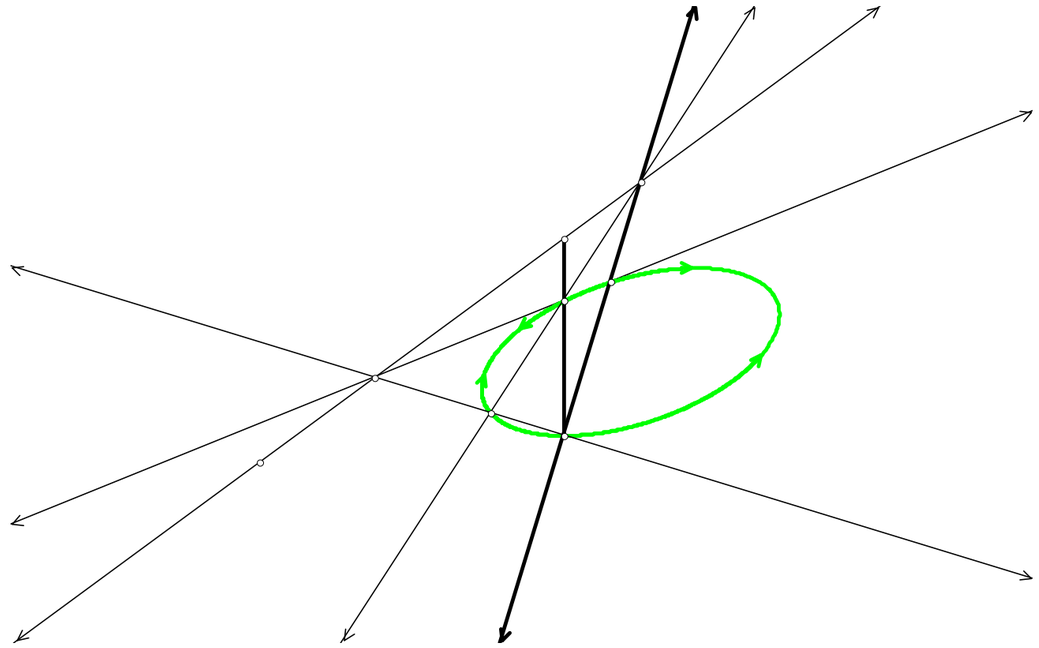
Let AC be some perpendicular on some line GH.



Unit.
Given.

Descriptions.
Definitions.

Parcing project for 032905b, using just straight lines.



Unit.

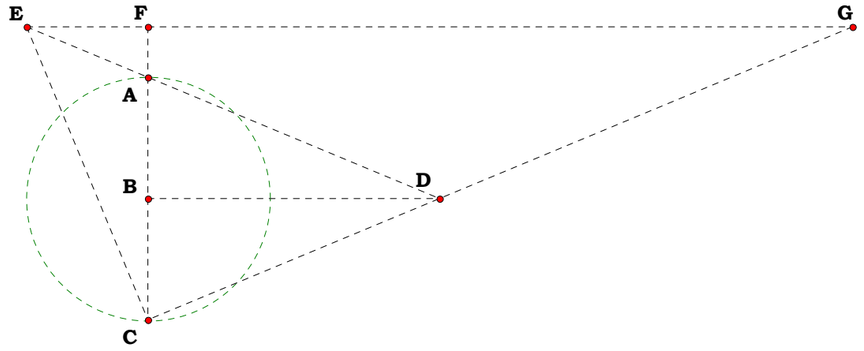
Given.

Writeup project for 032905e

Descriptions.

Definitions.

Another way to do cube roots, one which the ancients were looking for, is accomplished by crossing an isosceles triangle with a right triangle. I put off investigating it for some future date. What it does say is that cube roots are not impossible, but perhaps difficult. I found this while examining the figure of the preceeding ellipse.



EF = 2.04153 cm

AF = 0.84843 cm

FG = 11.82053 cm

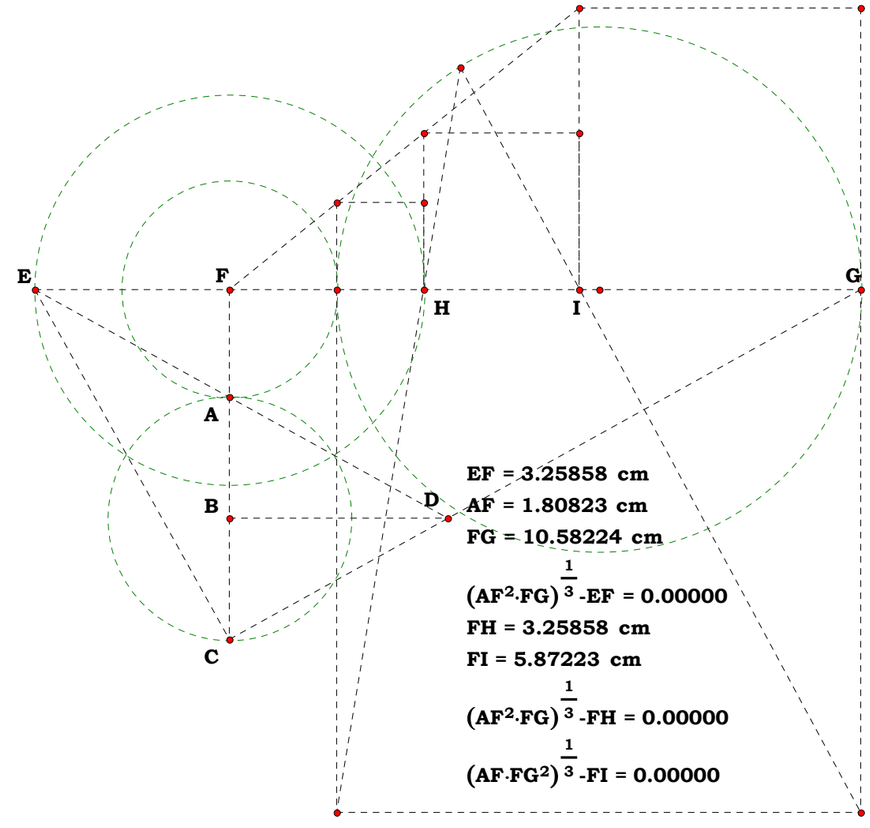
$$(AF^2 \cdot FG)^{\frac{1}{3}} - EF = 0.00000$$

What can be said, however, is that the figure can be used to prove my original construction which would be much, much shorter than the original. However, my interest was no longer on the Delian Quest, but it was cooking to arrive at BAM, and this took time but once it did happen, what unfolded is a work which I probably have no time to finish, it consists of thousands of pages and covers all of how to use it for even logical operations.

The Delian Quest, essentially conquered, is making me comprehend that it is only the door to a much bigger place. The Delian Quest contains the search, the climax, and the after glow, but the results is contained in the volumes of BAM.

One has to remark, though, it is a very beautiful figure. Very simple, straightforward, and reasonable.

Filling in the rest of the figure, we see it.



It might be said that a elliptial construct proved the head of the figure, and another proved its hands.

Unit.

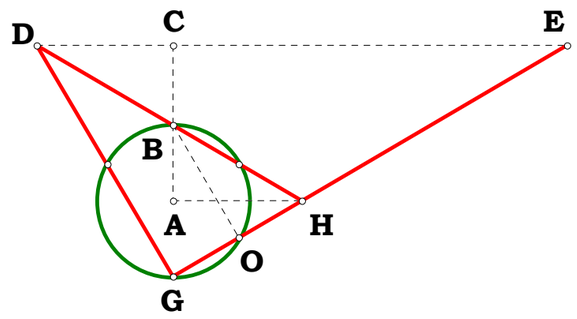
$AB := 1$

Given.

$N_1 := 2$ $\quad AH := N_1$

033005c

N^3 and More



Descriptions.

$BH := \sqrt{AB^2 + AH^2}$ $\quad GO := \frac{AB \cdot 2 \cdot AB}{BH}$ $\quad HO := BH - GO$

$DH := \frac{BH \cdot BH}{HO}$ $\quad AC := \frac{AB \cdot DH}{BH}$ $\quad BO := \sqrt{BH^2 - HO^2}$

$DG := \frac{BO \cdot BH}{HO}$ $\quad DE := \frac{BH \cdot DG}{AB}$ $\quad CD := \frac{DG^2}{DE}$

$BC := AC - AB$ $\quad CE := DE - CD$ $\quad N_1^3 - \frac{CE}{BC} = 0$

Definitions.

$BH - \sqrt{N_1^2 + 1} = 0$ $\quad GO - \frac{2}{\sqrt{N_1^2 + 1}} = 0$ $\quad HO - \frac{(N_1 - 1) \cdot (N_1 + 1)}{\sqrt{N_1^2 + 1}} = 0$

$DH - \frac{\left(\sqrt{N_1^2 + 1}\right)^3}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad AC - \frac{N_1^2 + 1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad BO - \frac{2 \cdot N_1}{\sqrt{N_1^2 + 1}} = 0$

$DG - \frac{2 \cdot N_1 \cdot \sqrt{N_1^2 + 1}}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad DE - \frac{2 \cdot N_1 \cdot (N_1^2 + 1)}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad CD - \frac{2 \cdot N_1}{(N_1 - 1) \cdot (N_1 + 1)} = 0$

$BC - \frac{2}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad CE - \frac{2 \cdot N_1^3}{(N_1 - 1) \cdot (N_1 + 1)} = 0$ $\quad N_1^3 - N_1^3 = 0$

Given.

$N_2 := 4$

Descriptions.

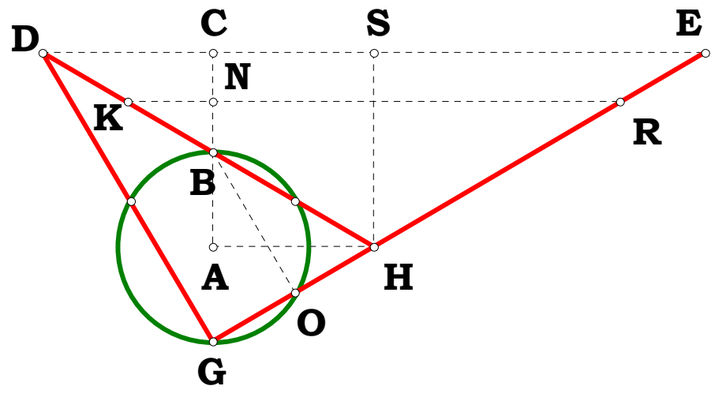
$BN := \frac{BC}{N_2}$ $KN := \frac{CD}{N_2}$

$CG := AC + AB$ $GN := 2 \cdot AB + \frac{BC}{N_2}$

$NR := \frac{CE \cdot GN}{CG}$

Definitions.

$\left[{N_1}^3 \cdot N_2 - N_1 \cdot \left(N_2 - 1 \right) \right] - \frac{NR}{BN} = 0$



${N_1}^3 \cdot N_2 - \left(N_2 - 1 \right) \cdot N_1 = 26$ $\frac{NR}{BN} = 26$

$BN - \frac{2}{N_2 \cdot \left(N_1 - 1 \right) \cdot \left(N_1 + 1 \right)} = 0$ $KN - \frac{2 \cdot N_1}{N_2 \cdot \left(N_1 - 1 \right) \cdot \left(N_1 + 1 \right)} = 0$

$CG - \frac{2 \cdot {N_1}^2}{\left(N_1 - 1 \right) \cdot \left(N_1 + 1 \right)} = 0$ $GN - \left[\frac{2 \cdot \left(N_2 \cdot {N_1}^2 - N_2 + 1 \right)}{N_2 \cdot \left(N_1 - 1 \right) \cdot \left(N_1 + 1 \right)} \right] = 0$

$NR - \frac{2 \cdot N_1 \cdot \left(N_2 \cdot {N_1}^2 - N_2 + 1 \right)}{N_2 \cdot \left(N_1 - 1 \right) \cdot \left(N_1 + 1 \right)} = 0$

033105

Unit.

$AB := 1$

Given.

$N_1 := .15$

$N_2 := .6$

Descriptions.

$CD := N_1$ $DE := 2 \cdot CD$ $AC := N_2$ $BC := AB - N_2$

$FG := DE + AB$ $AE := AC + CD$ $BD := BC + CD$

$AH := AE$ $BH := BD$

$AJ := \frac{AB^2 + AH^2 - BH^2}{2 \cdot AB}$ $HJ := \sqrt{AH^2 - AJ^2}$

Definitions.

$AE = 0.75$ $BD = 0.55$

$AJ = 0.63$ $HJ = 0.40694$

$CD - N_1 = 0$ $DE - 2 \cdot N_1 = 0$ $AC - N_2 = 0$ $BC - (1 - N_2) = 0$

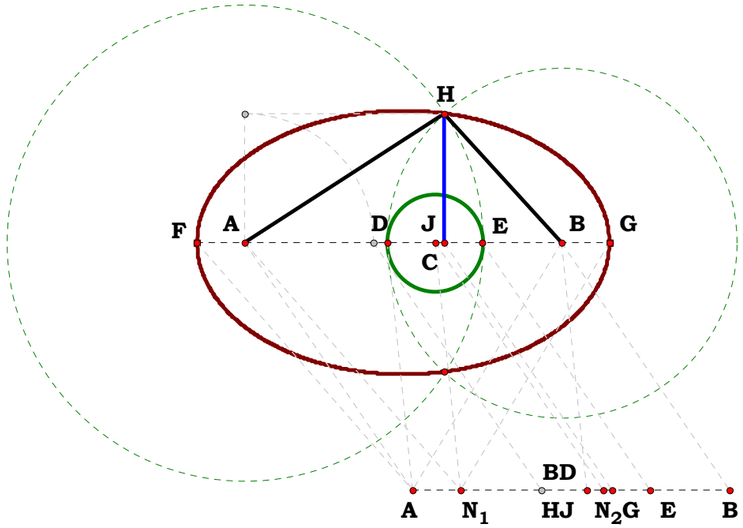
$FG - (2 \cdot N_1 + 1) = 0$ $AE - (N_2 + N_1) = 0$ $BD - (N_1 - N_2 + 1) = 0$

$AH - (N_2 + N_1) = 0$ $BH - (N_1 - N_2 + 1) = 0$

$AJ - (N_2 - N_1 + 2 \cdot N_1 \cdot N_2) = 0$ $HJ - \sqrt{4 \cdot N_1 \cdot N_2 \cdot (1 - N_2) \cdot (N_1 + 1)} = 0$

Just Another Ellipse

Given the difference between the foci and difference between the proportional radii, etc., etc.



Unit = 1.00000

AB = 1.00000

N_1 = 0.15000

X = 3.00000

Y = 20.00000

N_2 = 0.60000

X = 12.00000

Y = 20.00000

AE = 0.75000

BD = 0.55000

AJ = 0.63000

HJ = 0.40694

041205A

Unit.

$CD := 1$

Given.

$N_1 := .278 \quad EF := N_1$

$N_2 := 3.095$

Descriptions.

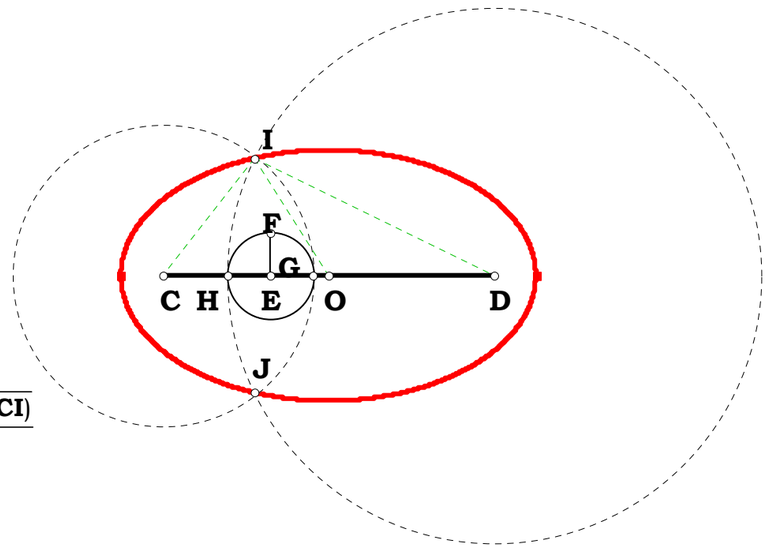
Mixing methods of naming

$$CO := \frac{CD}{2} \qquad CE := \frac{CD}{N_2} \qquad CG := CE + EF$$

$$DH := CD - CG + 2 \cdot EF \qquad CI := CG \qquad DI := DH$$

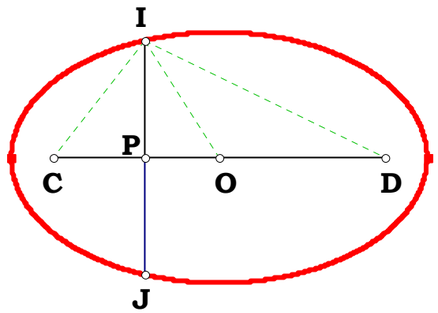
$$IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$



Definitions.

$$2 \cdot \frac{\sqrt{N_1 \cdot (1 + N_1) \cdot (N_2 - 1)}}{N_2} - IP = 0$$



A trip to IO

Unit.

$CD := 1$

Given.

$N_1 := .278$

$N_2 := .3$

041205B

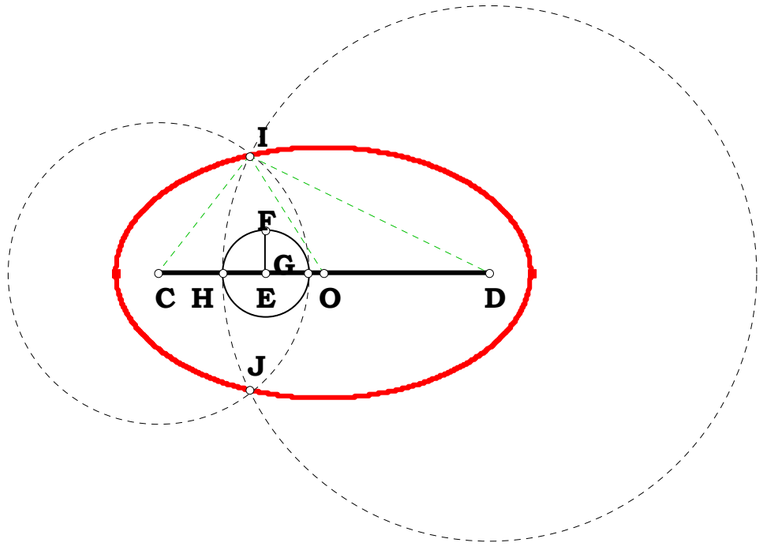
Descriptions.

$$CO := \frac{CD}{2} \qquad EF := N_1$$

$$CE := N_2 \qquad CG := CE + EF \qquad DH := CD - CG + 2 \cdot EF$$

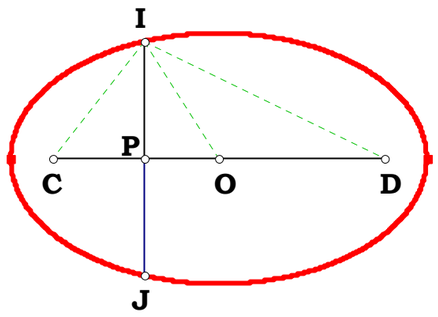
$$CI := CG \qquad DI := DH \qquad IO := \frac{\sqrt{2 \cdot CI^2 - CD^2 + 2 \cdot DI^2}}{2}$$

$$IP := \frac{\sqrt{(-CD + DI - CI)(CD + DI + CI)(CD - DI - CI)(CD + DI - CI)}}{2 \cdot CD}$$



Definitions.

$$2 \cdot \sqrt{\left[-N_1 \cdot N_2 \cdot (N_1 + 1) \cdot (N_2 - 1)\right]} - IP = 0$$



Unit.

$DH := 1$

Given.

$N_1 := 36 \qquad FH := N_1$

$\left(\sqrt{N_1}\right)^3$

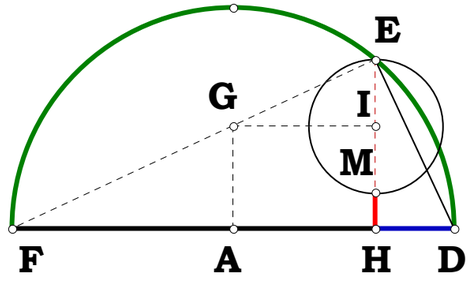
041305b

Descriptions.

$DF := DH + FH \qquad AD := \frac{DF}{2} \qquad EH := \sqrt{DH \cdot FH}$

$HI := \frac{EH \cdot AD}{FH} \qquad HM := EH - 2 \cdot (EH - HI)$

$DE := \sqrt{DH^2 + EH^2} \qquad EF := \sqrt{FH^2 + EH^2}$



Definitions.

$\frac{FH}{HM} = 216 \qquad \left(\frac{FH}{DH}\right)^{1.5} = 216 \qquad \frac{FH}{HM} - \left(\frac{DH}{HM}\right)^3 = 0 \qquad \frac{FH}{HM} - \left(\frac{EF}{DE}\right)^3 = 0$

$DF - (1 + N_1) = 0 \qquad AD - \frac{1 + N_1}{2} = 0 \qquad EH - \sqrt{N_1} = 0$

$HI - \frac{N_1 + 1}{2 \cdot \sqrt{N_1}} = 0 \qquad HM - \frac{1}{\sqrt{N_1}} = 0$

$DE - \sqrt{N_1 + 1} = 0 \qquad EF - \sqrt{\left[N_1 \cdot (N_1 + 1)\right]} = 0$

$N_1^{\frac{3}{2}} = 216 \qquad (N_1)^{1.5} = 216 \qquad N_1^{\frac{3}{2}} - \left(\sqrt{N_1}\right)^3 = 0 \qquad N_1^{\frac{3}{2}} - \left[\frac{\sqrt{N_1 \cdot (N_1 + 1)}}{\sqrt{N_1 + 1}}\right]^3 = 0$

Mathcad 15 will not reduce this last one.

One can see the taxing of logic in terms of precision on the computer.

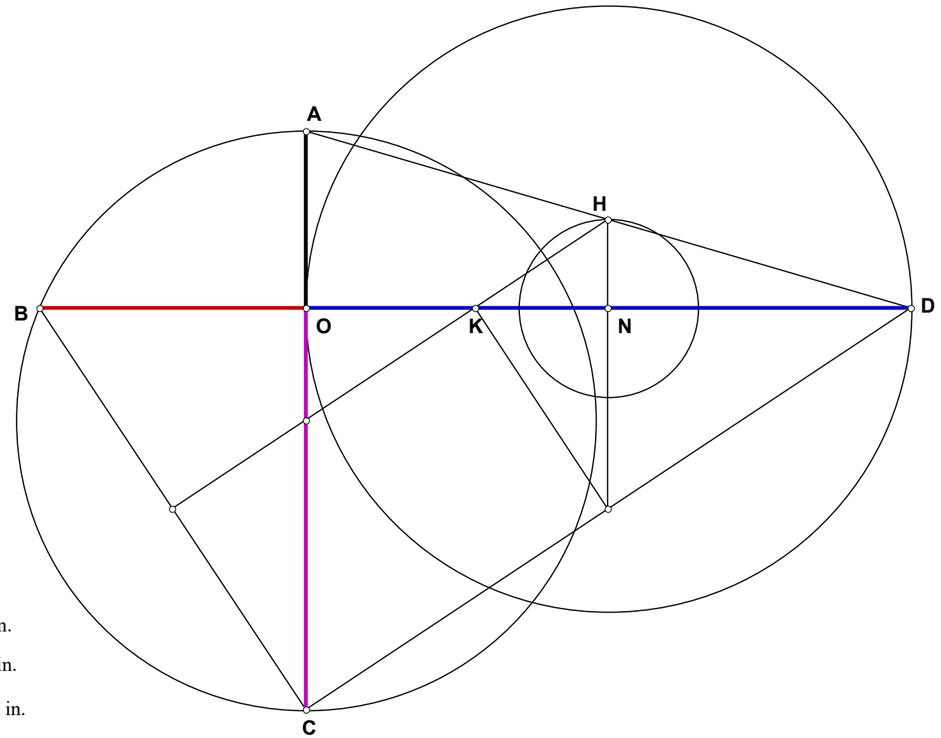
$N_1^{\frac{3}{2}} - \left(\frac{FH}{HM}\right) = 3.694822 \times 10^{-13} \qquad (N_1)^{1.5} - \left[\left(\frac{FH}{DH}\right)^{1.5}\right] = 0 \qquad N_1^{\frac{3}{2}} - \left(\frac{DH}{HM}\right)^3 = 1.13687 \times 10^{-12} \qquad N_1^{\frac{3}{2}} - \left(\frac{EF}{DE}\right)^3 = 0$

Unit.
Given.

041405

Another procrastinated writeup

Descriptions.
Definitions.



AO = 1.167 in.
BO = 1.759 in.
CO = 2.649 in.
DO = 3.991 in.

$\frac{BO}{AO} = 1.507$

$\frac{CO}{AO} = 2.270$

$\frac{CO^{1.5}}{AO} = 3.419$

$\frac{DO}{AO} = 3.419$

$\frac{BO^{3}}{AO} = 3.419$

$\frac{DO}{AO}^{\frac{1}{3}} - \frac{BO}{AO} = 0.000$

NK = 0.879 in.

ND = 1.996 in.

HN = 0.584 in.

$\frac{NK}{HN} = 1.507$

$\frac{ND}{HN} = 3.419$

$\frac{NK^{3}}{HN} = 3.419$

$\frac{ND}{HN} - \frac{NK^{3}}{HN} = 0.000$

042205

Unit.

Given.

$N_1 := 2.604$ $DE := N_1$

$N_2 := 2.234$ $FI := N_2$

Given the major axis and the difference between the two foci, whatis the minor axis?

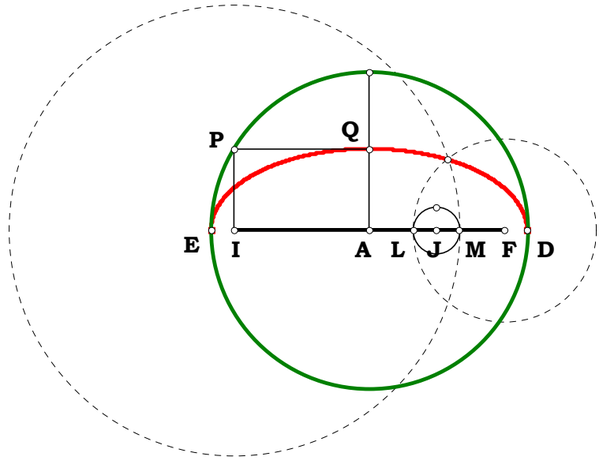
Descriptions.

$$EI := \frac{DE - FI}{2} \qquad DI := DE - EI$$

$$EP := \sqrt{EI \cdot DI} \qquad AQ := EP$$

Definitions.

$$AQ - \frac{1}{2} \cdot \sqrt{(N_1 + N_2) \cdot (N_1 - N_2)} = 0$$



ED = 2.083 in.

FI = 1.787 in.

AQ = 0.535 in.

$$AQ - \frac{\sqrt{(ED+FI) \cdot (ED-FI)}}{2} = 0.000 \text{ in.}$$

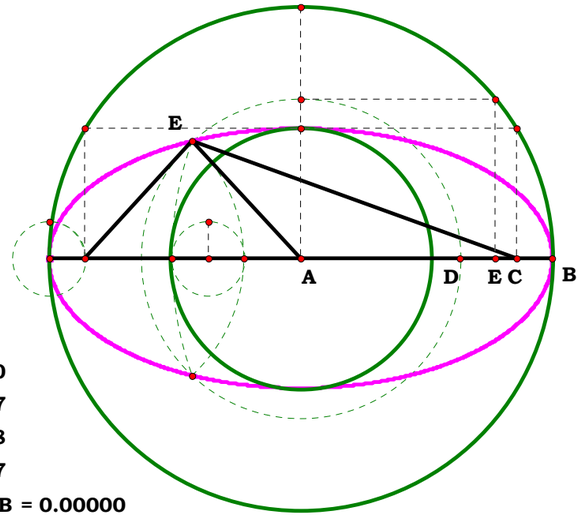
Sum of Area

042305A

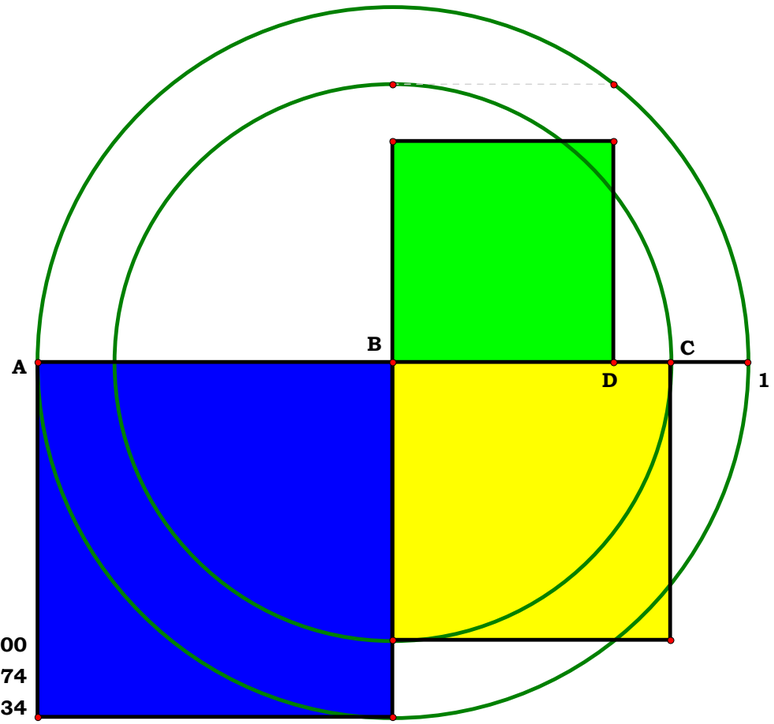
Descriptions.

It seems that the bottom two figures came to mind while I was examining an ellipse. This consist of three different applications. This write-will be Plate A.

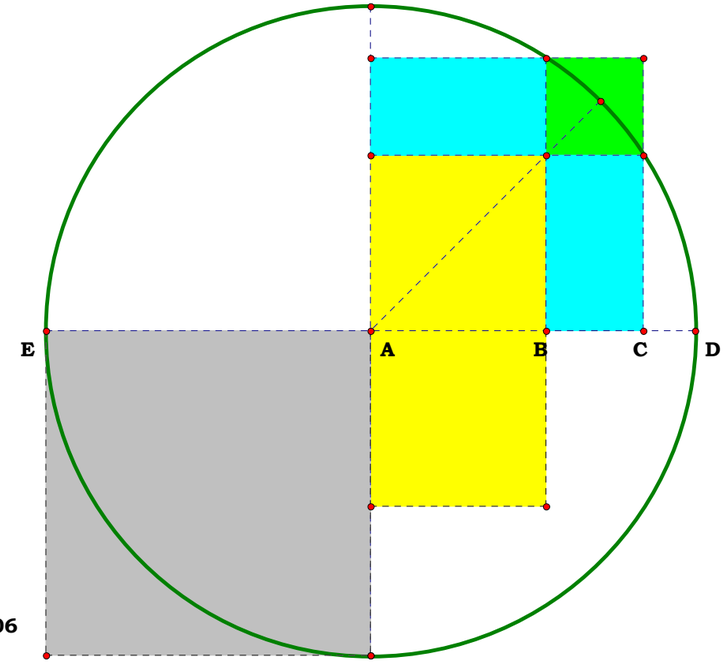
Is there a perfect way to write-up an ellipse? This whole ellipse is wholly determined by the ratio AB to AC.



AB = 1.00000
AC = 0.85647
AD = 0.63408
AE = 0.77327
$\sqrt{AD^2+AE^2}-AB = 0.00000$



AB = 1.00000
AC = 0.78274
AD = 0.62234
$AB^2-(AC^2+AD^2) = 0.00000$



Unit = 1.00000
XY = 0.54037
X = 10.80742
Y = 20.00000
AB = 0.54037
AC = 0.84143
AD = 1.00000
AC-AB = 0.30106
BC = 0.30106
$AD^2-(2\cdot AB^2+2\cdot AB\cdot BC+BC^2) = 0.00000$

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 17$

Plate A.

042305A

Descriptions of ellipse

$AC := \frac{X}{Y}$ $AC = 0.85$ $BC := AB - AC$ $BP := 2 \cdot AB$

$AF := \sqrt{(BP - BC) \cdot BC}$ $AF = 0.526783$ $CO := 2 \cdot AC$

$NM := 2 \cdot BC$

Descriptions AE, either CM or NO has to be given.

Given NO: $NO := .63455$

$CM := CO - NO + NM$ $DO := NO$ $CE := CM$

$AD := \frac{\sqrt{2 \cdot DO^2 - CO^2 + 2 \cdot CE^2}}{2}$ (Pythagoras Revisted) $AD = 0.641135$

$DP := AB + AD$ $BD := BP - DP$ $AE := \sqrt{DP \cdot BD}$ $AE = 0.767428$

$\sqrt{AD^2 + AE^2} - AB = 0$

Definitions. The equation for AD is indifferent as to which side, DO or CD, of the elliptical triangle given.

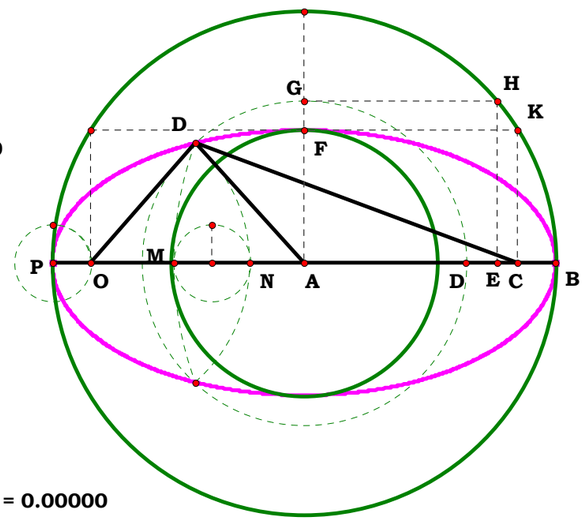
$AC - \frac{X}{Y} = 0$ $BC - \frac{Y - X}{Y} = 0$ $BP - 2 = 0$ $AF - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$ $CO - \frac{2 \cdot X}{Y} = 0$

$NM - \frac{2 \cdot (Y - X)}{Y} = 0$ $CM - (2 - NO) = 0$ $DO - NO = 0$ $CE - (2 - NO) = 0$ $AD - \frac{\sqrt{NO \cdot Y^2 \cdot (NO - 2) - X^2 + 2 \cdot Y^2}}{Y} = 0$

$DP - \frac{Y + \sqrt{NO^2 \cdot Y^2 - 2 \cdot NO \cdot Y^2 - X^2 + 2 \cdot Y^2}}{Y} = 0$ $BD - \frac{Y - \sqrt{NO^2 \cdot Y^2 - 2 \cdot NO \cdot Y^2 - X^2 + 2 \cdot Y^2}}{Y} = 0$ $AE - \frac{\sqrt{Y^2 + \left(2 \cdot NO \cdot Y^2 - NO^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y} = 0$

$\sqrt{\left[\frac{\sqrt{NO \cdot Y^2 \cdot (NO - 2) - X^2 + 2 \cdot Y^2}}{Y}\right]^2 + \left[\frac{\sqrt{Y^2 + \left(2 \cdot NO \cdot Y^2 - NO^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y}\right]^2} = 1$ $\sqrt{\left[\frac{\sqrt{CM \cdot Y^2 \cdot (CM - 2) - X^2 + 2 \cdot Y^2}}{Y}\right]^2 + \left[\frac{\sqrt{Y^2 + \left(2 \cdot CM \cdot Y^2 - CM^2 \cdot Y^2 + X^2 - 2 \cdot Y^2\right)}}{Y}\right]^2} = 1$

Unit = 1.00000
XY = 0.85000
X = 17.00000
Y = 20.00000
AB = 1.00000
AC = 0.85000
AD = 0.64114
AE = 0.76743
AF = 0.52678
CM = 1.36545
NO = 0.63455
$\sqrt{AD^2+AE^2}-AB$ = 0.00000



042305B

Descriptions.

Unit.

$AB := 1$

$BE := AB$

Given.

$Y := 20$

$X := 12$

$BC := \frac{X}{Y}$ $AE := 2 \cdot AB$ $MN := AE$ $FN := AB + BC$

$FG := \sqrt{(MN - FN) \cdot FN}$ $BD := FG$

$AB^2 - \left(BC^2 + BD^2\right) = 0$

Definitions.

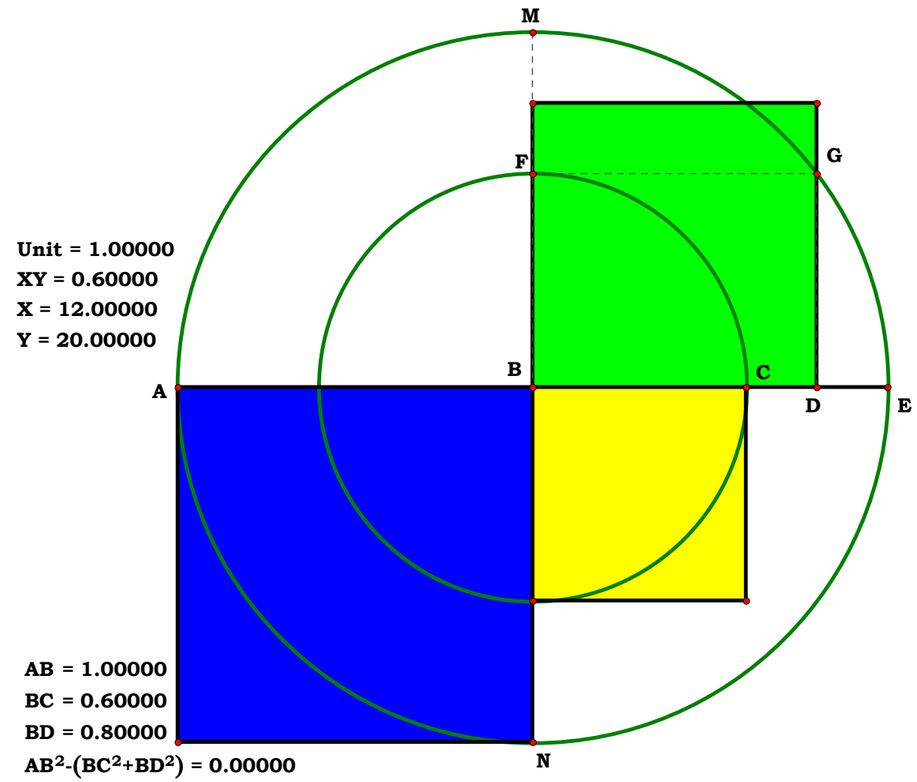
$BC - \frac{X}{Y} = 0$ $AE - 2 = 0$ $MN - 2 = 0$

$FN - \frac{X + Y}{Y} = 0$ $FG - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$

$BD - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$

Sum of Area

Plate B.



Unit.

$AD := 1$

$AE := AD$

Given.

$Y := 20$

$X := 12$

Sum of Area

Plate C.

Descriptions.

$AB := \frac{X}{Y}$ $AB = 0.6$ $BE := AB + AE$ $DE := 2 \cdot AD$

$BD := DE - BE$ $AC := \sqrt{BE \cdot BD}$ $BC := AC - AB$

$AD^2 - \left(2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2\right) = 0$

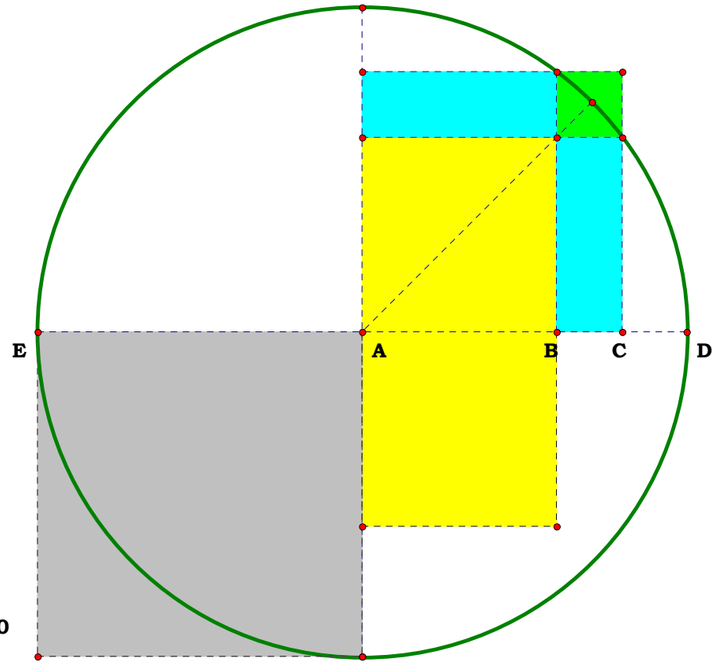
Definitions.

$AB - \frac{X}{Y} = 0$ $BE - \frac{X + Y}{Y} = 0$ $DE - 2 = 0$

$BD - \frac{Y - X}{Y} = 0$ $AC - \frac{\sqrt{Y^2 - X^2}}{Y} = 0$

$BC - \frac{\sqrt{Y^2 - X^2} - X}{Y} = 0$

$AD^2 - \left(2 \cdot AB^2 + 2 \cdot AB \cdot BC + BC^2\right) = 0$



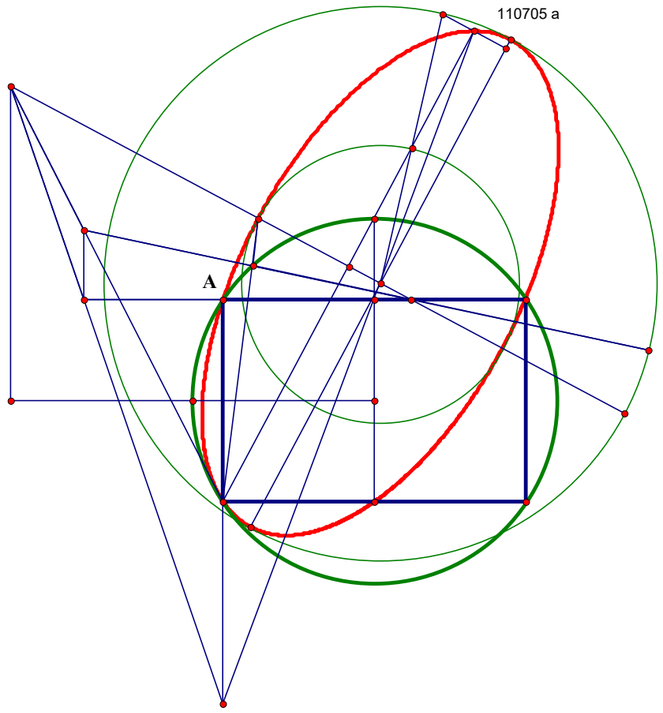
Unit = 1.00000
XY = 0.60000
X = 12.00000
Y = 20.00000
AB = 0.60000
AC = 0.80000
AD = 1.00000
AC-AB = 0.20000
BC = 0.20000
$AD^2-(2\cdot AB^2+2\cdot AB\cdot BC+BC^2)$ = 0.00000

Unit.
Given.

Descriptions.
Definitions.

Procrastinated Writeup for 110705

And by looking at it, maybe another twenty years it get done.

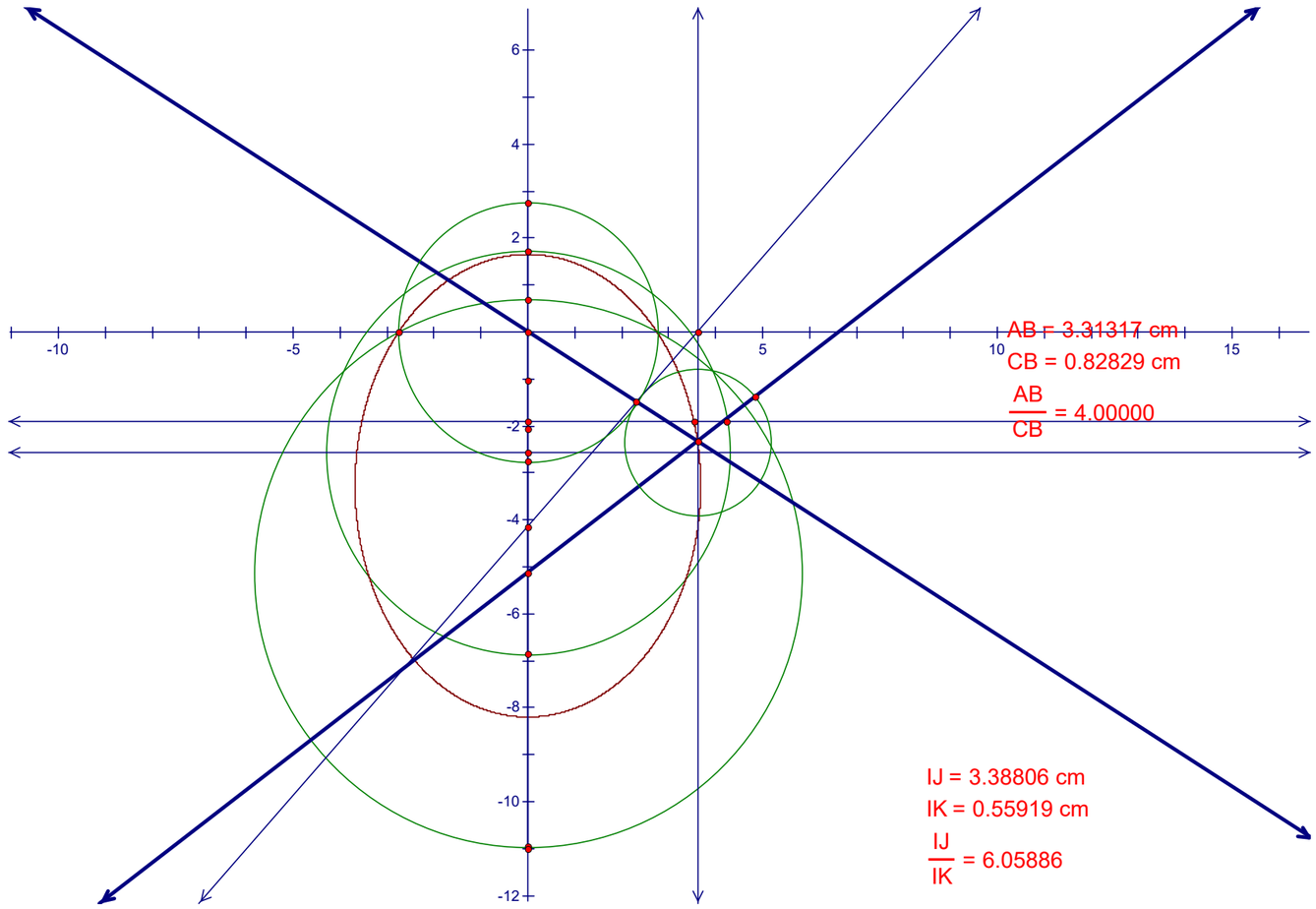


Unit.
Given.

Descriptions.
Definitions.

Parcing project for 012306

Three circle ellipse



012906

Descriptions.

Unit.

Given.

$N := 7.75095$

$CD := 5.55625$

Given AC what is CK?

This may be the first time I tried to get an equation in terms of N, so that given CK one could project AC. However, the version of Mathcad at that time could not reduce the equations, and I am not interested now to try.

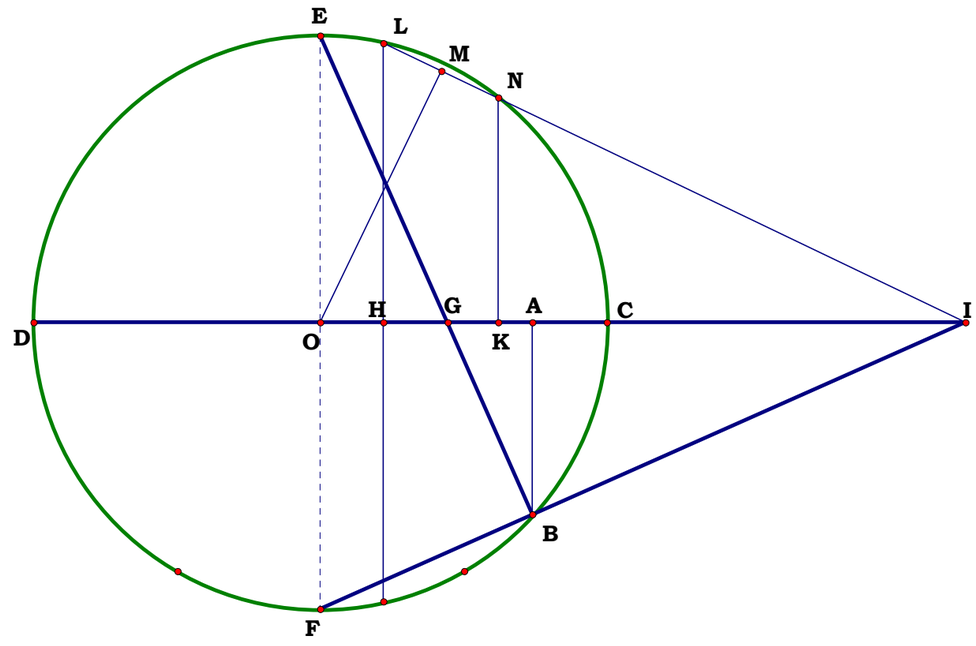
$$AC := \frac{CD}{N} \quad AD := CD - AC \quad AB := \sqrt{AC \cdot AD} \quad EO := \frac{CD}{2} \quad CO := EO$$

$$AO := CO - AC \quad GO := \frac{AO \cdot EO}{EO + AB} \quad HO := \frac{GO}{2} \quad AG := AO - GO \quad FO := EO$$

$$IO := \frac{AB \cdot FO}{AG} \quad IL := IO \quad CH := CO - HO \quad DO := EO \quad DH := DO + HO$$

$$HI := IO - HO \quad IK := \frac{HI \cdot (IL - 2 \cdot HO)}{IL} \quad KO := IO - IK$$

$$CK := CO - KO$$



Unit.

$AB := 1$

Given.

$Y := 20$

$X := 9$

102606A

Descriptions.

$AN := \frac{X}{Y}$ $AK := AB$ $AF := AB$ $KN := \sqrt{AN^2 + AK^2}$

$FG := \frac{AN \cdot 2 \cdot AB}{KN}$ $FG = 0.820729$

$GH := \frac{AB \cdot FG}{KN}$ $GH = 0.748441$ $GO := GH - AN$

$FH := \frac{AN \cdot FG}{KN}$ $AH := AF - FH$ $AH = 0.663202$

$CN := \frac{GO \cdot AB}{FH}$ $AC := CN + AN$ $AC = 1.336111$

$AJ := GH$ $CJ := AC - AJ$ $CE := CJ$ $CE = 0.58767$

$\sqrt{AC \cdot AJ} = 1$ Circles are to each other as their radii.

Definitions.

$AN - \frac{X}{Y} = 0$ $AK - 1 = 0$ $AF - 1 = 0$

$KN - \frac{\sqrt{X^2 + Y^2}}{Y} = 0$ $FG - \frac{2 \cdot X}{\sqrt{X^2 + Y^2}} = 0$

$GH - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $GO - \frac{X \cdot Y^2 - X^3}{Y \cdot (X^2 + Y^2)} = 0$ $FH - \frac{2 \cdot X^2}{X^2 + Y^2} = 0$ $AH - \frac{Y^2 - X^2}{X^2 + Y^2} = 0$ $CN - \frac{Y^2 - X^2}{2 \cdot X \cdot Y} = 0$

$AC - \frac{X^2 + Y^2}{2 \cdot X \cdot Y} = 0$ $AJ - \frac{2 \cdot X \cdot Y}{X^2 + Y^2} = 0$ $CJ - \frac{(X - Y)^2 \cdot (X + Y)^2}{2 \cdot X \cdot Y \cdot (X^2 + Y^2)} = 0$ $CE - \frac{(X - Y)^2 \cdot (X + Y)^2}{2 \cdot X \cdot Y \cdot (X^2 + Y^2)} = 0$

Angles and Area Plate A

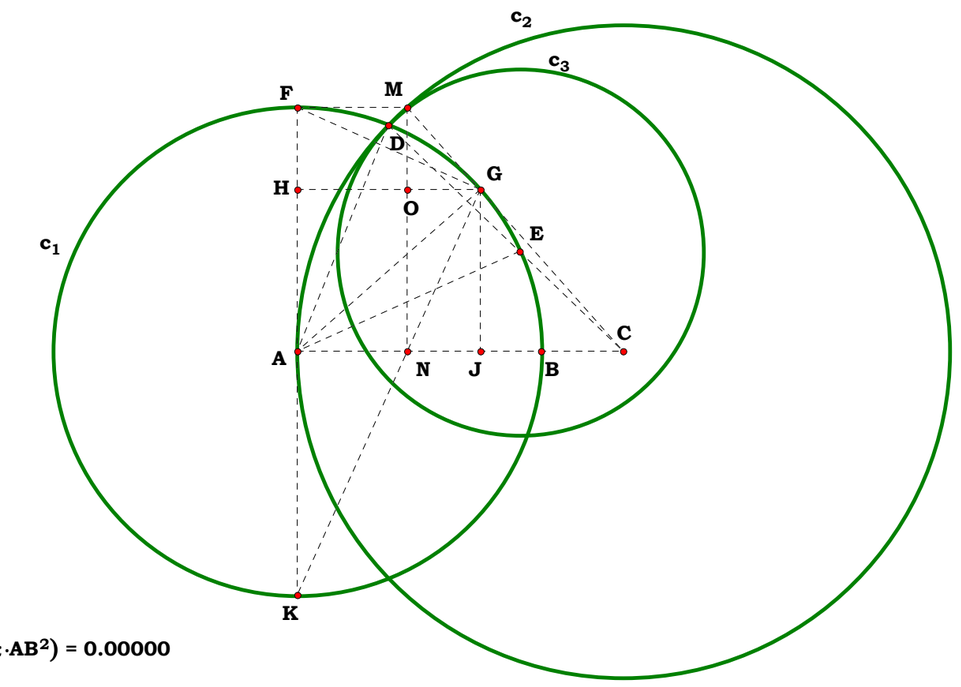
While exploring trisection, I found something else actually worth exploring. An entirely different way to think of an angle. An angle is takes the so called Pythagorean down to its elements, and it is wholly independent of any particular angle.

Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000
AB = 1.00000
AN = 0.45000
FG = 0.82073
GH = 0.74844
AJ = 0.74844
AH = 0.66320
CE = 0.58767
AC = 1.33611
$\sqrt{AC \cdot AJ}$ = 1.00000

$\frac{X}{Y}$ = 0.45000

$AN - \frac{X}{Y}$ = 0.00000

$\pi \cdot AB^2$ = 3.14159
$\pi \cdot AC^2$ = 5.60835
$\pi \cdot AJ^2$ = 1.75981
$\sqrt{(\pi \cdot AC^2) \cdot (\pi \cdot AJ^2)} - (\pi \cdot AB^2)$ = 0.00000



$\pi \cdot AB^2 = 3.141593$

$\pi \cdot AC^2 = 5.608349$

$\pi \cdot AJ^2 = 1.759806$

$\sqrt{\pi \cdot AJ^2 \cdot \pi \cdot AC^2} - \pi \cdot AB^2 = 0$

Unit.

$AB := 1$

Given.

$Y := 20$

$X := 9$

102606B

Descriptions.

$AC := \frac{X}{Y}$ $BC := AB - AC$

$BH := \frac{BC^2}{2 \cdot AB}$ **(Pythagoras Revisited)** $BH = 0.15125$

$BD := 2 \cdot BH$ $AD := AB - BD$ $FG := BD$

$\sqrt{AB \cdot BD} - BC = 0$

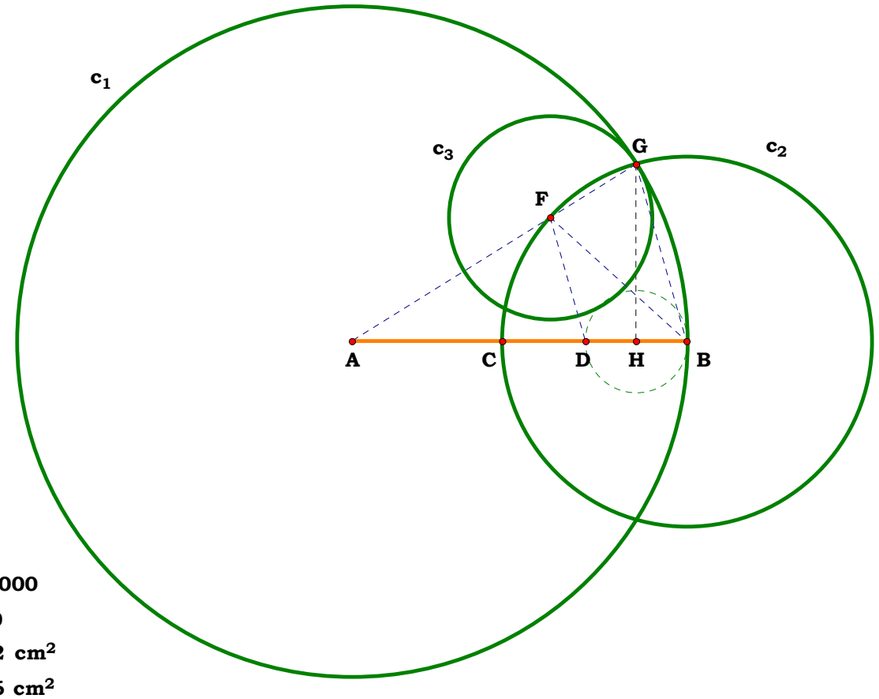
Definitions.

$AC - \frac{X}{Y} = 0$ $BC - \frac{Y - X}{Y} = 0$ $BH - \frac{(X - Y)^2}{2 \cdot Y^2} = 0$

$BD - \frac{(X - Y)^2}{Y^2} = 0$ $AD - \frac{X \cdot (2 \cdot Y - X)}{Y^2} = 0$

$FG - \frac{(X - Y)^2}{Y^2} = 0$

Angles and Area Plate B



Unit = 1.00000
XY = 0.45000
X = 9.00000
Y = 20.00000

AB = 1.00000
AC = 0.45000
AD = 0.69750
BC = 0.55000
BD = 0.30250
BH = 0.15125

$\sqrt{AB \cdot BD}$-BC = 0.00000
$\sqrt{BD}$-BC = 0.00000
Area c_1 = 99.02532 cm^2
Area c_2 = 29.95516 cm^2
Area c_3 = 9.06144 cm^2
$\sqrt{(\text{Area } c_1) \cdot (\text{Area } c_3)}$-(Area c_2) = 0.00000 cm^2

110706

Unit.

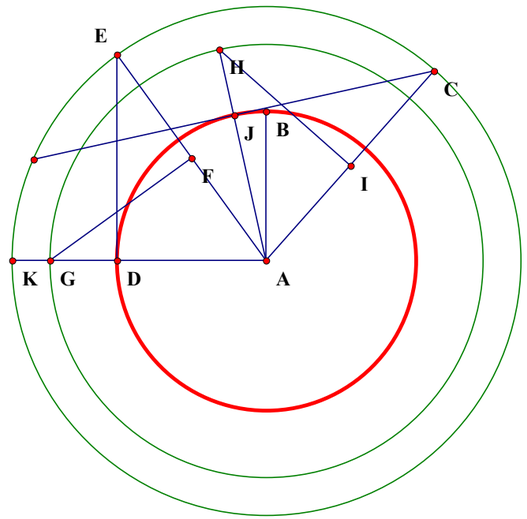
Given.

$N_1 := 3.12208$ $AD := N_1$

$N_2 := 5.31813$ $AC := N_2$

Going around in a circle

What is the tanget to AD from C? Although no one in their right mind would do this for a circle, it is essential as an introduction to finding the inverse ellilpse for any external point to it.



Descriptions.

$AE := AC$

$DE := \sqrt{AE^2 - AD^2}$ $AF := \frac{AE}{2}$

$AG := \frac{AE \cdot AF}{AD}$ $AH := AG$ $AI := \frac{AC}{2}$

$AJ := \frac{AI \cdot AC}{AH}$ $AJ - N_1 = 0$

$CJ := \sqrt{AC^2 - AJ^2}$ $CJ = 4.305244$

Definitions.

$AE - N_2 = 0$

$DE - \sqrt{N_2^2 - N_1^2} = 0$ $AF - \frac{N_2}{2} = 0$

$AG - \frac{N_2^2}{2 \cdot N_1} = 0$ $AH - \frac{N_2^2}{2 \cdot N_1} = 0$ $AI - \frac{N_2}{2} = 0$

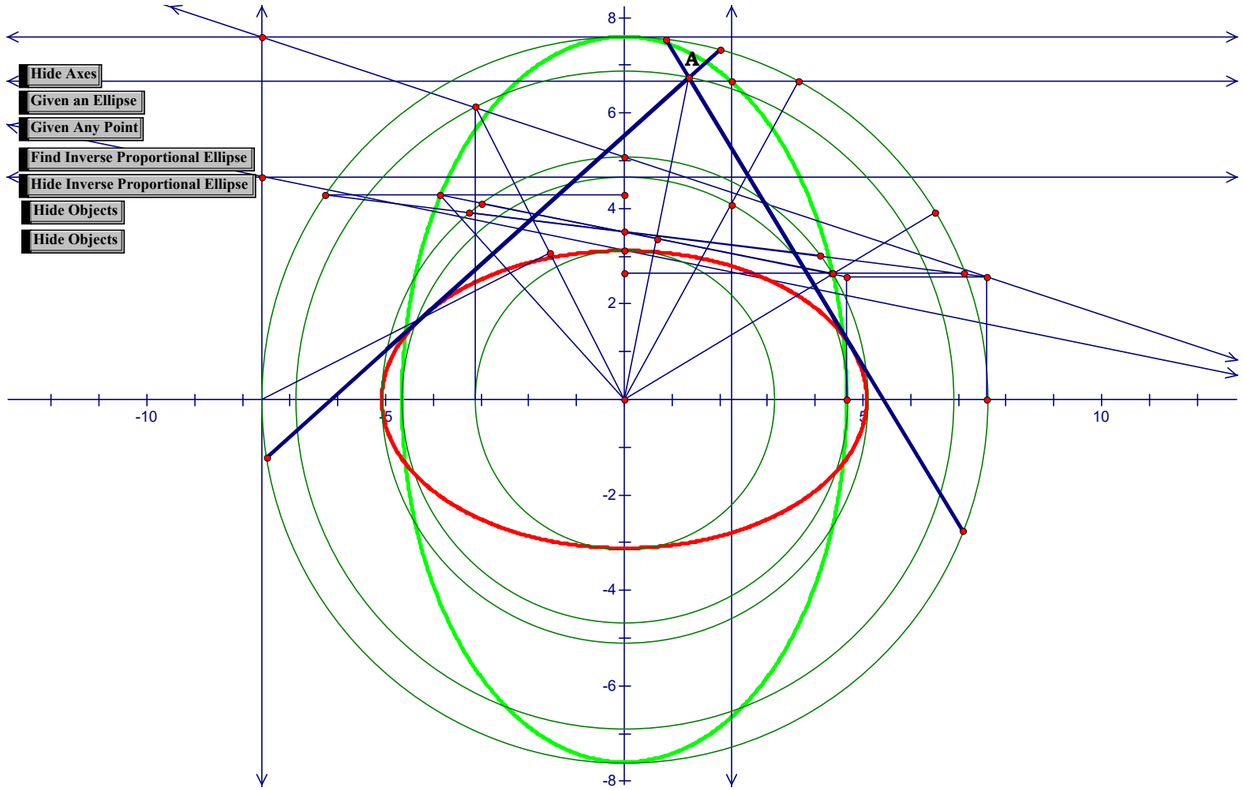
$AJ - N_1 = 0$ $CJ - \sqrt{N_2^2 - N_1^2} = 0$

Unit.
Given.

Parcing project for Step by Step

Descriptions.
Definitions.

From any point A find the tangents to an ellipse

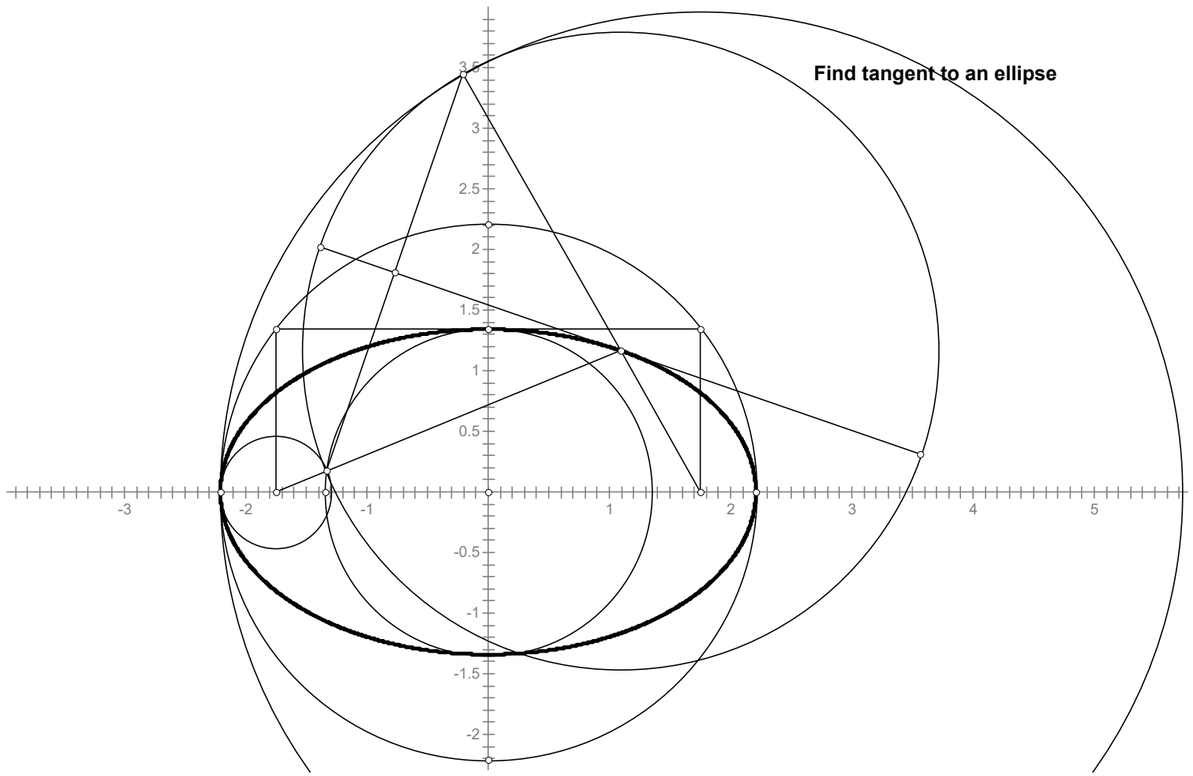


Unit.
Given.

Descriptions.
Definitions.

Parcing project for 110706u

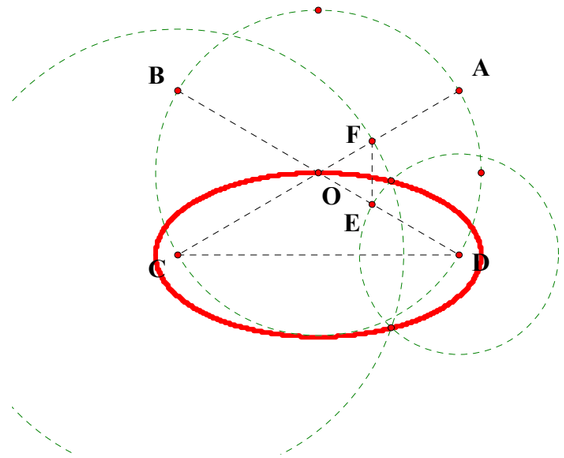
the tanget for any point on an ellipse



Unit.
Given.
Descriptions.
Definitions.

Really? Is this as far as you got on this project?

2008-0611



060807

Unit.

Given.

$N_1 := 3.38667$ $\quad AB := N_1$

$N_2 := 2.05199$ $\quad AI := N_2$

$N_3 := 3.57039$ $\quad CE := N_3$

$N_4 := .94919$ $\quad CD := N_4$

Equation for an Ellipse

Descriptions.

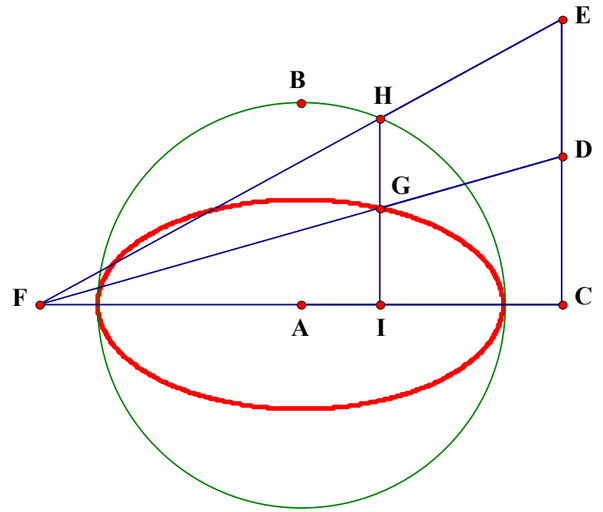
$$HI := \sqrt{(AB + AI) \cdot (AB - AI)}$$

$$GI := \frac{CD \cdot HI}{CE} \qquad AG := \sqrt{AI^2 + GI^2}$$

Definitions.

$$\frac{N_4 \cdot \sqrt{(N_1 + N_2) \cdot (N_1 - N_2)}}{N_3} - GI = 0$$

$$\frac{1}{N_3} \cdot \sqrt{N_2^2 \cdot N_3^2 + N_4^2 \cdot N_1^2 - N_4^2 \cdot N_2^2} - AG = 0$$



Unit. **Procrastinated Write up for 061307**

Given.

The blue circle is always tangent to both and its center is always on the ellipse, Therefore, it always resides in a cresant and, it switches between inclusion and exclusion at the intersection of all three for each of the other circles. This means that it is aways excluded from an area included in the other two.

Descriptions.

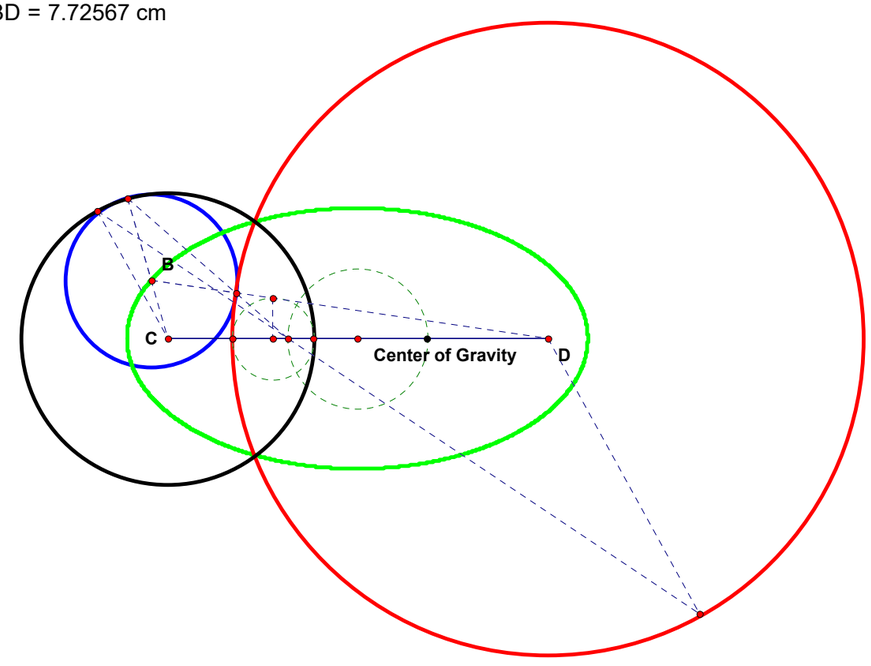
Definitions.

Animate Point

BC = 1.00975 cm

BD = 6.71592 cm

BC+BD = 7.72567 cm



062007C

Descriptions.

Unit.

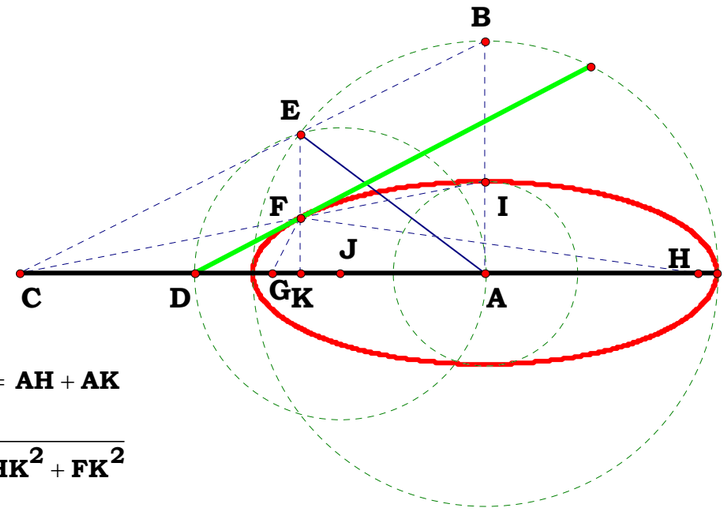
$AB := 1$

Given.

$N_1 := .39368$ $AI := N_1$

$N_2 := 1.25170$ $AD := N_2$

Tangent from Major Axis



$AJ := \frac{AD}{2}$ From 080193

$EK := \frac{AB \cdot \sqrt{(2 \cdot AJ - AB) \cdot (2 \cdot AJ + AB)}}{2 \cdot AJ}$

$AK := \sqrt{AB^2 - EK^2}$ $DK := AD - AK$ $FK := \frac{AI \cdot EK}{AB}$

$DF := \sqrt{DK^2 + FK^2}$ $AG := \sqrt{AB^2 - AI^2}$ $AH := AG$ $HK := AH + AK$

$GK := AG - AK$ $FG := \sqrt{FK^2 + GK^2}$ $FM := FG$ $FH := \sqrt{HK^2 + FK^2}$

$HM := FH - FM$ $HO := \frac{HK \cdot HM}{FH}$ $GH := 2 \cdot AH$ $GO := GH - HO$

$MO := \frac{FK \cdot HM}{FH}$ $\frac{GO}{MO} - \frac{DK}{FK} = 0$

Definitions.

$N_1 \cdot \frac{\sqrt{(N_2 - 1) \cdot (N_2 + 1)}}{N_2} - FK = 0$

062007D

Descriptions.

Unit.

$AB := 1$

Given.

$N_1 := .40636$ $AI := N_1$

$N_2 := .60804$ $AD := N_2$

$AJ := \frac{AD}{2}$ $PK := \frac{AI \cdot \sqrt{(2 \cdot AJ - AI) \cdot (2 \cdot AJ + AI)}}{2 \cdot AJ}$ $AK := \sqrt{AI^2 - PK^2}$

$DK := AD - AK$ $FK := \frac{PK \cdot AB}{AI}$ $DF := \sqrt{DK^2 + FK^2}$ $AG := \sqrt{AB^2 - AI^2}$

$AH := AG$ $HR := AH + FK$ $GR := AG - FK$ $FG := \sqrt{AK^2 + GR^2}$ $FM := FG$

$FH := \sqrt{HR^2 + AK^2}$ $HM := FH - FM$ $HO := \frac{HR \cdot HM}{FH}$ $GH := 2 \cdot AH$

$AK \cdot HM$ GO FK

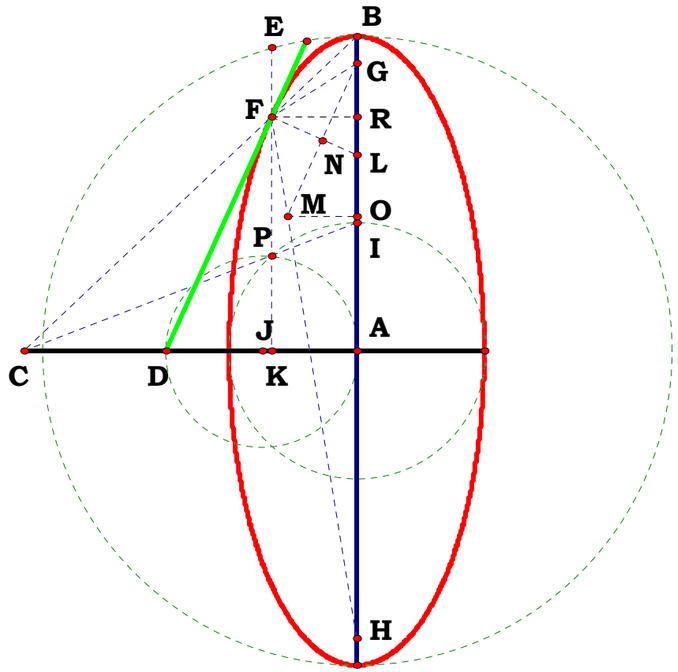
$GO := GH - HO$ $MO := \frac{AK \cdot HM}{FH}$ $\frac{GO}{MO} - \frac{FK}{DK} = 0$

Definitions.

$\frac{\sqrt{(N_2 - N_1) \cdot (N_2 + N_1)}}{N_2} - FK = 0$

$Major := N_1 \cdot \frac{\sqrt{(N_2 - 1) \cdot (N_2 + 1)}}{N_2}$

Tangent from Minor Axis



062407

Unit.

$AB := 1$

Given.

$N_1 := .64776$ $\quad AC := N_1$

$N_2 := .5444$ $\quad EJ := N_2$

Descriptions.

$AJ := \sqrt{AB^2 - AC^2}$ $\quad FI := (AB - EJ) + AB$ $\quad DE := \sqrt{AJ^2 - (AB - EJ)^2}$

$AN := \dfrac{\sqrt{DE^2 \cdot (DE + AB) \cdot (-DE + AB)}}{DE}$ $\quad DN := AB - AN$ $\quad GJ := EJ - DN$

$HI := FI - DN$ $\quad GL := \dfrac{GJ \cdot 2 \cdot DE}{GJ + HI}$ $\quad HL := 2 \cdot DE - GL$

$JL := \sqrt{GL^2 + GJ^2}$ $\quad IL := \sqrt{HI^2 + HL^2}$ $\quad (JL + IL) - 2 \cdot AB = 0$

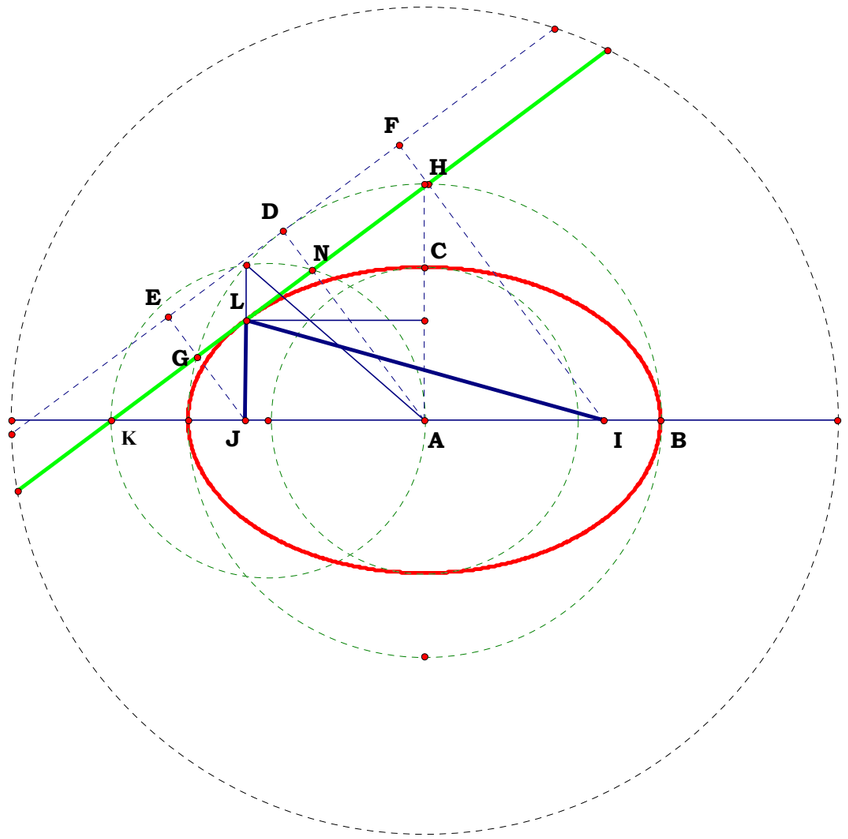
$IL = 1.575298$ $\quad JL = 0.424702$

Definitions.

Found on the Internet

Found the construction, now I explore it with Algebra.

The raw construction was found on the internet. I added the structures required for writing it up in my usual algebraic method.



AB = 3.95817 cm
AC = 2.56393 cm
EJ = 2.15482 cm
JL = 1.68103 cm
IL = 6.23530 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AC}{AB} = 0.64776$

$\frac{EJ}{AB} = 0.54440$

$\frac{JL}{AB} = 0.42470$

$\frac{IL}{AB} = 1.57530$

Definitions:

$$AJ - \sqrt{\left(1 - N_1^{\,2}\right)} = 0 \qquad FI - \left(2 - N_2\right) = 0$$

$$DE - \sqrt{\left(2 \cdot N_2 - N_2^{\,2} - N_1^{\,2}\right)} = 0 \qquad \sqrt{\left(N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1\right)} - AN = 0$$

$$1 - \sqrt{\left(N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1\right)} - DN = 0$$

$$N_2 + \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - 1 - GJ = 0$$

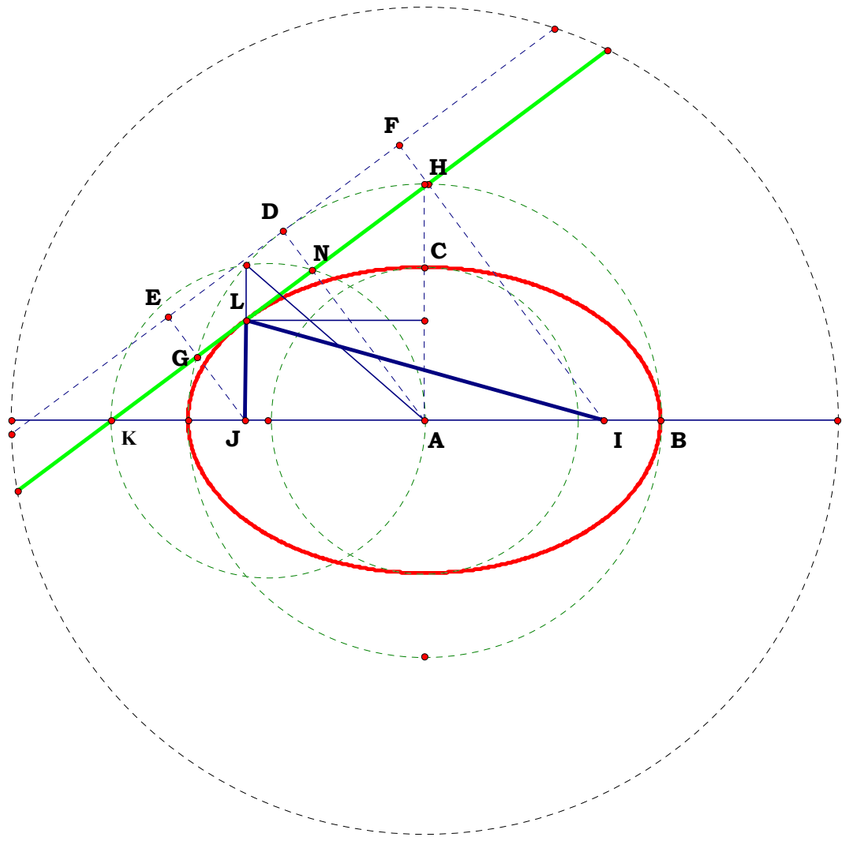
$$\left(\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - N_2 + 1\right) - HI = 0$$

$$\frac{\sqrt{2 \cdot N_2 - N_2^{\,2} - N_1^{\,2}} \cdot \left(N_2 + \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - 1\right)}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - GL = 0$$

$$\frac{\sqrt{2 \cdot N_2 - N_2^{\,2} - N_1^{\,2}} \cdot \left(\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} - N_2 + 1\right)}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - HL = 0$$

$$\frac{\sqrt{N_1^{\,2} - 4 \cdot N_2 + 2 \cdot N_2^{\,2} + 2 \cdot \left(N_2 - 1\right) \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} + 2}}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - JL = 0$$

$$\frac{\sqrt{N_1^{\,2} - 4 \cdot N_2 + 2 \cdot N_2^{\,2} - 2 \cdot \left(N_2 - 1\right) \cdot \sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1} + 2}}{\sqrt{N_1^{\,2} + N_2^{\,2} - 2 \cdot N_2 + 1}} - IL = 0$$



AB = 3.95817 cm
AC = 2.56393 cm
EJ = 2.15482 cm
JL = 1.68103 cm
IL = 6.23530 cm

$\frac{AB}{AB} = 1.00000$

$\frac{AC}{AB} = 0.64776$

$\frac{EJ}{AB} = 0.54440$

$\frac{JL}{AB} = 0.42470$

$\frac{IL}{AB} = 1.57530$

Unit.

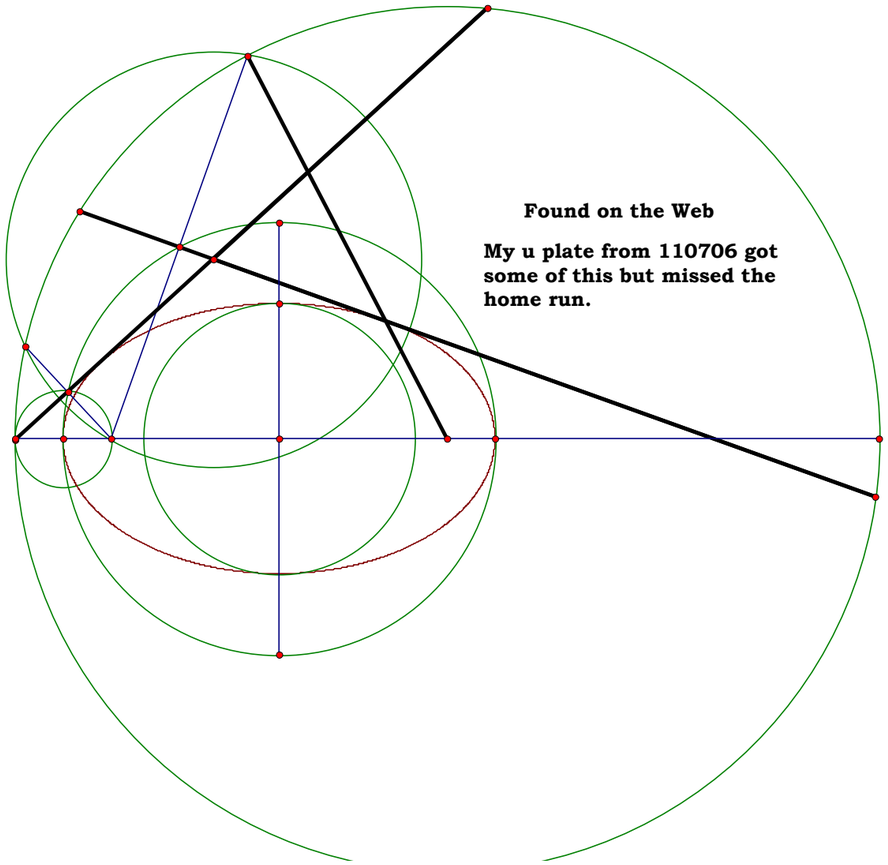
Given.

Parcing project for 062507

This is the second plate on the web I found for tangents, and it is quite good.

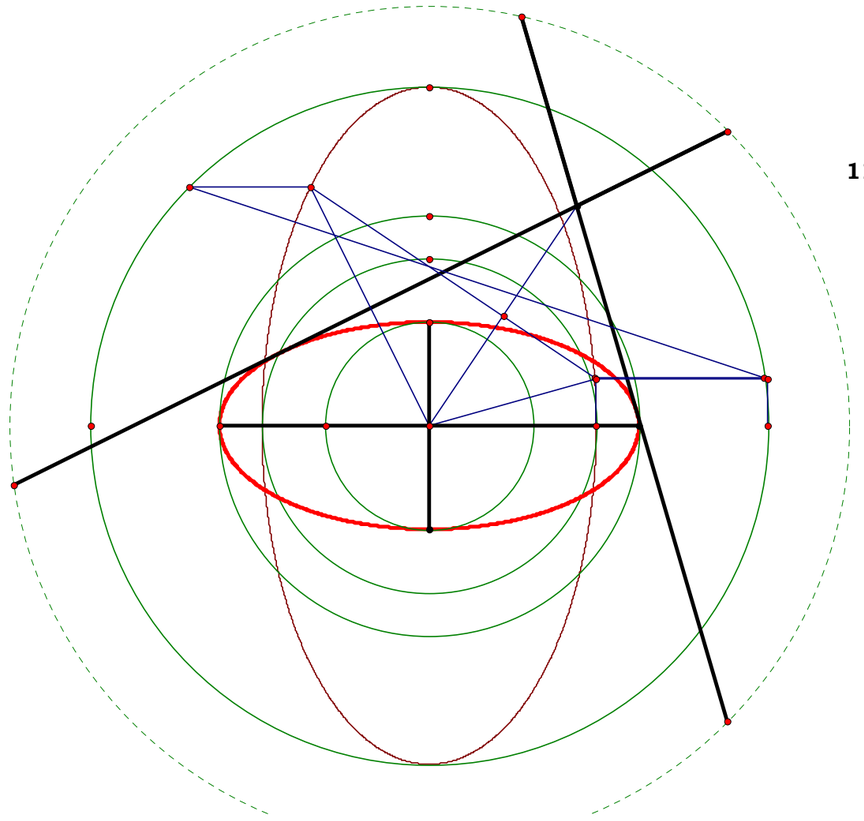
Descriptions.

Definitions.



I did find a cleaner plate for 110706 here

I call it the inverse ellipse method. I should at least example the stps in construction.



Unit.
Given.

Descriptions.
Definitions.

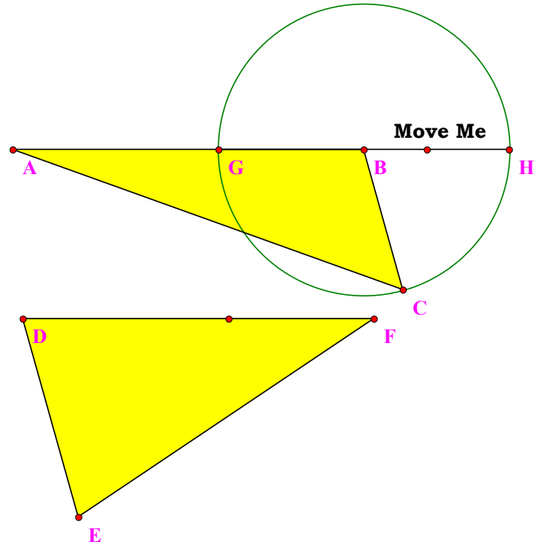
Parcing project for 072707

Area △ABC = 6.89734 cm^2
Area △DEF = 9.76888 cm^2
$\frac{\text{(Area △DEF)}}{\text{(Area △ABC)}}$ = 1.41633

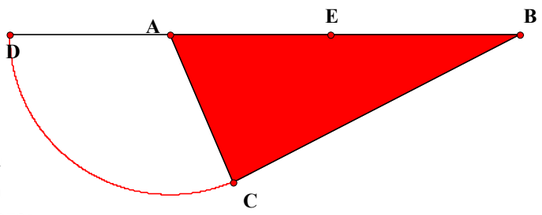
AB = 5.88433 cm
BG = 2.43524 cm

$\frac{AB}{BG}$ = 2.41633

m∠FDE = 74.29322°
m∠HBC = 74.29322°

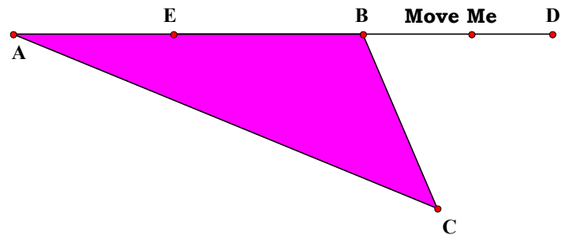


A series of plates exploring the relationship between an angle common to two figures which maintain a constant ratio in area.



AB = 5.86317 cm
BE = 3.17743 cm
m∠BAC = 66.89978°
m∠DBC = 66.89978°
$\frac{m\angle BAC}{m\angle DBC}$ = 1.00000

Area △ACB = 7.24215 cm^2
Area △ABC = 8.56804 cm^2
$\frac{\text{(Area △ACB)}}{\text{(Area △ABC)}}$ = 0.84525



My Name is John.

Hello. My name is John and I am going to explain how to multiply and divide a line by a line in Geometry. Now, if you are going to ask me if I am a geometer, I have to reply by myth. Explanation by myth is one the ancient Greek's methods of teaching by discourse.

Once upon a time, God created man; They created him male and female, in the image of God. Or one can say, male and female created They him, which is rather awkward, but it does have that ancient New England flair to it. At any rate, once upon a time is not this time. It came to pass as men multiplied on the earth that men started to work for a living and not being god's themselves needed a way to designate each other and so individuals, which are not, by definition man started calling each other by their craft. That is where we got Mr. Smith and Mr. Clark, etc. A vestige of this remains today. Not being man, we tend to think of each other by an assigned craft. I work in a factory, but my name is Clark. The conflict of course is why I spend the entirety of my wages in therapy.

Now this works to my advantage. I have learned that individuals calling themselves geometers (I am personally hoping for the day I become part of man) cannot multiply and divide a line by a line. So, I guess one could say, that a geometer is someone who cannot do the math, which is really a sign for some serious expenditure on therapy—and, if those in mind—field knew what they were doing, the outlay would be advantageous. Too bad they cannot define a man. Now a non-Euclidean Geometer is someone who not only cannot do the math, they demand, as part of their initiation rights, that one will never be able to do the math. So, in due respect to non-Euclidean Geometers, please stop reading and go back to your scribbling—and contradicting yourself. Doing geometry inside of or on the outside of a tennis ball, or a Frisbee, makes me think that one has spent way to many days on the court, spiking one's tea, and certainly missing the ball.

Now, if your like me, a factory worker, and someone were to give you two lines and say,

Hey, you (He is hairy and has a club). Here are two lines, show me how to multiply one by the other, and after that, show me how to divide one by the other.

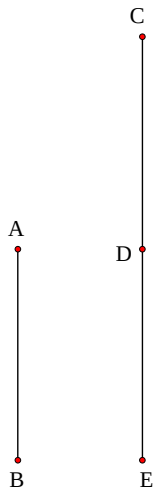
I would look at the man, think for a moment and draw a blank. What the heck does he mean? Then I would say, I am sorry, but I don't understand

what you mean. The man would leave off and I would go get another cup of coffee.

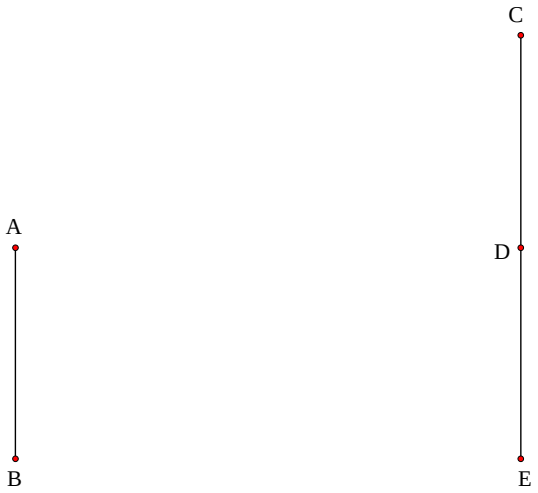
If I were a bit strange, I would consult Euclid's ***Elements*** and find to my dismay, the chap could do the math, but seems to have left this off for some reason, probably because it was too easy (so who don't lie for a friend?). Now, I happen to have in my possession a number of unpublished manuscripts which does have the answer in them and they are full of doing the math. I acquired them from the God's (and for those of you interested, the Delian Problem does have a solution—and it has something to do with Plato under extending himself). If it should be discovered that I am stealing a bit of fire, and giving it to man, please don't tell where you got it from. I have learned from first hand experience, you don't want to mess with Them—they be giants—really, really, big giants.

Now I am not going to explain this exactly as it was explained to me, as I have a poor memory. Please bear with me.

If I were given two lines, and asked to compare them, I would look at them and say;

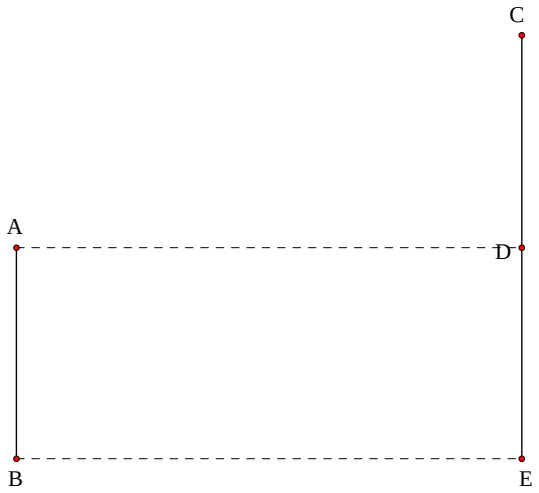


well, AB is shorter than CE. I mean, what can you do with two lines anyway. Reminds me of when I was a kid asking my mother what could I do with seven cents, realizing early on I was three cents short of a dime. If I were Euclid I would subtract one from the other and find that CE — AB = CD, or if you're a top down programmer, CE — AB = DE. If I move CE off a ways,

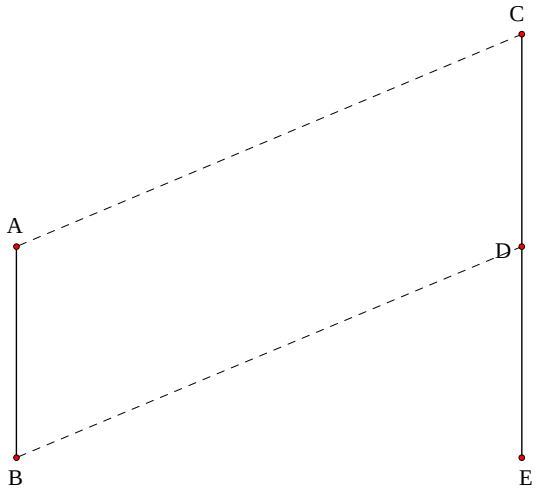


I would say that CE — AB = CD, or DE which ever you choose. Non-Euclidean Geometers, like Einstein, claim that this equality, this simultaneity, is not true and that at some point of moving AB and CE apart, as if it were part of the equation, does mysterious things to these segments. It amounts to a thief's logic—moving CE off sufficiently will make AB infinitely greater than CE 'cause we exact a kind of tribute on it and subtract that tribute as we go. It amounts to constructing a square say, of 25 square inches or so, and claiming if we repeat it enough, well, it just plain disappears—we wore it out. While on the other hand, there are those who claim that if I assert a point an infinite number of times, I can create a line. You know, like waving a knife in the air an infinite number of times an making a salad. This is the kind of mentality that makes credit card lenders rich. As I said, non-Euclidean Geometers are really crooked bankers in disguise—or really lousy cooks. A basic fact of abstraction, when you really know that a boundary is not the difference (a point is that which has not part), a form is in fact absolute, you know you can never attribute difference to that form, the form is applied as a boundary to any given difference—material. The cut is not the cutted! Wow, that was trashy!

Now if I had AB, and wanted to construct CE from it.



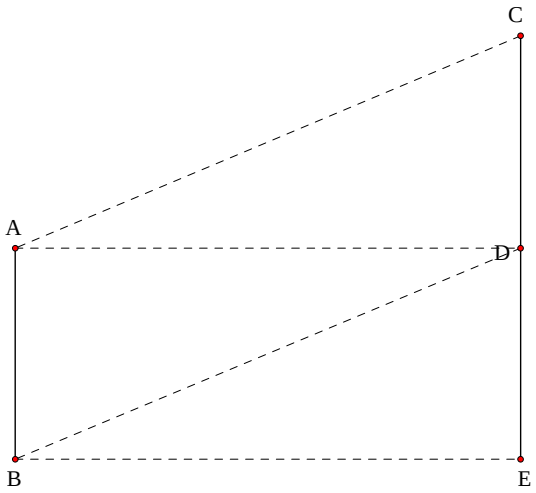
I could transfer one segment at a time



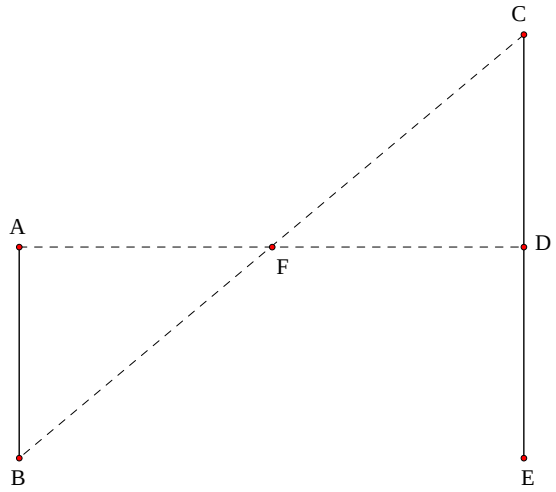
using parallel lines, but this is not multiplication, it is multiple processes, or simply addition. Parallel lines gives us the ability to do multiple additions, which is again not multiplication. One sign of that is that we have to assert each unit point in constructing CE. We have to assert each unit point just to do the parallels. Duh!

One of the things our ancient quibbling buddies, the Greeks, did tell us is that in order to multiply and divide, we have to have a unit. This is just part of plain simple Arithmetic. And they also said that when dealing with numbers in multiplication and division we were dealing with square and oblong (rectangular) numbers. Keep these ideas in mind. A square, an oblong, and a unit. Euclid drew a number of them. We will have need of them. For the moment let us learn what they did say about ratio,

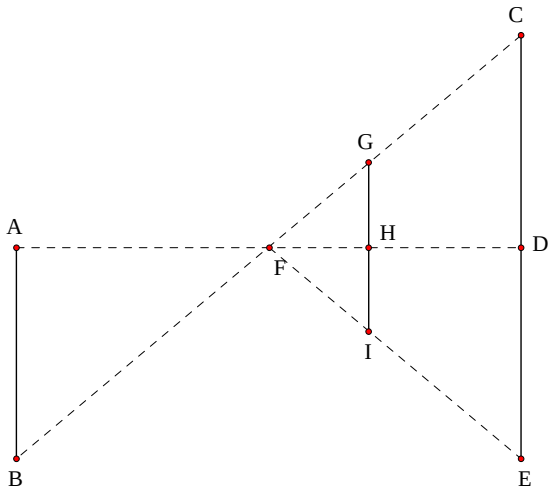
which we will also need. Now, if in constructing CE, we stayed up too late;—



and made a mistake in drawing—or were simply dyslexic;



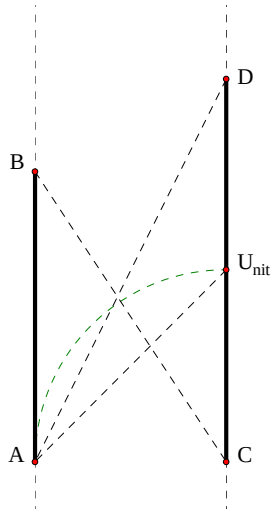
we would discover the ratio. As AB is to CD, so AF is to DF. And by George—(if you remember, he too was a hairy fellow and curious), One learns how to take any multiple and divide another segment of any length by the same multiple. From multiple addition, we have a kind of multiple division, but it is not division, it is still just a plain ratio, of anther segment.



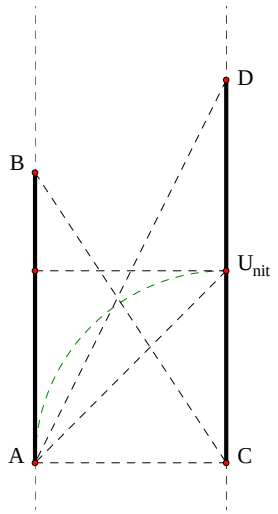
Now, as AB is to GH, so to DE is to HI, etc., etc. This is all fine and good, but, we still have not really learned how to multiply and divide. That is because these ratio's work regardless of the notion of unit, or square. Unless you are a crooked banker or a non-Euclidean Geometer, or a bad cook, this relationship is always true. There is one, and only one, difference between two points.

We are building our ideas up, one standard at a time. Intellectually, we fail, at the point we cannot abstract and use a standard—or what Plato called ***form*** because a boundary is not a difference and by definition (not a difference) always true. The divergence of language itself, starts with the inability to establish a standard even for a name. Many linguists call it the "growth" of language when meaning changes, but then they are non-Euclidean Geometers at heart also. What do they say of a government that has got its constitution saying exactly the opposite of what is written? If you want to reduce them to rubble, ask them outright, *Why can one word be or not be predicated of another*? Or again, if definition is conventional, and meaning can never be conventional, what in the heck does meaning have to do with definition? or even language? They will either get a funny look on their face mumbling to themselves, or start babbling non-sense to you. I have some books by the gods on that topic also. It is really simple, . . . but not here, not now.

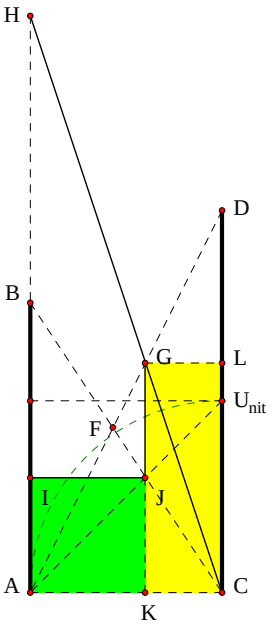
Multiplication and division rely on a standard in unit. So lets add that and see where we go.



At the outset the figure is very shy and unassuming. If you saw it laying in the street, you would hardly be pressed to pick it up. We have placed our segments the difference of our chosen unit apart, and we do have a square. No offence to Descartes who tried to find what I am doing, we don't have a number line, but a lined number. First time I ever seen a studious use of cross hairs actually miss the target.

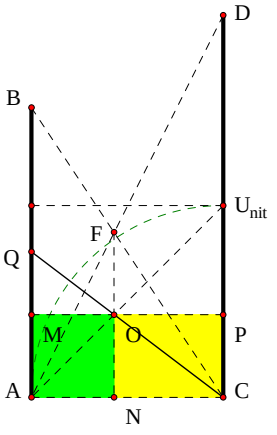


It don't look like much, but it can not only multiply and divide, one can use it to do much in the way of exponential manipulation as well. Let us take a closer look as to what the figure tells us.



$$\frac{\text{(Area } \triangle \text{BAC)}}{\text{(Area } \triangle \text{ADC)}} - \frac{\text{(Area IJKA)}}{\text{(Area KGLC)}} = 0.00000$$

This is how we perform multiplication. Given AC as our unit, AB × CD = AH. In order to see this using the Arithmetic Grammar system, We divide AC by AC and get 1, our Unit. We then divide AB by AC which gives AB in terms of our unit. We then divide CE by AC and acquire that in units, and again for AH. We will find that by using the notion of Unit, Square and Oblong Numbers, which is incorporated in the idea of ratio, we can Multiply. And we can do what no binary calculator will ever do, we do it exactly. What about division?



$$\frac{\text{(Area MANO)}}{\text{(Area ONCP)}} - \frac{\text{(Area } \triangle \text{BAC)}}{\text{(Area } \triangle \text{ADC)}} = 0.00000$$

Wouldn't you know it, there is a triplicate ratio in the figure! Right under our pencil. Didn't Euclid write that it was the hardest thing to do in geometry? Well, I have never taken geometry in school and set out to comprehend the triplicate ratio, guess I got somewhere. Going through our steps as before, we find that AB ÷ CD = AQ. Each of these steps is proven individually in Euclid. I suspect he was like Plato and wanted to see if his readers were smart enough to add and subtract ideas. And again, no binary computer will ever be up to Geometry, as Geometry is exact.

One can do a whole lot with this figure, through various projections. One can do a lot in the way of exponential manipulation. Try that with cross hairs! Some of the methods one will find in those unpublished books I was talking about. I don't know how long the gods will let me work on them, in fact, if it were not for Them, I would have been killed over thirty years ago. Imagine that, I am a walking contradiction, a living dead man. At any rate, I hope you have fun playing with the figure.

Now this is not the place to show the solution to the Delian Problem. My god, if one is just learning the simple four, by adding multiplication and division to our list of addition and subtraction, it may be too difficult realize a revolution in Euclidean Geometry based upon a standard long ago recognized but left unemployed—just like these. I will put the idea in the Geometer's Sketchpad file.

I hope I have made it clear that through multiple addition and subtraction, one leads into the understanding of ratio, just like Euclid did, but it is still a step away from multiplication and division. Those depend upon a respect for, and understanding of a standard in definition. We learn to add, and subtract. These teach us ratio—it is part of them. We learn about the units which is taught by them also. This then leads to multiplication and division and our primary four are thus established.

I do have some food for thought though. Using the facts of conventions in language, can you count the ways non-Euclidean geometries commit self-referential errors in simple logic? Apparently not, they are popular. Maybe it has something to do with linguist waving their knife in the air constructing sentences. What is prediction? Maybe I will read it to you sometime. The solution was once written on a Temple "Know Thyself." I will say this, as a sense system, the human mind is suppose to abstract form and create things with it. To deny form as the foundation for thought is simply a sign of dysfunction. I know, look at me.

Multiplication And Division of Lines

1. An unit is that by virtue of which each of the things that exist is called one.
Euclid's Elements

312

The Basic figures in this little thing are written up in my work Threee Pieces of Paper, or The Delian Quest. This is not a formal presentation, is a presentation of craft basics.

John Clark

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Printed via import to MS Word.

Following the Yellow Brick Road

Multiplication

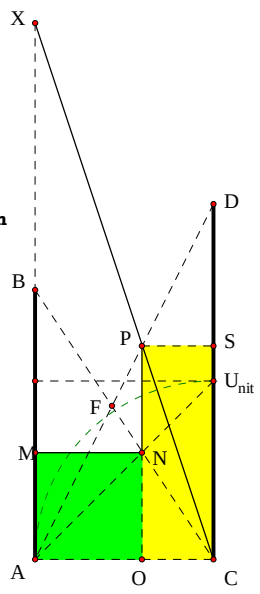
Unit = 1.00000

AB = 1.51095

CD = 1.99270

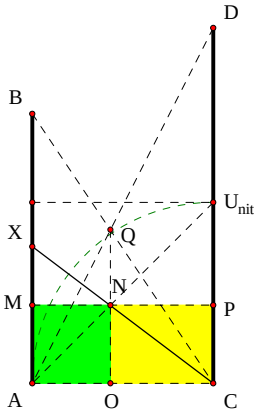
AX = 3.01087

AB·CD-AX = 0.00000



$$\frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} - \frac{\text{(Area MNOA)}}{\text{(Area OPSC)}} = 0.00000$$

In Ancient Geometry, one calls the Green Square Numbers, and one calls the yellow Oblong Numbers. Today we just do swatches.



$$\frac{\text{(Area MAON)}}{\text{(Area NOCP)}} - \frac{\text{(Area } \triangle\text{BAC)}}{\text{(Area } \triangle\text{ADC)}} = 0.00000$$

Division

Unit = 1.00000

AB = 1.48921

CD = 1.96403

AX = 0.75824

$$\frac{\text{AB}}{\text{CD}} - \text{AX} = 0.00000$$

Introduction

Maybe I am too dogmatic, but I think one should have geometry teach one something of basic math. One should be able to add, subtract, multiply and divide with lines. These can provide proofs and constructible.

The figures can be modified in various ways to produce various results. I present a few here. The main figure is composed of the notion of common unit, and that multiplication and division works with square numbers, which is distinct from squaring a number. The square thus constructed provides the properties needed for multiplication and division.

I once read, in an Algebra book, that exponential notation had nothing to do with Geometry, that it was a pure mental abstract. What am I, then, to do with all the figures I have come up with that display the principles?

I would also like to see how the four basic operations of Math hold up in "non-Euclidean" Geometries. In fact, as part of their presentation, I think the four basic operations of mathematics should be a requirement. Perhaps by teaching the remaining two in geometry, something about reality and standards of thought will be learned.

The material in this little flyer is not new to me, it is part of four works I am currently engaged in, The Delian Quest, which is essentially completed, it needs some lipstick and a dress, Three Pieces of Paper, Eloi, and something with a puny Latin name.

Oh, and no, I have never studied geometry in an institution-I have never seen ideas survive in an institution. I have and probably will be again, be institutionalized at my own request.

Contents

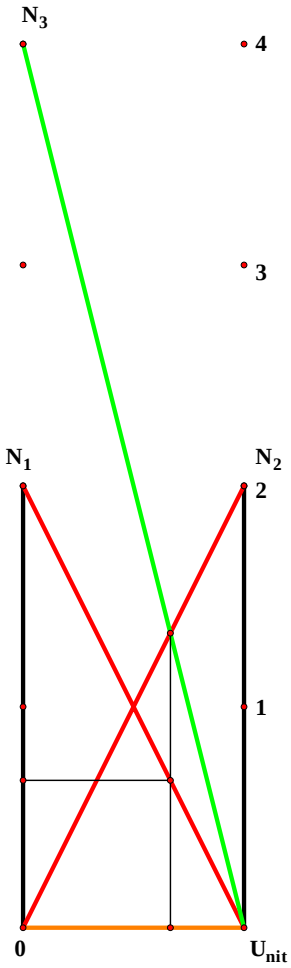
Function	Page
$(N_1 \cdot N_2) - N_3 = 0.00000$	Link to 1
$\frac{N_1}{N_2} = 1.13725$ $\frac{N_1+N_2}{N_1} = 1.87931$ $\frac{N_1+N_2}{N_2} = 2.13725$	Link to 2
$\frac{1}{N_1} - N_3 = 0.00000$	Link to 3
$\frac{N_1}{N_2^2} - N_4 = 0.00000$ $\frac{N_1}{N_2^3} - N_5 = 0.00000$	Link to 4
$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$	Link to 5
$\frac{N_1^2}{(N_2+N_1) \cdot N_2} - N_3 = 0.00000$	Link to 6
$(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$	Link to 7
$N_3^2 - \frac{BC}{BD} = 0.00000$ $N_3^3 - \frac{BC}{BE} = 0.00000$ $N_3^4 - \frac{BC}{BF} = 0.00000$	Link to 8
$\frac{N4_2}{N2_2} = 1.39420$ $\frac{N2_2}{N1_2} = 1.39420$ $\frac{N1_2}{Unit_2} = 1.39420$ $\frac{Unit_2}{N3_2} = 1.39420$	Link to 9
$\frac{N_1^2}{N_2} - N_5 = 0.00000$ $\frac{N_1^3}{N_2^2} - N_6 = 0.00000$ $\frac{N_2^2}{N_1} - N_7 = 0.00000$	Link to 10

Link to Introdution

Function	Page
$\frac{N_1^2}{N_2^2} - N_5 = 0.00000$ $N_2^2 - N_6 = 0.00000$	Link to 11
$\sqrt{2} - N_5 = 0.00000$ $\frac{\sqrt{2} \cdot N_1}{N_2} - N_6 = 0.00000$ $\frac{N_1 \cdot N_2}{\sqrt{2}} - N_7 = 0.00000$	Link to 12
$N_5 - 2^{0.75} = 0.00000$ $\left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$ $\frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$	Link to 13
$N_1^{0.5} - N_2 = 0.00000$ $N_1^{0.25} - N_3 = 0.00000$ $N_1^{0.125} - N_4 = 0.00000$	Link to 14
$\frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$ $\frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$ $N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$	Link to 15
$\frac{N_1^2}{(N_1+N_2) \cdot N_2} - L_1 = 0.00000$ $\frac{N_1^2 \cdot N_2}{N_1+N_2} - M_1 = 0.00000$	Link to 16
$\frac{N_1^2}{N_2 \cdot (N_1+1)} - L_1 = 0.00000$ $\frac{N_1^2 \cdot N_2}{N_1+1} - M_1 = 0.00000$	Link to 17
$\frac{N_1^2}{N_2^4} - N_7 = 0.00000$ $N_2^4 - N_8 = 0.00000$ $N_2^3 - N_{25} = 0.00000$	Link to 18
$\frac{N_2^4}{N_1} - N_{26} = 0.00000$ $\frac{N_1}{N_2} - N_{27} = 0.00000$ $\frac{N_2^7}{N_1} - N_8 = 0.00000$	

Contents

Function	Page	Function	Page
$N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$ $N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$	Link to 19		
$N_1 \cdot \left(\frac{N_1}{N_1+N_2}\right) - N_5 = 0.00000$ $N_1 \cdot \left(\frac{N_2}{N_1+N_2}\right) - N_6 = 0.00000$ $N_1 \cdot N_2 \cdot \left(\frac{N_1}{N_1+N_2}\right) - N_7 = 0.00000$	Link to 20		
$N_1^{\frac{1}{2}} - N_2 = 0.00000$ $N_1^{\frac{1}{4}} - N_3 = 0.00000$ $N_1^{\frac{1}{8}} - N_4 = 0.00000$	Link to 21		
$N_1^{\frac{8}{8}} - N_1 = 0.00000$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$	Link to 22		
$N_1^{\frac{5}{8}} - N_4 = 0.00000$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$			
$N_1^{\frac{2}{8}} - N_7 = 0.00000$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$			



$N_1 = 2.00000$

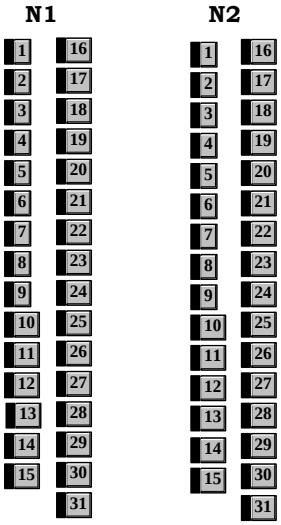
$N_2 = 2.00000$

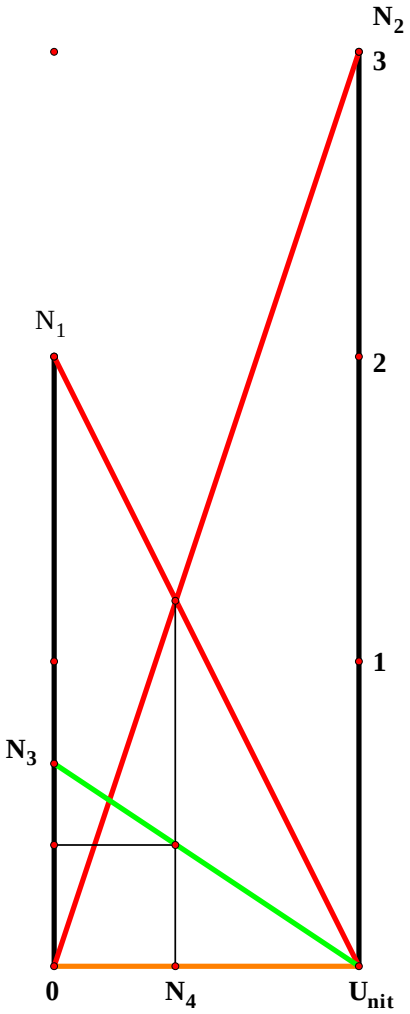
$N_1 \cdot N_2 = 4.00000$

$(N_1 \cdot N_2) - N_3 = 0.00000$

$N_3 = 4.00000$

Multiply N1 by N2





$N_1 = 2.00000$

$N_2 = 3.00000$

$\frac{N_1}{N_2} = 0.66667$

$N_3 = 0.66667$

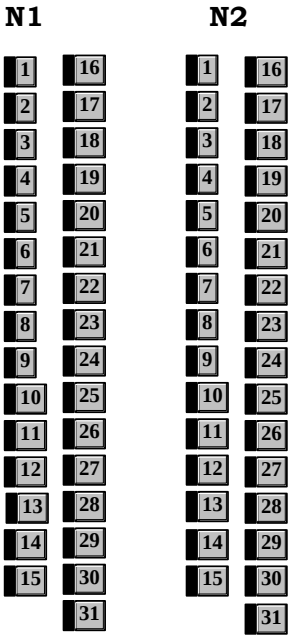
$\frac{U_{nit}}{0N_4} = 2.50000$

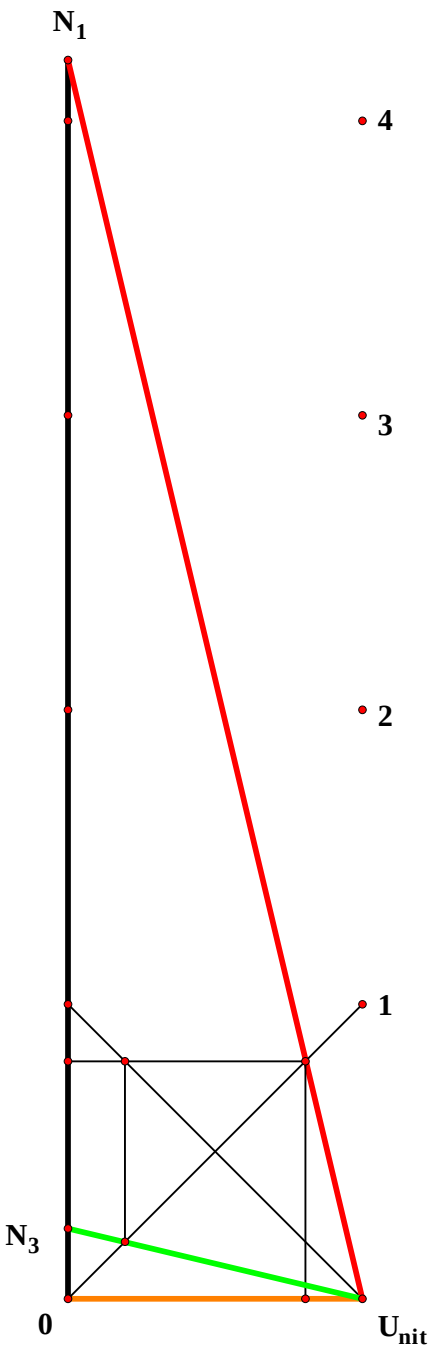
$\frac{N_1+N_2}{N_1} = 2.50000$

$\frac{U_{nit}}{U_{nit}N_4} = 1.66667$

$\frac{N_1+N_2}{N_2} = 1.66667$

Divide
N1 by N2





Find the Recprocal N1

$N_1 = 4.20930$

$\frac{1}{N_1} = 0.23757$

$N_3 = 0.23757$

$\frac{1}{N_1} - N_3 = 0.00000$

N1

1	16
2	17
3	18
4	19
5	20
6	21
7	22
8	23
9	24
10	25
11	26
12	27
13	28
14	29
15	30
	31

$N_1 = 3.00000$

$N_2 = 2.00000$

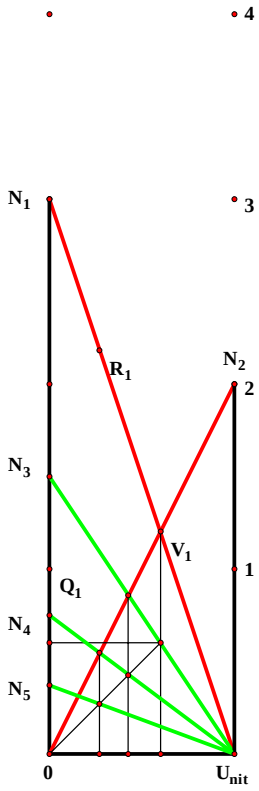
$\frac{N_1}{N_2} = 1.50000$

$N_3 = 1.50000$

$N_4 = 0.75000$ $\quad \frac{N_1}{N_2^2} - N_4 = 0.00000$

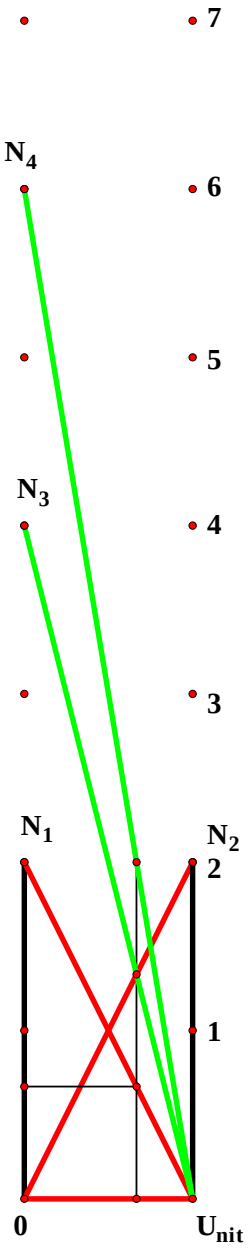
$N_5 = 0.37500$ $\quad \frac{N_1}{N_2^3} - N_5 = 0.00000$

etc.



Divide N1 by a Power of N2

	N1	N2
1	16	16
2	17	17
3	18	18
4	19	19
5	20	20
6	21	21
7	22	22
8	23	23
9	24	24
10	25	25
11	26	26
12	27	27
13	28	28
14	29	29
15	30	30
	31	31



$N_1 = 2.00000$

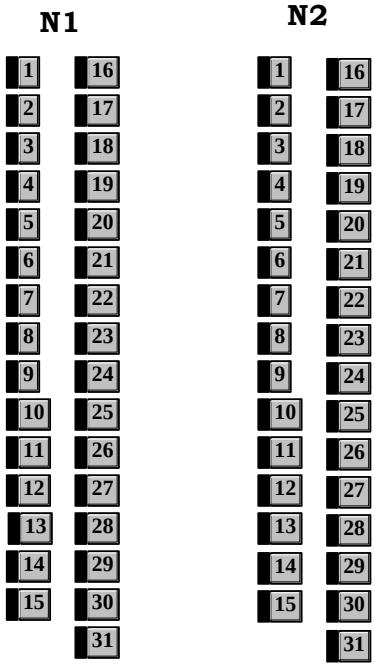
$N_2 = 2.00000$

$N_1 \cdot N_2 = 4.00000$

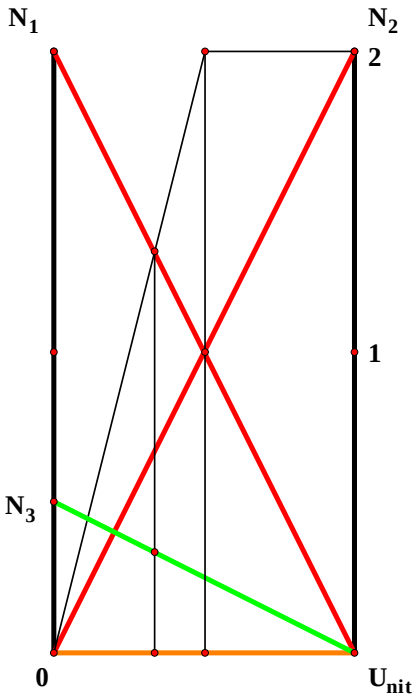
$N_3 = 4.00000$

$N_4 = 6.00000$

$2 \cdot N_1 \cdot N_2 - N_1 - N_4 = 0.00000$



3



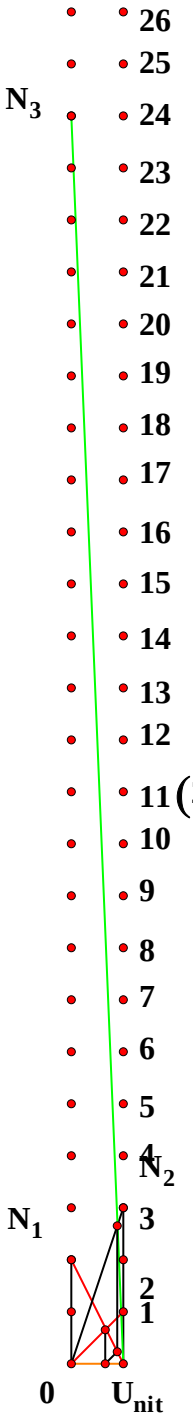
$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 0.50000$

$$\frac{N_1^2}{(N_2+N_1)\cdot N_2} - N_3 = 0.00000$$

N1		N2	
1	16	1	16
2	17	2	17
3	18	3	18
4	19	4	19
5	20	5	20
6	21	6	21
7	22	7	22
8	23	8	23
9	24	9	24
10	25	10	25
11	26	11	26
12	27	12	27
13	28	13	28
14	29	14	29
15	30	15	30
	31		31

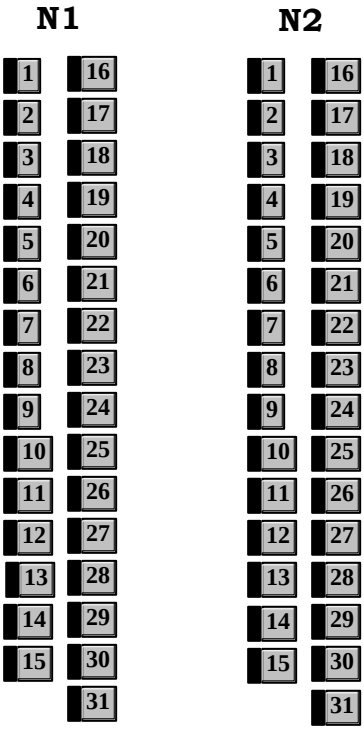


$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 24.00000$

$(2 \cdot N_1 \cdot N_2 + N_1^2 \cdot N_2) - N_3 = 0.00000$



$\frac{BC}{BV_1} = 0.81650$ $\quad N_3^{-0.5} = 0.81650$

$N_3^{0} = 1.00000$

$\frac{BC}{BU_1} = 1.22474$ $\quad N_3^{0.5} = 1.22474$

$\frac{BC}{BP_1} = 1.50000$

$\frac{BC}{BR_1} = 2.25000$ $\quad N_3^{2} - \frac{BC}{BR_1} = 0.00000$

$\frac{BC}{BS_1} = 3.37500$ $\quad N_3^{3} - \frac{BC}{BS_1} = 0.00000$

$\frac{BC}{BT_1} = 5.06250$ $\quad N_3^{4} - \frac{BC}{BT_1} = 0.00000$

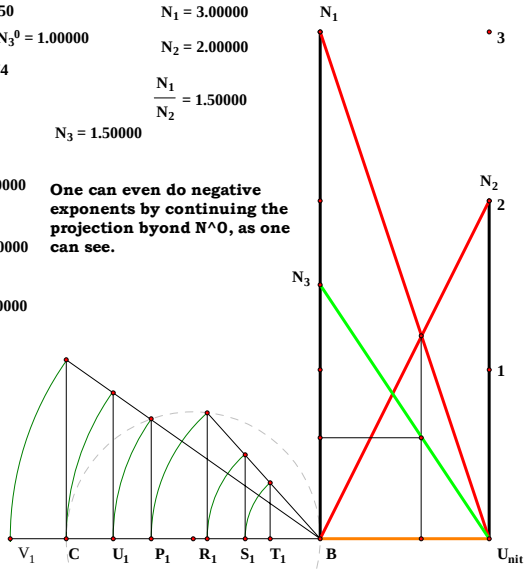
$N_1 = 3.00000$

$N_2 = 2.00000$

$\frac{N_1}{N_2} = 1.50000$

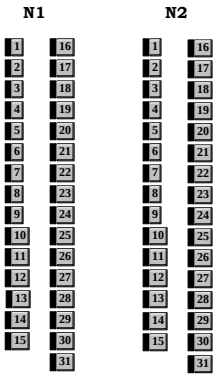
$N_3 = 1.50000$

One can even do negative exponents by continuing the projection beyond N^0, as one can see.



Adding a figure to a figure.

I tossed this together, so it is not perfect--it just looks good



How to find a Proportional Unit for a given pair of Magnitudes

$Unit_2 = 1.00000$

$N1_2 = 1.39420$

$N2_2 = 1.94380$

$N3_2 = 0.71726$

$N4_2 = 2.71004$

$\frac{N1_2}{N2_2} - N3_2 = 0.00000$

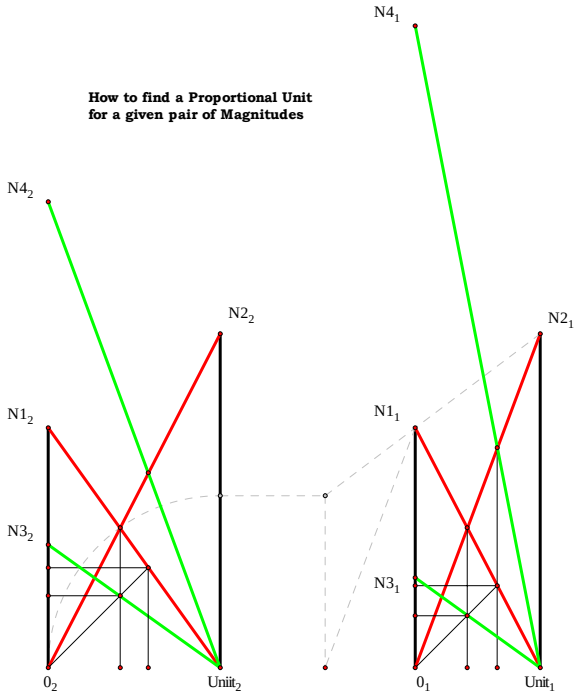
$N1_2 \cdot N2_2 - N4_2 = 0.00000$

$\frac{N4_2}{N2_2} = 1.39420$

$\frac{N2_2}{N1_2} = 1.39420$

$\frac{N1_2}{Unit_2} = 1.39420$

$\frac{Unit_2}{N3_2} = 1.39420$



$Unit_1 = 1.00000$

$N1_1 = 1.91861$

$N2_1 = 2.67492$

$N3_1 = 0.71726$

$N4_1 = 5.13213$

$\frac{N1_1}{N2_1} - N3_1 = 0.00000$

$N1_1 \cdot N2_1 - N4_1 = 0.00000$

$\frac{N4_1}{N2_1} = 1.91861$

$\frac{N2_1}{N1_1} = 1.39420$

$\frac{N1_1}{Unit_1} = 1.91861$

$\frac{Unit_1}{N3_1} = 1.39420$

Unit = 1.00000

$N_1 = 2.00000$

$N_2 = 3.00000$

$N_3 = 0.66667$

$N_4 = 6.00000$

$N_5 = 1.33333$

$N_6 = 0.88889$

$N_7 = 4.50000$

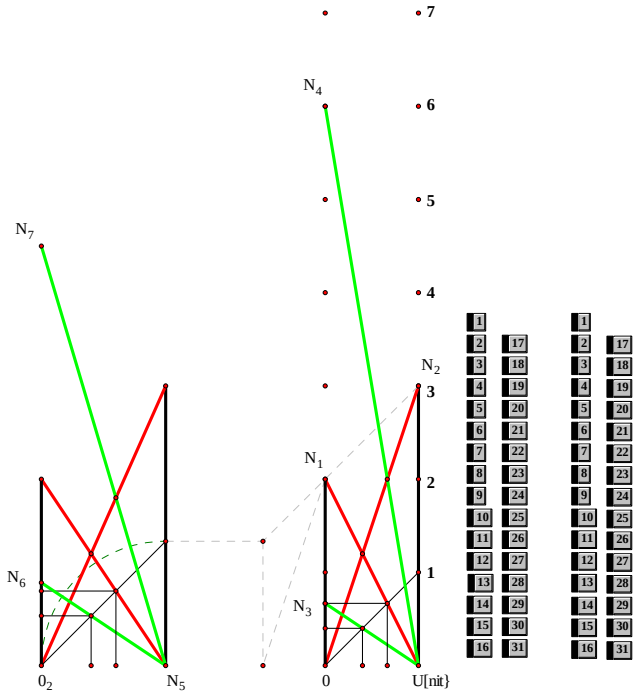
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1^2}{N_2} - N_5 = 0.00000$$

$$\frac{N_1^3}{N_2^2} - N_6 = 0.00000$$

$$\frac{N_2^2}{N_1} - N_7 = 0.00000$$



Unit = 1.00000

N_1 = 2.00000

N_2 = 2.30952

N_3 = 0.86598

N_4 = 4.61905

N_5 = 0.74992

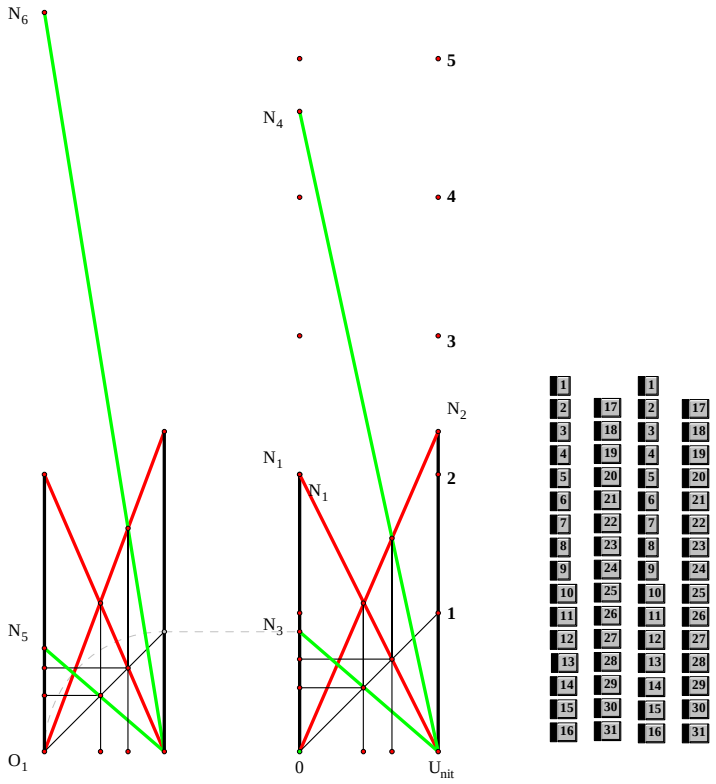
N_6 = 5.33390

$\frac{N_1}{N_2} - N_3 = 0.00000$

$N_1 \cdot N_2 - N_4 = 0.00000$

$\frac{N_1^2}{N_2^2} - N_5 = 0.00000$

$N_2^2 - N_6 = 0.00000$



Unit = 1.00000

N_1 = 2.00000

N_2 = 2.43939

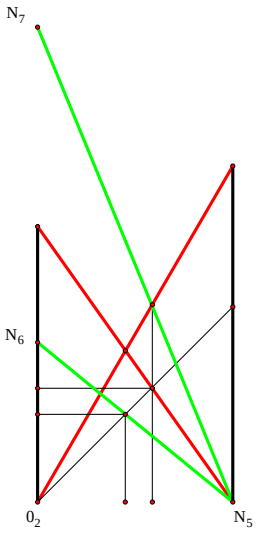
N_3 = 0.81988 $\frac{N_1}{N_2}-N_3 = 0.00000$

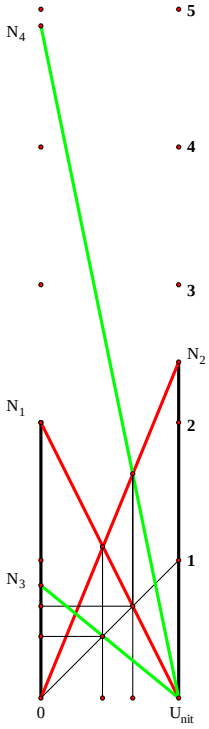
N_4 = 4.87879 $N_1 \cdot N_2 - N_4 = 0.00000$

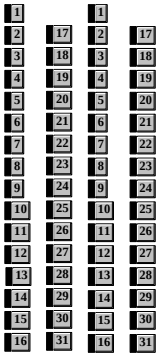
N_5 = 1.41421 $\sqrt{2}-N_5 = 0.00000$

N_6 = 1.15948 $\frac{\sqrt{2}\cdot N_1}{N_2}-N_6 = 0.00000$

N_7 = 3.44982 $\frac{N_1 \cdot N_2}{\sqrt{2}}-N_7 = 0.00000$







Unit = 1.00000

N_1 = 2.00000

N_2 = 2.00000 $\quad \frac{N_1}{N_2} - N_3 = 0.00000$

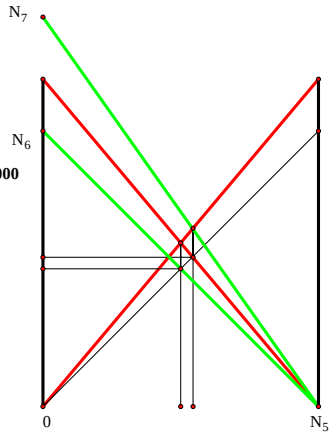
N_3 = 1.00000

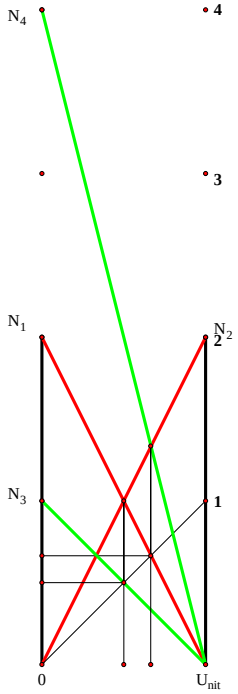
N_4 = 4.00000 $\quad N_1 \cdot N_2 - N_4 = 0.00000$

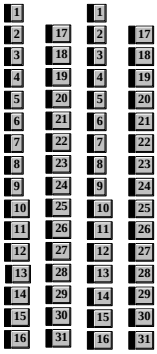
N_5 = 1.68179 $\quad N_5 - 2^{0.75} = 0.00000$

N_6 = 1.68179 $\quad \left(\frac{N_1}{N_2}\right) \cdot N_5 - N_6 = 0.00000$

N_7 = 2.37841 $\quad \frac{N_1 \cdot N_2}{N_5} - N_7 = 0.00000$







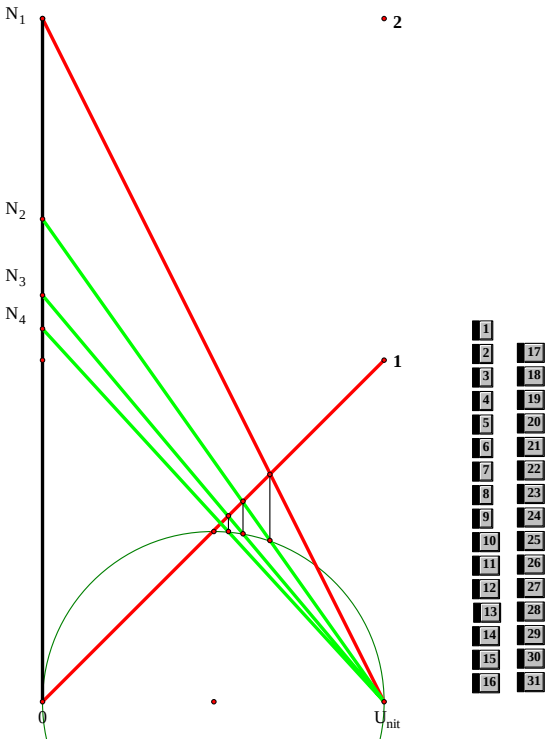
Unit = 1.00000

N_1 = 2.00000

N_2 = 1.41421 **$N_1^{0.5}$-N_2 = 0.00000**

N_3 = 1.18921 **$N_1^{0.25}$-N_3 = 0.00000**

N_4 = 1.09051 **$N_1^{0.125}$-N_4 = 0.00000**



Unit = 1.00000

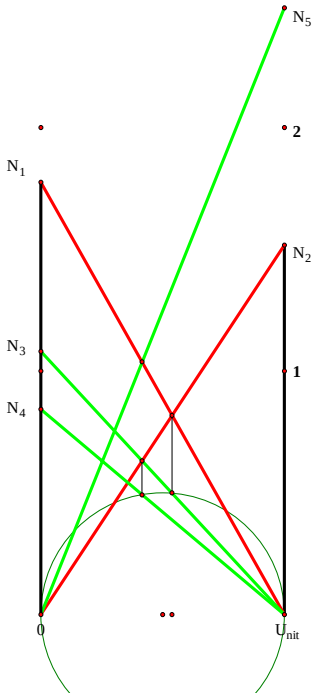
N_1 = 1.77542

N_2 = 1.51695

N_3 = 1.08185 $\quad \frac{N_1^{0.5}}{N_2^{0.5}} - N_3 = 0.00000$

N_4 = 0.84450 $\quad \frac{N_1^{0.25}}{N_2^{0.75}} - N_4 = 0.00000$

N_5 = 2.48947 $\quad N_1^{0.5} \cdot N_2^{1.5} - N_5 = 0.00000$



1	17	1	17
2	18	2	18
3	19	3	19
4	20	4	20
5	21	5	21
6	22	6	22
7	23	7	23
8	24	8	24
9	25	9	25
10	26	10	26
11	27	11	27
12	28	12	28
13	29	13	29
14	30	14	30
15	31	15	31
16		16	

Unit = 1.00000

N_1 = 1.00000

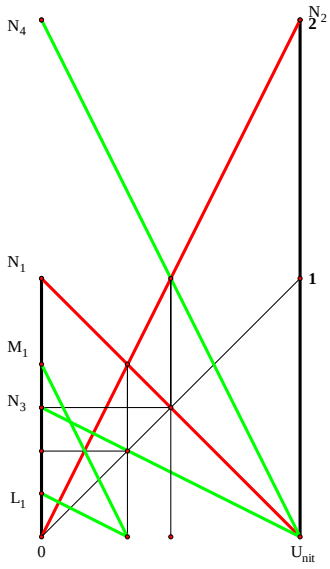
N_2 = 2.00000 $\frac{N_1}{N_2}-N_3 = 0.00000$

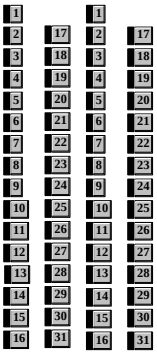
N_3 = 0.50000

N_4 = 2.00000 $N_1 \cdot N_2 - N_4 = 0.00000$

L_1 = 0.16667 $\frac{N_1^2}{(N_1+N_2)\cdot N_2}-L_1 = 0.00000$

M_1 = 0.66667 $\frac{N_1^2 \cdot N_2}{N_1+N_2}-M_1 = 0.00000$





Unit = 1.00000

N_1 = 2.00000

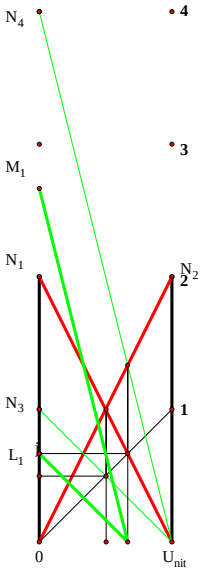
N_2 = 2.00000 $\frac{N_1}{N_2}$**-N_3 = 0.00000**

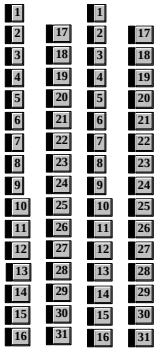
N_3 = 1.00000

N_4 = 4.00000 **$N_1 \cdot N_2 - N_4 = 0.00000$**

L_1 = 0.66667 $\frac{N_1^2}{N_2 \cdot (N_1+1)}$**-$L_1$ = 0.00000**

M_1 = 2.66667 $\frac{N_1^2 \cdot N_2}{N_1+1}$**-$M_1$ = 0.00000**





$U_1 = 1.00000$

$N_1 = 2.00000$

$N_2 = 2.00000$

$N_3 = 1.00000$ $\quad \frac{N_1}{N_2} = 1.00000$

$N_4 = 4.00000$ $\quad N_1 \cdot N_2 - N_4 = 0.00000$

$U[1] / U_2 = 0.25000$

$N_5 = 2.00000$

$N_6 = 2.00000$

$N_7 = 0.25000$ $\quad \frac{N_1^2}{N_2^4} - N_7 = 0.00000$

$N_8 = 16.00000$ $\quad N_2^4 - N_8 = 0.00000$

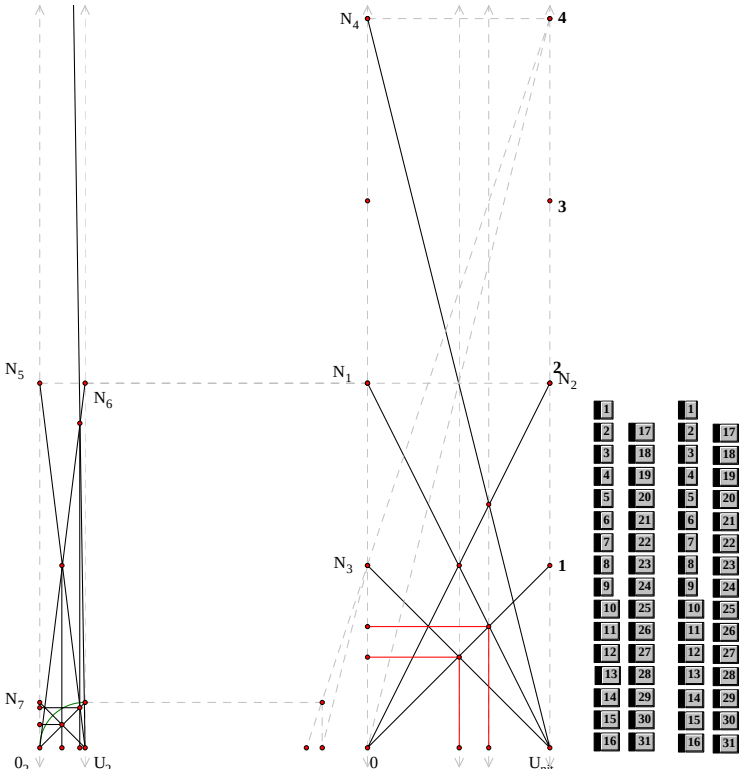
$U_2 = 1.00000$

$N_{25} = 8.00000$ $\quad N_2^3 - N_{25} = 0.00000$

$N_{26} = 8.00000$ $\quad \frac{N_2^4}{N_1} - N_{26} = 0.00000$

$N_{27} = 1.00000$ $\quad \frac{N_1}{N_2} - N_{27} = 0.00000$

$N_8 = 64.00000$ $\quad \frac{N_2^7}{N_1} - N_8 = 0.00000$



Unit = 1.00000

N_1 = 1.72159

N_2 = 2.12500

N_3 = 0.81016

N_4 = 3.65838

N_5 = 0.57386

N_6 = 1.14773

N_7 = 0.70833

N_8 = 1.41667

N_9 = 1.21946

N_{10} = 2.43892

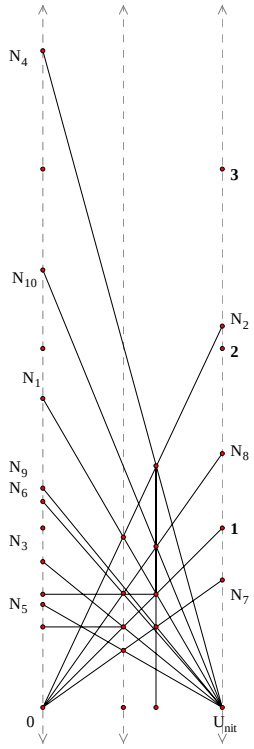
$$\frac{N_1}{N_2} - N_3 = 0.00000$$

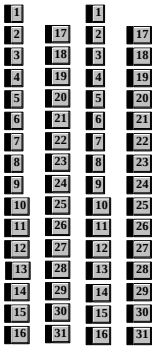
$$N_1 \cdot N_2 - N_4 = 0.00000$$

$$\frac{N_1}{N_2} = 0.81016$$

$$N_1 \cdot N_2 \cdot \left(\frac{1}{3}\right) - N_9 = 0.00000$$

$$N_1 \cdot N_2 \cdot \left(\frac{2}{3}\right) - N_{10} = 0.00000$$





Unit = 1.00000

N_1 = 1.37056

N_2 = 1.73096

N_3 = 0.79179 $\frac{N_1}{N_2} = 0.79179$

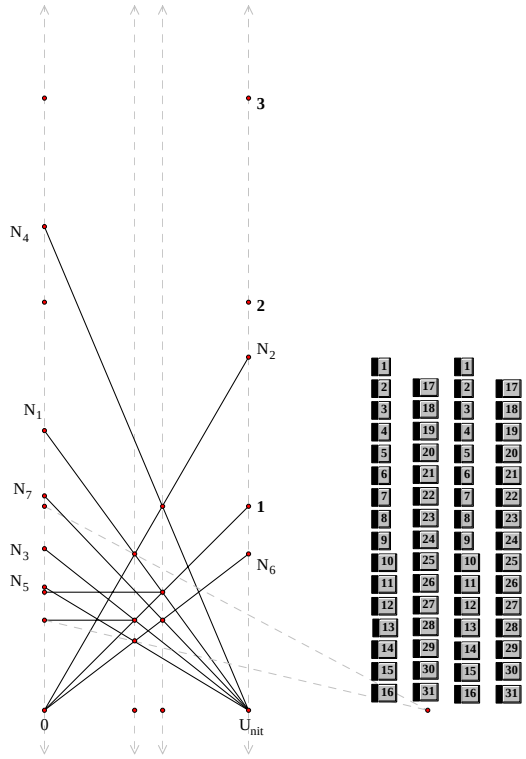
N_4 = 2.37239 $N_1 \cdot N_2 = 2.37239$

$\frac{N_1}{N_2} = 0.79179$

N_5 = 0.60565 $N_1 \cdot \left(\frac{N_1}{N_1+N_2} \right) - N_5 = 0.00000$

N_6 = 0.76491 $N_1 \cdot \left(\frac{N_2}{N_1+N_2} \right) - N_6 = 0.00000$

N_7 = 1.04835 $N_1 \cdot N_2 \cdot \left(\frac{N_1}{N_1+N_2} \right) - N_7 = 0.00000$



Unit = 1.00000

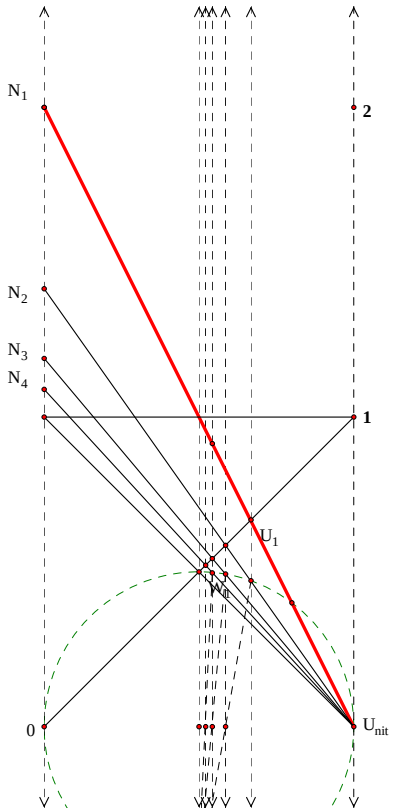
N_1 = 2.00000

N_2 = 1.41421 $\quad N_1^{\frac{1}{2}} - N_2 = 0.00000$

N_3 = 1.18921 $\quad N_1^{\frac{1}{4}} - N_3 = 0.00000$

N_4 = 1.09051 $\quad N_1^{\frac{1}{8}} - N_4 = 0.00000$

etc.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Unit = 1.00000

Eight root series.

One can project as long as they like.

$N_1 = 2.00000$ $N_1^{\frac{8}{8}} - N_1 = 0.00000$

$N_2 = 1.83401$ $N_1^{\frac{7}{8}} - N_2 = 0.00000$

$N_3 = 1.68179$ $N_1^{\frac{6}{8}} - N_3 = 0.00000$

$N_4 = 1.54221$ $N_1^{\frac{5}{8}} - N_4 = 0.00000$

$N_5 = 1.41421$ $N_1^{\frac{4}{8}} - N_5 = 0.00000$

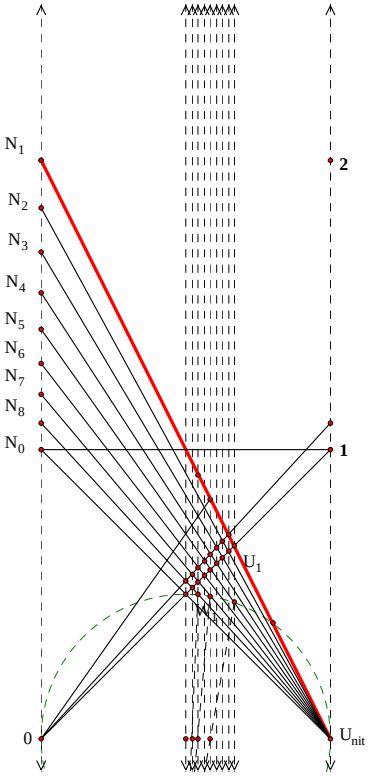
$N_6 = 1.29684$ $N_1^{\frac{3}{8}} - N_6 = 0.00000$

$N_7 = 1.18921$ $N_1^{\frac{2}{8}} - N_7 = 0.00000$

$N_8 = 1.09051$ $N_1^{\frac{1}{8}} - N_8 = 0.00000$

$N_0 = 1.00000$ $N_1^{\frac{0}{8}} - N_0 = 0.00000$

etc.



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The computational speed by straight edge and compass outdoes long hand by factors. The computational accuracy exceeds that of any binary computer. The understanding as to what numbers mean cannot be outdone. Yet, instead of improving Euclid, they made a mess of it.

What led me to this solution was not Euclid, it was my own geometry play-especially doing the formula's and solution to a power line In order to solve for the power line, I actually had to know how to divide a square by a line. That coupled with the feeling that one should know the basic mathematical operations in geometry, as a starter made me break down and simply do it.

Geometry is still undefined. It is undefined because, as we know, a set can be constructed in only two ways, by enumeration and by definition. By saying that Euclidean Geometry only uses two tools, the straight edge and compass, we have enumerated its set. To define it, one would have to say, Geometry is that language by which we speak where there is one, and only one difference between two points.

This change not only defines Euclidean Geometry, but we find that it has been short changed for a long time. A straight edge does indeed give us one and only one difference between two points, and so does a compass, these are the unit and universe of discourse in the subject. However, there is yet one more tool, that tool that gives us every ratio in-between the unit and the universe, the ellipes. There is indeed one and only one difference between the two points called the foci of an ellipse.

If one can accept that, one can then understand my solution to the Delian Problem. A figure that gives one every aspect of an ellipse and one simply has to lay it down. Accepting that definition also takes something that is implied in Euclidean Geometry and makes it explicit, the ability to add, to do the math.

I hope you have fun.

Unit.

$BE := 1$

Given.

$N_1 := 3.86292 \quad AB := N_1$

$N_2 := .74482 \quad BD := N_2$

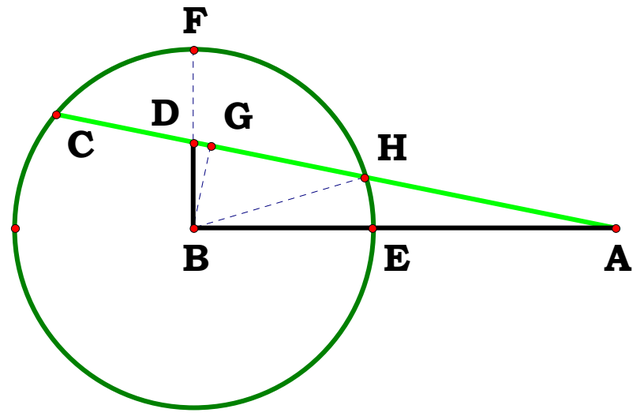
052108

Descriptions.

$AD := \sqrt{AB^2 + BD^2} \qquad DG := \frac{BD^2}{AD}$

$BH := BE \qquad BG := \frac{AB \cdot BD}{AD}$

$GH := \sqrt{BH^2 - BG^2} \qquad AC := AD + GH - DG$



Definitions.

$$\frac{\sqrt{AB^2 \cdot BE^2 + BD^2 \cdot BE^2 - AB^2 \cdot BD^2} + AB^2}{\sqrt{AB^2 + BD^2}} - AC = 0$$

$$AC - \frac{N_1^2 + \sqrt{N_1^2 - N_1^2 \cdot N_2^2 + N_2^2}}{\sqrt{N_1^2 + N_2^2}} = 0$$

052208

Unit.

Given.

$N_1 := 8.17825 \quad AB := N_1$

$N_2 := 2.36240 \quad BC := N_2$

$N_3 := .36912$

Descriptions.

$AD := N_1 - N_2 \quad DE := AD \cdot N_3 \quad AE := AD - DE$

$BE := AB - AE \quad AC := \sqrt{AB^2 + BC^2} \quad BH := \frac{AB \cdot BC}{AC}$

$BG := BE \quad CH := \frac{BC^2}{AC} \quad GH := \sqrt{BG^2 - BH^2} \quad FH := GH$

$AF := AC + FH - CH \quad AF = 11.753331$

Definitions.

$AD - (N_1 - N_2) = 0 \quad DE - (N_1 - N_2) \cdot N_3 = 0$

$AE - [(N_3 - 1) \cdot (N_2 - N_1)] = 0 \quad BE - (N_2 + N_1 \cdot N_3 - N_2 \cdot N_3) = 0$

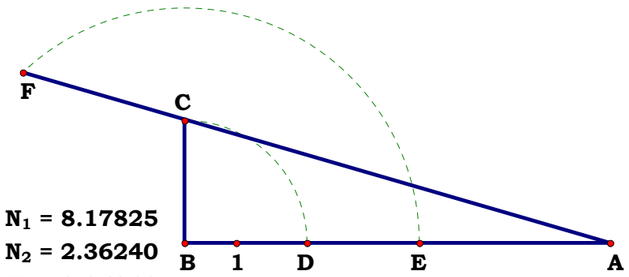
$AC - \sqrt{N_1^2 + N_2^2} = 0 \quad BH - \frac{N_1 \cdot N_2}{\sqrt{N_1^2 + N_2^2}} = 0$

$BG - (N_2 + N_1 \cdot N_3 - N_2 \cdot N_3) = 0 \quad CH - \frac{N_2^2}{\sqrt{N_1^2 + N_2^2}} = 0$

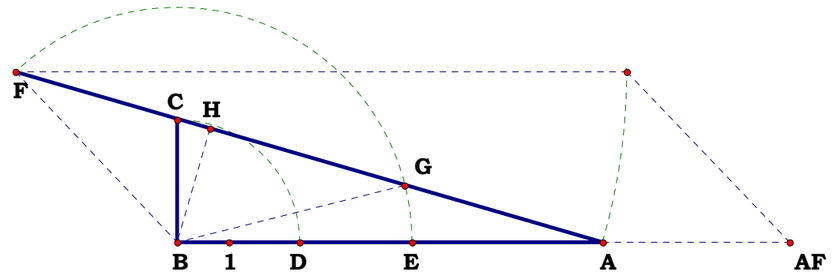
$GH - \frac{\sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$

$AF - \frac{N_1^2 + \sqrt{N_3^2 \cdot (N_1^2 + N_2^2) \cdot (N_1 - N_2)^2 + 2 \cdot N_3 \cdot N_2 \cdot (N_1 - N_2) \cdot (N_1^2 + N_2^2) + N_2^4}}{\sqrt{N_1^2 + N_2^2}} = 0$

Given AB, BC and AE as a portion of AD, what is AF?



$N_1 = 8.17825$
$N_2 = 2.36240$
$N_3 = 0.36912$
$X = 1.48192$
$Y = 4.01471$



$N_1 = 8.17825 \quad AF = 11.75336$
$N_2 = 2.36240$
$N_3 = 0.36912$
$X = 1.48192$
$Y = 4.01471$

060208A

Unit.

Given.

$N_1 := 1.40187$ $\quad AB := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := 1.13348$ $\quad CD := N_3$

For a straight line ellipse and three givens.

a: AB, AC, CD

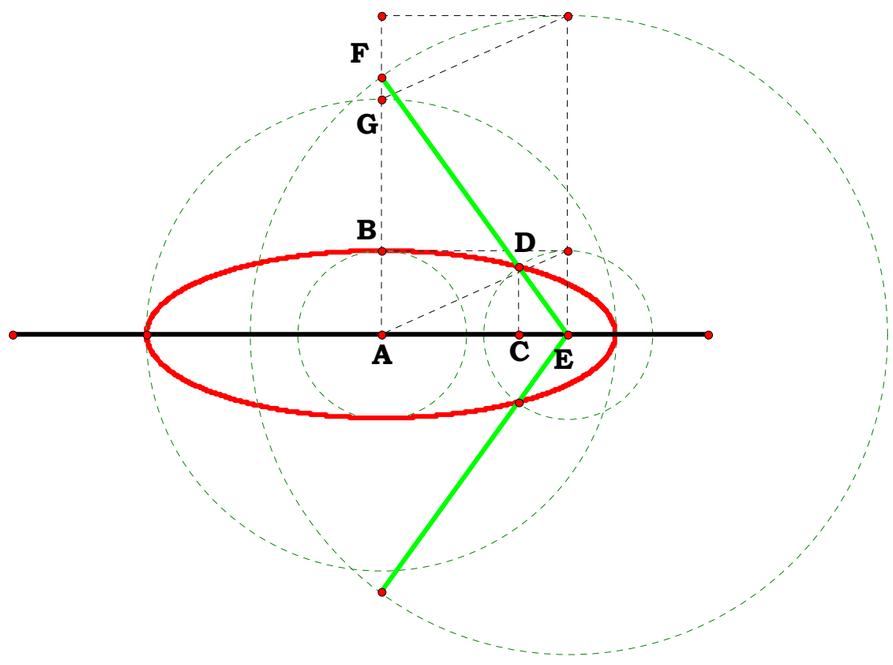
Descriptions.

$DE := AB \qquad CE := \sqrt{DE^2 - CD^2}$

$DF := \frac{DE \cdot AC}{CE} \qquad BG := DF - AB$

Definitions.

$$BG - N_1 \cdot \left(\frac{N_2}{\sqrt{N_1^2 - N_3^2}} - 1 \right) = 0$$



060208B

Unit.

Given.

$N_1 := .8249$ $\quad CE := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := 1.13348$ $\quad CD := N_3$

For a straight line ellipse and three givens.

b: CE, AC, CD.

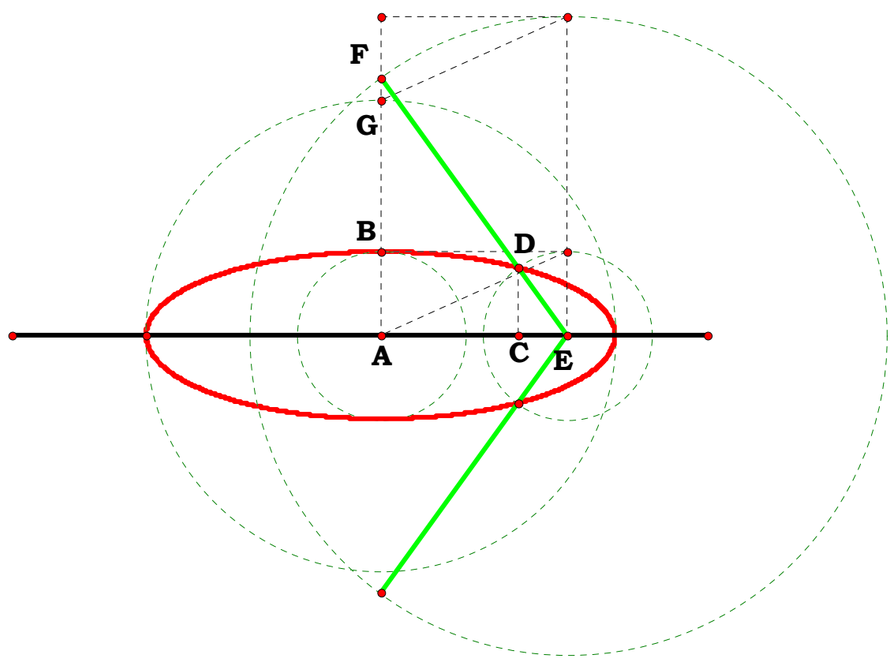
Descriptions.

$$AB := \sqrt{CD^2 + CE^2} \qquad DE := AB$$

$$DF := \frac{DE \cdot AC}{CE} \qquad BG := DF - AB$$

Definitions.

$$BG - \frac{\sqrt{N_1^2 + N_3^2} \cdot (N_2 - N_1)}{N_1} = 0$$



060208C

Unit.

Given.

$N_1 := 1.40187$ $\quad AB := N_1$

$N_2 := 2.31398$ $\quad AC := N_2$

$N_3 := .8249$ $\quad CE := N_3$

For a straight line ellipse and three givens.

Descriptions.

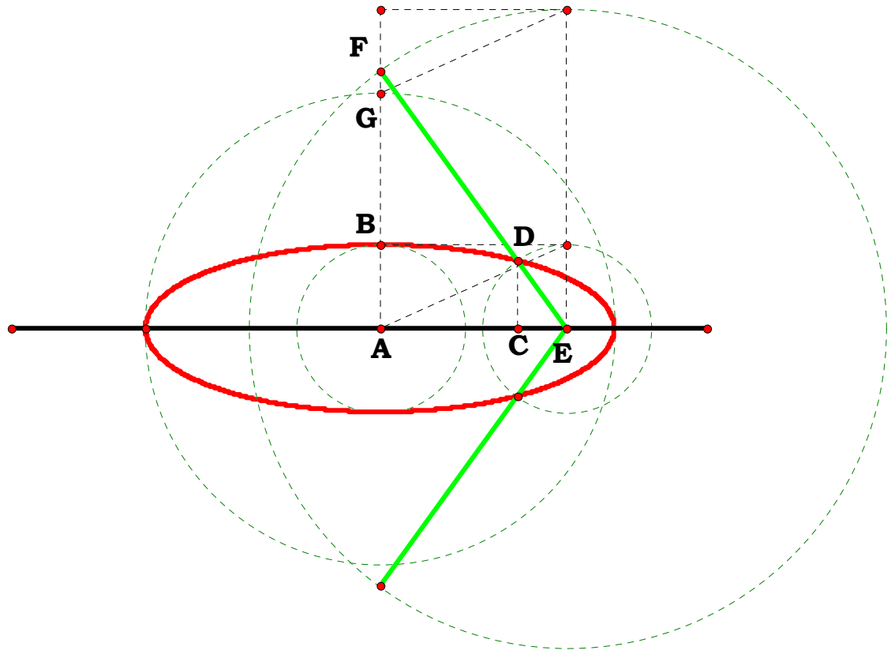
$DE := AB \qquad DF := \frac{DE \cdot AC}{CE}$

$BG := DF - AB$

Definitions.

$$BG - \frac{N_1 \cdot (N_2 - N_3)}{N_3} = 0$$

c: AB, AC, CE

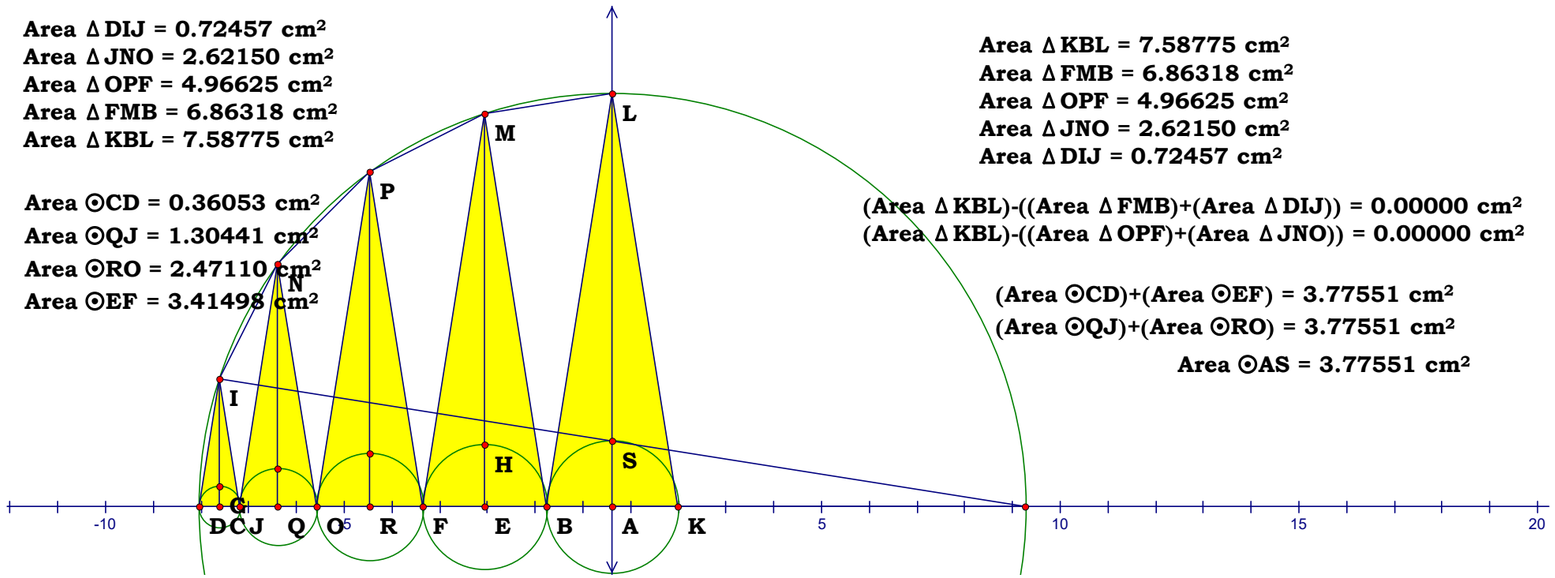


Unit.
Given.

Descriptions.
Definitions.

Procrastinated Write up for 060308

Angles are expressible as an elliptical progression, and they show very arithmetic properties to one another.



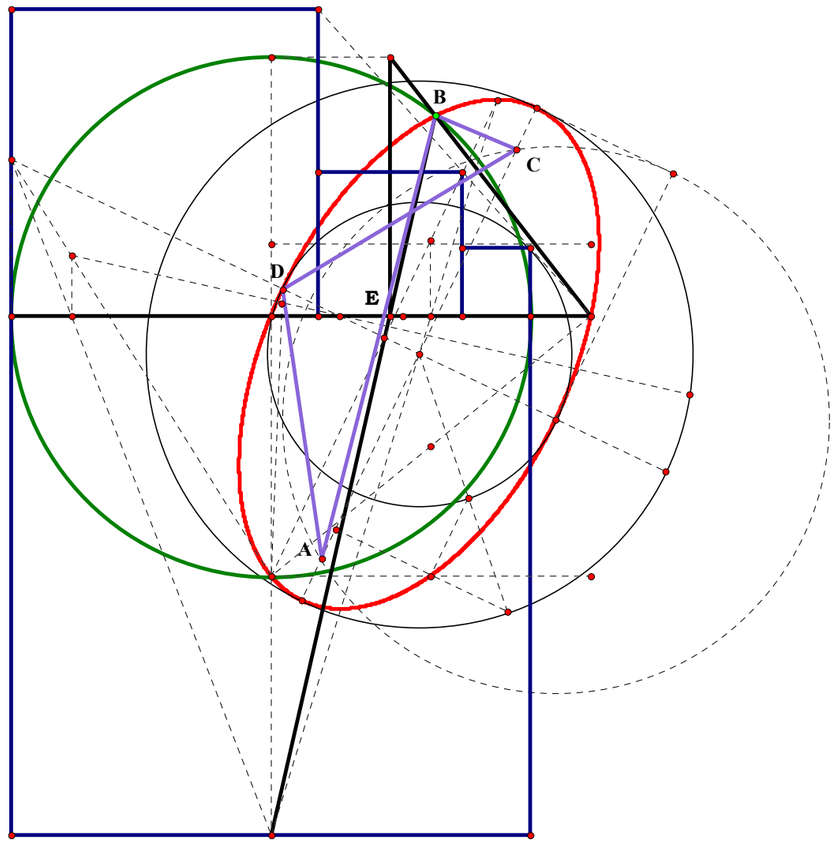
Unit.

Given. **Parcing project for 061308a**

Descriptions.

Definitions.

See about writing up a proof of the figure using the fact that from the center of the two roots, point E, a simple construct will produce the intersection B for the figure. And chect to see how the point E moves during Gemini roots.

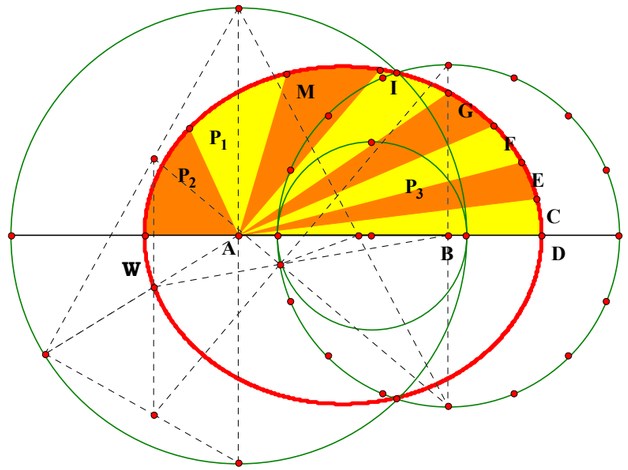


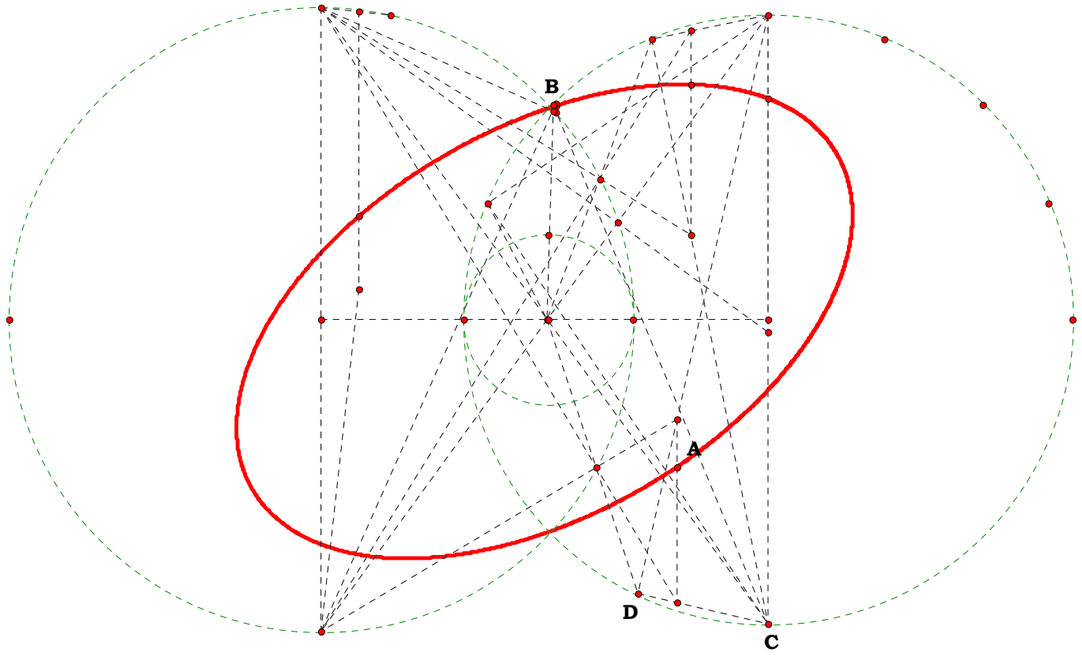
Unit.
Given.

Parcing project for 061308b

Looking at mass in eliptical motions from a different point of view

Descriptions.
Definitions.





How you understand the ellipes determines how you think an object moving in that orbit ought to be comprehended and written up as a law of nature, however ponder this fact. The velocity of D is a constant; it determines the velocity of A, the object you can see orbiting say the sun. However, the velocity of A exhibits the same characteristics of a planet or asteroid, slingshot effect and all. The velocity is still due to the constant velocity of D. The current understanding of planitary interaction is not correct.

Unit.
Given.

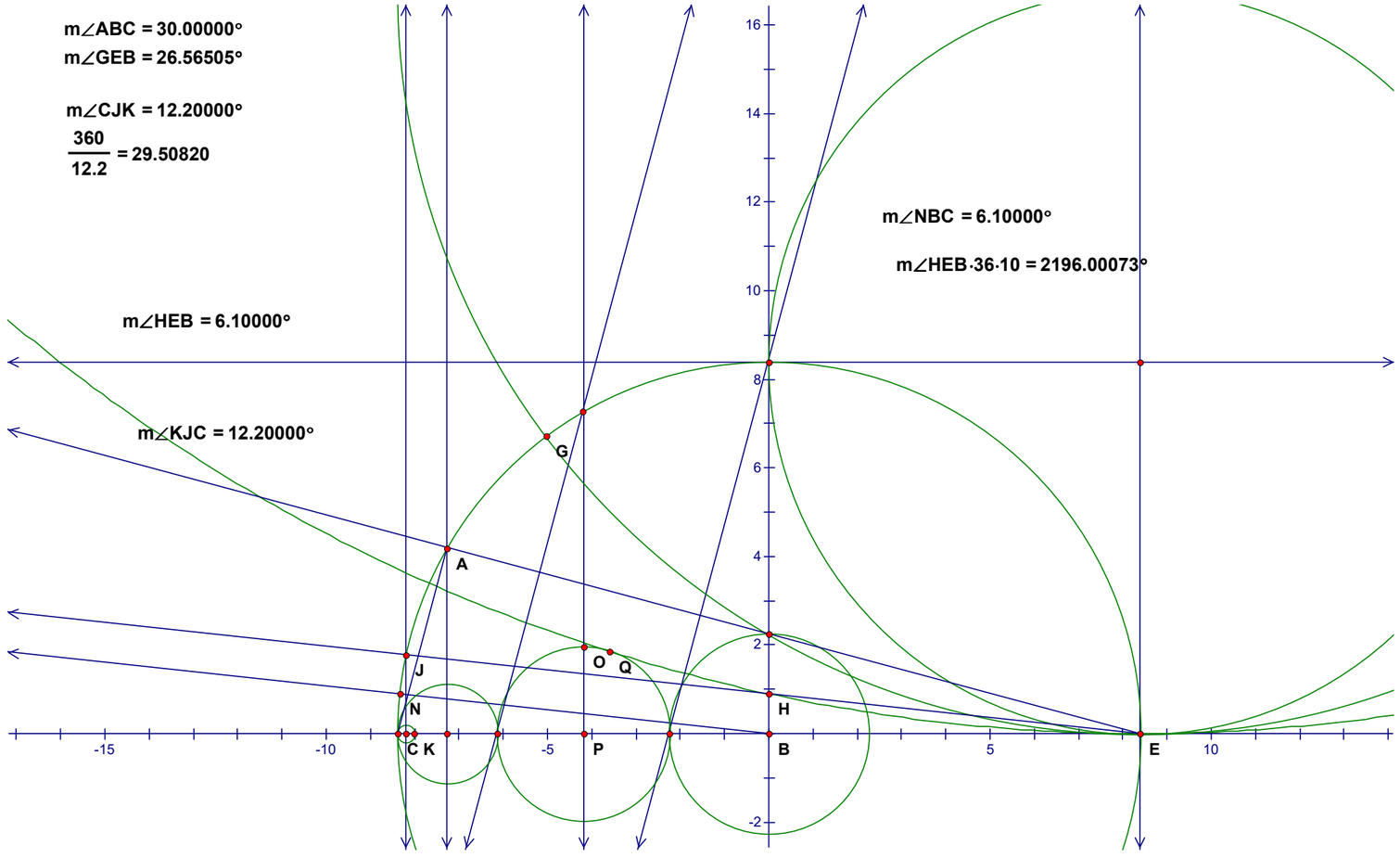
Descriptions.
Definitions.

Parcing project for 061708

It appears that there is a way to get a degree, by construction down to .1 of a degree, if this is true than compass constructrions throught impossible are not. Is it True, or only appears to be?

$m\angle ABC = 30.00000°$
$m\angle GEB = 26.56505°$

$m\angle CJK = 12.20000°$
$\frac{360}{12.2} = 29.50820$

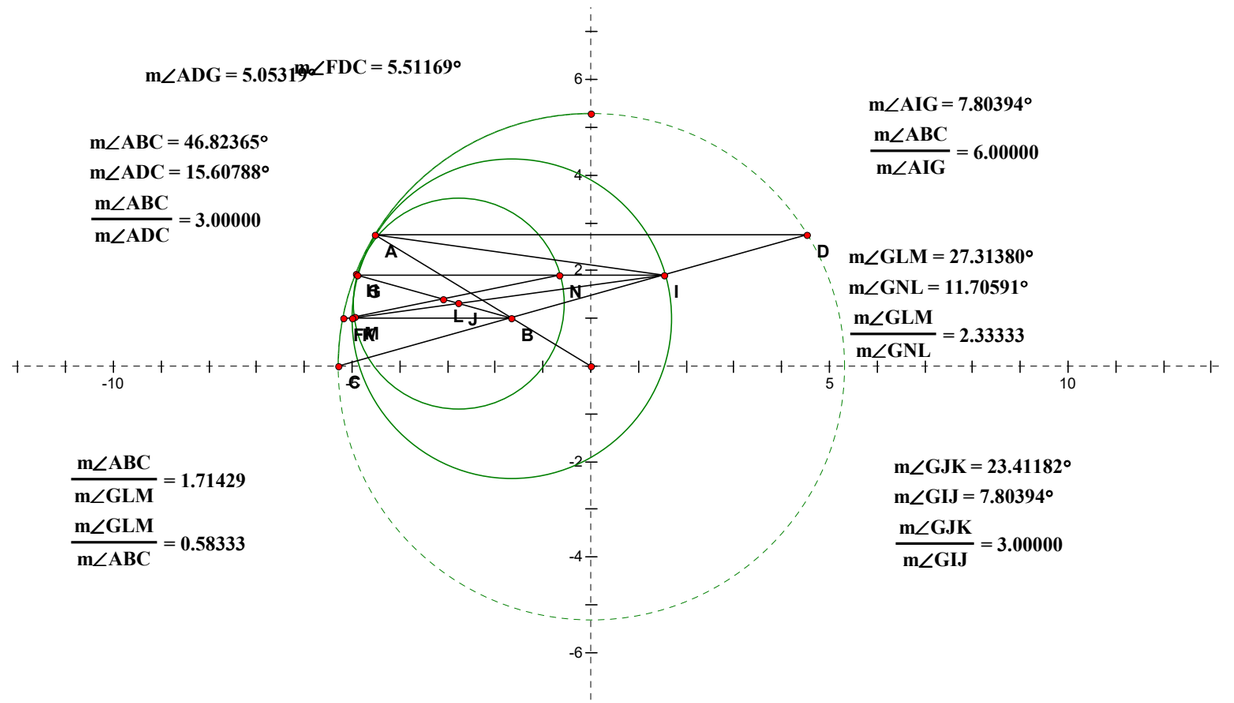


Unit.
Given.

Descriptions.
Definitions.

Parcing project for 062608

Another method of doing the paper trisector. AD is perpendictualar to the pole.



Unit.
Given.

On angle trisection.

061110

Descriptions.
Definitions.

Parcing project. There is a whole series of plates here, write them up.

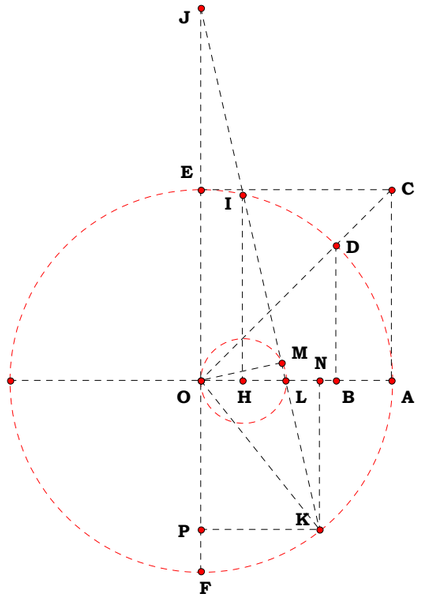
AO = 3.19617 cm
CO = 4.52006 cm
BO = 2.26003 cm

HO = 0.70753 cm

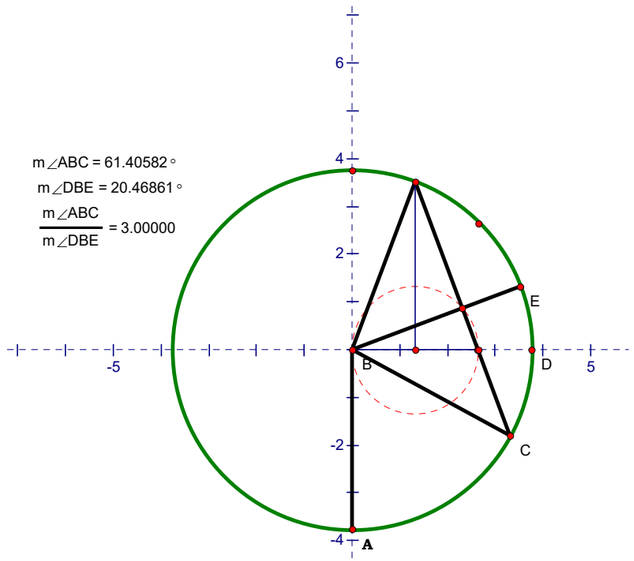
$\frac{BO}{HO}$ = 3.19427

LM = 0.31325 cm

MO = 1.37995 cm
KM = 2.88292 cm
JK = 8.96201 cm
KP = 1.98389 cm



m∠ABC = 61.40582°
m∠DBE = 20.46861°
$\frac{m\angle ABC}{m\angle DBE}$ = 3.00000



m∠ABC = 55.69074°

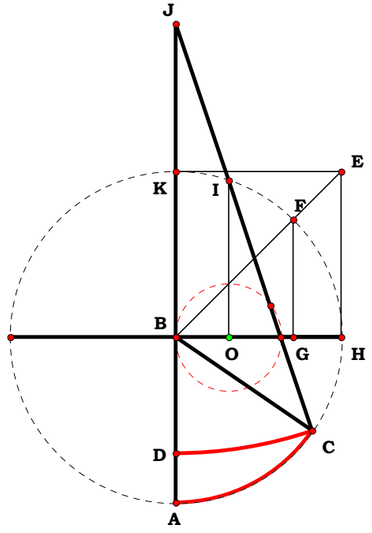
m∠IBK = 18.56358°

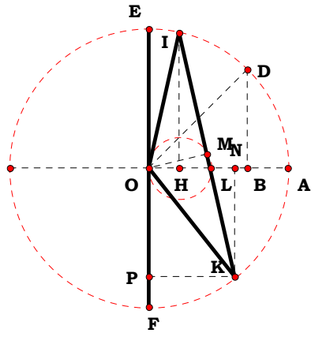
$\frac{m\angle ABC}{m\angle IBK}$ = 3.00000

BH = 2.77283 cm
BG = 1.96069 cm

$\frac{BH}{BG}$ = 1.41421

$\sqrt{2}$ = 1.41421





$\frac{m\angle FOK}{m\angle EOI}$ = 3.00000

020511

Descriptions.

Rant

Percentages, ratio's, proportions, currency conversion, number conversion, etc. Let us say we have a zillion and one items we which to tanslate from one system of measure to another. How can we do all of the items at one and the same time? Well, if you are a non-Euclidean Geometer, you cannot, you are screwed. In fact, if you are a non-Euclidean Geometer, you are claiming that a unit differs from itself and are way too stupid to realize an obvios fact. So, no, I am not entrusting anything to them. And since they are supported by, if not every educational institution, then almost every-one, I had to drop out of school at an early age. The explicit and the tacit admission of their doctrines by our social structure even made me a social outcast. I say, they can argue with the foundation of their own psychology, Language.

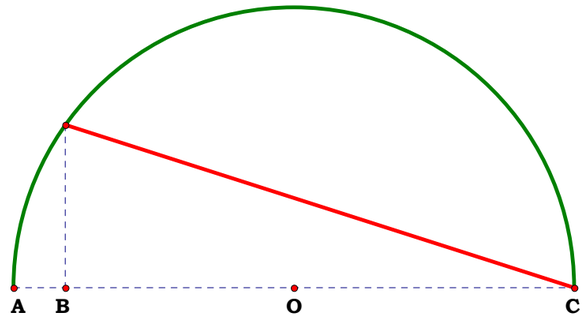
Therefore, I do not believe that what is needed is a write-up demonstrating proportion, again. Maybe I was just entertaining a rant.

What do you think the sentence, As A is to B, so too, C is to D, means? And if you claim that is is not true, citing the mentally lame as authorities for that claim, why is it you prefer a moron over countless examples in your daily life?

Why do you suffer your religious leaders, your political leaders, your teachers and your schools to teach the lies?

It is not because you are mentally functional. All of my ranting can never change the fact that man is still being made, that his mind is still incapable of doing simple operations.

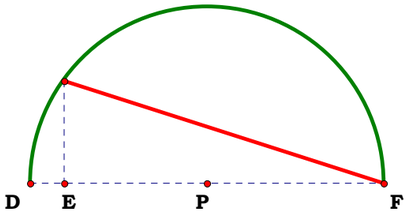
Therefore, in order to put something constructive here, I will put a digitalization of The Science of Absolute Space, a title which is wholly indicative of someone who is illiterate.



AC = 2.00000

AB = 0.18650

$\frac{AC}{AB}$ = 10.72378



DF = 5.91633 cm

DE = 0.55170 cm

$\frac{DF}{DE}$ = 10.72378

$\frac{AC}{AB} - \frac{DF}{DE}$ = 0.00000

THE SCIENCE ABSOLUTE OF SPACE

Bolyai János

THE SCIENCE ABSOLUTE OF SPACE

Independent of the Truth or Falsity of Euclid's Axiom XI

(*which can never be decided a priori*).

JOHN BOLYAI

TRANSLATED FROM THE LATIN

BY DR. GEORGE BRUCE HALSTED

PRESIDENT OF THE TEXAS ACADEMY OF SCIENCE

FOURTH EDITION.

VOLUME THREE OF THE NEOMONIC SERIES

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U. S. A. 1896

TRANSLATOR'S INTRODUCTION.

The immortal *Elements* of Euclid was already in dim antiquity a classic, regarded as absolutely perfect, valid without restriction.

Elementary geometry was for two thousand years as stationary, as fixed, as peculiarly Greek, as the Parthenon. On this foundation pure science rose in Archimedes, in Apollonius, in Pappus; struggled in Theon, in Hypatia; declined in Proclus; fell into the long decadence of the Dark Ages.

The book that monkish Europe could no longer understand was then taught in Arabic by Saracen and Moor in the Universities of Bagdad and Cordova.

To bring the light, after weary, stupid centuries, to western Christendom, an Englishman, Adelhard of Bath, journeys, to learn Arabic, through Asia Minor, through Egypt, back to Spain. Disguised as a Mohammedan student, he got into Cordova about 1120, obtained a Moorish copy of Euclid's *Elements*, and made a translation from the Arabic into Latin.

The first printed edition of Euclid, published in Venice in 1482, was a Latin version from the Arabic. The translation into Latin from the Greek, made by Zamberti from a MS. of Theon's revision, was first published at Venice in 1505.

Twenty - eight years later appeared the *editio princeps* in Greek, published at Basle in 1533 by John Hervagius, edited by Simon Grynaeus. This was for a century and three-quarters the only printed Greek text of all the books, and from it the first English translation (1570) was made by "Henricus Billingsley," afterward Sir Henry Billingsley, Lord Mayor of London in 1591.

And even today, 1895, in the vast system of examinations carried out by the British Government, by Oxford, and by Cambridge, no proof of a theorem in geometry will be accepted which infringes Euclid's sequence of propositions.

Nor is the work unworthy of this extraordinary immortality.

Says Clifford : "This book has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state.

"The encouragement; for it contained a body of knowledge that was really known and could be relied on.

"The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained."

But Euclid stated his assumptions with the most painstaking candor, and would have smiled at the suggestion that he claimed for his conclusions any other truth than perfect deduction from assumed hypotheses. In favor of the external reality or truth of those assumptions he said no word.

Among Euclid's assumptions is one differing from the others in prolixity, whose place fluctuates in the manuscripts.

Peyrard, on the authority of the Vatican MS., puts it among the postulates, and it is often called the parallel-postulate. Heiberg, whose edition of the text is the latest and best (Leipzig, 1883–1888), gives it as the fifth postulate.

James Williamson, who published the closest translation of Euclid we have in English, indicating, by the use of italics, the words not in the original, gives this assumption as eleventh among the Common Notions.

Bolyai speaks of it as Euclid's Axiom XI. Todhunter has it as twelfth of the Axioms.

Clavius (1574) gives it as Axiom 13.

The Harpur Euclid separates it by forty-eight pages from the other axioms.

It is not used in the first twenty-eight propositions of Euclid. Moreover, when at length used, it appears as the inverse of a proposition already demonstrated, the seventeenth, and is only needed to prove the inverse of another proposition already demonstrated, the twenty-seventh.

Now the great Lambert expressly says that Proklus demanded a proof of this assumption because when inverted it is demonstrable.

All this suggested, at Europe's renaissance, not a doubt of the necessary external reality and exact applicability of the assumption, but the possibility of deducing it from the other assumptions and the twenty-eight propositions already proved by Euclid without it.

Euclid demonstrated things more axiomatic by far. He proves what every dog knows, that any two sides of a triangle are together greater than the third.

Yet after he has finished his demonstration, that straight lines making with a transversal equal alternate angles are parallel, in order to

prove the inverse, that parallels cut by a transversal make equal alternate angles, he brings in the unwieldy assumption thus translated by Williamson (Oxford, 1781) :

"11. And if a straight line meeting two straight lines make those angles which are inward and upon the same side of it less than two right angles, the two straight lines being produced indefinitely will meet each other on the side where the angles are less than two right angles."

As Staeckel says, "it requires a certain courage to declare such a requirement, alongside the other exceedingly simple assumptions and postulates." But was courage likely to fail the man who, asked by King Ptolemy if there were no shorter road in things geometric than through his *Elements*? answered, "To geometry there is no special way for kings!"

In the brilliant new light given by Bolyai and Lobachevski we now see that Euclid understood the crucial character of the question of parallels.

There are now for us no better proofs of the depth and systematic coherence of Euclid's masterpiece than the very things which, their cause unappreciated, seemed the most noticeable blots on his work.

Sir Henry Savile, in his Praelectiones on Euclid, Oxford, 1621, p. 140, says : "In pulcherrimo Geometriae corpore duo sunt naevi, duae labes . . . " etc., and these two blemishes are the theory of parallels and the doctrine of proportion; the very points in the Elements which now arouse our wondering admiration. But down to our very nineteenth century an ever renewing stream of mathematicians tried to wash away the first of these supposed stains from the most beauteous body of Geometry.

The year 1799 finds two extraordinary young men striving thus

"To gild refined gold, to paint the lily,
To cast a perfume o'er the violet."

At the end of that year Gauss from Braunschweig writes to Bolyai Farkas in Klausenburg (Kolozsvár) as follows : [Abhandlungen der Koeniglichen Gesellschaft der Wissenschaften zu Goettingen, Bd. 22, 1877.]

"I very much regret, that I did not make use of our former proximity, to find out *more* about your investigations in regard to the first grounds of geometry; I should certainly thereby have spared myself much vain labor, and would have become more restful than any one, such

as I, can be, so long as on such a subject there yet remains so much to be wished for.

In my own world thereon I myself have advanced far (though my other wholly heterogeneous employments leave me little time therefore) but *the* way, which I have hit upon, leads not so much to the goal, which one wishes, as much more to making doubtful the truth of geometry.

Indeed I have core upon much, which with most no doubt would pass for a proof, but which in my eyes proves as good as *nothing*.

For example, if one could prove, that a rectilineal triangle is possible, whose content may be greater, than any given surface, then I am in condition, to prove with perfect rigor all geometry.

Most would indeed let that pass as an axiom; I not; it might well be possible, that, how far apart soever one took the three vertices of the triangle in space, yet the content was always under a given limit.

I have more such theorems, but in none do I find anything satisfying."

From this letter we clearly see that in 1799 Gauss was still trying to prove that Euclid's is the only non-contradictory system of geometry,

and that it is the system regnant in the external space of our physical experience.

The first is false; the second can never be proven.

Before another quarter of a century, Bolyai János, then unborn, had created another possible universe; and, strangely enough, though nothing renders it impossible that the space of our physical experience may, this very year, be satisfactorily shown to belong to Bolyai János , yet the same is not true for Euclid.

To decide our space is Bolyai's, one need only show a single rectilineal triangle whose angle-sum measures less than a straight angle. And this could be shown to exist by imperfect measurements, such as human measurements must always be. For example, if our instruments for angular measurement could be brought to measure an angle to within one millionth of a second, then if the lack were as great as two millionths of a second, we could make certain its existence.

But to prove Euclid's system, we must show that a triangle's angle-sum is *exactly* a straight angle, which nothing human can ever do.

However this is anticipating, for in 1799 it seems that the mind of the elder Bolyai, Bolyai Farkas, was in precisely the same state as

that of his friend Gauss. Both were intensely trying to prove what now we know is indemonstrable. And perhaps Bolyai got nearer than Gauss to the unattainable. In his "Kurzer Grundriss eines Versuchs," etc., p. 46, we read : "Koennten jede 3 Punkte, die nicht in einer Geraden sind, in eine Sphaere fallen, so waere das Eucl. Ax. XI. bewiesen." Frischauf calls this "das anschaulichste Axiom." But in his Autobiography written in Magyar, of which my Life of Bolyai contains the first translation ever made, Bolyai Farkas says : "Yet I could not become satisfied with my different treatments of the question of parallels, which was ascribable to the long discontinuance of my studies, or more probably it was due to myself that I drove this problem to the point which robbed my rest, deprived me of tranquillity."

It is well-nigh certain that Euclid tried his own calm, immortal genius, and the genius of his race for perfection, against this self-same question. If so, the benign intellectual pride of the founder of the mathematical school of the greatest of universities, Alexandria, would not let the question cloak itself in the obscurities of the infinitely great or the infinitely small. He would say to himself : "Can I prove

this plain, straightforward, simple theorem : "those straights which are produced indefinitely from less than two right angles meet." [This is the form which occurs in the Greek of Eu. I. 29.]

Let us not underestimate the subtle power of that old Greek mind. We can produce no Venus of Milo. Euclid's own treatment of proportion is found as flawless in the chapter which Stolz devotes to it in 1885 as when through Newton it first gave us our present continuous number-system.

But what fortune had this genius in the fight with its self-chosen simple theorem? Was it found to be deducible from all the definitions, and the nine "Common Notions," and the five other Postulates of the immortal Elements? Not so. But meantime Euclid went ahead without it through twenty-eight propositions, more than half his first book. But at last came the practical pinch, then as now the triangle's angle-sum.

He gets it by his twenty-ninth theorem : "A straight falling upon two parallel straights makes the alternate angles equal."

But for the proof of this he needs that recalcitrant proposition which has how long been keeping him awake nights and waking

him up mornings? Now at last, true man of science, he acknowledges it indemonstrable by spreading it in all its ugly length among his postulates.

Since Schiaparelli has restored the astronomical system of Eudoxus, and Hultsch has published the writings of Autolycus, we see that Euclid knew surface-spherics, was familiar with triangles whose angle-sum is more than a straight angle. Did he ever think to carry out for himself the beautiful system of geometry which comes from the contradiction of his indemonstrable postulate; which exists if there be straights produced indefinitely from less than two right angles yet nowhere meeting; which is real if the triangle's angle-sum is less than a straight angle?

Of how naturally the three systems of geometry flow from just exactly the attempt we suppose Euclid to have made, the attempt to demonstrate his postulate fifth, we have a most romantic example in the work of the Italian priest, Saccheri, who died the twenty-fifth of October, 1733. He studied Euclid in the edition of Clavius, where the fifth postulate is given as Axiom 13. Saccheri says it should not be called an axiom, but ought to be demonstrated. He tries this seemingly simple

task; but his work swells to a quarto book of 101 pages.

Had he not been overawed by a conviction of the absolute necessity of Euclid's system, he might have anticipated Bolyai János, who ninety years later not only discovered the new world of mathematics but appreciated the transcendent import of his discovery.

Hitherto what was known of the Bolyais came wholly from the published works of the father Bolyai Farkas, and from a brief article by Architect Fr. Schmidt of Budapest "Aus dem Leben zweier ungarischer Mathematiker, Johann und Wolfgang Bolyai von Bolya." Grunert's Archiv, Bd. 48, 1868, p. 217.

In two communications sent me in September and October 1895, Herr Schmidt has very kindly and graciously put at my disposal the results of his subsequent researches, which I will here reproduce. But meantime I have from entirely another source come most unexpectedly into possession of original documents so extensive, so precious that I have determined to issue them in a separate volume devoted wholly to the life of the Bolyais; but these are not used in the sketch here given.

Bolyai Farkas was born February 9th, 1775, at Bolya, in that part of Transylvania

(Erdély) called Székelyföld. He studied first at Enyed, afterward at Klausenburg (Kolozsvár), then went with Baron Simon Kemény to Jena and afterward to Goettingen. Here he met Gauss, then in his 19th year, and the two formed a friendship which lasted for life.

The letters of Gauss to his friend were sent by Bolyai in 1855 to Professor Sartorius von Walterhausen, then working on his biography of Gauss. Everyone who met Bolyai felt that he was a profound thinker and a beautiful character.

Benzenberg said in a letter written in 1801 that Bolyai was one of the most extraordinary men he had ever known.

He returned home in 1799, and in January, 1804, was made professor of mathematics in the Reformed College of Maros-Vásárhely. Here for 47 years of active teaching he had for scholars nearly all the professors and nobility of the next generation in Erdély.

Sylvester has said that mathematics is poesy.

Bolyai's first published works were dramas.

His first published book on mathematics was an arithmetic :

Az arithmetica eleje. 8vo. i- xvi, 1–162 pp. The copy in the library of the Reformed College is enriched with notes by Bolyai János .

Next followed his chief work, to which he constantly refers in his later writings. It is in Latin, two volumes, 8vo, with title as - follows :

TENTAMEN | JUVENTUTEM STUDIOSAM IN ELEMENTA MATHESEOS PURAE, ELEMENTARIS AC | SUBLIMIORIS, METHODO INTUITIVA, EVIDENTIA— | QUE HUIC PROPRIA, INTRODUCENDI. |

CUM APPENDICE TRIPLICI. | Auctore Professore Matheseos et Physices Chemiaeque | Publ. Ordinario. | Tomus Primus. | *Maros Vasarhelyini*. 1832. | Typis Collegii Reformatorum per JOSEPHUM, et | SIMEONEM KALI de felsö Vist. | At the back of the title : Imprimatur. | M. Vásárhelyini Die | 12 Octobris, 1829. [Paulus Horváth m. p. | Abbas, Parochus et Censor | Librorum.

Tomus Secundus. | *Maros Vasarhelyini*. 1833.

The first volume contains :

Preface of two pages : *Lectori salutem*.

A folio table : *Explicatio signorumn*.

Index rerum (I–XXXII). *Errata* (XXXIII–XXXVII).

Pro tyronibus prima vice legentibus notanda sequentia (XXXVIII–LII).

Errores (LIII–LXVI).

Scholion (LXVII - LXXIV).

Pluriium errorum haud animadversorum numerous minuitur (LXXV–LXXVI). Recensio per auctorem ipsum facta (LXXVII–LXXVIII).

Errores recentius detecti (L X X V–XCVIII).

Now comes the body of the text (pages 1–502).

Then, with special paging, and a new title page, comes the immortal Appendix, here given in English.

Professors Staeckel and Engel make a mistake in their "Parallellinien" in supposing that this Appendix is referred to in the title of " Tentamen." On page 241 they quote this title, including the words "Cum appendice triplici," and say : "In dem dritten Anhange, der nur 28 Seiten umfasst, hat Johann Bolyai seine neue Geometrie entwickelt."

It is not a third Appendix, nor is it referred to at all in the words "Cum appendice triplici."

These words, as explained in a prospectus in the Magyar language, issued by Bolyai Farkas, asking for subscribers, referred to a real triple Appendix, which appears, as it

should, at the end of the book Tomus Secundus, pp. 265 - 322.

The now world renowned Appendix by Bolyai János was an afterthought of the father, who prompted the son not "to occupy himself with the theory of parallels," as Staeckel says, but to translate from the German into Latin a condensation of his treatise, of which the principles were discovered and properly appreciated in 1823, and which was given in writing to Johann Walter von Eckwehr in 1825.

The father, without waiting for Vol. II, inserted this Latin translation, with separate paging (1 - 26), as an Appendix to his Vol. I, where, counting a page for the title and a page "Explicatio signorum," it has twenty-six numbered pages, followed by two unnumbered pages of Errata.

The treatise itself, therefore, contains only twenty-four pages—the most extraordinary two dozen pages in the whole history of thought! Milton received but a paltry £5 for his Paradise Lost; but it was at least plus £5.

Bolyai János, as we learn from Vol. II, p. 384, of "*Tentamen*," contributed for the

printing of his eternal twenty-six pages, 104 florins 50 kreuzers.

That this Appendix was finished considerably before the Vol. I, which it follows, is seen from the references in the text, breathing a just admiration for the Appendix and the genius of its author.

Thus the father says, p. 452 : Elegans est conceptus *similium*, quem J. B. *Appendicis Auctor* dedit. Again, p. 489 : *Appendicis Actor*, rem acumine singulari aggressus, Geometriam pro omni casu absolute veram posuit; quamvis e magna mole, tantum summe necessaria, in Appendice hujus tomi exhibuerit, multis (ut tetraedri resolutione generali, pluribusque aliis disquisitionibus elegantibus) brevitatis studio omissis.

And the volume ends as follows, p. 502 : Nec operae pretium est plura referre; quum res tota exaltiori contemplationis puncto, in ima penetranti oculo, tractetur in Appendice sequente, a quovis fideli veritatis purae alumno diagna legi.

The father gives a brief resumé of the results of his own determined, life-long, desperate efforts to do that at which Saccheri, J. H. Lambert, Gauss also had failed, to establish Euclid' theory of parallels *a priori*.

He says, p. 490 : "Tentamina idcirco quae olim feceram, breviter exponenda veniunt; ne saltem alius quis operam eandem perdat." He anticipates J. Delboeuf's "Prolégoménes philosophiques de la géométrie et solution des postulats," with the full consciousness in addition that it is *not* the solution,—that the final solution has crowned not his own intense efforts, but the genius of his son.

This son's Appendix which makes all preceding space only a special case, only a species under a genus, and so requiring a descriptive adjective, *Euclidean*, this wonderful production of pure genius, this strange Hungarian flower, was saved for the world after more than thirty-five years of oblivion, by the rare erudition of Professor Richard Baltzer of Dresden, afterward professor in the University of Giessen. He it was who first did justice publicly to the works of Lobachevski and Bolyai.

Incited by Baltzer, in 1866 J. Hoüel issued a French translation of Lobachevski's Theory of Parallels, and in a note to his Preface says : "M. Richard Baltzer, dans la seconde édition de ses excellents *Elenents de Geometrie*, a, le premier, introduit ces notions exactes à la place qu'elles doivent occuper," Honor to

Baltzer! But alas! father and son were already in their graves!

Fr. Schmidt in the article cited (1868) says : "It was nearly forty years before these profound views were rescued from oblivion, and Dr. R. Baltzer, of Dresden, has acquired imperishable titles to the gratitude of all friends of science as the first to draw attention to the works of Bolyai, in the second edition of his excellent *Elemente der Mathematik* (1866–67). Following the steps of Baltzer, Professor Hoüel, of Bordeaux, in a brochure entitled, Essai critique sur les principes fondamentaux de la Géométrie élémentaire, has given extracts from Bolyai's book, which will help in securing for these new ideas the justice they merit."

The father refers to the son's Appendix again in a subsequent book, Urtan elemei kezdöknek [Elements of the science of space for beginners] (1850 – 51), pp. 48. In the College are preserved three sets of figures for this book, two by the author and one by his grandson, a son of János.

The last work of Bolyai Farkas; the only one composed in German, is entitled,

Kurzer Grundriss eines Versuchs

I. Die Arithmetik, durch zvekmässig konstruirte

Begriffe, von eingebildeten und unendlich-kleinen Grössen gereinigt, anschaulich und logisch-streng darzustellen.

II. In der Geometrie, die Begriffe der geraden Linie, der Ebene, des Winkels allgemein, der winkellosen Formen, und der Krummen, der verschiedenen Arten der Gleichheit u. d. gl. nicht nur scharf zu bestimmen; sondern auch ihr Seyn im Raume zu beweisen : und da die Frage, *ob zwey von der dritten geschnittene Geraden, wenn die summe der inneren Winkel nicht = 2R, sich schneiden oder nicht?* neimand auf der Erde ohne ein Axiom (wie Euklid das XI) aufzustellen, beantworten wird; die davon unabhängige Geometrie abzusondern; und eine auf die *Ja*— Antwort, andere auf das *Nein* so zu bauen, das die Formeln der letzten, auf ein Wink auch in der ersten gültig seyen.

Nach ein lateinischen Werke von 1829, M. Vásérhely, und eben daselbst gedruckten ungrischen.

Maros Vásárhely 1851. 8vo. pp. 88.

In this book he says, referring to his son's Appendix : "Some copies of the work published here were sent at that time to Vienna, to Berlin, to Goettingen . . . . From Goettingen the giant of mathematics, who from

his pinnacle embraces in the same view the stars and the abysses, wrote that he was surprised to see accomplished what he had begun, only to leave it behind in his papers."

This refers to 1832. The only other record that Gauss ever mentioned the book is a letter from Gerling, written October 31st, 1851, to Wolfgang Boylai, on receipt of a copy of "Kurzer Grundriss." Gerling, a scholar of Gauss, had been from 1817 Professor of Astronomy at Marburg. He writes : "I do not mention my earlier occupation with the theory of parallels, for already in the year 1810–1812 with Gauss, as earlier 1809 with J. F. Pfaff I had learned to perceive how all previous attempts to prove the Euclidean axiom had miscarried. I had then also obtained preliminary knowledge of your works, and so, when I first [1820] had to print something of my view thereon, I wrote it exactly as it yet stands to read on page 187 of the latest edition.

''We had about this time [1819] here a law professor, Schweikart, who was formerly in Charkov, and had attained to similar ideas, since without help of the Euclidean axiom he developed in its beginnings a geometry which he called Astralgeometry. What he communicated to me thereon I sent to Gauss, who

then informed me how much farther already had been attained on this way, and later also expressed himself about the great acquisition, which is offered to the few expert judges in the Appendix to your book."

The "latest edition" mentioned appeared in 1851, and the passage referred to is : "This proof [of the parallel-axiom] has been sought in manifold ways by acute mathematicians, but yet until now not found with complete sufficiency. So long as it fails, the theorem, as all founded on it, remains a hypothesis, whose validity for our life indeed is sufficiently proven by *experience*, whose *general*, *necessary exactness*, however, could be doubted without absurdity."

Alas! that this feeble utterance should have seemed sufficient for more than thirty years to the associate of Gauss and Schweikart, the latter certainly one of the independent discoverers of the non-Euclidean geometry. But then, since neither of these sufficiently realized the transcendent importance of the matter to publish any of their thoughts on the subject, a more adequate conception of the issues at stake could scarcely be expected of the scholar and colleague. How different with Bolyai János and Lobachévski, who claimed

at once, unflinchingly, that their discovery marked an epoch in human thought so momentous as to be unsurpassed by anything recorded in the history of philosophy or of science, demonstrating as had never been proven before the supremacy of pure reason at the very moment of overthrowing what had forever seemed its surest possession, the axioms of geometry.

On the 9th of March, 1832, Bolyai Farkas was made corresponding member in the mathematics section of the Magyar Academy.

As professor he exercised a powerful influence in his country.

In his private life he was a type of true originality. He wore roomy black Hungarian pants, a white flannel jacket, high boots, and a broad hat like an old-time planter's. The smoke-stained wall of his antique domicile was adorned by pictures of his friend Gauss, of Schiller, and of Shakespeare, whom he loved to call the child of nature. His violin was his constant solace.

He died November 20th, 1856. It was his wish that his grave should bear no mark. The mother of Bolyai János , née, Arkosi Benkö Zsuzsanna, was beautiful, fascinating,

of extraordinary mental capacity, but always nervous.

János , a lively, spirited boy, was taught mathematics by his father. His progress was marvelous. He required no explanation of theorems propounded, and made his own demonstrations for them, always wishing his father to go on. "Like a demon, he always pushed me on to tell him more."

At 12, having passed the six classes of the Latin school, he entered the philosophic-curriculum, which he passed in two years with great distinction.

When about 13, his father, prevented from meeting his classes, sent his son in his stead. The students said they liked the lectures of the son better than those of the father. He already played exceedingly well on the violin.

In his fifteenth year he went to Vienna to K. K. Ingenieur-Akademie.

In August, 1823, he was appointed "souslieutenant" and sent to Temesvár, where he was to present himself on the 2nd of September.

From Temesvár, on November 3rd, 1823, János wrote to his father a letter in Magyar, of which a French translation was sent me by Professor Koncz József on February 14th,

1895. This will be given in full in my life of Bolyai; but here an extract will suffice :

'"*My Dear and Good Father*. "I have so much to write about my new inventions that it is impossible for the moment to enter into great details, so I write you only on one-fourth of a sheet. I await your answer to my letter of two sheets; and perhaps I would not have written you before receiving it, if 1 had not wished to address to you the letter I am writing to the Baroness, which letter I pray you to send her.

"First of all I reply to you in regard to the binominal.

* * * * * * * * * *

"Now to something else, so far as space permits. I intend to write, as soon as I have put it into order, and when possible to publish, a work on parallels.

"At this moment it is not yet finished, but the way which I have followed promises me with certainty the attainment of the goal, if it in general is attainable. It is not yet attained, but I have discovered such magnificent things that I am myself astonished at them.

"It would be damage eternal if they were

lost. When you see them, my father, you yourself will acknowledge it. Now I can not say more, only so much : *that from nothing I have created another wholly new world*. All that I have hitherto sent you compares to this only as a house of cards to a castle.

"P. S.—I dare to judge absolutely and with conviction of these works of my spirit before you, my father; I do not fear from you any false interpretation (that certainly I would not merit), which signifies that, in certain regards, I consider you as a second self."

Prom the Bolyai MSS., now the property of the College at Maros-Vásárhely, Fr. Schmidt has extracted the following statement by János :

"First in the year 1823 have I pierced through the problem in its essence, though also afterwards completions yet were added.

"I communicated in the year 1825 to my former teacher, Herr Johann Walter von Eckwehr (later k. k. General) [in the Austrian Army], a written treatise, which is still in his hands.

"On the prompting of my father I translated my treatise into the Latin language, and

it appeared as *Appendix* to the *Tentamen*, 1832."

The profound mathematical ability of Bolyai János showed itself physically not only in his handling of the violin, where he was a master, but also of arms, where he was unapproachable.

It was this skill, combined with his haughty temper, which caused his being retired as Captain on June 16th, 1833, though it saved him from the fate of a kindred spirit, the lamented Galois, killed in a duel when only 19. Bolyai, when in garrison with cavalry officers, was provoked by thirteen of them and accepted all their challenges on condition that he be permitted after each duel to play a bit on his violin. He came out victor from his thirteen duels, leaving his thirteen adversaries on the square.

He projected a universal language for speech as we have it for music and for mathematics.

He left parts of a book entitled : Principia doctrinae novae quantitatum imaginariarum perfectae uniceque satisfacientis, aliaeque disquisitiones analyticae et analytico-geometricae cardinales gravissimaeque; auctore

Johan. Bolyai de eadem, C. R. austriaco castrensium captaneo pensionato.

Vindobonae vel Maros Vásárhelyini, 1853.

Bolyai Farkas was a student at Goettingen from 1796 to 1799.

In 1799 he returned to Kolozsvár, where Bolyai János was born December 18th, 1802.

He died January 27th, 1860, four years after his father.

In 1894 a monumental stone was erected on his long-neglected grave in Maros-Vásárhely by the Hungarian Mathematico-Physical Society.

APPENDIX.

SCIENTIAM SPATII *absolute veram* exhibens :

a veritate aut falsitate Axiomatis XI *Euclidei*

(*a priori haud unquam decidenda*)

independentemn. adjecta ad casum falsitatis,

quadratura circuli

geometrica.

Auctore JOHANNE BOLYAI de eadem, Geometrarum

in Exercitu Caesareo Regio Austriaco

Castrensium Capitaneo.

EXPLANATION OF SIGNS.

The straight AB means the aggregate of all points situated in the same straight line with A and B.

The sect AB means that piece of the straight AB between the points A and B.

The ray AB means that half of the straight AB which commences at the point A and contains the point B.

The plane ABC means the aggregate of all points situated in the same plane as the three points (not in a straight) A, B, C.

The hemi-plane ABC means that half of the plane *ABC* which starts from the straight AB and contains the point C.

ABC means the smaller of the pieces into which the plane ABC is parted by the rays BA, BC, or the non-reflex angle of which the sides are the rays BA, BC.

ABCD (the point D being situated within ∠ABC, and the straights BA, CD not intersecting) means the portion of ∠ABC comprised between ray BA, sect BC, ray CD; while BACD designates the portion of the plane ABC comprised between the straights AB and CD.

⊥ is the sign of perpendicularity.

∥ is the sign of parallelism.

∠ means angle.

rt.∠ is right angle.

st.∠ is straight angle.

≅ is the sign of congruence, indicating that two magnitudes are superposable.

AB Π CD means ∠CAB = ∠ACD.

x Y a means x converges toward the limit a.

Δ is triangle.

⊙r means the [circumference of the] circle of radius r.

area ⊙r means the area of the surface of the circle of radius r.

[Not Mentioned: □ , ∞]

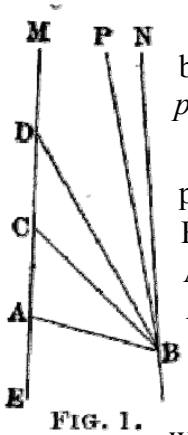

FIG. 1.

§1. If the ray AM is not cut by the ray [3] BN, situated in the same plane, but is cut by every ray BP comprised in the angle ABN, we will call ray BN *parallel* to ray AM; this is designated by BN ‖ AM;

It is evident that there *is one such ray BN*, *and only one*, passing through any point B (taken outside of the straight AM), and that the sum of the angles BAM, ABN can not exceed a st. ∠; for in moving BC around B until BAM + ABC = st. ∠, somewhere ray BC *first* does not cut ray AM, and it is then BC ‖ AM. It is clear that BN ‖ EM, wherever the point E be taken on the straight AM (supposing in all such cases AM > AE).

If while the point C goes away to infinity on ray AM, always CD = CB, we will have constantly CDB = (CBD < NBC); but NBC Y 0; and so also ADB Y 0.

§ 2. If BN ‖ AM, we will have also CN ‖ AM. For take D anywhere in MACN. If C is on ray BN, ray BD cuts ray AM, since BN ‖ AM, and so also ray CD cuts ray AM. But if C is on ray BR take BQ ‖ CD; BQ falls within the ∠ABN (§1), and cuts ray AM; and so also ray CD cuts ray AM. Therefore every ray CD (in ACN) cuts, in each case, the ray AM, without CN itself cutting ray AM. Therefore always CN ‖ AM.

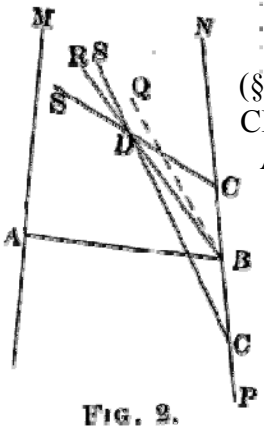

FIG. 2.

§ 3. (Fig. 2.) If BR and CS and each ‖ AM, and C is not on the ray BR, then ray BR and ray CS do not intersect. For if ray BR and ray CS had a common point D, then (§ 2) DR and DS would be each ‖ AM, and ray DS (§ 1) would fall on ray DR, and C on the ray BR (contrary to the hypothesis).

§ 4. If MAN > MAB, we will have for every point B of ray AB, a point C of ray AM, such that BCM = NAM.

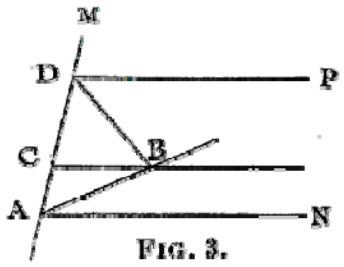

FIG. 3.

For (by § 1) is granted BDM > NAM, and so that MDP = MAN, and B falls in

NADP. If therefore NAM is carried along AM until ray AN arrives on ray DP, ray AN will somewhere have necessarily passed through B, and some BCM = NAM.

§ 5. If BN ‖ AM, there is on the straight [4] AM a point F such that FM Π BN. For by § 1 is granted BCM > CBN; and if CE = CB, and so EC Π BC; evidently BEM < EBN. The point P is moved on EC, the angle BPM always being called *u*, and the angle PBN always *v*, evidently *u* is at first less than the corresponding *v*, but afterwards greater. Indeed *u* increases *continuously* from BEM to BCM; since (by ~ 4) there exists no angle > BEM and < BCM, to which u does not at some time become equal. Likewise v decreases continuously from EBN to CBN. There is therefore on EC a point F such that BFM = FBN.

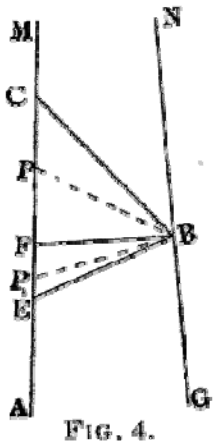

FIG. 4.

§ 6. If BN ‖ AM and E anywhere in the straight AM, and G in the straight BN; then GN ‖ EM and EM ‖ GN. For (by § 1) BN ‖ EM, whence (by § 2) GN ‖ EM. If moreover FM Π BN (§ 5); then MFBN ≅ NBFM, and consequently (since BN ‖ FM) also FM ‖ BN, and (by what precedes) EM ‖ GN.

§ 7. If BN and CP are each ‖ AM, and C not on the straight BN; also BN ‖ CP. For the rays BN and CP do not intersect (§ 3); but AM, BN and CP either are or are not in the same plane; and in the first case, AM either is or is not within BNCP.

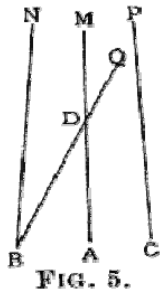

FIG. 5.

If AM, BN, CP are coplanar, and AM falls within BNCP; then every ray BQ (in NBC) cuts the ray AM in some point D (since BN ‖ AM); moreover, since DM ‖ CP (§ 6), the ray DQ will cut the ray CP, and so BN ‖ CP.

But if BN and CP are on the same side of AM; then one of them, for example CP, falls between the two other straights BN, AM : but every ray BQ (in NBA) cuts the ray AM, and so also the straight CP. Therefore BN ‖ CP.

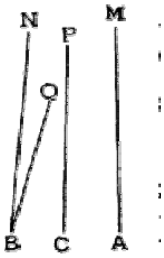

FIG. 6.

If the planes MAB, MAC make *an angle*; then CBN and ABN have in common nothing but the ray BN, while the ray AM (in ABN) and the ray BN, and so also NBC and the ray AM have nothing in common.

But hemi-plane BCD, drawn through any ray BD (in NBA), cuts the ray AM, since ray

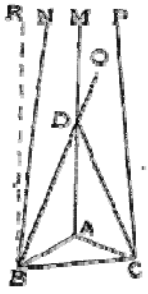

Fig. 7.

BQ cuts ray AM (as BN ‖ AM). Therefore in revolving the hemi-plane BCD around BC until it *begins* to leave the ray AM, the hemi-plane BCD at last will fall upon the hemi-plane BCN. For the same reason this same will fall upon hemi-plane BCP. Therefore BN falls in BCP. Moreover, if BR ‖ CP; then (because also AM ‖ CP) by like reasoning, BR falls in BAM, and also (since BR ‖ CP) in BCP. Therefore the straight BR, being common to the two planes MAB, PCB, of course is the straight BN, and hence BN ‖ CP.*

If therefore CP ‖ AM, and B exterior to the plane CAM; then the intersection BN of the planes BAM, BCP is ‖ as well to AM as to CP.

§ 8. If BN ‖ and Π CP (or more briefly BN ‖ Π CP), and AM (in NBCP) bisects ⊥ the sect BC; then BN ‖ AM.

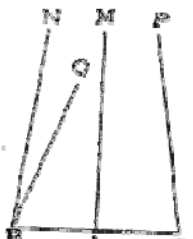

Fig. 8.

For if ray BN cut ray AM, also -ray CP would cut ray AM at the same point (because MABNB ≅ MACP), and this would be common to the rays BN, CP themselves,

* The third case being put before the other two, these can be demonstrated together with more brevity and elegance, like case 2 of ~10. [Author's note.]

although BN ‖ CP. But every ray BQ (in CBN) cuts ray CP; and so ray BQ cuts also ray AM. Consequently BN ‖ AN.

§ 9. If BN II AM, and MAP ⊥ MAB, and the ∠, which NBD makes with NBA (on that side of MABN, where MAP is) is < rt.∠; then MAP and NBD intersect.

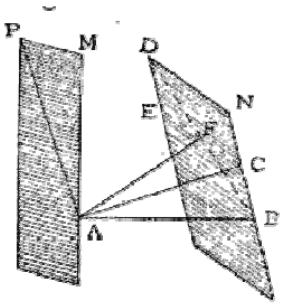

FIG. 9.

For let ∠BAM = rt.∠, and AC ⊥ BN (whether or not C falls on B), and CE ⊥ BN (in NBD); by hypothesis ∠ACE < rt.∠, and AF (⊥ CE) will fall in ACE.

Let ray AP be the intersection of the hemi-planes ABF, AMP (which have the point A common); since BAM ⊥ MAP, ∠BAP = ∠BAM = rt.∠.

If finally the hemi-plane ABF is placed upon the hemi-plane ABM (A and B remaining), ray AP will fall on ray AM; and since AC ⊥ BN, and sect AF < sect AC, evidently sect AF will terminate within ray BN, and so BF falls in ABN. But in *this* position, ray BF cuts ray AP (because BN ‖ AM); and so ray AP and ray BF intersect also *in the* original position; and the point of section is common to the hemi-planes MAP and NBD. Therefore the hemi-planes MAP and NBD intersect. Hence follows

easily that the hemi-planes MAP and NBD intersect if the sum of the interior angles which they make with MABN is < st.∠.

§ 10. If both BN and CP ‖ Π AM; also is BN ‖ Π CP.

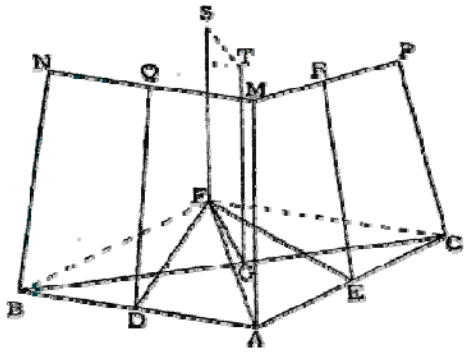

FIG. 10.

For either MAB and MAC make an angle, or they are in a plane.

If the first; let the hemi-plane QDF bisect ⊥ sect AB; then DQ ⊥ AB, and so DQ ‖ AM (§ 8); likewise if hemi-plane ERS bisects ⊥ sect AC, is ER ‖ AM; whence (§ 7) DQ ‖ ER.

Hence follows easily (by § 9), the hemi-planes QDF and ERS intersect, and have (§ 7) their intersection FS ‖ DQ, and (on account of BN ‖ DQ) also FS ‖ BN. Moreover (for any point of FS) FB = FA = FC, and the straight FS falls in the plane TGF, bisecting ⊥ sect BC. But (by § 7) (since FS ‖ BN) also GT ‖ BN. In the same way is proved GT ‖ CP. Meanwhile GT bisects ⊥ sect BC; and so TGBN ≅ TGCP (§ 1), and BN ‖ Π CP.

If BN, AM and CP are in a plane, let (falling without this plane) FS ‖ Π AM; then (from

what precedes) FS ‖ ⊥ both to BN and to CP, and so also BN ‖ ⊥ CP.

§ 11. Consider the aggregate of the point A, and *all* points of which any one B is such, that if BN ‖ AM, also BN Π AM; call it F; but the intersection of F with any plane containing the sect AM call L.

F has a point, and one only, on any straight ‖ AM; and evidently L is divided by ray AM into two congruent parts.

Call the ray AM *the axis* of L. Evidently also, in any plane containing the sect AM, there is for the *axis* ray AM a single L. Call any L of this sort the L of this ray AM (in the plane considered, being understood). Evidently by revolving L around AM we describe the F of which ray AM is called the axis, and in turn F *may be ascribed to the axis ray AM*.

§ 12. If B is anywhere on the L of ray AM, and BN ‖ Π AM (§ 11); then the L of ray AM and the L of ray BN *coincide*. For suppose, in distinction, L′ the L of ray BN. Let C be anywhere in L′, and CP ‖ Π BN (§ 11). Since BN ‖ Π AM, so CP ‖ Π AM (§ 10), and so C also will fall on L. And if C is anywhere on L, and CP ‖ Π AM; then CP ‖ Π BN (§ 10); and C also falls on L′ (§ 11). Thus L and L′ are the

same; and every ray BN is also axis of L, and between all axes of this L, is Π.

The same is evident in the same way of F.

§ 13. If BN ‖ AM, and CP ‖ DQ, and ∠BAM + ∠ABN = st.∠; then also ∠DCP + ∠CDQ = st.∠.

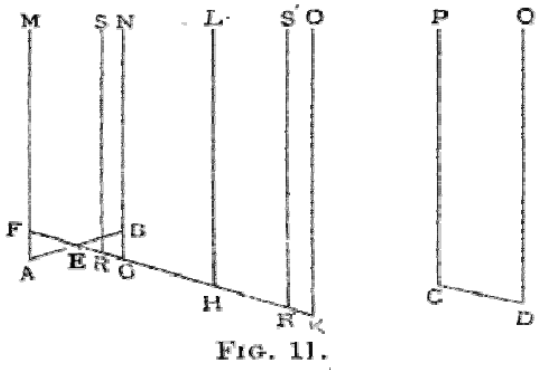

FIG. 11.

For let EA = EB, and EFM = DCP (§ 4). Since ∠BAM + ∠ABN = st. ∠ = ∠ABN + ∠ ABG, we have ∠EBG = ∠EAF; and so if also BG = AF, then ΔEBG ≅ ΔEAF, ∠BEG = ∠AEF and G will fall on the ray FE. Moreover ∠GFM + ∠FGN = st. ∠ (since ∠EGB = ∠EFA).

Also GN ‖ FM (§ 6).

Therefore if MFRS ≅ PCDQ, then RS ‖ GN (§ 7), and R falls within or without the sect FG (unless sect CD = sect FG, where the thing now is evident).

I. In the first case ∠FRS is not > (st. ∠ – ∠RFIM = ∠FGN), since RS ‖ FM. But as RS ‖ GN, also ∠FRS is not < ∠FGN; and so ∠FRS = ∠FGN, and ∠RFM + ∠FRS = ∠GFM +

$\angle FGN = \text{st.}\angle$. Therefore also $\angle DCP + \angle CDQ = \text{st.}\angle$.

II. If R falls without the sect FG; then $\angle NGR = \angle MFR$, and let $MFGN \cong NGHL \cong LHKO$, and so on, until FK = FR or begins to be > FR. Then KO ‖ HL ‖ FM (§7).

If K falls on R, then KO falls on RS (§ 1); and so $\angle RFM + \angle FRS = \angle KFM + \angle FKO = \angle KFM + \angle FGN = \text{st.}\angle$; but if R falls within the sect HK, then (by I) $\angle RHL + \angle KRS = \text{st.}\angle = \angle RFM + \angle FRS = \angle DCP + \angle CDQ$.

§ 14. If BN ‖ AM, and CP ‖ DQ, and $\angle BAM + \angle ABN < \text{st.}\angle$; then also $\angle DCP + \angle CDQ < \text{st.}\angle$.

For if $\angle DCP + \angle CDQ$ were not < st.∠, and so (by § 1) were = st.∠, then (by § 13) also $\angle BAM + \angle ABN = \text{st.}\angle$ (contra hyp.).

15. Weighing §§ 13 and 14, *the System of Geometry resting on the hypothesis of the truth of Euclid's Axiom XI is called Σ*; *and the system founded on the contrary hypothesis is S*.

All things which are not expressly said to be in Σ or in S, *it is understood are enunciated absolutely*, *that is are asserted true whether Σ or S is reality*.

§16. If AM is the axis of any L; then L, in Σ is a straight ⊥ AM.

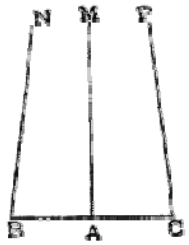

Fig. 12.

For suppose BN an axis from any point B of L; in Σ, ∠BAM + ∠ABN = st.∠, and so ∠BAM = rt.∠.

And if C is any point of the straight AB, and CP ‖ AM; then (by § 13) CP Π AM, and so C on L (§ 11).

But in S, no three points A, B, C on L or on F are in a straight. For some one of the axes AM, BN, CP (e.g. AM) falls between the two others; and then (by § 14) ∠BAM and ∠CAM are each < rt.∠.

§ 17. *L in S also is a line*, *and F a surface*. For (by § 11) any plane ⊥ to the axis ray AM (through any point of F) cuts F in [the circumference of] a circle, of which the plane (by § 14) is ⊥ to no other axis ray BN. If we revolve F about BN, any point of F (by § 12) will remain on F, and the section of F with a plane not ⊥ ray BN will describe a surface; and whatever be the points A, B taken on it, F can so be congruent to itself that A falls upon B (by § 12); therefore F is a *uniform surface*.

Hence evidently (by §§ 11 and 12) L is a uniform line.*

§ 18. *The intersection* with F of *any plane*, drawn through a point A of F obliquely to the axis AM, is, in S, *a circle*.

For take A, B, C, three points of this section, and BN, CP, axes; AMBN and AMCP make an angle, for otherwise the plane determined by A, B, C (from § 16) would contain AM, (contra hyp.). Therefore the planes bisecting ⊥ the sects AB, AC intersect (§ 10) in some *axis* ray FS (of F), and FB = FA = FC.

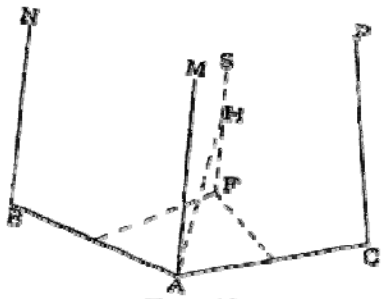

FIG. 13.

Make AH ⊥ FS, and revolve FAH about FS; A will describe a circle of radius HA, passing, through B and C, and situated *both* in F and in the plane ABC; nor have F and the plane ABC anything in common but ⊙ HA (§ 16).

It is also evident that in revolving the portion FA of the line L (as radius) in F around F, its extremity will describe ⊙ HA.

* It is not necessary to restrict the demonstration to the system S; since it may easily be so set forth, that it holds absolutely for S and for Σ.

§ 19. *The perpendicular* BT to the axis BN of L (falling in the plane of L) is, in S, N *tangent* to L. For L has in ray BT no point except B (§ 14), but if BQ falls in TBN, then the center of the section of the plane through BQ perpendicular to TBN with the F of ray BN (§ 18) is evidently located on ray BQ; and if sect BQ is a diameter, evidently ray BQ cuts in Q the line L of ray BN.

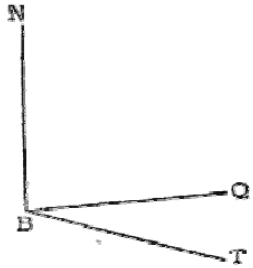

FIG. 14.

§ 20. Any two points of F determine a line L (§§ 11 and 18); and since (from §§ 16 and 19) L is ⊥ to all its axes, every ∠ of lines L in F is equal to the ∠ of the planes drawn through its sides perpendicular to F.

21. Two L form lines, ray AP and ray BD, in the same F, making with a third L form AB, a sum of interior angles < st.∠, intersect.

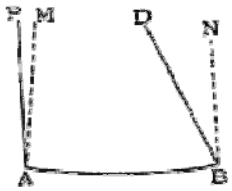

FIG. 15.

(By line AP in F, is to be understood the line L drawn through A and P, but by ray AP that half of this line beginning at A, in which P falls.)

For if AM, BN are axes of F, then the hemi-planes AMP, BND intersect (§ 9); and F cuts

their intersection (by §§ 7 and 11); and so also ray AP and ray BD intersect.

From this it is evident that Euclid's Axiom XI and all things which are claimed in geometry and plane trigonometry hold good *absolutely* in F, L lines being substituted in place of straights : therefore the trigonometric functions are taken here in the same sense as in Σ; and the circle of which the L form radius = r in F, is $2\pi r$; and likewise area of $\odot r$ (in F) = πr^2 (by π understanding $\frac{1}{2} \odot 1$ in F, or the known 3.1415926 . . .)

§ 22. If ray AB were the L of ray AM, and C on ray AM; and the ∠CAB (formed by the straight ray AM and the L form line ray AB), carried first along the ray AB, then along the ray BA, always forward to infinity : the path CD of C will be the line L of CM.

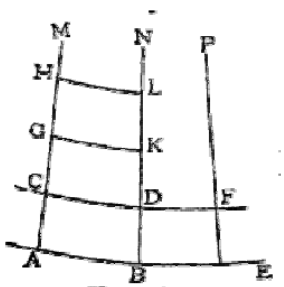

FIG. 16.

For let D be any point in line CD (called later L′, let DN be ‖ CM, and B the point of L falling on the straight DN. We shall have BN Π AM, and sect AC = sect BD, and so DN Π CM, consequently D in L′. But if D in L′ and DN ‖ CM, and B the point of L on the straight DN; we shall have AM Π BN and CM Π DN, whence manifestly sect BD = sect AC,

and D will fall on the path of the point C, and L′ and the line CD are the same. Such an L′ is designated by L′ 8 L.

§ 23. If the L form line CDF 8 ABE (§ 22), and AB = BE, and the rays AM, BN, EP are axes; manifestly CD = DF; and if any three points A, B, E are of line AB, and AB = n.CD, we shall also have AE = n.CF; and so (manifestly even for AB, AE, DC incommensurable), AB : CD = AE : CF, and AB : CD is *independent of AB*, *and completely determined by AC*.

This ratio AB : CD is designated by the capital letter (as X) corresponding to the small letter (as X) by which we represent the sect AC.

§ 24. Whatever be *x* and *y*, (§23), $Y = X^{\frac{x}{y}}$.

For, one of the quantities *x*, *y* is a multiple of the other (e. g. *y* of *x*), or it is not.

If *y* = n.*x*, take *x* = AC = CG = GH = &c., until we get AH = *y*.

Moreover, take CD 8 GK 8 HL.

We have ((§ 23) X = AB : CD – CD : GK = GK : HL; and so

$$\frac{AB}{HL} = \left(\frac{AB}{CD}\right)^n$$

or $Y = X^n = X^{\frac{y}{x}}$.

If *x*, *y* are multiples of *i*, suppose *x* = *mi*, and *y* = *ni*; (by the preceding) $X = I^m$, $Y = I^n$, consequently

$$Y = X^{\frac{n}{m}} = X^{\frac{y}{x}}$$

The same is easily extended to the case of the incommensurability of x and y.

But if $q = y - x$, manifestly $Q = Y : X$. It is also manifest that in Σ, for any x, we have $X = 1$, but in S is X > 1, and for any AB and ABE there is such a CDF 8 AB, that CDF = AB, whence AMBN ≅ AMEP, though the first be any multiple of the second; which indeed is singular, but evidently does not prove the absurdity of S.

§ 25. *In any rectilineal triangle, the circles with radii equal to its sides are as the sines of the opposite angles*.

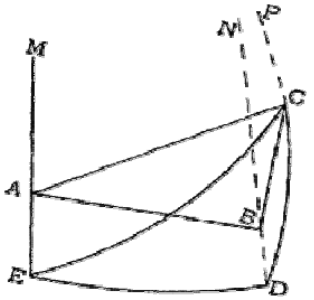

FIG. 17.

For take ∠ABC = rt.∠, and AM ⊥ BAC, and BN and CP ‖ AM; we shall have CAB ⊥ AMBN, and so (since CB ⊥ AMBN, consequently CPBN ⊥ AMBN.

Suppose the F of ray CP cuts the straights BN, AM respectively in D and E, and the bands CPBN, CPAM, BNAM along the L form lines CD, CE, DE. Then (§ 20) ∠CDE = the angle of NDC, NDE, and so = rt.∠; and by like reasoning ∠CED = CAB. But (by § 21) in the L line ΔCDE (supposing always here the radius = 1),

EC : DC = 1 : sin DEC = : sin CAB.

Also (by § 21)

EC : DC = ⊙EC : ⊙DC (in F) = ⊙AC : ⊙BC (§ 18); and so is also

⊙AC : ⊙BC - 1 : sin CAB;

whence the theorem is evident for any triangle.

§ 26. *In any spherical triangle*, *the sines of the sides are as the sines of the angles opposite*.

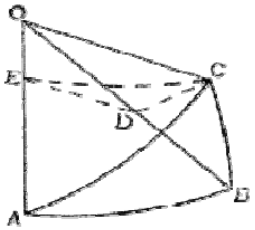

FIG. 18.

For take ∠ABC = rt.∠, and CED ⊥ to the radius OA of the sphere. We shall have CED ⊥ AOB, and (since also BOC ⊥ BOA), CD ⊥ OB. But in the triangles CEO, CDO (by § 25) ⊙EC : ⊙OC : ⊙DC = sin COE : 1 : sin COD = sin AC : 1 : sin BC; meanwhile also (§ 25) ⊙EC : ⊙DC = sin CDE : sin CED. Therefore, sin AC : sin BC = sin CDE : sin CED; but CDE = rt.∠ = CBA, and CED = CAB. Consequently

sin AC : sin BC = 1 : sin A.

Spherical trigonometry, *lowing from this*, *is thus established independently of Axiom XI*.

§ 27. If AC and BD are ⊥ AB, and CAB is carried along the straight AB; we shall have, designating by CD the path of the point C,

CD : AB = sin *u* : sin *v*.

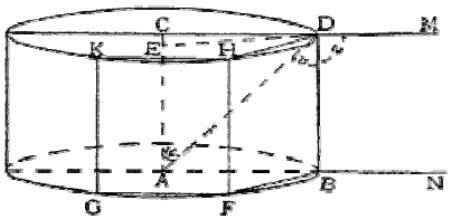

FIG. 19.

For take DE ⊥ CA; in the triangles ADE, ADB (by § 25) ⊙ED : ⊙AD : ⊙AB = sin *u* : 1 : sin *v*.

In revolving BACD about AC, B describes ⊙AB, and D describes ⊙ED; and designate here by *s*⊙CD the path of the said CD. Moreover, let there be any polygon BFG . . . inscribed in ⊙AB.

Passing through all the sides BF, FG, &c., planes ⊥ to ⊙AB we form also a polygonal figure of the same number of sides in *s*⊙CD, and we may demonstrate, as in § 23, that CD : AB = DH : BF = HK : FG, &c., and so

DH + HK &c. : BF + FG &c. : = CD : AB.

If each of the sides BF, FG . . . approaches the limit zero, manifestly

BF + FG + . . . Y⊙AB and

DH + HK + . . . ⊙ED.

Therefore also ⊙ED : ⊙AB = CD : AB. But we had ⊙ED : ⊙AB = sin *u* : sin *v*. Consequently

CD : AB = sin *u* : sin *v*.

If AC goes away from BD to infinity, CD : AB, and so also sin *u* : sin *v* remains *constant*; but *u* Y rt.∠ (§1), and if DM ‖ BN, *v* Y *z*; whence CD : AB = 1 : sin *z*.

The path called CD will be denoted by CD 8 AB.

§ 28. If BN ‖ Π AM, and C in ray AM, and AC = x. we shall have (§ 23)

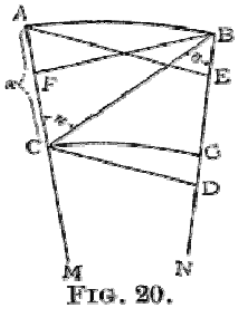

FIG. 20.

$$X = \sin u : \sin v.$$

For if CD and AE are ⊥ BN, and BF ⊥ AM; we shall have (as in § 27)

$$\odot BF : \odot DC = \sin u : \sin v.$$

But evidently BF = AE : therefore

$$\odot EA : \odot CD = \sin u : \sin v.$$

But in the F form surfaces of AM and CM (cutting AMBN in AB and CG) (by § 21)

$$\odot EA : \odot DC = AB : CG = X.$$

Therefore also

$$X = \sin u : \sin v.$$

§ 29. If ∠BAM – rt.∠, and sect AB = y, and BN ‖ AM, we shall have in S

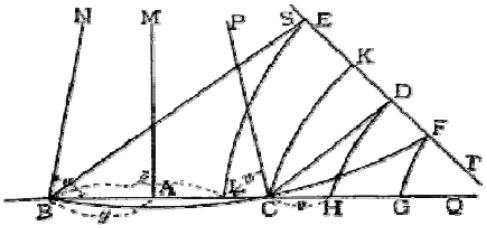

FIG. 21.

$$Y = \text{cotan } ½ u.$$

For, if sect AB = sect AC, and CP ‖ AM (and so BN ‖ CP), and ∠PCD = ∠QCD; there is given (§19) DS ⊥ ray CD, so that DS ‖ CP, and so (§ 1) DT ‖ CQ. Moreover, if BE ⊥ ray DS, then (§ 7) DS ‖ BN, and so (§ 6)

BN ‖ ES, and (since DT ‖ CG) BQ ‖ ET; consequently (§1) ∠EBN = ∠EBQ. Let BCF be an L-line of BN, and FG, DH, CK, EL, L form lines of FT, DT, CQ and ET; evidently (§ 22) HG = DF = DK = HC; therefore,

$$CG = 2CH = 2v.$$

Likewise it is evident $BG - 2BL = 2z$.

But BC = BG – CG; wherefore $y = z - v$, and so (§24) Y = Z : V.

Finally (§ 28)

Z = : sin ½ u,

and V = : sin (rt.∠ – ½ *u*),

consequently Y - cotan ½ *u*.

§ 30. However, it is easy to see (by § 25) that the solution of the problem of Plane Trigonometry, in S, requires the expression of the circle in terms of the radius; but this can by obtained by the rectification of L.

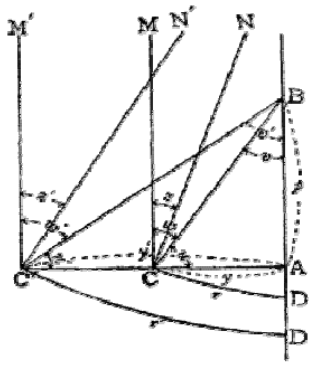

FIG. 22.

Let AB, CM, C'M′ be ⊥ ray AC, and B anywhere in ray AB; we shall have (§ 25)

$$\sin u : \sin v = \odot p : \odot y,$$

$$\text{and } \sin u' : \sin v' = \odot p' : \odot y';$$

$$\text{and so } \frac{\sin u}{\sin v} \cdot \odot y = \frac{\sin u'}{\sin v'} \cdot \odot y'.$$

But (by § 27) $\sin v : \sin v' = \cos u : \cos u'$;

consequently $\frac{\sin u}{\sin u} \cdot \odot y = \frac{\sin u'}{\sin u'} \cdot \odot y'$; or $\odot y : \odot y' = : \tan u' : \tan u = \tan w : \tan w'$.

Moreover, take CN and C'N′ ‖ AB, and CD, C'D' L-form lines ⊥ straight AB; we shall have also (§21)

$$\odot y : \odot y' = r : r', \text{ and so}$$

$$r : r' = \tan w : \tan w'.$$

Now let p beginning from A increase to infinity; then w Y z, and w' Y z', whence also $r : r' = \tan z : \tan z'$.

Designate by i the *constant*

$r : \tan z$ (independent of r);

whilst y Y 0,

$\frac{r}{y} = \frac{i \tan z}{y}$ Y 1, and so

$\frac{y}{\tan z}$ Y i. From §29, tan z = ½ (Y – Y $^{-1}$);

therefore $\frac{2y}{\text{Y} - \text{Y}^{-1}}$ Y i,

or (§ 24). $\frac{2y \cdot \text{I}^{\frac{\text{y}}{\text{I}}}}{\text{I}^{\frac{2\text{y}}{\text{I}}} - 1}$ Y i.

But we know the limit of this expression (where y Y 0) is.

$\frac{i}{\text{nat. log I}}$ Therefore

$$\frac{i}{\text{nat. log I}} = i, \text{ and}$$

$$\text{I} = e = 2.7182818 \ldots,$$

which noted quantity shines forth here also.

If obviously henceforth i denote that sect of which the I = e, we shall have

$$r = i \tan z.$$

But (§ 21) $\odot y = 2\pi r$; therefore

$$\odot y = 2\pi i \tan z = \pi i \,(\text{Y} - \text{Y}^{-1}) = \pi i \left(e^{\frac{\text{y}}{\text{i}}} - e^{\frac{\text{y}}{-\text{i}}}\right)$$

$$= \frac{\pi y}{\text{nat. log Y}} (\text{Y} - \text{Y}^{-1}) \text{ (by § 24).}$$

§ 31. For the trigonometric solution of all right-angled rectilineal *triangles* (whence the resolution of all *triangles* is easy, in S, three equations suffice : indeed (a, b denoting the sides, c the hypothenuse, and α, β the angles opposite the sides) an equation expressing the relation

1st, between a, c, α;

2d, between a, α, β;

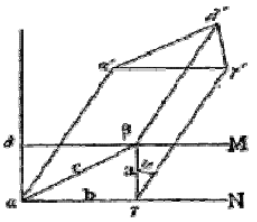

FIG. 23.

3d, between a, b, c;

of course from these equations emerge three others by elimination.

From §§ 25 and 30

$$1 : \sin a = (\text{C} - \text{C'}) : (\text{A} - \text{A}^{-1}) = \left(e^{\frac{\text{e}}{\text{i}}} - e^{\frac{-\text{e}}{\text{i}}}\right) : \left(e^{\frac{\text{a}}{\text{i}}} - e^{\frac{-\text{a}}{\text{i}}}\right)$$

(equation for c, a and α).

II. From § 27 follows (if β M ‖ γ N)

cos α : sin β = 1 : sin u, but from § 29

$$1 : \sin u = \tfrac{1}{2}(A + A^{-1});$$

therefore cos α : sin $\beta = \tfrac{1}{2}(A + A^{-1}) = \tfrac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)$

(equation for α, β and a).

III. If $\alpha\alpha' \perp \beta\alpha\gamma$, and $\beta\beta'$ and $\gamma\gamma'$ ‖ $\alpha\alpha'$ (§ 27), and i $\beta'\alpha'\gamma' \perp \alpha\alpha'$; manifestly (as in § 27),

$$\frac{\beta\beta'}{\gamma\gamma'} = \frac{1}{\sin u} = \tfrac{1}{2}(A + A^{-1});$$

$$\frac{\gamma\gamma'}{\alpha\alpha'} = \tfrac{1}{2}(B + B^{-1});$$

$$\text{and } \frac{\beta\beta'}{\alpha\alpha'} = \tfrac{1}{2}(C + C^{-1}); \text{ consequently}$$

$$\tfrac{1}{2}(C + C^{-1}) = \tfrac{1}{2}(A + A^{-1}) \cdot \tfrac{1}{2}(B + B^{-1}), \text{ or}$$

$$\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right) = \tfrac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right) \cdot \left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)$$

(equation for a, b and c).

If $\gamma\alpha\delta$ = rt.∠, and $\beta\delta \perp \alpha\delta$;

$\odot c : \odot a = 1 : \sin \alpha$, and

$\odot c : \odot (d = \beta\delta) = 1 : \cos \alpha$,

and so (denoting by $\odot x^2$, for any x, the product $\odot x \cdot \odot x$) manifestly

$\odot a^2 + \odot d^2 - \odot c^2$.

But (by § 27 and II)

$$\odot d = \odot b \,.\, \tfrac{1}{2}(A + A^{-1}), \text{ consequently}$$

$$\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right)^2 = \tfrac{1}{4}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)^2 \cdot \left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)^2 + \left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)^2$$

another equation for a, b and c (the second

member of which may be easily reduced to a form *symmetric* or *invariable*).

Finally, from

$$\frac{\cos\alpha}{\sin\beta} = \tfrac{1}{2}(A + A^{-1}), \text{ and } \frac{\cos\beta}{\sin\alpha} = \tfrac{1}{2}(B + B^{-1}), \text{ we get}$$

(by III)

$$\cot\alpha \cot\beta = \tfrac{1}{2}\left(e^{\frac{c}{i}} + e^{\frac{-c}{i}}\right)$$

(equation for α, β, and c.)

§ 32. It still remains to show briefly the mode of resolving *problems* in S, which being accomplished (through the more obvious examples), finally will be candidly said what this theory shows.

I. Take AB a line in a plane, and $y = f(x)$ its equation in rectangular coordinates, call dz any increment of z, and respectively dx, dy, du the increments of x, of y, and of the area u, corresponding to this dz; take BH 8 CF, and express (from § 31) $\frac{BH}{dx}$ by means of y, and seek the *limit* of $\frac{dy}{dx}$ when dx tends towards the limit zero (which is understood where a limit of this sort is sought) : then will become known also the limit of $\frac{dy}{BH}$ and so tan HBG; and

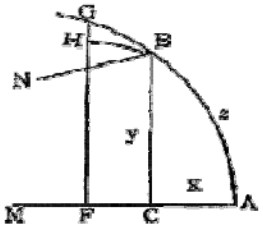

FIG. 24.

(since HBC manifestly is neither > nor < , and so = rt∠.), the *tangent* at B of BG will be determined by *y*.

II. It can be demonstrated

$$\frac{dz^2}{dy^2 + \overline{\mathrm{BH}}^2}\,\mathrm{Y1}.$$

Hence is found the *limit* of $\frac{dz}{dx}$ and thence, by integration, *z* (expressed in terms of *x*.)

And of any line *given in the concrete*, the equation in *S* can be found; e. g., of L. For if ray AM be the axis of L; then any ray CB from ray AM cuts L [since (by § 19) any straight from A except the straight AM will cut L]; but (if BN is axis)

$$\mathrm{X} = 1 : \sin \mathrm{CBN}\ (\S 28),$$

and Y = cotan ½ CBN (§ 29), whence

$$\mathrm{Y} = \mathrm{X} + \sqrt{\mathrm{X}^2 - 1}.$$

or $e^{\frac{y}{i}} = e^{\frac{x}{i}} + \sqrt{e^{\frac{2x}{i}} - 1}$,

the equation sought.

Hence we get

$$\frac{dy}{dy}\,\mathrm{Y}\ \mathrm{X}(\mathrm{X}^2 - 1)^{\frac{-1}{2}};$$

and $\frac{\mathrm{BH}}{dx}$ Y1 : sin CBN = X; and so

$$\frac{dy}{\mathrm{BH}}\ \mathrm{Y}\ (\mathrm{X}^2 - 1)^{\frac{-1}{2}};$$

$$1 + \frac{dy^2}{BH^2} \text{ Y } X^2(X^2 - 1)^{-1},$$

$$\frac{dz^2}{BH^2} \text{ Y } X^2(X^2 - 1)^{-1},$$

and $$\frac{dz}{BH} \text{ Y } X (X^2 - 1)^{\frac{1}{2}} \text{ and}$$

$$\frac{dz}{dx} \text{ Y } X^2 (X^2 - 1)^{\frac{-1}{2}},$$ whence, by integration, we get (as in § 30)

$$z = i (X^2 - 1)^{\frac{1}{2}} = i \cot \text{CBN}.$$

III. Manfestly

$$\frac{du}{dx} \text{ Y } \frac{\text{HFCBH}}{dx}$$

which (unless given in y) now first is to be expressed in terms of y; whence we get u by integrating.

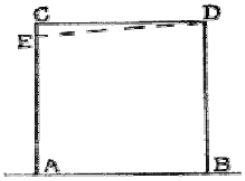

FIG. 25.

If AB = p, AC = q, CD = r, and CABDC = s; we might show (as in II) that

$$\frac{ds}{dq} \text{ Y } r, \text{ which } = -\tfrac{1}{2} p \left(e^{\frac{q}{i}} - e^{\frac{-q}{i}} \right), \text{ and, integrating, } s = \tfrac{1}{2} pi \left(e^{\frac{q}{i}} - e^{\frac{-q}{i}} \right).$$

This can also be deduced apart from integration.

For example, the equation of the circle (from § 31, III), of the straight (from § 31, II), of a conic (by what precedes), being expressed, the

areas bounded by these lines could also be expressed.

We know, that a surface *t*, 8 to a plane figure *p* (at the distance *q*), is to *p* in the ratio of the second powers of homologous lines, or as

$$¼\left(e^{\frac{q}{i}} - e^{\frac{-q}{i}}\right)^2 : 1.$$

It is easy to see, moreover, that the calculation of volume, treated in the same manner, requires two integrations (since the differential itself here is determined only by integration); and before all must be investigated the volume contained between *p* and *t*, and the aggregate of all the straights $\perp p$ and joining the boundaries of *p* and *t*.

We find for the volume of this solid (whether by integration or without it)

$$⅛\, pi\left(e^{\frac{2q}{i}} - e^{\frac{-2q}{i}}\right) + ½\, pq.$$

The surfaces of bodies may also be determined in S, as well as the *curvatures*, the *involutes*, and *evolutes* of any lines, etc.

As to curvature; this in S either is the curvature of L, or is determined either by the radius of a circle, or by the *distance* to a straight from the curve 8 to this straight; since from what precedes, it may easily be shown, that in a plane there are no uniform lines other than L-lines, circles and curves 8 to a straight.

IV. For the circle (as in III) $\frac{d \text{ area } \odot x}{dx}$ Y $\odot x$, whence (by § 29), integrating,

$$\text{area } \odot x = \pi i^2 \left(e^{\frac{x}{i}} - 2 + e^{\frac{-x}{i}} \right).$$

V. For the area CABDC = u (inclosed by an L form line AB = r, the 8 to this, CD = y, and the sects AC = BD = x) $\frac{du}{dx}$ Y y; and (§ 24) $y = re^{\frac{-x}{i}}$, and so (integrating)

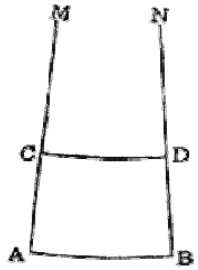

FIG. 26.

$$u = ri\left(1 - e^{\frac{x}{i}}\right).$$

If x increases to infinity, then, in S, $e^{\frac{-x}{i}}$ Y 0, and so u Y ri. By the *size* of MABN, in future this limit is understood.

In like manner is found, if p is a figure on F, the space included by p and the aggregate of axes drawn from the boundaries of p is equal to ½ pi.

VI. If the angle at the center of a segment z of a sphere is $2u$, and a great circle is p, and x the arc FC (of the angle u); (§25)

$$1 : \sin u = p : \odot \text{BC},$$

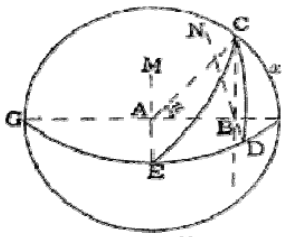

FIG. 27.

and hence $\odot \text{BC} = p \sin u$.

Meanwhile $x = \frac{pu}{2\pi}$, and $dx = \frac{pdu}{2\pi}$.

Moreover, $\frac{dz}{dx}$ Y ⊙ BC, and hence

$$\frac{dz}{du} \text{ Y } \frac{p^2}{2\pi} \sin u, \text{ whence (integrating)}$$

$$z = \frac{\text{ver sin } u}{2\pi} p^2.$$

The F may be conceived on which P falls (passing through the middle F of the segment); through AF and AC the planes FEM, CEM are placed, perpendicular to F and cutting F along FEG and CE; and consider the L form CD (from C ⊥ to FEG), and the L form CF; (§ 20) CEF = u, and (§ 21)

$$\frac{\text{FD}}{p} = \frac{\text{ver sin } u}{2\pi}, \text{ and so } z - \text{FD} \cdot p.$$

But (§ 21) $p = \pi \cdot$ FGD; therefore

$z = \pi \cdot$ FD $\cdot$ FDG. But (§ 21)

FD $\cdot$ FDG = FC $\cdot$ FC; consequently

$z = \pi \cdot$ FC $\cdot$ FC = area ⊙ FC, in F.

Now let BJ = CJ = r; (§ 30) $2r = i$ (Y – Y $^{-1}$), and so (§ 21)

area ⊙ $2r$ (in F) = πi^2(Y – Y $^{-1}$)^2.

Also (IV) area ⊙ $2y = \pi i^2$(Y^2 – 2 + Y $^{-2}$); therefore, area ⊙ $2r$ (in F) = area ⊙ $2y$, and so *the surface z of a segment of a sphere is equal to the surface of the circle described with the chord* FC *as a radius*.

Hence the whole surface of the sphere

$$= \text{area} \odot \text{FG} = -\text{FDG} \cdot p - \frac{p^2}{\pi},$$

and the surfaces of spheres are to each other as the second powers of their great circles.

VII. In like manner, in S, the volume of the sphere of radius x is found

$$= \tfrac{1}{2}\pi i^3 (\text{X}^2 - \text{X}^{-2}) - 2\pi i^2 x;$$

the surface generated by the revolution of the line CD about AB

$$= \tfrac{1}{2}\pi i p (\text{Q}^2 - \text{Q}^{-2}),$$

and the body described by CABDC

$$= \tfrac{1}{4}\pi i^2 p (\text{Q}^2 - \text{Q}^{-2})^2.$$

But in what manner all things treated from (IV) *even to here*, *also may be reached apart from integration*, *for the sake of brevity is suppressed*.

It can be demonstrated that the limit of every expression containing the letter i (and so resting upon the hypothesis that *i* is given), *when i increases to infinity*, *expresses the quantity simply for* Σ (and so for the hypothesis of no *i*), if indeed the equations do not become identical.

But beware lest you understand to be supposed, that the system itself may be varied (for it is entirely determined in itself and by itself); but only *the hypothesis*, which may be

done successively, as long as we are not conducted to an absurdity. *Supposing* therefore that, in *such* an expression, the letter *i*, in case S is reality, designates that unique quantity whose I = *e*; but if Σ is actual, the said limit is supposed to be taken in place of the expression : manifestly *all the expressions originating from the hypothesis of the reality of* S (*in this sense*) *will be true absolutely*, a*lthough it be completely unknown whether or not* Σ *is reality*.

So e. g. from the expression obtained in § 30 easily (and as well by aid of differentiation as apart from it) emerges the known value in Σ,

$$\odot x = 2\pi x;$$

from I (§ 31) suitably treated, follows

$$1 : \sin\alpha = c : a,$$

but from II

$$\frac{\cos\alpha}{\sin\beta} = 1, \text{ and so}$$

$$\alpha + \beta = \text{rt.}\angle;$$

the first equation in III becomes identical, and so is true in Σ, although it there determines nothing; but from the second follows

$$c^2 = a^2 + b^2.$$

These are the known fundamental equations of plane trigonometry in Σ.

Moreover, we find (from § 32) in Σ, the area and the volume in III each = pq; from IV area $\odot x = \pi x^2$;

(from VII) the globe of radius x

$$= \tfrac{4}{3}\pi x^3, \text{ etc.}$$

The theorems enunciated at the end of VI are manifestly *true unconditionally*.

§ 33. It still remains to set forth (as promised in § 32) what this theory means.

I. Whether Σ or some one S is reality, remains undecided.

II. All things deduced from the hypothesis of the falsity of Axiom *XI* (always to be understood in the sense of § 32) are *absolutely true*, and so in this sense, depend upon no hypothesis.

There is therefore *a plane trigonometry a priori*, *in which the system alone really remains unknown*; and so where remain unknown solely the *absolute* magnitudes in the expressions, but where a *single* known case would manifestly fix the whole system. But spherical trigonometry is established absolutely in § 26.

(And we have, on F, a geometry wholly analogous to the plane geometry of Σ.)

III. If it were agreed that Σ exists, nothing more would be unknown in this respect; but

if it were *established* that Σ does not exist, then (§ 31), (e. g.) from the sides *x*, *y*, and the rectilineal angle they include being given in a special case, manifestly it would be impossible in itself and by itself to solve absolutely the triangle, that is, to determine *a priori* the other angles and *the ratio of the third side* to the two given; unless X, Y were determined, for which it would be necessary to have in concrete form a certain sect *a* whose A was known; and then *i* would be *the natural unit for length* (just as *e* is the base of *natural* logarithms).

If the existence of this *i* is determined, it will be evident how it could be constructed, at least very exactly, for practical use.

IV. In the sense explained (I and II), it is evident that all things in space can be solved by the modern analytic method (within just limits strongly to be praised).

V. Finally, to friendly readers will not be unacceptable; that for that case wherein not Σ but S is reality, a rectilineal figure is constructed equivalent to a circle.

§ 34. Through D we may draw DM ∥ AN in the following manner. From D drop DB ⊥ AN; from any point A of the straight AB erect AC ⊥ AN (in DBA), and let fall DC ⊥ AC. We

will have (§ 27) ⊙ CD : ⊙ AB – 1 : sin *z*, provided that DM ‖ BN. But sin *z* is not > 1; and so AB is not > DC. Therefore a quadrant described from the center A in BAC, with a radius = DC, will have a point B or O in common with ray BD. In the first case, manifestly $z = rt.\angle$; but in the second case (§ 25)

$$(\odot AO = \odot CD) : \odot AB = 1 \sin AOB,$$

and so *z* = AOB.

If therefore we take *z* = AOB, then DM will be ‖ BN.

§ 35. If S were reality; we may, as follows, draw a straight ⊥ to one arm of an acute angle, which is ‖ to the other.

Take AM ⊥ BC, and suppose AB = BC so small (by § 19), that if we draw BN ‖ AM (§ 34), ABN > the given angle.

Moreover draw CP ‖ AM (§ 34); and take NBG and PCD each equal to the given angle; rays BG and CD will cut; for if ray BG (falling *by construction* within NBC) cuts ray CP in E; we shall have (since BN Π CP), ∠EBC < ∠ECB, and so EC < EB. Take EF = EC, EFR

= ECD, and FS ‖ EP; then FS will fall within BFR. For since BN ‖ CP, and so BN ‖ EP, and BN ‖ FS; we shall have (§ 14)

$$\angle FBN + \angle BFS < (\text{st.} \angle = FBN + BFR);$$

therefore, BFS < BFR. Consequently, ray FR cuts ray EP, and so ray CD also cuts ray EG in some point D. Take now DG = DC and DGT = DCP = GBN; we shall have (since CD Π GD) BN Π GT Π CP. Let K (§ 19) be the point of the L-form line of BN falling in the ray BG, and KL the axis; we shall have BN Π KL, and so BKL = BGT = DCP; but also KL Π CP : therefore manifestly K fall on G, and GT ‖ BN. But if HO bisects ⊥ BG, we shall have constructed HO ‖ BN.

§36. Having given the ray CP and the plane MAB, take CB ⊥ the plane MAB, BN (in plane BCP) ⊥ BC, and CQ ‖ BN (§ 34); the intersection of ray CP (if this ray falls within BCQ) with ray BN (in the plane CBN), and so with the plane MAB is found. And if we are given the two planes PCQ, MAB, and we have CB ⊥ to plane MAB, CR ⊥ plane PCQ; and (in plane BCR) BN ⊥ BC, CS ⊥ CR, BN will fall in plane MAB, and CS in plane PCQ; and the

intersection of the straight BN with the straight CS (if there is one) having been found, the perpendicular drawn through this intersection, in PCQ, to the straight CS will manifestly be the intersection of plane MAB and plane PCQ.

§ 37. On the straight AM ‖ BN, is found such an A, that AM Π BN. If (by § 34) we construct outside of the plane NBM, GT ‖ BN, and make BG ⊥GT, GC = GB, and CP ‖ GT; and so place the hemi-plane TGD that it makes with hemi-plane TGB an angle equal to that which hemi-plane PCA makes with hemi-plane PCB; and is sought (by § 36) the intersection straight DQ of hemi-plane TGD with hemi-plane NBD; and BA is made ⊥ DQ.

We shall have indeed, on account of the similitude of the triangles of L lines produced on the F of BN (§ 21), manifestly DB = DA, and AM Π BN.

Hence easily appears (L-lines being given by their extremities alone) we may also find a fourth proportional, or a mean proportional, and execute in this way in F, apart from Axiom XI, all the geometric constructions made

on the plane in Σ. Thus e. g. a perigon can be geometrically divided into any special number of equal parts, if it is permitted to make this special partition in Σ.

§ 38. If we construct (by § 37) for example, NBQ = ⅓ rt.∠, and make (by. § 35), in S, AM ⊥ ray BQ and ‖ BN, and determine (by §37) IM Π BN; we shall have, if IA = *x*, (§ 28), X = 1 : sin ⅓ rt. ∠ = 2, and *x* will be constructed *geometrically*.

And NBQ may be so computed, that IA differs from *i* less than by anything given, which happens for sin NBQ = 1/*e*.

§ 39. If (in a plane) PQ and ST are 8 to the straight MN (§27), and AB, CD are equal perpendiculars to MN; manifestly ΔDEC ≅ ΔBEA; and so the angles (perhaps mixtilinear) ECP, EAT will fit, and EC = EA. If, moreover, CF = AG, then ΔACF ≅ ΔCAG, and each is half of the *quadrilateral* FAGC.

If FAGC, HAGK are two quadrilaterals of this sort on AG, between PQ and ST; their equivalence (as in Euclid) is evident, as also

the equivalence of the triangles AGC, AGH, standing on the same AG, and having their vertices on the line PQ. Moreover, ACF = CAG, GCQ = CGA, and ACF + ACG + GCQ = st.∠ (§ 32); and so also CAG + ACG + CGA = st.∠; therefore, in any triangle ACG of this sort, the sum of the three angles = st.∠. But whether the straight AG may have fallen upon AG (which 8 MN), or not; *the equivalence* of the rectilineal triangles AGC, AGH, as well of themselves, *as of the sums of their angles*, is evident.

§ 40. *Equivalent triangles* ABC, ABD, (henceforth rectilineal), *having one side equal*, *have the sums of their angles equal*. For let MN bisect AC and BC, and take (through C) PQ 8 MN; the point D will fall on line PQ.

For, if ray BD cuts the straight MN in the point E, and so (§ 39) the line PQ at the distance EF = EB; we shall have ΔABC = ΔABF, and so also ΔABD = ΔABF, whence D falls at F.

But if ray BD has not cut the straight MN, let C be the point, where the perpendicular bisecting the straight AB cuts the line PQ, and

let GS = HT, so, that the line ST meets the ray BD prolonged in a certain K (which it is evident can be made in a way like as in § 4); moreover take SR = SA, RO 8 ST, and O the intersection of ray BK with RO; then ΔABR = ΔABO (§39), and so ΔABC > ΔABD (contra hyp.).

§ 41. *Equivalent triangles ABC*, *DEF have the sums of their triangles equal*.

For let MN bisect AC and BC, and PQ bisect DF and FE; and take RS 8 MN, and TO 8 PQ; the perpendicular AG to RS will equal the perpendicular DH to TO, or one for example DH will be the greater.

In each case, the ⊙ DF, from center A, has with line-ray GS some point K in common, and (§ 39) ΔABK = ΔABC = ΔDEF. But the ΔAKB (by § 40) has the same angle-sum as ΔDFE, and (by § 39) as ΔABC. Therefore also the triangles ABC, DEF have each the same angle-sum.

In S the inverse of this theorem is true.

For take ABC, DEF two triangles having equal angle-sums, and ΔBAL = ΔDEF; these will have (by what precedes) equal angle-sums,

and so also will ΔABC and ΔABL, and hence manifestly

$$\text{BCL} + \text{BLC} + \text{CBL} = \text{st.}\angle.$$

However (by § 31), the angle-sum of any triangle, in S, is < st.∠.

Therefore L falls on C.

§ 42. Let u be the supplement of the angle-sum of the ΔABC, but v of ΔDEF; then is ΔABC : ΔDEF = u : v. F For if p be the area of each of the triangles ACG, GCH, HCB, DFK, KFE; and ΔABC = $m \cdot p$, and ΔDEF = $n \cdot p$; and s the angle-sum of any triangle equivalent to p, manifestly

st. ∠ – u = $m \cdot s$ – (m – 1)st. ∠ = st. ∠ – m(st. ∠ – s); and u = m(st.∠ – s); and in like manner v = n(st.∠ – s).

Therefore ΔABC : ΔDEF = m : n = u : v. It is evidently also easily extended to the case of the incommensurability of the triangles ABC, DEF.

In the same way is demonstrated that triangles on a sphere are as the excesses of the sums of their angles above a st. < .

If two angles of the spherical Δ are right, the third z will be the said excess. But

(a great circle being called p) this Δ is manifestly

$$= \frac{z}{2\pi} \frac{p^2}{2\pi} \text{ (§ 32, VI);}$$

consequently, any triangle of whose angles the excess is z, is

$$= \frac{zp^2}{4\pi^2}.$$

§ 43. Now, in S, the area of a rectilineal Δ is expressed by means of the sum of its angles.

If AB increases to infinity; (§ 42) Δ ABC : (rt.∠ – *u* – *v*) will be constant. But Δ ABC Y BACN (§ 32, V), and rt.∠ – *u* – *v* Y *z* (§ 1); and so BACN : *z* = Δ ABC : (rt. ∠– *u* – *v*) = BAC'N' : *z*'.

Moreover, manifestly (§ 30) BDCN : BD'C'N' = *r* : *r*' tan *z* : tan *z*'.

But for *y*' Y 0, we have

$$\frac{\text{BD'C'N'}}{\text{BAC'N'}} \doteq 1 \text{ and also } \frac{\tan z'}{z'} \text{ Y } 1;$$

consequently,

$$\text{BDCN} : \text{BACN} = \tan z : z.$$

But (§ 32)

$$\text{BDCN} = r \cdot i = i^2 \tan z;$$

therefore,

$$\text{BACN} = z \cdot i^2.$$

Designating henceforth, for brevity, any triangle the supplement of whose angle-sum is z by Δ, we will therefore have $\Delta = z \cdot i^2$.

Hence it readily flows that, if OR ‖ AM and RO ‖ AB, the *area* comprehended between the straights OR, ST, BC (which is manifestly the absolute limit of the area of rectilineal triangles increasing without bound, or of Δ for z Y st. $\angle$), is $= \pi i^2 =$ area $\odot i$, in F.

This limit being denoted by $\square$, moreover (by § 30) $\pi r^2 = \tan^2 z \cdot \square =$ area $\odot r$ in F (§ 21) = area $\odot s$ (by §32, VI) if the chord CD is called s.

If now, bisecting at right angles the given radius s of the circle in a plane (or the L form radius of the circle in F), we construct (by § 34) DB ‖ Y CN; by dropping CA $\perp$ DB, and erecting CM $\perp$ CA, we shall get z; whence (by § 37), assuming at pleasure an L form radius for unity, $\tan^2 z$ *can be determined geometrically by means of two uniform lines of the same curvature* (which, their extremities alone being given and their axes

constructed, manifestly may be compared like straights, and in this respect considered equivalent to straights). Moreover, a quadrilateral, ex. gr. regular = □ is constructed as follows :

Take ABC = rt.∠, BAC = ½ rt. ∠, ACB = ¼ rt. ∠, and BC = x.

By mere square roots, X (from § 31, II) can be expressed and (by 37) constructed; and having X (by § 38 or also §§ 29 and 35), x itself can be determined. And octuple Δ ABC is manifestly = □, and by this *a plane circle of radius s is geometrically squared by means of a rectilinear figure and uniform lines of the same species* (*equivalent to straights as to comparison inter se*); *but an F form circle is planified in the same manner*: *and we have either the Axiom* XI *of Euclid true or the geometric quadrature of the circle*, although thus far it has remained undecided, which of these two has place in reality.

Whenever $\tan^2 z$ is either a whole number, or a rational fraction, whose denominator (reduced to the simplest form) is either a prime number of the form $2^m + 1$ (of which is also $2 = 2^0 + 1$), or a product of however many prime numbers of this form, of which each (with the

exception of 2, which alone may occur any number of times) occurs only once as factor, we can, by the theory of polygons of the illustrious Gauss (remarkable invention of our, nay of every age) (and only for such values of z), construct a rectilineal figure = $\tan^2 z \, \square$ = area $\odot s$. For the division of $\square$ (the theorem of § 42 extending easily to any polygons) manifestly requires the partition of a st. $\angle$, which (as can be shown) can be achieved geometrically only under the said condition.

But in all such cases, what precedes conducts easily to the desired end. And any rectilineal figure can be converted geometrically into a regular polygon of n sides, if n falls under the Gaussian form. It remains, finally (that the thing may be completed in every respect), to demonstrate the impossibility (apart from any supposition), of deciding *a priori*, whether Σ, or some S (and which one) exists. This, however, is reserved for a more suitable occasion.

APPENDIX I.

REMARKS ON THE PRECEDING TREATISE, BY BOLYAI FARKAS.

[From Vol. II of Tentamen, pp. 380 – 383.]

Finally it may be permitted to add something appertaining to the author of the *Appendix* in the first volume, who, however, may pardon me if something I have not touched with his acuteness.

The thing consists briefly in this : *the formulas of spherical trigonometry* (demonstrated in the said *Appendix* independently of Euclid's Axiom XI) *coincide with the formulas of plane trigonometry*, *if* (in a way provisionally speaking) *the sides of a spherical triangle are accepted as reals*, *but of a rectilineal triangle as imaginaries*; so that, as to trigonometric formulas, the plane may be considered as an imaginary sphere, if for real, that is accepted in which sin rt. $\angle$ = 1.

Doubtless, of the Euclidean axiom has been said in volume first enough and to spare : for

the case if it were not true, is demonstrated (Tom. I. App., p. 13), that there is given a certain *i*, for which the I there mentioned is = e (the base of natural logarithms), and for this case are established also (*ibidem*, p. 14) the formulas of plane trigonometry, and indeed so, that (by the side of p. 19, ibidem) the formulas are still valid for the case of the verity of the said axiom; indeed if the limits of the values are taken, supposing that *i* Y ∞; truly the Euclidean system is as if the limit of the anti-Euclidean (for *i* Y ∞).

Assume for the case of *i* existing, the unit = *i*, and extend the concepts sine and cosine also to imaginary arcs, so that, *p* designating an arc whether real or imaginary,

$$\frac{e^{p\sqrt{-1}} + e^{-p\sqrt{-1}}}{2}$$ is called the *cosine* of *p*, and

$$\frac{e^{p\sqrt{-1}} - e^{-p\sqrt{-1}}}{2\sqrt{-1}}$$ is called the *sine* of *p* (as Tom. I., p. 177).

Hence for *q* real

$$\frac{e^{q} - e^{-q}}{2\sqrt{-1}} = \frac{e^{-q\sqrt{-1}\cdot\sqrt{-1}} - e^{q\sqrt{-1}\cdot\sqrt{-1}}}{2\sqrt{-1}} = \sin(-q\sqrt{-1})$$

$$= -\sin(q\sqrt{-1}).$$

$$\frac{e^{q} + e^{-q}}{2} = \frac{e^{-q\sqrt{-1}\cdot\sqrt{-1}} + e^{q\sqrt{-1}\cdot\sqrt{-1}}}{2} = \cos(-q\sqrt{-1})$$

$$= \cos(q\sqrt{-1});$$

if of course also in the imaginary circle, the sine of a negative arc is the same as the sine of a positive arc otherwise equal to the first, except that it is negative, and the cosine of a positive arc and of a negative (if otherwise they be equal) the same.

In the said Appendix, § 25, is demonstrated absolutely, that is, independently of the said axiom; that, in any rectilineal triangle *the sines of the circles are as the circles of radii equal to the sides opposite*.

Moreover is demonstrated for the case of *i* existing, that the circle of radius *y* is

$$= \pi i \left(e^{\frac{y}{i}} - e^{\frac{-y}{i}} \right) \text{ which, for } i = 1, \text{ becomes}$$

$$\pi (e^{y} - e^{-y}).$$

Therefore (§ 31 *ibidem*), for a right-angled rectilineal triangle of which the sides are *a* and *b*, the hypothenuse *c*, and the angles opposite to the sides *a*, *b*, *c* are α, β, rt. ∠, (for i = 1), in I,

$$1 : \sin \alpha = \pi (e^{c} - e^{-c}) : \pi (e^{a} - e^{-a});$$

and so

$$1 : \sin \alpha = \frac{e^{c} - e^{-c}}{2\sqrt{-1}} : \frac{e^{a} - e^{-a}}{2\sqrt{-1}}.$$

Whence

$$1 : \sin \alpha = - \sin (c\sqrt{-1}) : - \sin (a\sqrt{-1}).$$

And hence

$$1 : \sin \alpha = \sin (c\sqrt{-1}) : \sin (a\sqrt{-1}).$$

In II becomes

$$\cos \alpha : \sin \beta = \cos (a\sqrt{-1}) : 1;$$

in III becomes

$$\cos (c\sqrt{-1}) = \cos (a\sqrt{-1}) \cdot \cos (b\sqrt{-1}).$$

These, as all the formulas of plane trigonometry deducible from them, coincide completely with the formulas of spherical trigonometry; except that if, ex. gr., also the sides and the angles opposite them of a right-angled spherical triangle and the hypothenuse bear the same names, the sides of the rectilineal triangle are to be divided by $\sqrt{-1}$ to obtain the formulas for the spherical triangle.

Obviously we get (clearly as Tom.,II., p. 252),

from I, $1 : \sin \alpha = \sin c : \sin a$;

from II, $1 : \cos a = \sin \beta : \cos \alpha$;

from III, $\cos c = \cos a \cos b$.

Though it be allowable to pass over other things; yet I have learned that the reader may be offended and impeded by the deduction omitted, (Tom. I., App., p. 19) [in § 32 at end] : it will not be irrelevant to show how, ex. gr., from

$$e^{\frac{c}{i}} + e^{\frac{-c}{i}} = \frac{1}{2}\left(e^{\frac{a}{i}} + e^{\frac{-a}{i}}\right)\left(e^{\frac{b}{i}} + e^{\frac{-b}{i}}\right)$$

follows

$$c^2 = a^2 + b^2.$$

(the theorem of Pythagoras for the Euclidean system); probably thus also the author deduced it, and the others also follow in the same manner.

Obviously we have, the powers of e being expressed by series (like Tom. I., p. 168),

$$e^{\frac{k}{i}} = 1 + \frac{k}{i} + \frac{\cancel{k^2}}{2i^2} + \frac{k^3}{2\cdot3\cdot i^3} + \frac{k^4}{2\cdot3\cdot4\cdot i^4} \cdot \cdot \cdot \cdot,$$

$$e^{\frac{-k}{i}} = 1 - \frac{k}{i} + \frac{\cancel{k^2}}{2i^2} - \frac{k^3}{2\cdot3\cdot i^3} + \frac{k^4}{2\cdot3\cdot4\cdot i^4} \cdot \cdot \cdot \cdot, \text{ and so}$$

$$e^{\frac{k}{i}} + e^{\frac{-k}{i}} = 2 + \frac{\cancel{k^2}}{i^2} + \frac{k^4}{3\cdot4\cdot i^4} + \frac{k^6}{3\cdot4\cdot5\cdot6\cdot i^6} \cdot \cdot \cdot \cdot,$$

$$= 2 + \frac{\cancel{k^2 + u}}{i^2}, \text{ (designating by}$$

$\frac{u}{i^2}$ the sum of all the terms after $\frac{k^2}{i^2}$); and we have u Y 0, while i Y ∞. For all the terms which follow $\frac{k^2}{i^2}$, are divided by i^2; the first term will be $\frac{k^4}{3\cdot4\cdot i^4}$; and any ratio $< \frac{k^2}{i^2}$; and though the ratio everywhere should remain this, the sum would be (Tom. I., p. 131),

$$\frac{k^4}{3\cdot4\cdot i^2} : \left(1 - \frac{k^2}{i^2}\right) = \frac{k^4}{3\cdot4\cdot(i^4 - k^2)}$$

which manifestly Y 0, while i Y ∞.

And from

$$e^{\frac{c}{i}} + e^{\frac{-c}{i}} = \frac{1}{2}\left(e^{\frac{(a+b)}{i}} + e^{\frac{-(a+b)}{i}} + e^{\frac{(a-b)}{i}} + e^{\frac{-(a-b)}{i}}\right)$$

follows (for w, v, λ taken like u)

$$2 + \frac{\cancel{c^2 + w}}{i^2} = \frac{1}{2}\left(2 + \frac{\cancel{(a+b)^2 + v}}{i^2} + 2 + \frac{\cancel{(a+b)^2 + \lambda}}{i^2}\right)$$

And hence

$$c^2 = \frac{a^2 + 2ab + b^2 + a^2 - 2ab + b^2 + v + \lambda - w}{2}$$

which Y $a^2 + b^2$.

APPENDIX II.

SOME POINTS IN JOHN BOLYAI'S

COMPARED WITH LOBACHEVSKI,

BY WOLFGANG BOLYAI.

[From *Kurzer Grundriss*, p. 82.]

Lobachevski and the author of the Appendix each consider two points A, B, of the sphere-limit, and the corresponding axes ray AM, ray BN (§ 23).

They demonstrate that, if α, β, γ designate the arcs of the circle limit AB, CD, HL, separated by segments of the axis AC = 1, AH = x, we have

$$\frac{\alpha}{\gamma} = \left(\frac{\alpha}{\beta}\right)^x .$$

Lobachevski represents the value of $\frac{\gamma}{\alpha}$ by e^{-x}, e having some value > 1, dependent on the unit for length that we have chosen, and able to be supposed equal to the Naperian base.

The author of the Appendix is led directly to introduce the base of natural logarithms.

If we put $\frac{\alpha}{\beta} = \delta$, and γ, γ' are arcs situated at the distances y, i from α, we shall have

$$\frac{\alpha}{\gamma} = \delta^{y} = \mathrm{Y}, \frac{\alpha}{\gamma'} = \delta^{i} = \mathrm{I}, \text{ whence } \mathrm{Y} = \mathrm{I}^{\frac{y}{i}}.$$

He demonstrates afterward (§ 29) that, if u is the angle which a straight makes with the perpendicular y to its parallel, we have

$$\mathrm{Y} = \cot \tfrac{1}{2} u.$$

Therefore, if we put $z = \frac{\pi}{2} - u$, we have

$$\mathrm{Y} = \tan (z + \tfrac{1}{2} u) = \frac{\tan z + \tan - \tfrac{1}{2} u}{1 - \tan z \tan \tfrac{1}{2} u},$$

whence we get, having regard to the value of $\tan \frac{1}{2} - u = \mathrm{Y}^{-1}$,

$$\tan \mathrm{z} - \tfrac{1}{2} (\mathrm{Y} - \mathrm{Y}^{-1}) = \tfrac{1}{2} \left(\mathrm{I}^{\frac{y}{i}} - \mathrm{I}^{\frac{-y}{i}} \right) (\S 30).$$

If now y is the semi-chord of the arc of circle-limit $2r$, we prove (§30) that $\frac{r}{\tan z} = \text{constant}$.

Representing this constant by i, and making y tend toward zero, we have

$$\frac{2r}{2y} \mathrm{Y}\ 1, \text{ whence}$$

$$2y \mathrm{Y}\ 2i \tan z \mathrm{Y}\ i \frac{\mathrm{I}^{\frac{2y}{i}} - 1}{\mathrm{I}^{\frac{y}{i}}},$$

or putting $\frac{2y}{i} = k$, $\mathrm{I} = el$,

$$k \mathrm{I} \mathrm{I} \tfrac{\mathrm{y}}{\mathrm{i}} \mathrm{Y}\, e^{\mathrm{kl}} - 1 \mathrm{Y}\, kl\,(1 + \lambda),$$

λ being infinitesimal at the same time as k. Therefore, for the limit, $1 = l$ and consequently $\mathrm{I} = e$.

The circle traced on the sphere-limit with the arc r of the curve-limit for radius, has for length $2\pi r$. Therefore,

$$(\odot y = 2\pi r = 2\pi i \tan z = \pi i\,(\mathrm{Y} - \mathrm{Y}^{-1}).$$

In the rectilineal Δ where α, β designate the angles opposite the sides a, b, we have (§ 25)

$$\sin\alpha : \sin\beta = \odot\, a : \odot\, b = \pi\, i\,(\mathrm{A} - \mathrm{A}^{-1}) : \pi\, i\,(\mathrm{B} - \mathrm{B}^{-1})$$
$$= \sin(a\sqrt{-1}) : \sin(b\sqrt{-1}).$$

Thus in plane trigonometry as in spherical trigonometry, the sines of the angles are to each other as the sines of the opposite sides, only that on the sphere the sides are reals, and in the plane we must consider them as imaginaries, just as if the plane were an imaginary sphere.

We may arrive at this proposition without a preceding determination of the value of I.

If we designate the constant $\frac{r}{\tan z}$ by q, we shall have, as before

$$\odot\, y = \pi\, q\,(\mathrm{Y} - \mathrm{Y}^{-1}),$$

whence we deduce the same proportion as above, taking for *i* the distance for which the ratio I is equal to *e*.

If axiom XI is not true, there exists a determinate, which must be substituted in the formulas.

If, on the contrary, this axiom is true, we must make in the formulas $i = \infty$. Because, in this case, the quantity $\frac{\alpha}{\gamma} = Y$ is always = 1, the sphere-limit being a plane, and the axes being parallel in Euclid's sense.

The exponent $\frac{y}{i}$ must therefore be zero, and consequently $i = \infty$.

It is easy to see that Bolyai's formulas of plane trigonometry are in accord with those of Lobachevski.

Take for example the formula of § 37,

$$\tan \parallel (a) = \sin B \tan \parallel (p),$$

a being the hypothenuse of a right-angled triangle, *p* one side of the right angle, and B the angle opposite to this side.

Bolyai's formula of § 31, I, gives

$$1 : \sin B = (A - A^{-1}) : (P - P^{-1}).$$

Now, putting for brevity, $\frac{1}{2} \parallel (k) = k'$, we have $\tan 2p' : \tan 2a' = (\cot a' - \tan a') : (\cot p' - \tan p') = (A - A^{-1}) : (P - P^{-1}) : \sin B$.

APPENDIX III.

LIGHT FROM NON-EUCLIDEAN SPACES ON THE

TEACHING OF ELEMENTARY GEOMETRY.

BY G. B. HALSTED.

As foreshadowed by Bolyai and Riemann, founded by Cayley, extended and interpreted for hyperbolic, parabolic, elliptic spaces by Klein, recast and applied to mechanics by Sir Robert Ball, projective metrics may be looked upon as characteristic of what is highest and most peculiarly modern in all the bewildering range of mathematical achievement. Mathematicians hold that number is wholly a creation of the human intellect, while on the contrary our space has an empirical element. Of possible geometries we can not say a priori which shall be that of our actual space, the space in which we move. Of course an advance so important, not only for mathematics but for philosophy, has had some metaphysical opponents, and as long ago as 1878 I mentioned in my Bibliography of Hyper-Space

and Non-Euclidean Geometry (American Journal of Mathematics, Vol. I, 1878, Vol. II, 1879) one of these, Schmitz-Dumont, as a sad paradoxer, and another, J. C. Becker, both of whom would ere this have shared the oblivion of still more antiquated fighters against the light, but that Dr. Schotten, praiseworthy for the very attempt at a comparative planimetry, happens to be himself a believer in the *a priori* founding of geometry, while his American reviewer, Mr. Ziwet, was then also an anti-non-Euclidean, though since converted.

He says, "we find that some of the best German text books do not try at all to define what is space, or what is a point, or even what is a straight line." Do any German geometries define space? I never remember to have met one that does.

In experience, what comes first is a bounded surface, with its boundaries, lines, and their boundaries, points. Are the points whose definitions are omitted anything different or better?

Dr. Schotten regards the two ideas "direction" and "distance" as intuitively given in the mind and as so simple as to not require definition.

When we read of two jockeys speeding

around a track in opposite directions, and also on page 87 of Richardson's Euclid, 1891, read, "The sides of the figure must be produced in the same direction of rotation; . . . going round the figure always in the same direction," we do not wonder that when Mr. Ziwet had written : "he therefore bases the definition of the straight line on these two ideas," he stops, modifies, and rubs that out as follows, "or rather recommends to elucidate the intuitive idea of the straight line possessed by any well-balanced mind by means of the still simpler ideas of direction" [in a circle] "and distance" [on a curve.

But when we come to geometry as a science, as foundation for work like that of Cayley and Ball, I think with Professor Chrystal : "It is essential to be careful with our definition of a *straight line*, for it will be found that virtually the properties of the straight line determine the nature of space.

"Our definition shall be that two points in *general* determine a straight line."

We presume that Mr. Ziwet glories in that unfortunate expression "a straight line is the shortest distance between two points," still occurring in Wentworth (New Plane Geometry, page 33), even after he has said, page 5,

"the length of the straight line is called the *distance* between two points." If the *length* of the one straight line between two points is the distance between those points, how can the straight line itself be the *shortest* distance? If there is only one distance, it is the longest as much as the shortest distance, and if it is the *length* of this shorto-longest distance which is the distance then it is not the straight line itself which is the longo-shortest distance. But Wentworth also says : "Of all lines joining two points the *shortest* is the straight line."

This general comparison involves the measurement of curves, which involves the theory of limits, to say nothing of ratio. The very ascription of length to a curve involves the idea of a limit. And then to introduce this general axiom, as does Wentworth, only to prove a very special case of itself, that two sides of a triangle are together greater than the third, is surely bad logic, bad pedagogy, bad mathematics.

This latter theorem, according to the first of Pascal's rules for demonstrations, should not be proved at all, since every dog knows it. But to this objection, as old as the sophists, Simson long ago answered for the science of

geometry, that the number of assumptions ought not to be increased without necessity; or as Dedekind has it : "*Was beweisbar ist*, *soll in der Wissenschaft nicht ohne Beweis geglaubt werden*."

Professor W. B. Smith (Ph. D., Goettingen), has written : "Nothing could be more unfortunate than the attempt to lay the notion of Direction at the bottom of Geometry."

Was it not this notion which led so good a mathematician as John Casey to give as a demonstration of a triangle's angle-sum the procedure called " a practical demonstration" on page 87 of Richardson's Euclid, and there described as "laying a 'straight edge' along one of the sides of the figure, and then turning it round so as to coincide with each side in turn."

This assumes that a segment of a straight line, a sect, may be translated without rotation, which assumption readily comes to view when you try the procedure in two-dimensional spherics. Though this fallacy was exposed by so eminent a geometer as Olaus Henrici in so public a place as the pages of 'Nature,' yet it has just been solemnly reproduced by Professor G. C. Edwards, of the University of California, in his Elements of Geometry : MacMillan, 1895.

It is of the greatest importance for every teacher to know and connect the commonest forms of assumption equivalent to Euclid's Axiom XI. If in a plane two straight lines perpendicular to a third nowhere meet, are there others, not both perpendicular to any third, which nowhere meet? Euclid' s Axiom XI is the assumption *No*. Playfair's answers *no* more simply. But the very same answer is given by the common assumption of our geometries, usually unnoticed, that a circle may be passed through any three points not costraight.

This equivalence was pointed out by Bolyai Parkas, who looks upon this as the simplest form of the assumption. Other equivalents are, the existence of any finite triangle whose angle-sum is a straight angle; or the existence of a plane rectangle; or that, in triangles, the angle-sum is constant.

One of Legendre's forms was that through every point within an angle a straight line may be drawn which cuts both arms.

But Legendre never saw through this matter because he had hot, as we have, the eyes of Bolyai and Lobachevski to see with. The same lack of their eyes has caused the author of the charming book " Euclid and His Modern

Rivals," to give us one more equivalent form : " In any circle, the inscribed equilateral tetragon is greater than any one of the segments which lie outside it." (A New Theory of Parallels by C. L. Dodgson, 3d. Ed., 1890.)

Any attempt to define a straight line by means of "direction" is simply a case of "argumentum in circulo." In all such attempts the loose word "direction" is used in a sense which presupposes the straight line. The directions from a point in Euclidean space are only the ∞^2 rays from that point.

Rays not costraight can be said to have the same direction only after a theory of parallels is presupposed, assumed.

Three of the exposures of Professor G. C. Edwards' fallacy are here reproduced. The first, already referred to, is from Nature, Vol. XXIX, p. 453, March 13, 1884.

"I select for discussion the 'quaternion proof" given by Sir William Hamilton. . . . Hamilton's proof consists in the following : "One side AB of the triangle ABC is turned about the point B till it lies in the continuation of BC; next, the line BC is made to slide along BC till B comes to C, and is then turned about C till it comes to lie in the continuation of AC.

" It is now again made to slide along CA till the point B comes to A, and is turned about A till it lies in the line AB. Hence it follows, *since rotation is independent of translation*, that the line has performed a whole revolution, that is, it has been turned through four right angles. But it has also described in succession the three exterior angles of the triangle, hence these are together equal to four right angles, and from this follows at once that the interior angles are equal to two right angles.

"To show how erroneous this reasoning is—in spite of Sir William Hamilton and in spite of quaternions—I need only point out that it holds exactly in the same manner for a triangle on the surface of the sphere, from which it would follow that the sum of the angles in a spherical triangle equals two right angles, whilst this sum is known to be always greater than two right angles. The proof depends only on the fact, that any line can be made to coincide with any other line, that two lines do so coincide when they have two points in common, and further, that a line may be turned about any point in it without leaving the surface. But if instead of the plane we take a spherical surface, and instead of a line a great

circle on the sphere, all these conditions are again satisfied.

"The reasoning employed must therefore be fallacious, and the error lies in the words printed in italics; for these words contain an assumption which has not been proved.

"O. HENRICI."

Perronet Thompson, of Queen's College, Cambridge, in a book of which the third edition is dated 1830, says:

"Professor Playfair, in the Notes to his 'Elements of Geometry' [1813], has proposed another demonstration, founded on a remarkable *non causa pro causa*.

"It purports to collect the fact [Eu. I., 32, Cor., 2] that (on the sides being successively prolonged to the same hand) the exterior angles of a rectilinear triangle are together equal to four right angles, from the circumstance that a straight line carried round the perimeter of a triangle by being applied to all the sides in succession, is brought into its old situation again; the argument being, that because this line has made the sort of somerset it would do by being turned through four right angles about a fixed point, the exterior

angles of the triangle have necessarily been equal to four right angles.

"The answer to which is, that there is no connexion between the things at all, and that the result will just as much take place where the exterior angles are avowedly not equal to four right angles.

"Take, for example, the plane triangle formed by three small arcs of the same or equal circles, as in the margin; and it is manifest that an arc of this circle may be carried round precisely in the way described and return to its old situation, and yet there be no pretense for inferring that the exterior angles were equal to four right angles.

"And if it is urged that these are curved lines and the statement made was of straight; then the answer is by demanding to know, what property of straight lines has been laid down or established, which determines that what is not true in the case of other lines is

true in theirs. It has been shown that, as a general proposition, the connexion between a line returning to its place and the exterior angles having been equal to four right angles, is a *non sequitur*; that it is a thing that may be or may not be; that the notion that it returns to its place *because* the exterior angles have been equal to four right angles, is a mistake. From which it is a legitimate conclusion, that if it had pleased nature to make the exterior angles of a triangle greater or less than four right angles, this would not have created the smallest impediment to the line's returning to its old situation after being carried round the sides; and consequently the line's returning is no evidence of the angles not being greater or less than four right angles."

Charles L. Dodgson, of Christ Church, Oxford, in his "Curiosa Mathematica," Part I, pp. 70 – 71, 3d Ed., 1890, says:

"Yet another process has been invented—quite fascinating in its brevity and its elegance—which, though involving the same fallacy as the Direction-Theory, proves Euc. I, 32, without even mentioning the dangerous word 'Direction.'

"We are told to take any triangle ABC; to produce CA to D; to make part of CD, viz., AD, revolve, about A, into the position ABE; then to make part of this line, viz., BE, revolve, about B, into the position BCF; and lastly to make part of this line, viz., CF, revolve, about C, till it lies along CD, of which it originally formed a part. We are then assured that it must have revolved through four right angles : from which it easily follows that the interior angles of the triangle are together equal to two right angles.

"The disproof of this fallacy is almost as brief and elegant as the fallacy itself. We first quote the general principle that we can not reasonably be told to make a line fulfill two conditions, either of which is enough by itself to fix its position : e. g., given three points X, Y, Z, we can not reasonably be told to draw a line from X which shall pass through Y and Z : we can make it pass through Y, but it must then take its chance of passing through Z; and *vice versa*.

" Now let us suppose that, while one part of

AE, viz., BE, revolves into the position BF, another little bit of it, viz., AG, revolves, through an equal angle, into the position AH; and that, while CF revolves into the position of lying along CD, AH revolves—and here comes the fallacy.

"You must not say 'revolves, through an equal angle, into the position of lying along AD,' for this would be to make AH *fulfill two conditions at once*.

"If you say that the one condition involves the other, you are virtually asserting that the lines CF, AH are equally inclined to CD—and this in *consequence* of AH having been so drawn that these same lines are equally inclined to AE.

"That is, you are asserting, 'A pair of lines which are equally inclined to a certain transversal, are so to any transversal.' [Deducible from Euc. I, 27, 28, 29.]"

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I found this in Chapter 2 of THE PARABOLA, ELLIPSE, AND HYPERBOLA, TREATED GEOMETRICALLY. BY
ROBERT WILLIAM GRIFFIN, A.M., LL.D.,
Now a Lemma is a given, but why? Could the author not show the construction? I remembered the figure from Heath's translation of The Elements by Euclid, but alas, the construction was not there either. So, I did a work up of the figure.

LEMMA.

If a right line AB be divided internally at O in any ratio, and externally at O' in the same ratio, and a circle be described on OO' as diameter, the right lines joining any point P on this circle with the extremities of the line AB will have the same ratio.

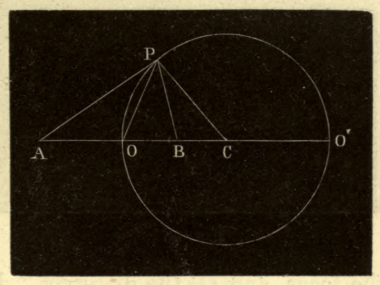


Fig. 1.

Bisect OO' in C; join CP, PO.

Then $AO' : O'B = AO : OB$;

$\therefore AO' + AO : AO' - AO = O'B + OB : O'B - OB$;

$\therefore 2AC : 2OC = 2OC : 2BC$;

$\therefore AC : CP = CP : CB$;

$\therefore \Delta ACP$ is similar to ΔPCB; (6 VI. Euclid.)

$\therefore \angle CPB = \angle CAP$;

but $\angle CPO = \angle COP$ (5 I. Euclid.)

$= \angle OAP + \angle OPA$ (32 I. Euclid.)

$= \angle CPB + \angle OPA$;

$\therefore \angle BPO = \angle OPA$;

$\therefore AP : PB = AO : OB$. (3 VI. Euclid.)

Pages from Heath's Translation of Euclid's Elements

and also, as BD is to DC, so is BA to AE: for AD has been drawn parallel to EC, one of the sides of the triangle BCE: [VI. 2]

therefore also, as BA is to AC, so is BA to AE. [V. 11]

Therefore AC is equal to AE, [V. 9]

so that the angle AEC is also equal to the angle ACE. [I. 5]

But the angle AEC is equal to the exterior angle BAD, [I. 29]

and the angle ACE is equal to the alternate angle CAD; [*id.*]

therefore the angle BAD is also equal to the angle CAD.

Therefore the angle BAC has been bisected by the straight line AD.

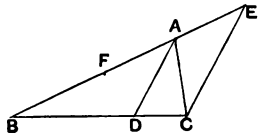
Therefore etc.

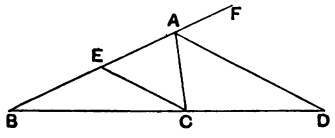
Q. E. D.

The demonstration assumes that CE *will* meet BA produced in some point E. This is proved in the same way as it is proved in VI. 4 that BA, ED will meet if produced. The angles ABD, BDA in the figure of VI. 3 are together less than two right angles, and the angle BDA is equal to the angle BCE, since DA, CE are parallel. Therefore the angles ABC, BCE are together less than two right angles; and BA, CE must meet, by I. Post. 5.

The corresponding proposition about the segments into which BC is divided *externally* by the bisector of the *external angle* at A when that bisector meets BC produced (i.e. when the sides AB, AC are not equal) is important. Simson gives it as a separate proposition, A, noting the fact that Pappus assumes the result without proof (Pappus, VII. p. 730, 24).

The best plan is however, as De Morgan says, to combine Props. 3 and A in one proposition, which may be enunciated thus: *If an angle of a triangle be bisected internally or externally by a straight line which cuts the opposite side or the opposite side produced, the segments of that side will have the same ratio as the other sides of the triangle; and, if a side of a triangle be divided internally or externally so that its segments have the same ratio as the other sides of the triangle, the straight line drawn from the point of section to the angular point which is opposite to the first mentioned side will bisect the interior or exterior angle at that angular point.*

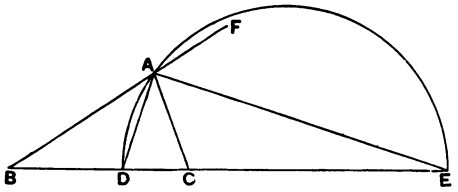




Let AC be the smaller of the two sides AB, AC, so that the bisector AD of the exterior angle at A may meet BC produced beyond C. Draw CE through C parallel to DA, meeting BA in E.

Then, if FAC is the exterior angle bisected by AD in the case of external bisection, and if a point F is taken on AB in the figure of VI. 3, the proof of

VI. 3 can be used almost word for word for the other case. We have only to speak of the angle "FAC" for the angle "BAC," and of the angle "FAD" for the angle "BAD" wherever they occur, to say "let BA, or BA produced, meet CE in E," and to substitute "BA or BA produced" for "BAE" lower down.



If AD, AE be the internal and external bisectors of the angle A in a triangle of which the sides AB, AC are unequal, AC being the smaller, and if AD, AE meet BC and BC produced in D, E respectively,

the ratios of BD to DC and of BE to EC are alike equal to the ratio of BA to AC.

Therefore BE is to EC as BD to DC,

that is, BE is to EC as the difference between BE and ED is to the difference between ED and EC,

whence BE, ED, EC are in *harmonic progression*, or DE is a *harmonic mean* between BE and EC, or again B, D, C, E is a *harmonic range*.

Since the angle DAC is half of the angle BAC,

and the angle CAE half of the angle CAF,

while the angles BAC, CAF are equal to two right angles,

the angle DAE is a right angle.

Hence the circle described on DE as diameter passes through A.

Now, if the ratio of BA to AC is given, and if BC is given, the points D, E on BC and BC produced are given, and therefore so is the circle on D, E as diameter. Hence *the locus of a point such that its distances from two given points are in a given ratio (not being a ratio of equality) is a circle.*

This locus was discussed by Apollonius in his *Plane Loci*, Book II., as we know from Pappus (VII. p. 666), who says that the book contained the theorem that, if from two given points straight lines inflected to another point are in a given ratio, the point in which they meet will lie on either a straight line or a circumference of a circle. The straight line is of course the locus when the ratio is one of equality. The other case is quoted in the following form by Eutocius (Apollonius, ed. Heiberg, II. pp. 180—4).

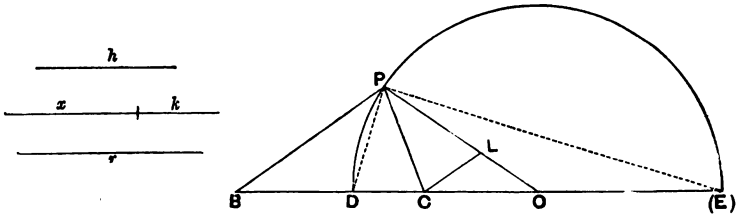
Given two points in a plane and a proportion between unequal straight lines, it is possible to describe a circle in the plane so that the straight lines inflected from the given points to the circumference of the circle shall have a ratio the same as the given one.

Apollonius' construction, as given by Eutocius, is remarkable because he makes no use of either of the points D, E. He finds O, the centre of the required circle, and the length of its radius directly from the data BC and the given ratio which we will call $h : k$. But the construction was not discovered by Apollonius; it belongs to a much earlier date, since it appears in exactly

the same form in Aristotle, *Meteorologica* III. 5, 376 a 3 sqq. The analysis leading up to the construction is, as usual, not given either by Aristotle or Eutocius. We are told to take three straight lines x, CO (a length measured along BC produced beyond C, where B, C are the points at which the greater and smaller of the inflected lines respectively terminate), and r, such that, if $h:k$ be the given ratio and $h > k$,

$$k : h = h : k + x, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\alpha)$$

$$x : BC = k : CO = h : r. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta)$$



This determines the position of O, and the length of r, the radius of the required circle. The circle is then drawn, *any* point P is taken on it and joined to B, C respectively, and it is proved that

$$PB : PC = h : k.$$

We may conjecture that the analysis proceeded somewhat as follows.

It would be seen that B, C are "conjugate points" with reference to the circle on DE as diameter. (Cf. Apollonius, *Conics*, I. 36, where it is proved, in terms, for a circle as well as for an ellipse and a hyperbola, that, if the polar of B meets the diameter DE in C, then $EC : CD = EB : BD$.)

If O be the middle point of DE, and therefore the centre of the circle, D, E may be eliminated, as in the *Conics*, I. 37, thus.

Since $$EC : CD = EB : BD,$$

it follows that $$EC + CD : EC \sim CD = EB + BD : EB \sim BD,$$

or $$2OD : 2OC = 2OB : 2OD,$$

that is, $$BO \cdot OC = OD^2 = r^2, \text{ say.}$$

If therefore P be any point on the circle with centre O and radius r,

$$BO : OP = OP : OC,$$

so that BOP, POC are similar triangles.

In addition, $h : k = BD : DC = BE : EC$

$$= BD + BE : DE = BO : r.$$

Hence we require that

$$BO : r = r : OC = BP : PC = h : k. \quad \ldots\ldots\ldots\ldots\ldots (\delta)$$

Therefore, alternately,

$$k : CO = h : r,$$

which is the second relation in (β) above.

Now assume a length x such that each of the last ratios is equal to $x : BC$, as in (β).

Then $x : BC = k : CO = h : r.$

Therefore $x + k : BO = h : r,$

and, alternately, $x + k : h = BO : r$

$= h : k$, from (δ) above;

and this is the relation (α) which remained to be found.

Apollonius' proof of the construction is given by Eutocius, who begins by saying that it is manifest that r is a mean proportional between BO and OC. This is seen as follows.

From (β) we derive

$$x : BC = k : CO = h : r = (k + x) : BO,$$

whence $BO : r = (k + x) : h$

$= h : k$, by (α),

$= r : CO$, by (β),

and therefore $r^2 = BO \cdot CO.$

But the triangles BOP, POC have the angle at O common, and, since $BO : OP = OP : OC$, the triangles are similar and the angles OPC, OBP are equal.

[Up to this point Aristotle's proof is exactly the same; from this point it diverges slightly.]

If now CL be drawn parallel to BP meeting OP in L, the angles BPC LCP are equal also.

Therefore the triangles BPC, PCL are similar, and

$$BP : PC = PC : CL,$$

whence $BP^2 : PC^2 = BP : CL$

$= BO : OC$, by parallels,

$= BO^2 : OP^2$ (since $BO : OP = OP : OC$).

Therefore $BP : PC = BO : OP$

$= h : k$ (for $OP = r$).

[Aristotle infers this more directly from the similar triangles POB, COP. Since these triangles are similar,

$$OP : CP = OB : BP,$$

whence $BP : PC = BO : OP$

$= h : k.$]

Apollonius proves lastly, by *reductio ad absurdum*, that the last equation cannot be true with reference to any point P which is not on the circle so described.

Proposition 4.

In equiangular triangles the sides about the equal angles are proportional, and those are corresponding sides which subtend the equal angles.

052211 Plate 1

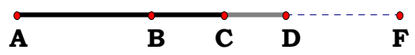
Unit.

Given.

$N_1 := 5 \quad AC := N_1$

$N_2 := 2 \quad CD := N_2$

Given AC is not one half of AD, find AF, DF such that AC : CD :: AF : DF.



Descriptions.

The author of The PAKABOLA, ELLIPSE, AND HYPEKBOLA by R. W. Griffin, gave this figure as a Lemma, and Book VI Prop. 6 of Euclid only has it in the notes, but no construction.

Basically it is how to construct a circle with a given ratio, which is why it is probably the first thing in Chapt. 2 of Griffin's work. So, I will first demonstrate how to construct the figure from the internal ratio of AC : CD.

$CK := CD$

$AK := AC - CK \qquad AG := \sqrt{2 \cdot AC^2}$

$AD := AC + CD \qquad AJ := \dfrac{AG \cdot AD}{AK}$

$AF := \dfrac{AC \cdot AJ}{AG} \qquad DF := AF - AD$

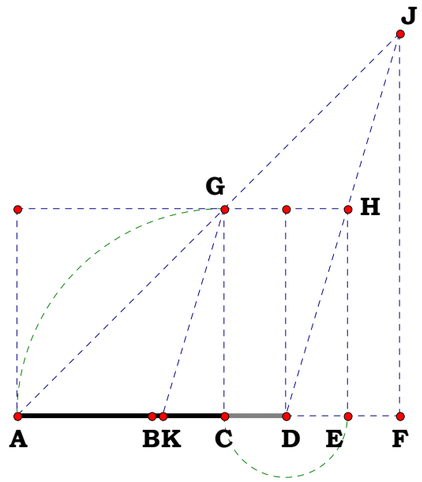
$CF := AF - AC$

Definitions.

$$AF - \left(N_1 \cdot \frac{N_1 + N_2}{N_1 - N_2} \right) = 0 \qquad \frac{AC}{CD} - \frac{AF}{DF} = 0$$

$$DF - \left(N_1 \cdot \frac{N_1 + N_2}{N_1 - N_2} - N_1 - N_2 \right) = 0$$

$$\left(\frac{N_1 \cdot \dfrac{N_1 + N_2}{N_1 - N_2}}{N_1 \cdot \dfrac{N_1 + N_2}{N_1 - N_2} - N_1 - N_2} \right) - \frac{N_1}{N_2} = 0$$



Next demonstrate that from the circumference of CM, the proportion remains.

$N_3 := 1$ $N_4 := 4$

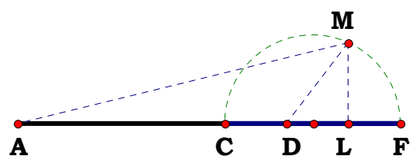
$CL := CF \cdot \frac{N_3}{N_4}$ $FL := CF - CL$

$AL := AC + CL$ $ML := \sqrt{CL \cdot FL}$

$AM := \sqrt{ML^2 + AL^2}$

$DL := CL - CD$

$DM := \sqrt{ML^2 + DL^2}$



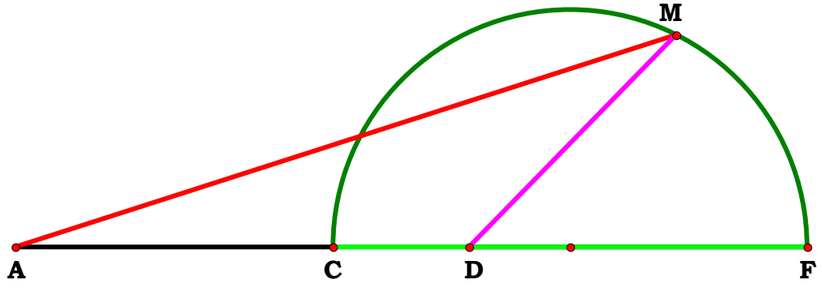
$\frac{AC}{CD} - \frac{AM}{DM} = 0$

$DM - \left[\left| \frac{N_2}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}} \right| \right] = 0$

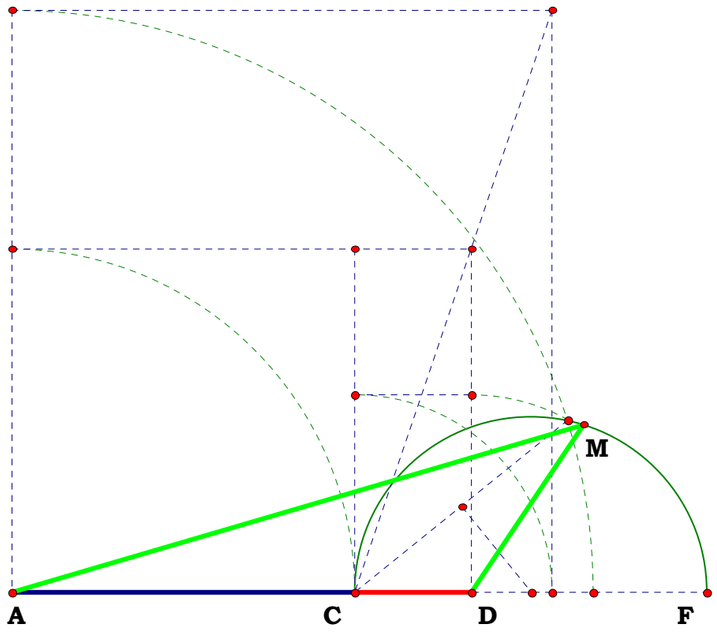
$AM - \left[\left| \frac{N_1}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}} \right| \right] = 0$

$$\left[\frac{\frac{N_1}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}}}{\frac{N_2}{(N_1 - N_2)} \cdot \left(\frac{4 \cdot N_3 \cdot N_1 \cdot N_2 + N_4 \cdot N_1^2 - 2 \cdot N_2 \cdot N_4 \cdot N_1 + N_2^2 \cdot N_4}{N_4} \right)^{\frac{1}{2}}} \right] - \frac{N_1}{N_2} = 0$$

Therefore the three ratios are equal. AC : CD :: AF : DF :: AM : DM, and one can construct a circle using a given ratio.



Alternate Construction


M
A
C
D
F

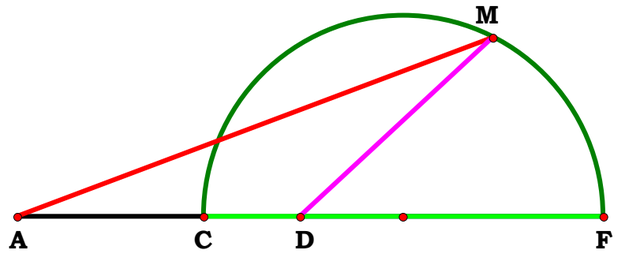
052211 Plate 2

Unit.

Given.

$N_1 := 3.22792$ $AC := N_1$

$N_2 := 8.65187$ $AF := N_2$



Descriptions.

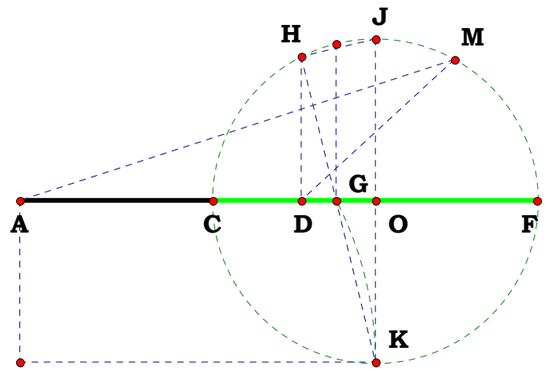
For the given figure, find D if only AC and AF are given.

$CF := AF - AC$

$AG := \sqrt{AC \cdot AF}$ $OK := \frac{CF}{2}$

$AO := AC + OK$

$GO := AO - AG$ $GK := \sqrt{OK^2 + GO^2}$



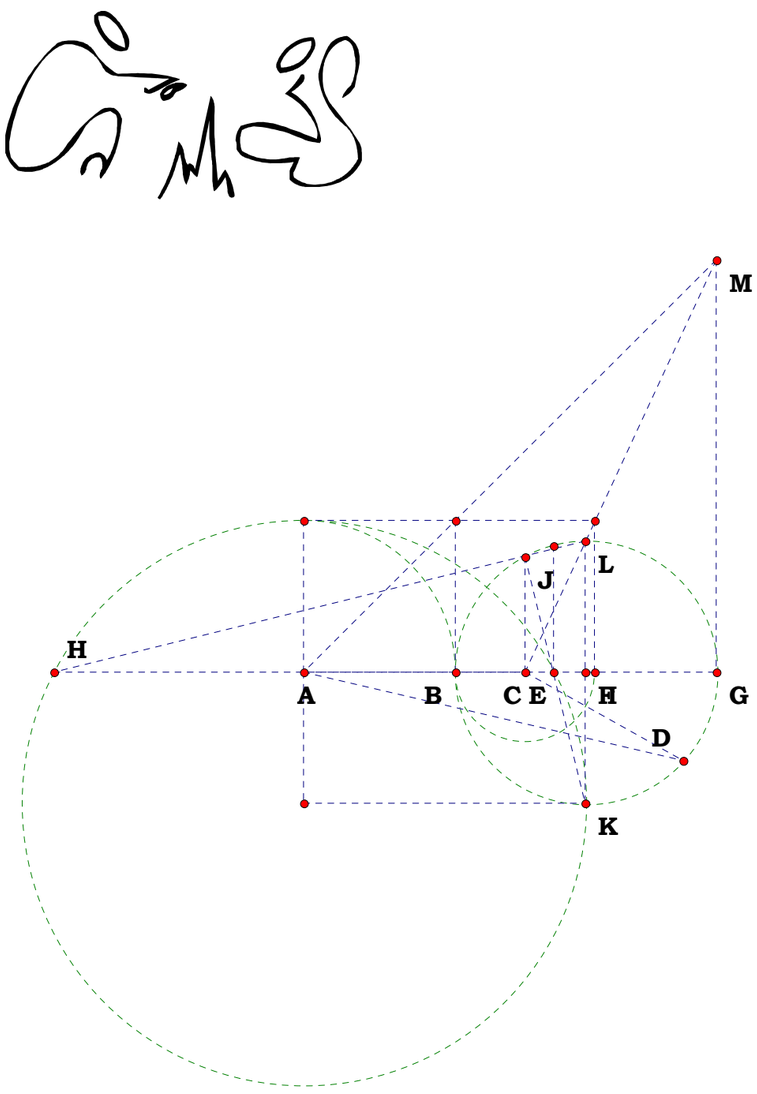
$HK := \frac{OK \cdot CF}{GK}$ $GH := HK - GK$ $DG := \frac{GO \cdot GH}{GK}$

$AD := AG - DG$ $DF := AF - AD$ $CD := AD - AC$

Definitions.

$\frac{AC}{CD} - \frac{AF}{DF} = 0$ $AD - \frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} = 0$

$DF - \frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$ $CD - \frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} = 0$


M
L
J
H
A
B
C
E
I
G
D
K

Unit.

Given.

$N_1 := 3.86292$ $AB := N_1$

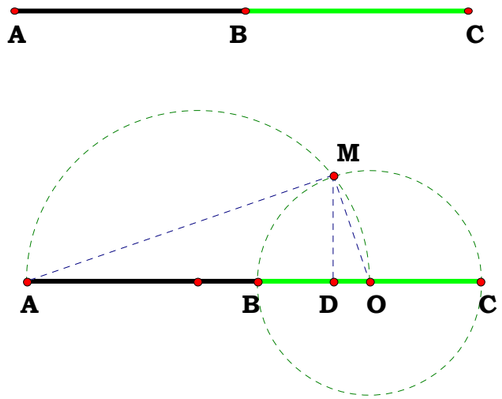
$N_2 := 7.59354$ $AC := N_2$

052211

Descriptions.

Lemma Plate 3

Simplify Plate 2.



$BC := AC - AB$ $BO := \frac{BC}{2}$ $MO := BO$

$AO := AB + BO$ $AM := \sqrt{AO^2 - MO^2}$

$DO := \frac{MO^2}{AO}$ $BD := BO - DO$

$CD := BC - BD$ $AD := AB + BD$

Definitions.

$\frac{N_1 \cdot (N_2 - N_1)}{N_1 + N_2} - BD = 0$ $\frac{N_2 \cdot (N_2 - N_1)}{N_1 + N_2} - CD = 0$

$\frac{2 \cdot N_1 \cdot N_2}{N_1 + N_2} - AD = 0$

Unit.

Given.

$AB := 6.20183$

$AC := 4.89358$

$BC := 9.20468$

08092015

Descriptions.

$$GA := \frac{AC^2}{AB} \qquad HB := \frac{BC^2}{AB} \qquad GH := AB - (GA + HB) \qquad JA := GA + \frac{GH}{2}$$

$$JB := HB + \frac{GH}{2} \qquad CJ := \sqrt{AC^2 - JA^2} \qquad CD := \sqrt{\left(\frac{AB}{2} - JA\right)^2 + CJ^2}$$

Definitions.

$$CJ - \frac{\sqrt{(AB + AC - BC) \cdot (AB - AC + BC) \cdot (AC - AB + BC) \cdot (AB + AC + BC)}}{2 \cdot AB} = 0$$

$$CD - \frac{\sqrt{2 \cdot AC^2 - AB^2 + 2 \cdot BC^2}}{2} = 0 \qquad JA - \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB} = 0$$

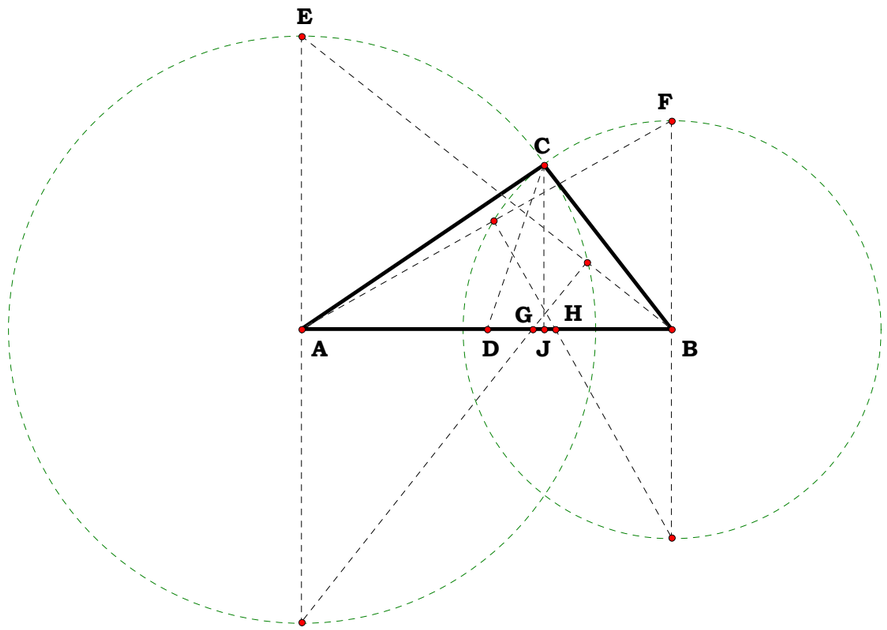
$$JB - \frac{AB^2 - AC^2 + BC^2}{2 \cdot AB} = 0$$

Now the above is just what I got with Pythagoras revisited, however, it does not yet comply with the naming convention. There is no such thing as a negative name.

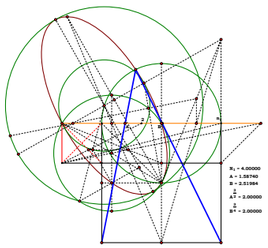
$$AJ := \sqrt{AC^2 - CJ^2} \qquad AJ - \frac{\sqrt{\left(AB^2 + AC^2 - BC^2\right)^2}}{2 \cdot AB} = 0$$

$$BJ := \sqrt{BC^2 - CJ^2} \qquad BJ - \frac{\sqrt{\left(AB^2 - AC^2 + BC^2\right)^2}}{2 \cdot AB} = 0$$

One of the items often missed, as I aptly demonstrated, is thinking a process through. In order to insure the process is complete, use a Law as a standard to complete the equation. In this case, the Pythagorean Theorem.



Pythagoras Revisited Again!

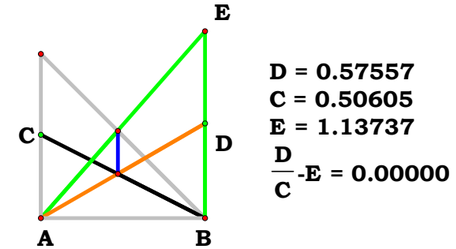


Two Triangles

Saturday, April 6, 2019

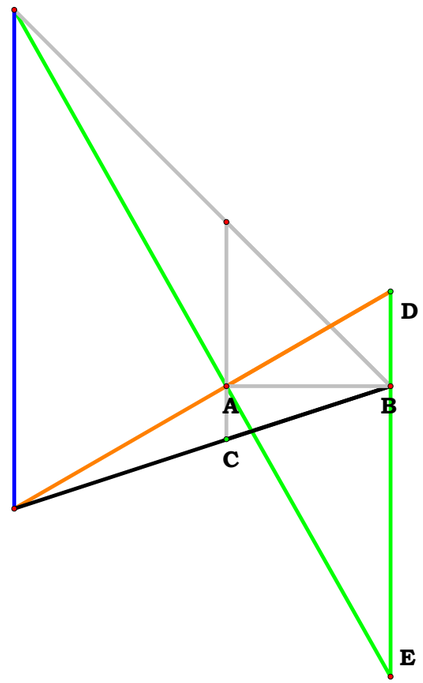
Just the first book or two of Euclid's *Elements* should give one enough information to discover *Basic Analog Mathematics*. However, it apparently did not happen. The reason being is that it is a whole lot easier to repeat and memorize perceptible information than to see the intelligible being expressed.

In this little essay, I am not going to say much, I am just going to present a little figure which I call Two Triangles. Just imagine what you can do with two right angles on the same base. I can call one ACB and the other ADB. I will simply present a series of plates which only differ in so far as a point go.

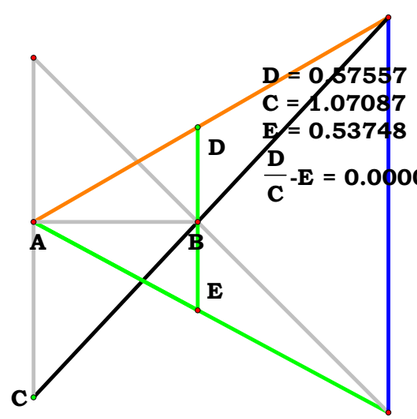


Let us call AB the standard unit by which C and D and their produce E is named. We simply have a right triangle to perform our operation. Call it AB. The intersection of the two triangles perpendicular to the base AB fall on what we can call the segment known to be the square root of 2. We are not, however, interested where it intersects the segment, but only that it intersects the line which contains the segment. We are interested in it only insofar as it expresses and projects a ratio.

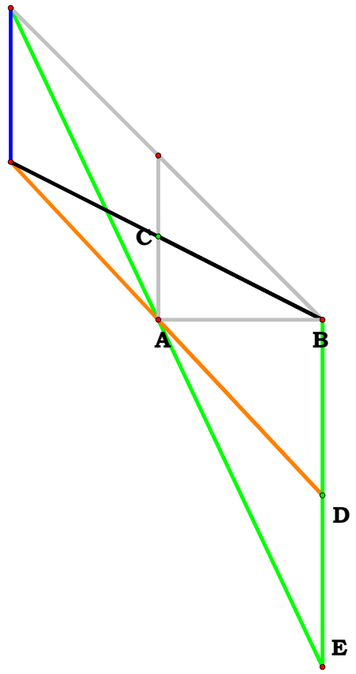
Now, we can give C and D any values we like. We can imagine them as triangles ACB and ADB, however, we have no wish to think or speak in terms of the mystical angle. We adhere to the notion that a two-dimensional plane is expressible as a ratio between two units.



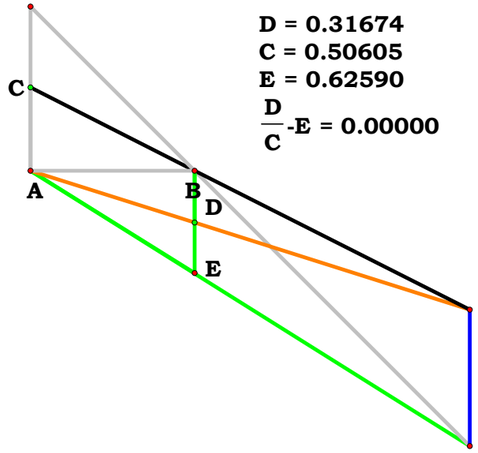
D = 0.57557
C = 0.32472
E = 1.77253
$\frac{D}{C}$-E = 0.00000



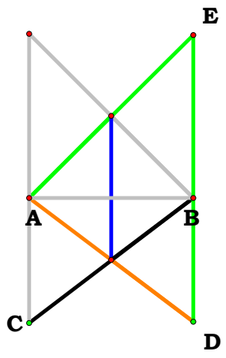
D = 0.57557
C = 1.07087
E = 0.53748
$\frac{D}{C}$-E = 0.00000



D = 1.07058
C = 0.50605
E = 2.11556
$\frac{D}{C}$-E = 0.00000



D = 0.31674
C = 0.50605
E = 0.62590
$\frac{D}{C}$-E = 0.00000



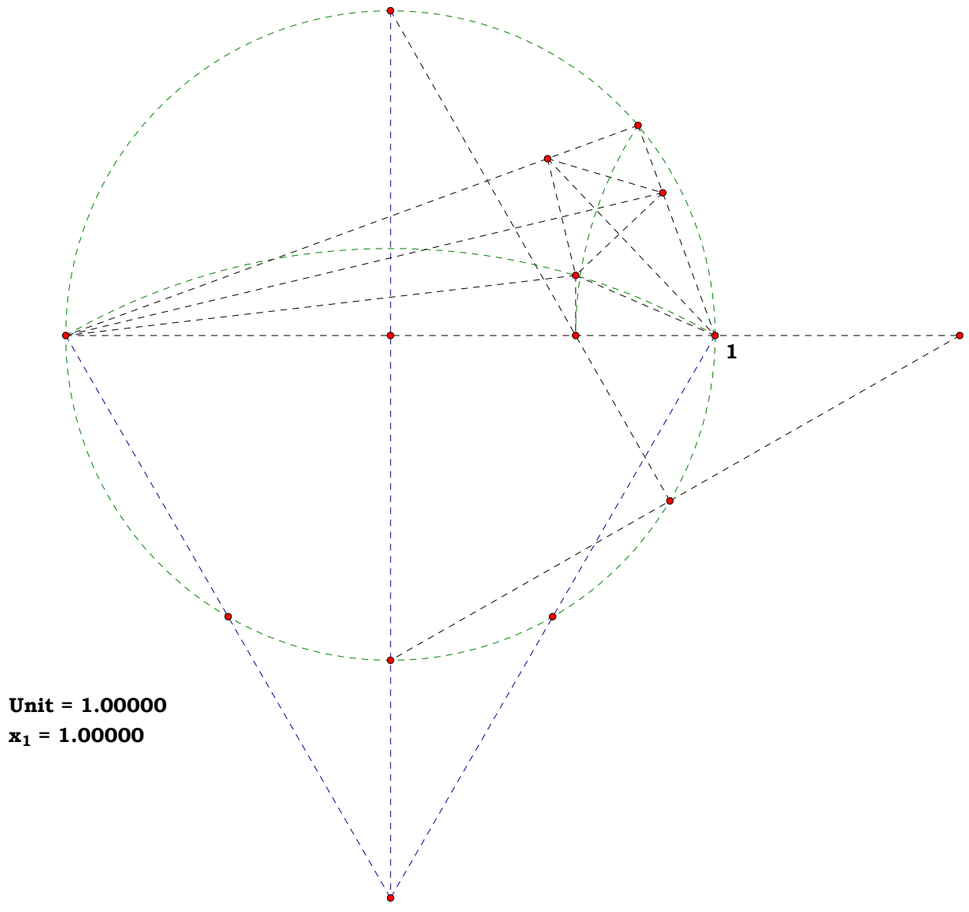
D = 0.75520
C = 0.76318
E = 0.98955
$\frac{D}{C}$-E = 0.00000

One of the things which BAM helps one with, or one of the things Geometry helps one with, is not to set the standard of understanding a figure based on the perceptible, which can be very confusing, but on the intelligible content established by standards. The standard references what is intersected while the mind is looking at where.

One may notice, even in the *Elements* as we have them today, propositions written up by a weaker mind writes up the same proposition in terms of cases based on perceptible location. One can see here, if they were done correctly, the equation never changes. Notice also that any and every other type of triangle can be found using in the figure. I am not interested in obtuse, acute, or any other name one can give to any other expression of a triangle. I am only interested in the fact that a two-dimensional matrix can produce results using an unit whatsoever when compared to another and I do not need Cartesian Geometry, Trigonometry, or Calculus to do it.

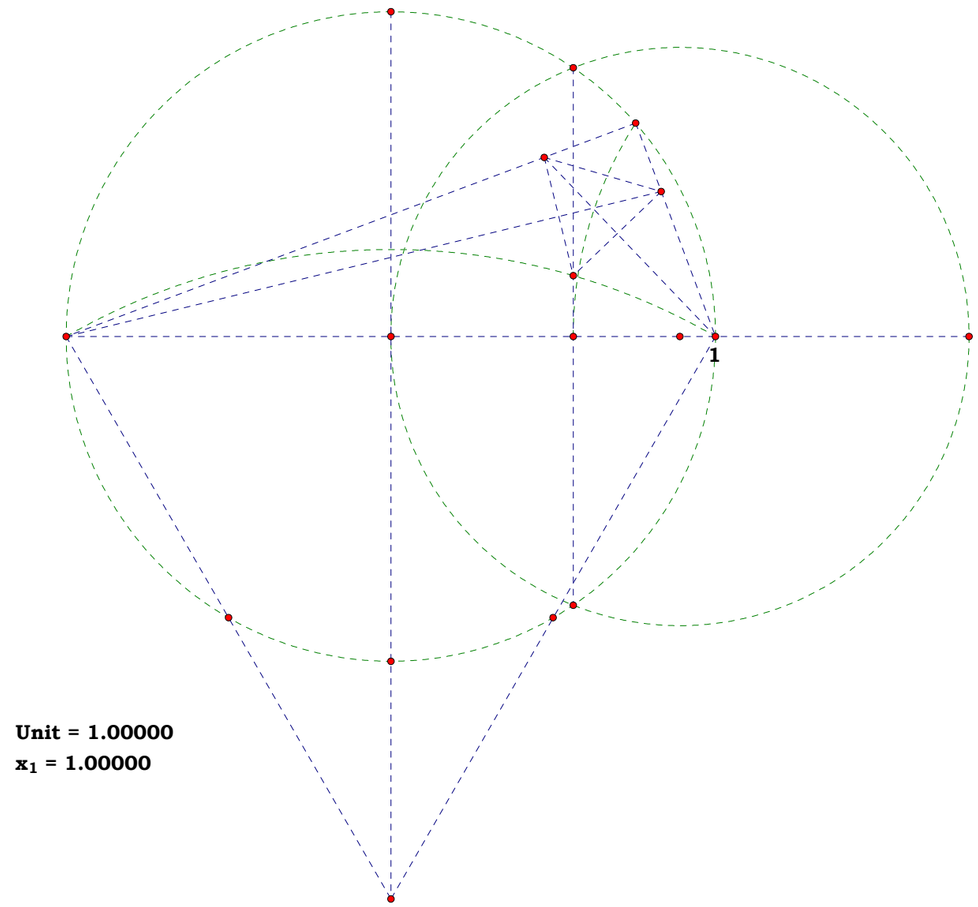
The ability to equate an analog to its logical name is not, in any wise apparent to the eye. One has to find and use standards to express it, and comprehend it in the mind. You can call an analog an isosceles right triangle in gray, or a method of dividing two given things of the same relative difference in accordance with a standard unit. All of the other so called triangles are simply parts of a much bigger and better ordered universe.

Let us take our little figure, Lay it on its side and imagine that C and D are on two parallels and AB is just a unit.



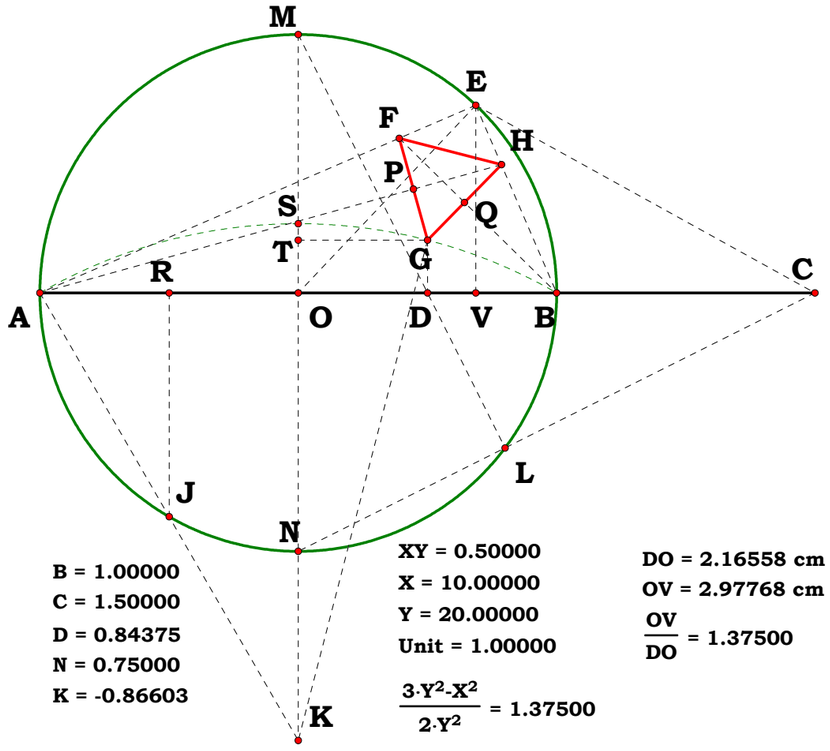
Unit = 1.00000

x_1 = 1.00000



Unit = 1.00000

x_1 = 1.00000



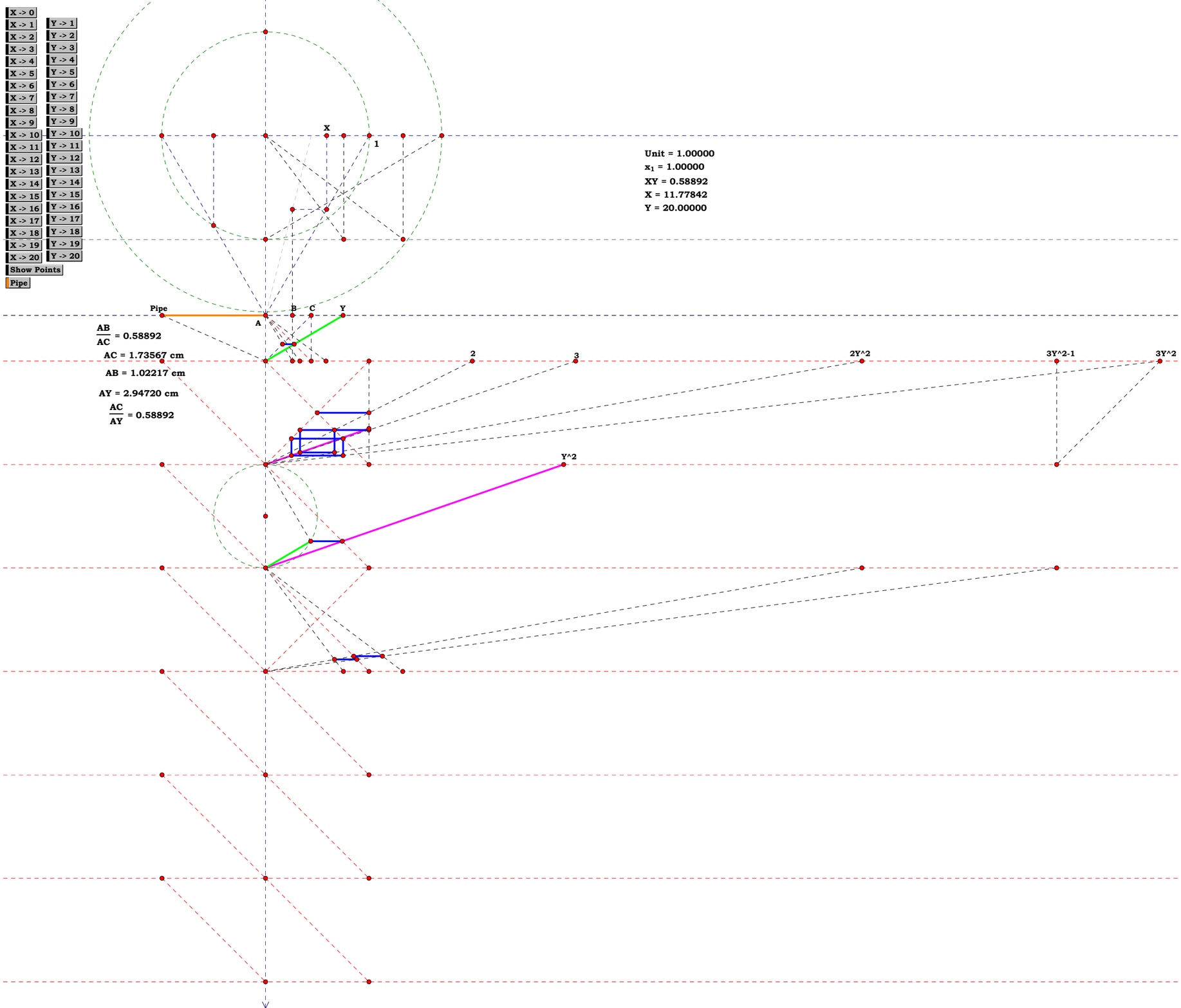
B = 1.00000
C = 1.50000
D = 0.84375
N = 0.75000
K = -0.86603

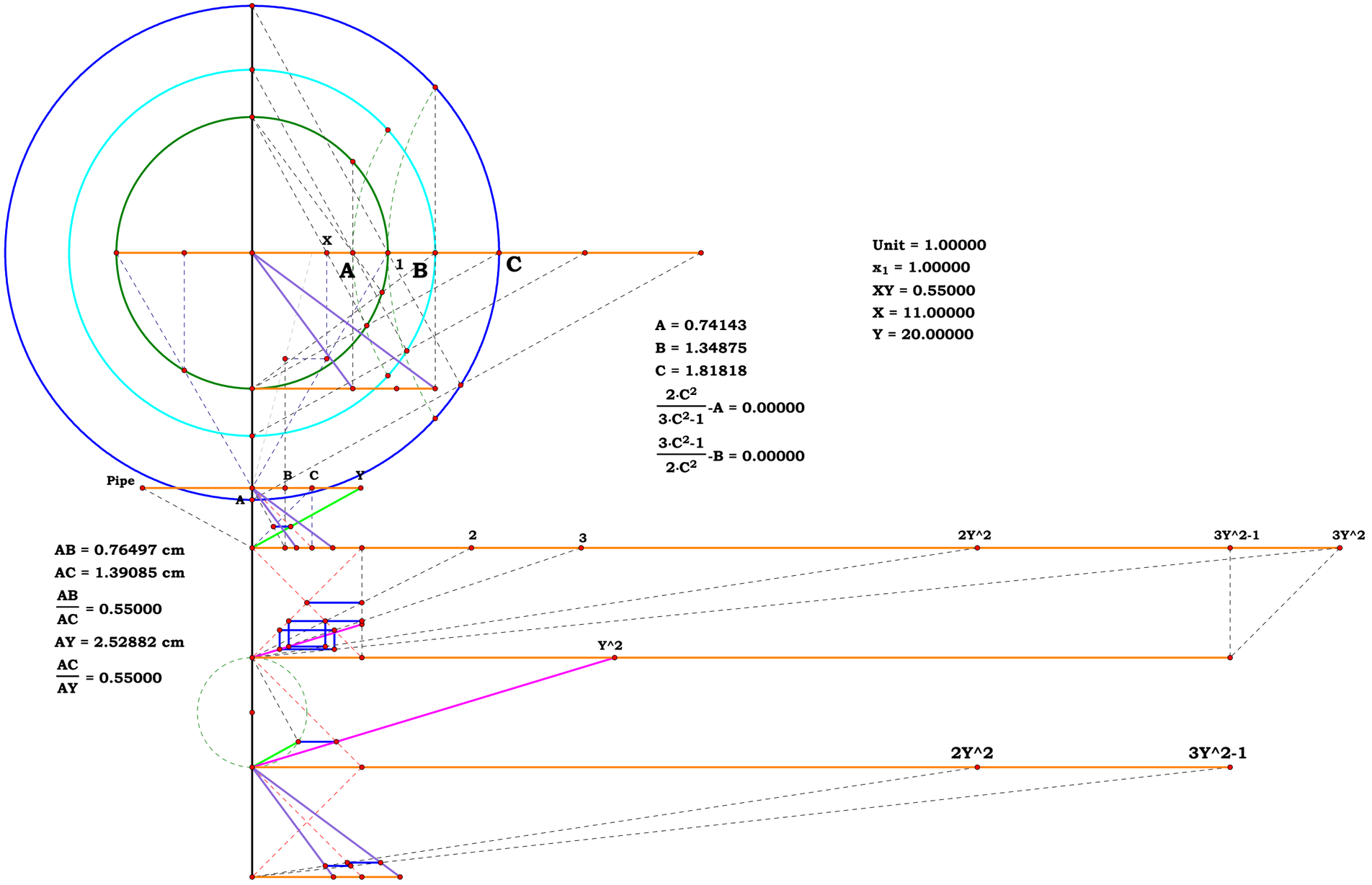
XY = 0.50000
X = 10.00000
Y = 20.00000
Unit = 1.00000

$\frac{3 \cdot Y^2 - X^2}{2 \cdot Y^2} = 1.37500$

DO = 2.16558 cm
OV = 2.97768 cm

$\frac{OV}{DO} = 1.37500$


X -> 0
X -> 1
X -> 2
X -> 3
X -> 4
X -> 5
X -> 6
X -> 7
X -> 8
X -> 9
X -> 10
X -> 11
X -> 12
X -> 13
X -> 14
X -> 15
X -> 16
X -> 17
X -> 18
X -> 19
X -> 20
Y -> 1
Y -> 2
Y -> 3
Y -> 4
Y -> 5
Y -> 6
Y -> 7
Y -> 8
Y -> 9
Y -> 10
Y -> 11
Y -> 12
Y -> 13
Y -> 14
Y -> 15
Y -> 16
Y -> 17
Y -> 18
Y -> 19
Y -> 20
Show Points
Pipe
X
1
Unit = 1.00000
x1 = 1.00000
XY = 0.58892
X = 11.77842
Y = 20.00000
Pipe
A
B
C
Y
AB/AC = 0.58892
AC = 1.73567 cm
AB = 1.02217 cm
AY = 2.94720 cm
AC/AY = 0.58892
2
3
2Y^2
3Y^2-1
3Y^2
Y^2



Unit = 1.00000

x_1 = 1.00000

XY = 0.55000

X = 11.00000

Y = 20.00000

A = 0.74143

B = 1.34875

C = 1.81818

$\frac{2 \cdot C^2}{3 \cdot C^2 - 1}$ -A = 0.00000

$\frac{3 \cdot C^2 - 1}{2 \cdot C^2}$ -B = 0.00000

AB = 0.76497 cm

AC = 1.39085 cm

$\frac{AB}{AC}$ = 0.55000

AY = 2.52882 cm

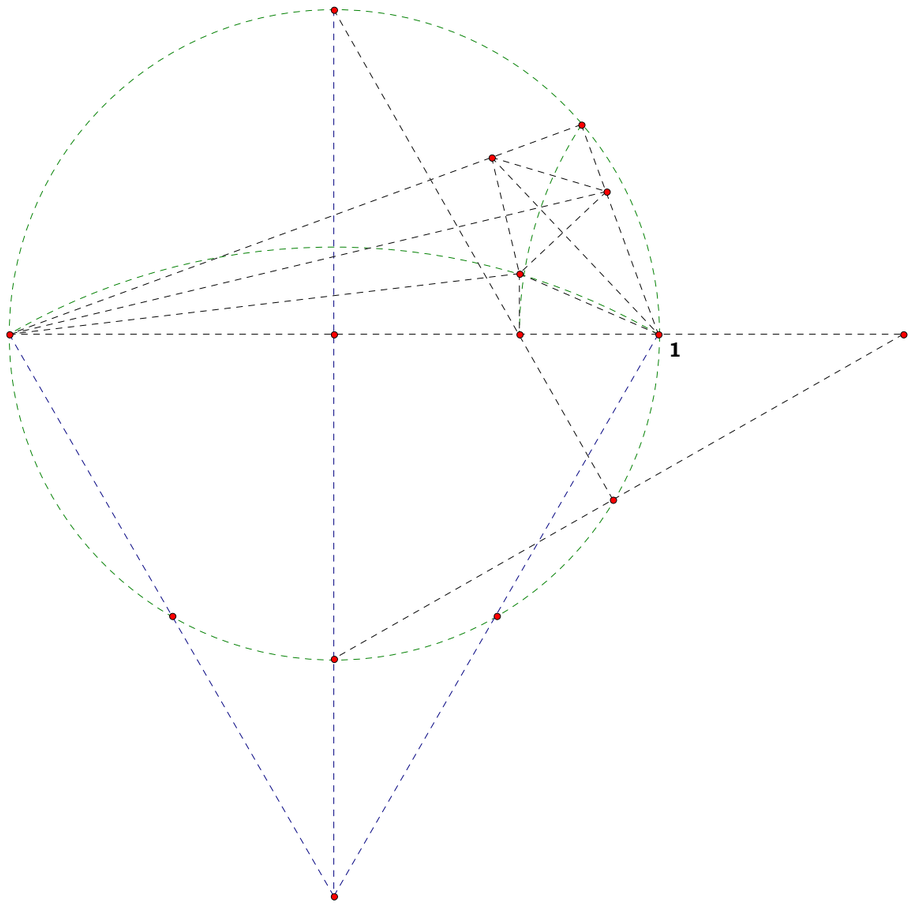
$\frac{AC}{AY}$ = 0.55000

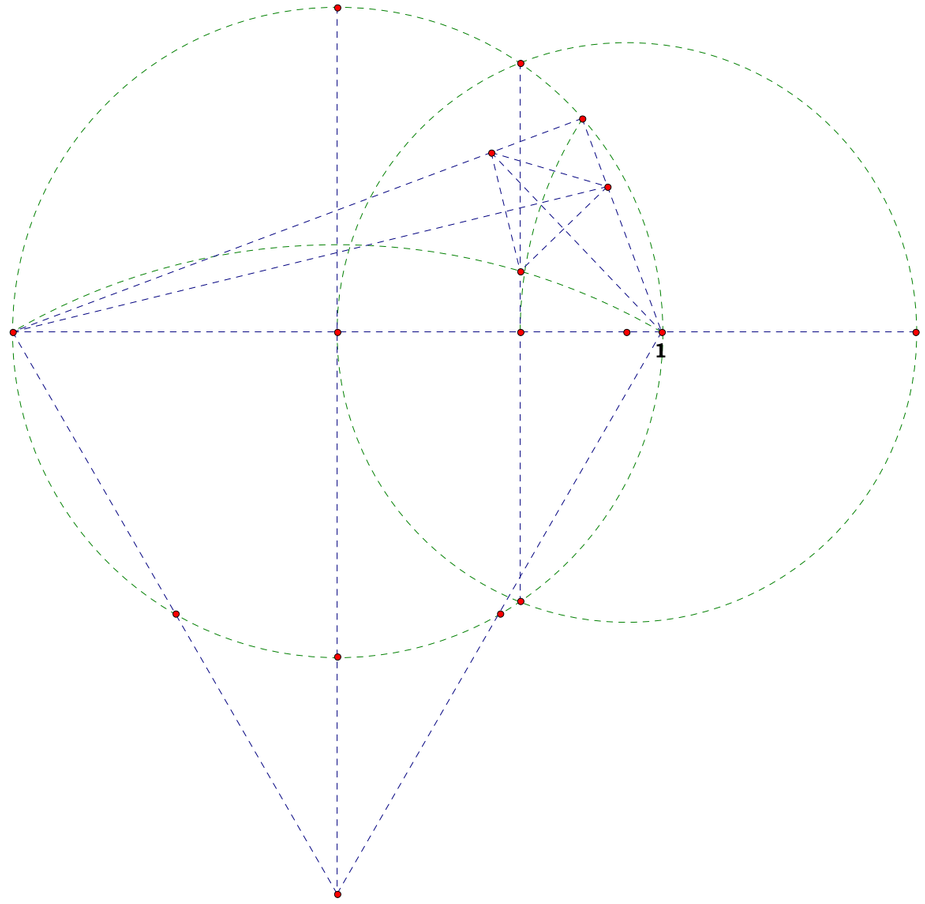
012220

Redo 080400

Construction variation.

Trisecting any angle in Geometry is only possible when you know the difference between the perceptible and the intelleligible.





Unit.

$AB := 1$

Given.

$X := 10$

$Y := 20$

012220

Descriptions.

What is the ratio of DO to OV?

$AO := \frac{AB}{2}$ $AD := AO + AO \cdot \frac{X}{Y}$ $AR := \frac{AO}{2}$ $RJ := \sqrt{AR \cdot (AB - AR)}$

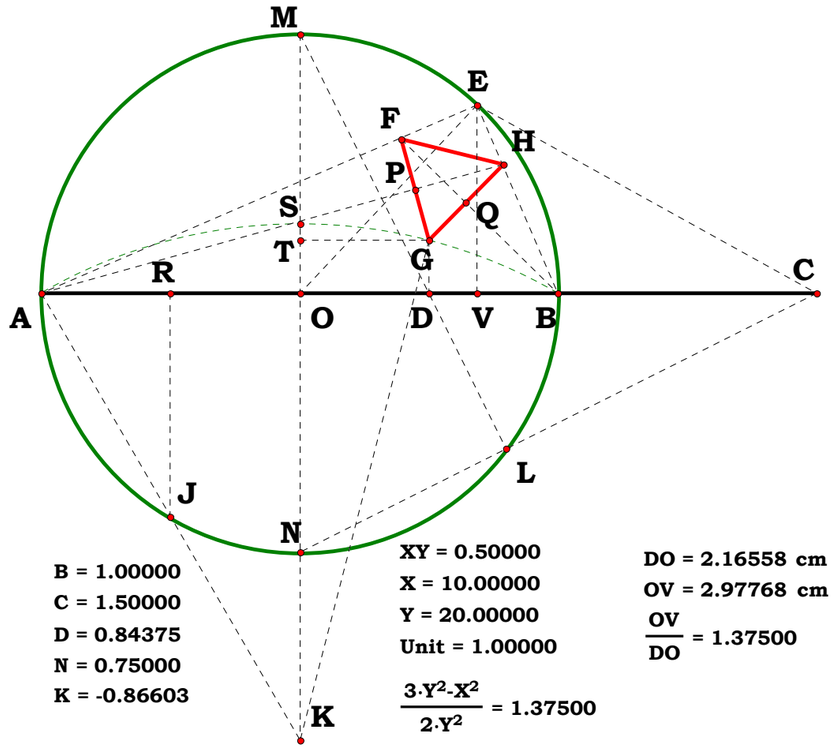
$KO := 2 \cdot RJ$ $DO := AD - AO$ $AJ := AO$ $AK := 2 \cdot AJ$ $EO := AO$

$KS := AK$ $GK := AK$ $KT := \sqrt{GK^2 - DO^2}$ $TO := KT - KO$

$DG := TO$ $MN := AB$ $MO := AO$ $DM := \sqrt{MO^2 + DO^2}$

$LN := \frac{DO \cdot MN}{DM}$ $CO := \frac{MO^2}{DO}$ $CD := CO - DO$ $CE := CD$

$OV := \frac{CO^2 + EO^2 - CE^2}{2 \cdot CO}$ $\frac{OV}{DO} = 1.375$ $\frac{DO}{OV} = 0.727273$



Definitions.

$AO - \frac{1}{2} = 0$ $AD - \frac{X + Y}{2 \cdot Y} = 0$ $AR - \frac{1}{4} = 0$ $RJ - \frac{\sqrt{3}}{4} = 0$

$KO - \frac{\sqrt{3}}{2} = 0$ $DO - \frac{X}{2 \cdot Y} = 0$ $AJ - \frac{1}{2} = 0$ $AK - 1 = 0$

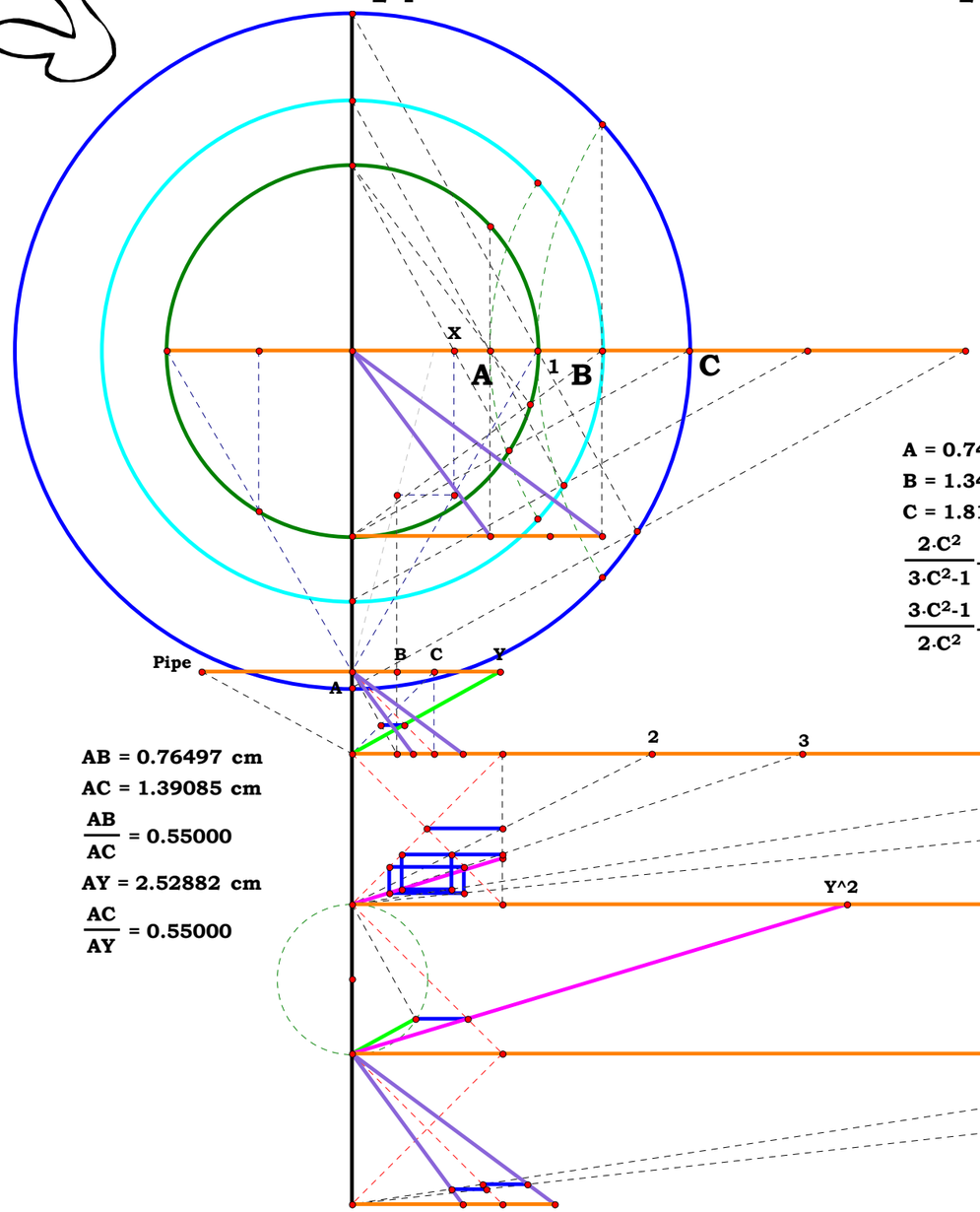
$EO - \frac{1}{2} = 0$ $KS - 1 = 0$ $GK - 1 = 0$ $KT - \frac{\sqrt{(2 \cdot Y - X) \cdot (X + 2 \cdot Y)}}{2 \cdot Y} = 0$ $TO - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{2 \cdot Y} = 0$

$DG - \frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{2 \cdot Y} = 0$ $MN - 1 = 0$ $MO - \frac{1}{2} = 0$ $DM - \frac{\sqrt{X^2 + Y^2}}{2 \cdot Y} = 0$ $LN - \frac{X}{\sqrt{X^2 + Y^2}} = 0$

$CO - \frac{Y}{2 \cdot X} = 0$ $CD - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $CE - \frac{(Y - X) \cdot (X + Y)}{2 \cdot X \cdot Y} = 0$ $OV - \frac{X \cdot \left(3 \cdot Y^2 - X^2\right)}{4 \cdot Y^3} = 0$ $\frac{OV}{DO} - \frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} = 0$ $\frac{DO}{OV} - \frac{2 \cdot Y^2}{\left(3 \cdot Y^2 - X^2\right)} = 0$

One can say now that the length of the sides of an equalateral triangle in a right triangle used in trisection is $\frac{\sqrt{4 \cdot Y^2 - X^2} - \sqrt{3} \cdot Y}{Y}$

$\frac{X}{Y} = 0.5$ $\frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} = 1.375$ $OV = 0.34375$ $DO \cdot \frac{\left(3 \cdot Y^2 - X^2\right)}{2 \cdot Y^2} - OV = 0$ $DO = 0.25$ $OV \cdot \frac{2 \cdot Y^2}{\left(3 \cdot Y^2 - X^2\right)} - DO = 0$



Unit = 1.00000
x_1 = 1.00000
XY = 0.55000
X = 11.00000
Y = 20.00000

A = 0.74143
B = 1.34875
C = 1.81818
$\frac{2 \cdot C^2}{3 \cdot C^2 - 1} - A = 0.00000$
$\frac{3 \cdot C^2 - 1}{2 \cdot C^2} - B = 0.00000$

AB = 0.76497 cm
AC = 1.39085 cm
$\frac{AB}{AC} = 0.55000$
AY = 2.52882 cm
$\frac{AC}{AY} = 0.55000$

Side Jobs

Monday, February 3, 2020

While I am working on the finishing up of the Delian Quest, it seems that I will run into problems I want to explore on the side, Musings. Will these plates make it in the finished DQ? I do not know.

I also want to do a couple of compilations or so, one The Footstool, a compilation of structures used to examine the DQ figure, Three pieces of Paper, for trisection figures. And maybe even Eloi, all the ways to draw an ellipse found in the work.

Musings

020320 Easy Cube

020320 Easy Trisection

020320 Squaring a Rectangle

Eloi

On the ellipse.

Three Pieces of Paper

On trisection.

The Footstool

On cube roots.

Descriptions. **020320 Easy Cube**

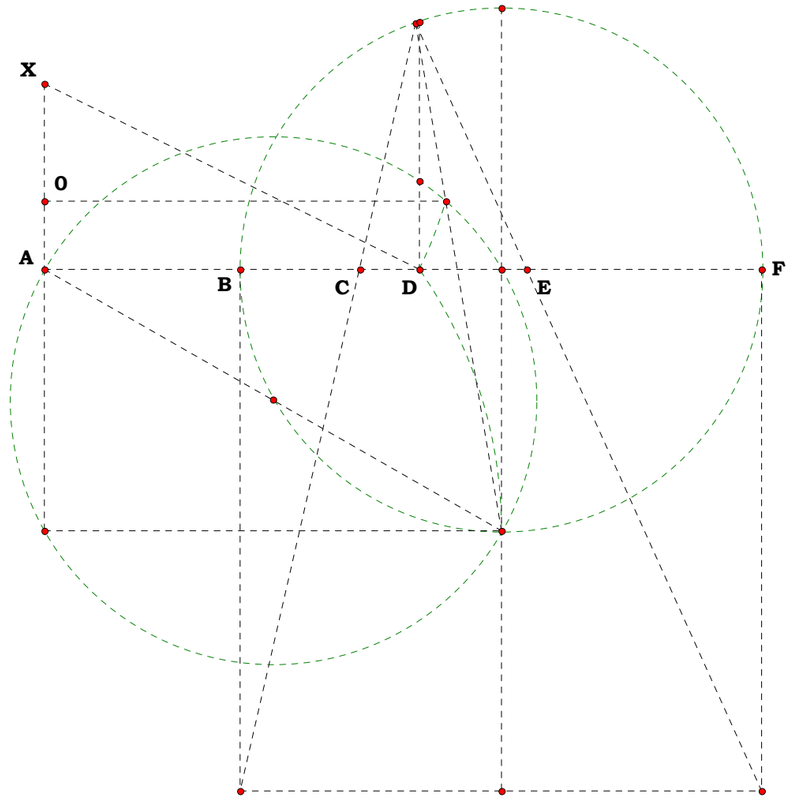
Often, one would like to create an easy plate for cube roots, instead of doing the whole figure. Here is the simplest and most accurate way to achive it.

Draw X to A anywhere and then construct 0 parallel to AF. Have your macro make X seek 0.
You might believe that simple geometry can out Calculus Calculus, maybe in these plates you will change your mind. Calculus is a work frought with grammatical contradictions, Cartesian Geometry, Calculus, Trigonometry are not even grammatically correct. They are not derived from a correct concept of grammar as any possible grammar is afforded by complete induction and deduction of a simple binary unit.

Unit = 1.00000
XY = 3.66072
X = 18.10729
Y = 4.94638

F = 3.66072
E = 2.46101
C = 1.60738
D = 1.91330

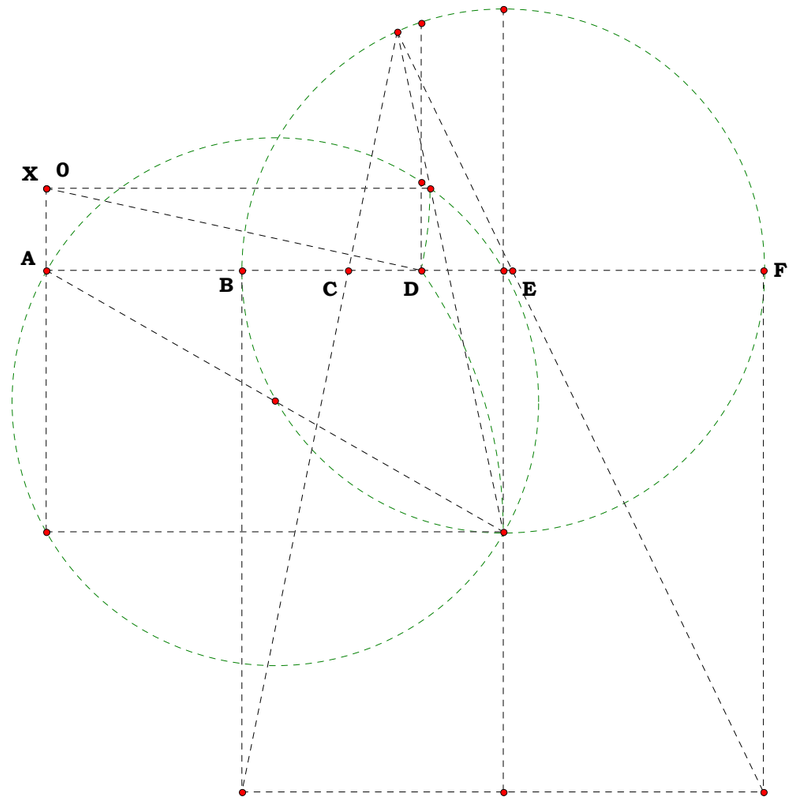
$F^{\frac{1}{2}}$-D = 0.00000
$F^{\frac{1}{3}}$-C = -0.06619
$F^{\frac{2}{3}}$-E = -0.08575



Unit = 1.00000
XY = 3.66072
X = 18.10729
Y = 4.94638

F = 3.66072
E = 2.37526
C = 1.54119
D = 1.91330

$F^{\frac{1}{2}}$-D = 0.00000
$F^{\frac{1}{3}}$-C = 0.00000
$F^{\frac{2}{3}}$-E = 0.00000

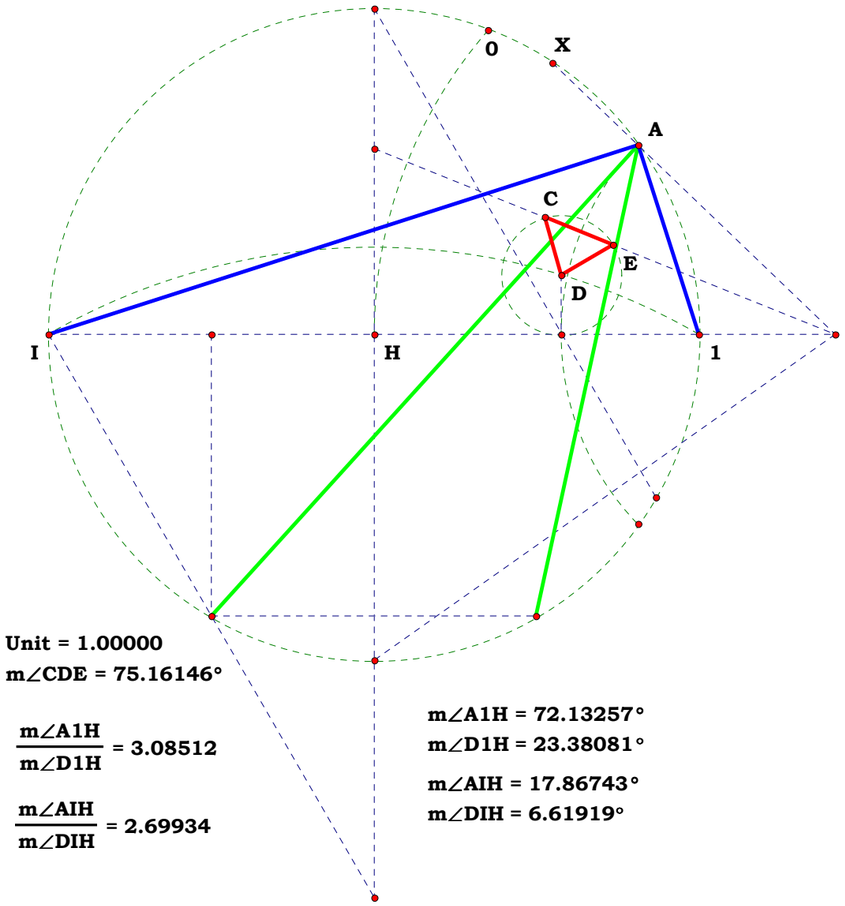


020320 Easy Trisection

Descriptions.

Sometimes one is want to trisect some particular angle the easy way. Here it is. Draw it up with X anywhere and project 0. Have your macro make X seek 0. It is a whole lot neater than sliding a piece of paper.

By knowing the geometric end results, one can write algorithms which are a whole lot more accurate and efficient than by just using the traditional methods.

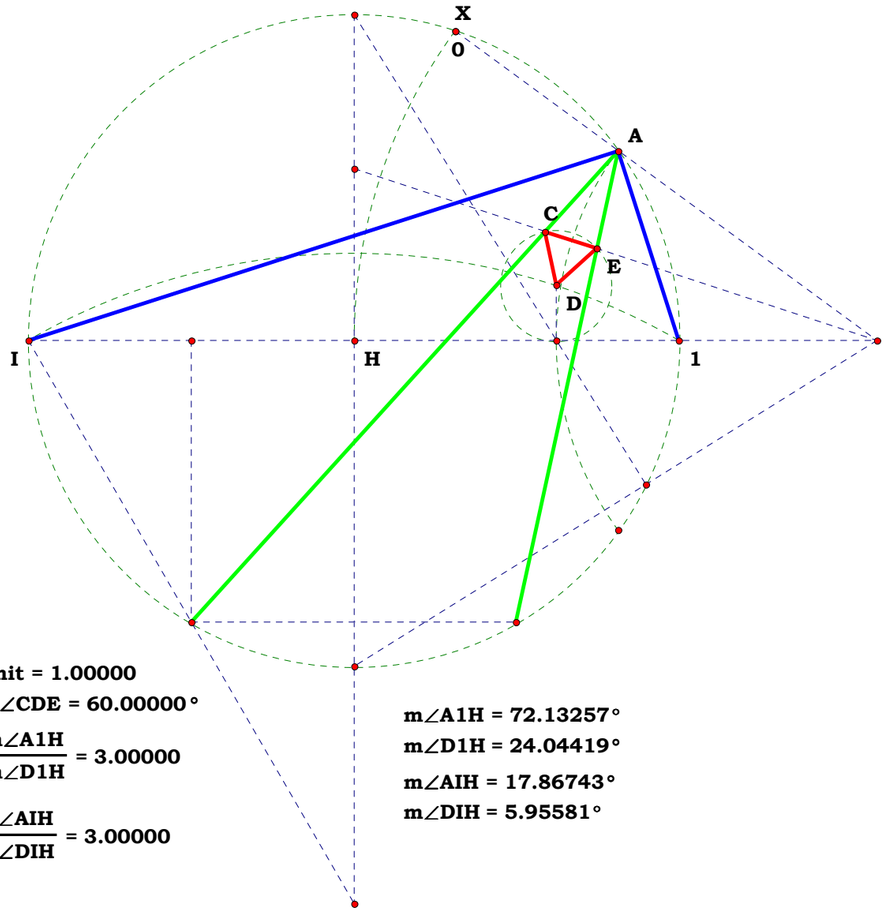


Unit = 1.00000
m∠CDE = 75.16146°

$$\frac{m\angle A1H}{m\angle D1H} = 3.08512$$

$$\frac{m\angle AIH}{m\angle DIH} = 2.69934$$

m∠A1H = 72.13257°
m∠D1H = 23.38081°
m∠AIH = 17.86743°
m∠DIH = 6.61919°



Unit = 1.00000
m∠CDE = 60.00000°

$$\frac{m\angle A1H}{m\angle D1H} = 3.00000$$

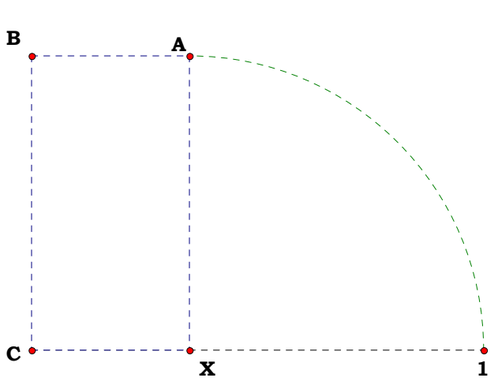
$$\frac{m\angle AIH}{m\angle DIH} = 3.00000$$

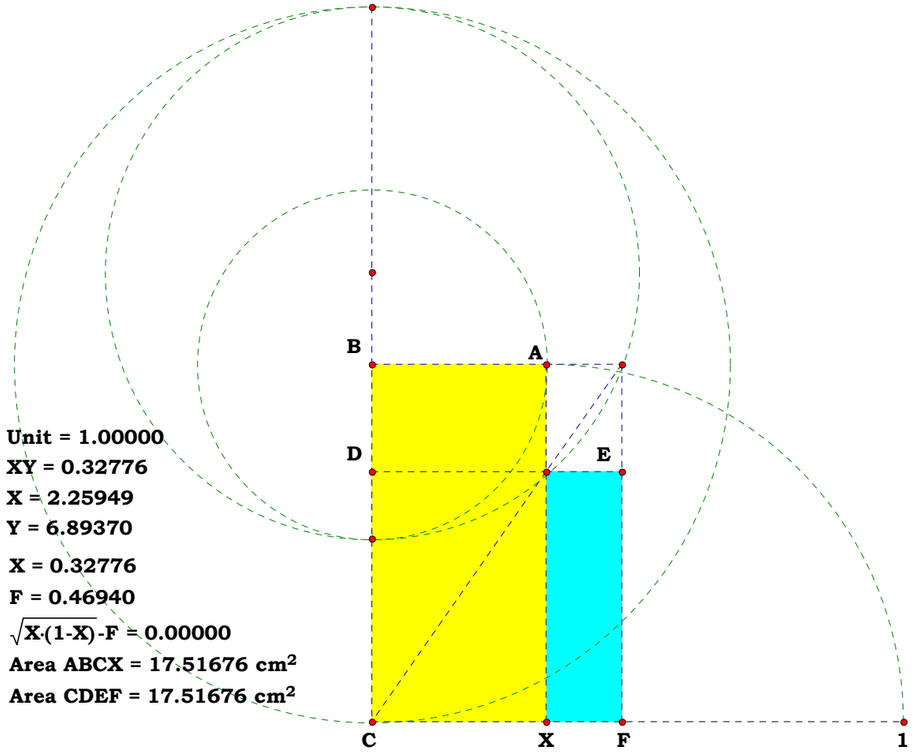
m∠A1H = 72.13257°
m∠D1H = 24.04419°
m∠AIH = 17.86743°
m∠DIH = 5.95581°

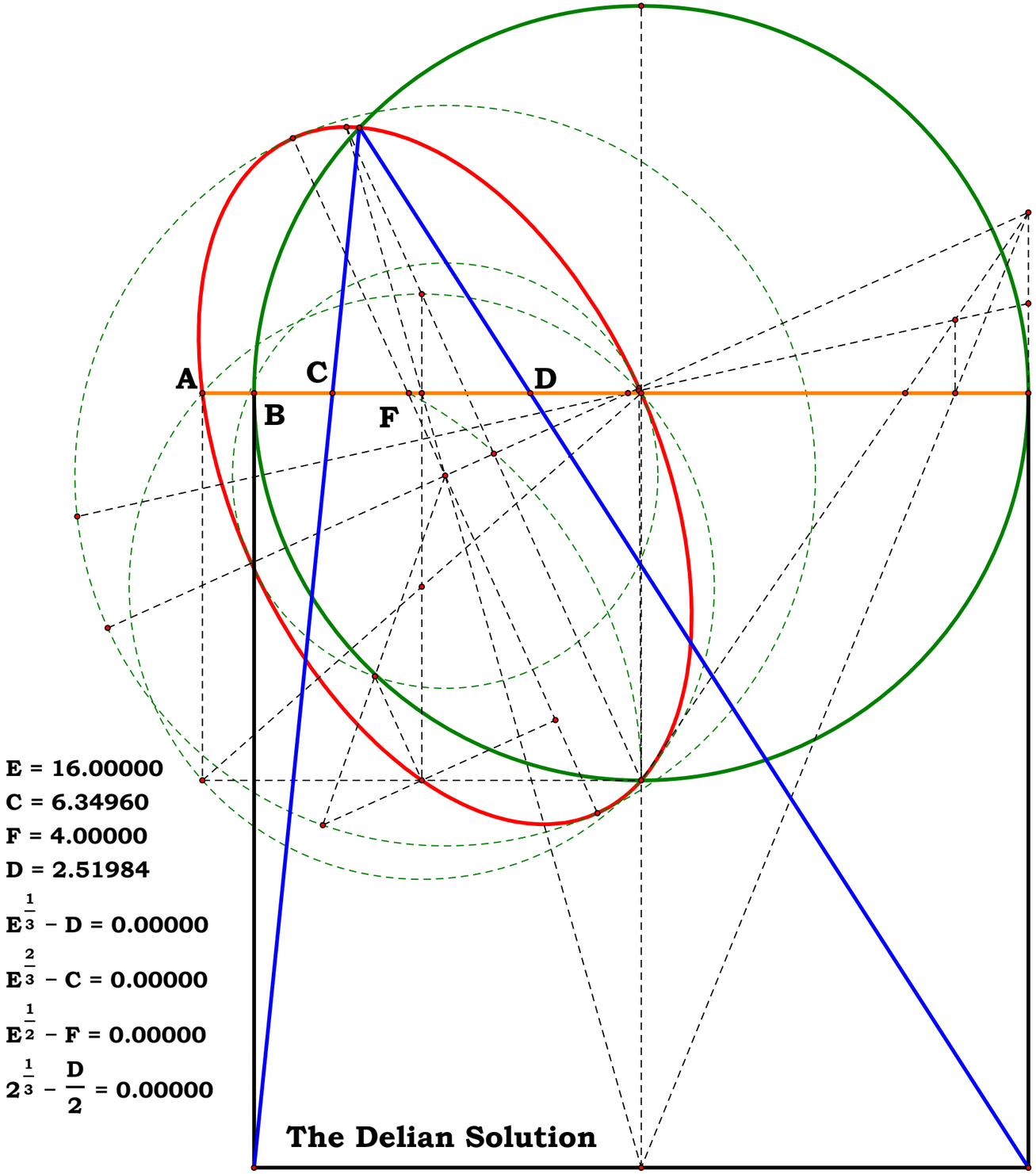
Descriptions.

This is just another way to study rectangles, complements and square roots.

020320 Squaring a Rectangle






A
B
C
F
D
E
1
E = 16.00000
C = 6.34960
F = 4.00000
D = 2.51984
$E^{\frac{1}{3}} - D = 0.00000$
$E^{\frac{2}{3}} - C = 0.00000$
$E^{\frac{1}{2}} - F = 0.00000$
$2^{\frac{1}{3}} - \frac{D}{2} = 0.00000$
The Delian Solution